

European sources for research of actuarial history and the scope for translation

by Trevor Sibbett FIA

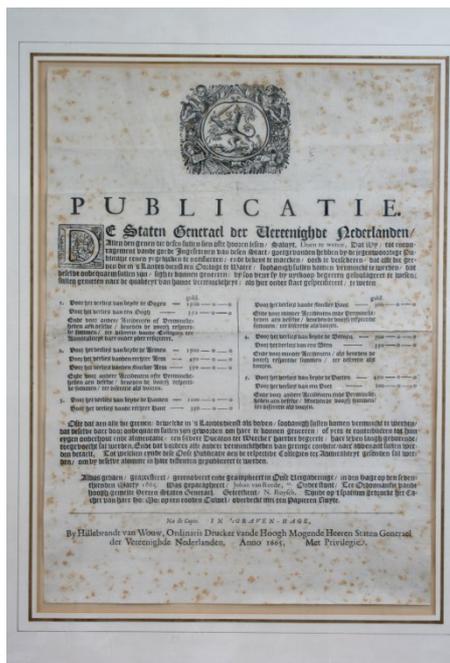
There is a large amount of research material in the continent of Europe and there are good people to write it up. I am selecting a few for illustration purposes. Peter Koch held, and possibly still holds, a chair of insurance and actuarial history at the University of Aachen in Germany. A 30-page list of his books and articles is at the end of *Beiträge zur Geschichte des deutschen Versicherungswesens*¹ ('Contributions to the history of the German insurance business') published in 1995 by his friends to celebrate his 65th birthday. The same work lists ten major German libraries with insurance material. Clearly not all of it is antiquarian. Since 1995 Peter Koch has remained active and his friends published another *festschrift* to celebrate his 70th birthday.

In France, there is currently an upsurge of monographs in demographic history. These are published by the French government funded Institute for Demographic Studies (Institut National d'Études Démographiques or INED), which is housed in a glass palace in a shabby suburb of Paris. This organisation is focused on researching current demographic issues but also has two history departments (*Histoire et Populations* and *Histoire, Critique des Sources et des Méthodes en Démographie*) with appointed historical demographers.

In Copenhagen, there is a laboratory of actuarial science at the University that publishes historical articles from time to time.

Dutch Document: Publicatie, 1665²

I'd like start close to home with a Dutch document from the year 1665. The International Association of Consulting Actuaries presented it to the Institute in 1973 to mark the Institute's 125th anniversary. It hung on a wall in Staple Inn for a while, but now it is kept in a safe. This poster is written in seventeenth century Dutch.



Netherlands. Staten Generael der Vereenighde Nederlanden (1665). *Publicatie. De Staten Generael der Vereenighde Nederlanden* (© Institute of Actuaries Library)

Work with a modern Dutch dictionary suggests that it starts with a formulaic sentence like, “To all who read these presents, Greetings!” It then sets out benefits payable (possibly to Dutch sailors) by the Dutch government in the event of various accidents that result in the loss of limbs or part of limbs. The amount of benefit varies according to the estimated degree of disability suffered, e.g. for the loss of one eye the payment is 350 guilders but for the loss of both eyes it is 1,500 guilders. Further, the loss of both hands the payment is 1500 guilders, but for the loss of the right arm only or the left arm only it is 450 guilders and 350 guilders respectively. There are broadly similar benefits for the loss of legs and feet.

This type of accident benefit then seems to disappear for a couple of centuries. It reappears in the United Kingdom in the second half of the nineteenth century after the disablements arising from railway accidents. Perhaps there are similar benefits offered by friendly societies, insurance companies or other bodies on the continent of Europe after 1665 and before the second half of the nineteenth century, but I have been unable to trace any.

So there is a need for a proper translation of this document if any further research into this type of insurance benefit is contemplated.

Emmanuel Etienne Duvillard

Duvillard’s first published actuarial book was in 1787 on the subject of researches on annuities, loans and repayments (*Recherches sur les rentes, les emprunts et les remboursements*³). He shows how loans can be repaid with annuities certain both constant and variable or annuities on lives. The publicity material for the work included a broadsheet showing by detailed calculations how the burden of annuities on lives and other financial instruments on the state could be reduced by use of the principles of the book. In the United Kingdom John Finlaison showed how the government was giving too generous terms for UK annuitants in the 1820s and any similarities may be interesting. In Duvillard’s book on page 56 in a footnote on the population of Geneva and mortality tables he shows his first interest in the mathematics of deaths from smallpox and measles.

What is desirable for this work, and all the other works I shall mention, would be to have a summary of the work describing the problems tackled, the mathematics used and any notable points.

A year later, in 1788, the first French life assurance company was formed. Its first prospectus⁴ has a number of references to Richard Price, e.g., “the observations of Dr Price have guided the company”⁵ (*Prospectus*, 1788, page 53). “Following the advice of Dr Price, the company has already appointed good and skilful mathematician accurate in his calculations”⁶ (page 59). It is not clear if the advice from Richard Price was taken from *Observations on Reversionary Payments* (e.g. page 130 of the third edition of 1773) where Richard Price wrote, “... it is of great importance to the safety of such a society, that its affairs should be under the inspection of able mathematicians.”⁷ or if there was a direct consultation. Some researchers have said that Richard Price gave advice to the company but, as far as I can tell, there is no surviving evidence. The appointed mathematician was Duvillard. The life policies offered to the French public were much like those on offer by the Equitable Life. I do not know the mortality table used in the calculation of life assurance premiums, but possibly it was Price’s Northampton table. This company is the second to be set up to offer scientific life assurance, although in the

United Kingdom the Royal Exchange Assurance (and possibly the London Assurance) had added scientific life assurance to an existing life portfolio earlier.

This first French life assurance company became involved in the politics of the French revolution because life assurance was regarded as speculation. The company was abolished by decree in 1793 and its manager committed suicide.

In 1790, Duvillard published a prospectus for a mutual provident association operating rather like a life assurance company (*Plan d'une Association de Prévoyance*⁸). The association opened to business and had an office on the left bank of the river Seine in Paris. How long it survived I do not know. There is no algebra in this work, but he gives specimen premiums. The mortality table used was Price's Northampton table at 4% interest, although some of the calculations involving the order of death of 3 lives use De Moivre's geometric progression hypothesis. Why was there a need for policies in France paying on the death of C provided A dies first and B dies second at that time?

In 1806 Duvillard published *Analyse et Tableaux de l'Influence de la Petite Vérole sur la Mortalité à chaque Age*⁹ ('Analysis and tables of the influence of smallpox on mortality at each age'). This work has been discussed in the examination of the mathematics of smallpox and multiple decrement tables. Others who worked on the subject include Daniel Bernoulli, D'Alembert, J H Lambert and others. R H Daw published an article¹⁰ discussing smallpox mortality in *JIA* 106, page 299. If you have not read Daw's article, I'd suggest that you do.

After he had published his article, Daw wrote to the Institute and asked that Lambert's book, *Beiträge zum Gebrauche der Mathematik und deren Anwendungen*¹¹ ('Contributions to the use of mathematics and its applications'), be added to the "wants list" for the library. This work was published in three parts from 1765 to 1772. Daw said it deals with osculatory interpolation as well as multiple decrement tables. Lambert is possibly best known for being the first to prove that 'pi' and 'e' are irrational and he expressed them as unit continued fractions. Some Germans compare Lambert with Leibniz.

Now I'd like to digress a little and talk about old books. The books with actuarial content that we are talking about were never best sellers and were produced in small quantities. Some were sold in sheets and you had to get a bookbinder to fold and bind the sheets into a book for you. Any unsold sheets were presumably dumped, as they do not seem to survive. Some sheets were folded and bound in temporary sugar paper wraps, often blue, and it was expected that the purchaser would put on his own binding. Some works in sugar paper bindings do survive. The books then had to survive war, fire and flood. Further some were probably worn out with use or thrown away as having little value on the death of the owner. The books that have survived still come onto the market. If an institution buys one, that copy does not usually come onto the market again. If a private collector buys them, they reappear on the market again at roughly 30-year intervals.

The Institute library has some biographical work in French on Duvillard, written by Albert Quiquet¹², a twentieth-century French actuary and a small amount of other material in Italian. Bosredon's book *Histoire des assurances sur la vie, origines, développements en France*¹³ published in Bordeaux in 1900 is an accepted early source for Duvillard's work, but is very difficult to find. Duvillard had an interesting life. His parents left France after the revocation of the Edict of Nantes in 1685. You may remember that Abraham De Moivre left France for the same reason. The edict was introduced by Henry IV of France in the year 1597 to end religious wars and it granted religious freedom to Protestants. Duvillard became a Member of Parliament for the Léman area of France during the revolution; he held a mathematical post in the revolutionary government and survived all problems to die of cholera in 1832.

It seems to me that there is scope for an English language article on Duvillard's life and work.

Wilhelm Gottfried Leibniz

About the only actuarial information we have on Leibniz in the United Kingdom is that Leibniz received some bills of mortality tabulations from Casper Neumann and sent them on to Justell, Secretary of the Royal Society, who in turn passed them to Edmund Halley. Halley then produced his Royal Society papers¹⁴ on annuities in 1693 in the form of a modern mortality table.

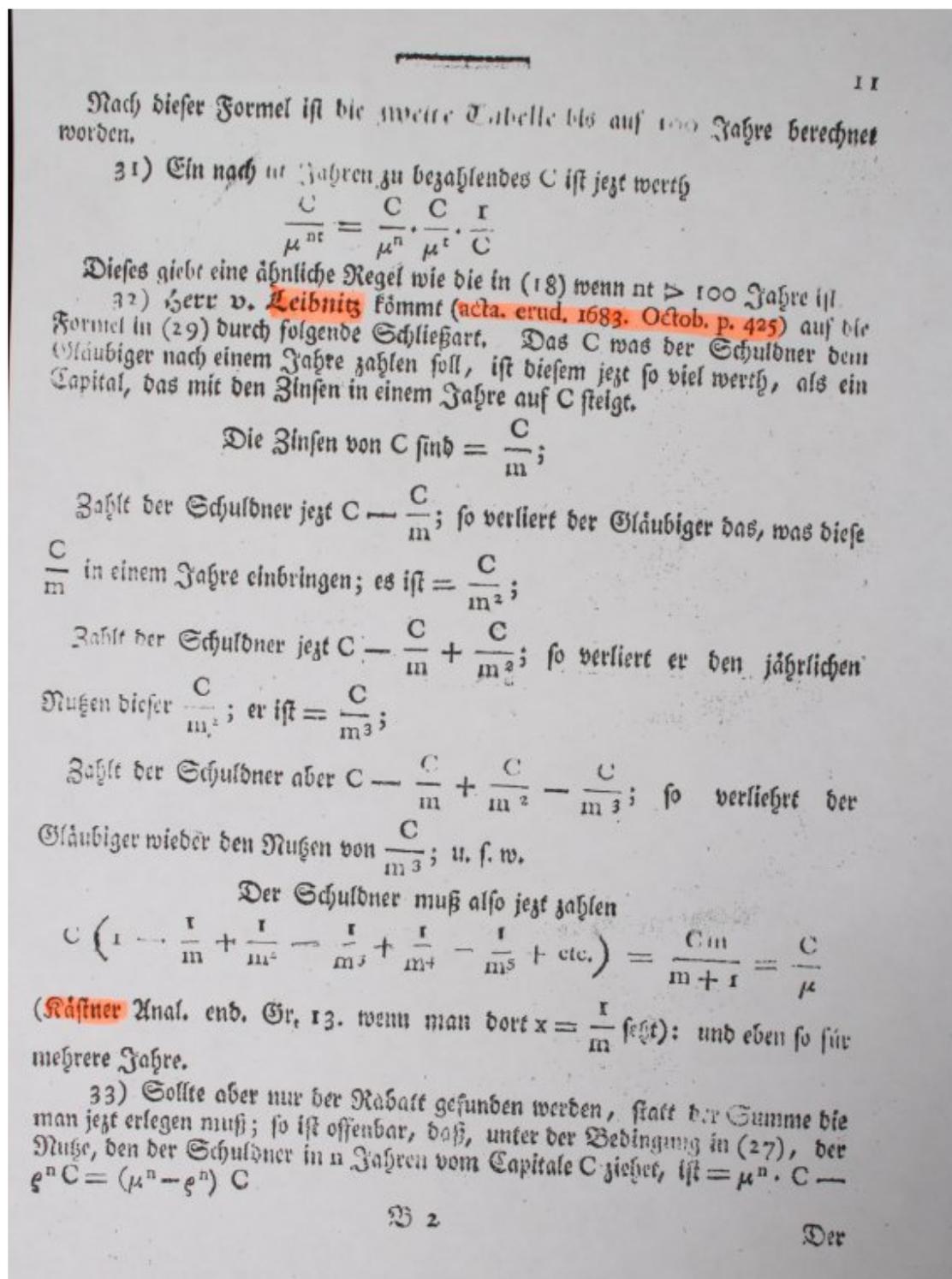
However, Leibniz became involved in more than that. He was involved in early insurance. In 1678, Leibniz proposed the formation of an insurance company against the misfortunes of life¹⁵. In 1697 Leibniz wrote a discussion paper (*Denkschrift*) that repeated the proposal for an insurance company against all the misfortunes of life or at least against damage from fire and water¹⁶. Leibniz realised that the misfortunes of life did not occur very often and when they did whole families were suddenly left poor and unable to look after themselves. It fell to the state to feed and look after beggars. Leibniz was happy to regard the state as a large family with thousands of members who would look after each other. It followed that the state should make a contribution to help those whose bad luck was no fault of their own¹⁷. There was a discussion on whether Leibniz was inventing a new tax, which he denied.

Now I'd like to turn to compound interest. Compound interest is thought to have originated in Italy as a natural result of double entry bookkeeping, which also developed in that country. It is not clear when double entry bookkeeping began. The earliest printed work on the subject is Pacioli's *Summa de Arithmetica* dated 1494, but it is said that the double entry technique preceded Pacioli by at least two centuries¹⁸.

That ties in well with the earliest reference I can find to compound interest. It is in C M Waller Zeper's *De oudste Intresttafels in Italië, Frankrijk en Nederland met een Herdruk van Stevins "Tafeln van Interest"*¹⁹ – 'The oldest interest tables in Italy, France and the Netherlands with a reprint of Van Stevin's "Tables of Interest"'. This work is a PhD thesis.) The reference is to a manuscript, written for a trader going to China in the year 1340. It is overwritten. Presumably it became unwanted and perhaps faded. As writing materials were scarce and expensive, it was the practice to use them again, turning them sideways and writing over the original script. The manuscript is in the Bibliotheca Riccardiana, a library in Florence – at least it was in 1937. I do not know if it is still there. As compound interest became known, it then spread northwards from Italy throughout Europe.

The first printed work containing compound interest problems is the 1489 arithmetic book by Johannes Widmann: *Behende und hubsche Rechenung auff allen kaufmannschafft*²⁰ ('Handy and beautiful arithmetic for merchants' – see Heinrich Braun page 36 below) Widmann's work went to several editions. There must be a limited number of early arithmetic works and manuscripts in various languages containing compound interest problems. It ought to be possible to list these with the place and date of publication in order to see how printed compound interest ideas spread and also to see what problems were being solved. The earliest work devoted wholly to interest is by Simon Stevin of Bruges. It was written in the year 1582 (in sixteenth-century Flemish). There is a French arithmetic by Stevin containing similar compound interest tables printed three years later in 1585. There is a reference to the French work of Stevin's and plenty of seventeenth-century and other early compound interest works ably discussed in two articles by Chris Lewin²¹ in issues of *Journal of the Institute of Actuaries* of 1970 and 1981.

As I was looking through Carl Chassot de Flourencourt's *Abhandlungen aus der juristischen und politischen Rechenkunst*²² ('Treatises on legal and political arithmetic') I came across a reference on page 11 to Leibniz and compound interest which he published in the *Acta Eruditorum* in 1683. The *Acta Eruditorum* is a scientific journal. There is a number such journals in Europe. Leibniz was answering the question of how much should be paid now instead of a sum in one year's time. The answer is easy but it was Leibniz's approach that is different. He suggested that all that is needed is to deduct 'i' from the capital of 1. But that is too much as there is a loss of interest on interest, so i^2 has to be added back. But that is also too much to add back, so by the same reasoning i^3 has to be deducted etc. Leibniz then summed his infinite series and came to $1/(1+i)$. As can be seen from the text on page 11 of Florencourt's work, Leibniz did not actually work with 'i' but with its reciprocal i^{-1} . What other compound interest problems has Leibniz solved? I do not know and further research in the *Acta* is necessary to ascertain that.



Carl Chassot de Florencourt. *Abhandlung aus der juristischen und politischen Rechenkunst*, 1781, p.11

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But that is not all Leibniz's work in the actuarial field. Mark Parmentier, an academic of Lille University in France has examined some 21 of Leibniz's manuscripts and he published his results in 1995 in *L'Estime des Apparences*²³. Leibniz wrote in Latin, German and French. These manuscripts cover probability including dice throwing problems, games, expectation of life, life annuities and pensions. They date from around 1680, i.e. after Johan de Witt but before Halley. Some of Leibniz's Latin and German manuscripts are transcribed in Parmentier and then also translated into French. Parmentier has added his explanations. Parmentier's work on Leibniz was followed by a further study by J-M Rohrbasser and J Véron, *Leibniz et les raisonnements sur la vie humaine*, in 2001²⁴. It seems to me there is a need for a summary of their findings in the English language and also room for further study to see how Leibniz's work fits into the actuarial history perspective.

Portugal

The first book on insurance was published in Portugal in the Latin language in the year 1542. The Chartered Insurance Institute owns a copy. There are several later editions and in the twentieth century at least two translations into English. The author is Pedro de Santarem and the title of the work is *Tractus de Assecurationibus et Sponsionibus*²⁵. The subject is marine insurance law, although the principles are still valid in current insurance law generally.

Sweden

The first life table using the ratio of deaths to living together with a radix was produced by Pehr Wargentin in Sweden in 1766 from census data. The first British table to use this method came 49 years later and was Milne's Carlisle table of 1815²⁶. Before then, mortality tables were merely the sum of the deaths, the d_x column, at each age from the highest age downwards.

Wargentin's paper was published in the *Kongl. Vetenskaps Academiens Handlingar*²⁷ ('Transactions of the Swedish Royal Scientific Academy') for the year 1766 and there was another similar paper published in 1767. The 1766 Swedish table was noticed and published by Richard Price in the fourth edition of *Observations on Reversionary Payments*²⁸ in 1783. It is in the 1783 work of Price's that the popular version of the Northampton table was published and where the l_x table consisted of the sum of deaths d_x . In 1785 Price recommended to the Massachusetts Congregational Society for the provision of annuities and orphans of the Congregational Ministers and University Presidents and Professors that they used the Swedish tables of Wargentin that Price had published two years earlier²⁹. Price's advice was accepted.

A copy of Wargentin's paper 'Mortality in Sweden according to the «Tabell-Verket» (General Register Office) for the year 1766' and an English language translation were given to delegates of the International Congress of Actuaries in Stockholm in 1930³⁰. In addition to the two papers I have mentioned, there are nearly twenty other papers, many by Henrik Nicander, dealing with mortality and the Swedish population right up to the year 1854. As far as I know, they are not researched and there is no English language summary of them.

Italy

There is a 1776 translation by Roberto Gaeta and Gregorio Fontana into Italian entitled *La Dottrina degli Azzardi applicata ai Problemi della Probabilita dell Vita, delle Pensioni Vitalizie, Reverzioni, Tontine, ec. Di Abramo Moivre*³¹ of the main text (i.e. excluding the dedication and the preface) of Abraham de Moivre's *Annuities upon Lives*³². The translation is of the second edition of 1743 or perhaps the third edition as they are identical. It is preceded by a 53-page history and discussion of actuarial science and also a bibliography of a further ten pages listing actuarial books in 7 languages.

The table on page liii of the history³³ section shows in the first column deaths from Corbyn Morris's work³⁴ on Bills of Mortality (extracted from Thomas Birch's collection of the yearly bills of mortality of 1759 quoted in Sußmilch's second edition³⁵). The second column is a table of l_x derived from the first column and the third column is the second column expressed with a radix of 1,000,000. The bills are for the 30 years 1728-1757. The equation at the bottom of the page is the same l_x column reduced to a radix of 10,000.

P A R T E III. liij

Età Anni	Morti	Vivi	Numero de' Vivi ridotto
0	272903	750332	1000000
2	64745	477419	636286
5	25912	412674	549996
10	22891	386762	515414
20	58474	363871	484884
30	71502	305397	407021
40	73228	223895	311726
50	59783	160675	214117
60	47269	100875	134423
70	33679	53606	71444
80	16948	19927	26555
90	2979	2979	3970

Quindi congiungendo insieme una Parabola, e due Logistiche, dopo molti e varj tentativi e pruove fatte a tafone [com' è verisimile dover essere accaduto ; giacchè egli sopprime interamente la strada per dove ha camminato] s' incontra finalmente nella bella semplicissima equazione esponenziale

$$y = 10000 \left(\frac{96-x}{96} \right)^2 - 6176 \left(e^{-x/13,632} - e^{-x/2,4514} \right),$$

nella quale indica il numero, che ha per logaritmo iperbolico l'unità.

e in conseguenza per tutt' i valori di x si ritrovano con tutta speditezza i corrispondenti valori di y , e si giugne ad ottenere la seguente Tavola ;

x	y calcolato	y osservato	Differenza
0	10000	10000	0
2	6407	6363	+44
5	5490	5500	-10
10	5153	5154	-1
20	4838	4849	-11
30	4038	4070	-32
40	3099	3117	-18
50	2136	2141	-5
60	1329	1344	-15
70	697	714	-17
80	260	266	-6
90	30	40	-10

Roberto Gaeta and Gregorio Fontana (1776). *La Dottrina degli Azzardi applicata ai Problemi della Probabilita dell Vita, delle Pensioni Vitalizie, Reverzioni, Tontine, ec. Di Abramo Moivre*. Milan, 1776, pages liii, liv. (Pages reproduced by permission from a privately owned copy).

Page lv shows how the equation fitted the data. The columns are calculated numbers, the observed numbers and the difference respectively. All this is extracted from Johann Heinrich Lambert's *Beyträge zum Gebrauche der Mathematik und deren Anwendung: Teil 3*³⁶ ('Contribution to the use of mathematics and its applications part 3') of 1772. In 1765 in part 1 of the same work, he produced a degree 5 polynomial to fit part of the mortality curve from

age 45 to age 90 from a similar set of data. This curve fitting is the earliest I know of in actuarial science. The empirical exponential equation in part 3 of Lambert's work is worked on further in a very long footnote starting on page 8 in the 1787 work on annuities, loans and repayments by E E Duvillard³⁷ that I mentioned earlier.

At the end of the Gaeta translation of De Moivre there are 105 pages of largely mathematical explanation of De Moivre's book. As far as I can tell, no part of the Gaeta and Fontana book has yet been researched in the English-speaking world.

Holland

There is already a nineteenth century manuscript translation in the Institute of Actuaries library of Kersseboom's mortality tables and demographic treatises on the population of Holland³⁸ published in the first part of the eighteenth century. You need to be careful when using this, as the translator has added a few comments and in the past I've fallen into the trap of assuming that the comments were original Kersseboom. I think the translator of was John Rickman. There is also a translation of Deparcieux's work *Essai sur les Probabilités de la Durée de la Vie humaine* ('Essay on the probabilities of the duration of human life')³⁹ of 1746. Rickman is known to have translated both works, but if these are in his handwriting and how they got into the Institute library is a mystery. Rickman was the organiser of the first four UK censuses from 1801.

You may recall that Nicholas Struyck produced the first mortality tables for males and females separately in 1740, as an appendix to a work on geography that also contains material on astronomy. (*Inleiding tot de algemeene geographie* – 'Introduction to general geography'⁴⁰). He also wrote on probability, lotteries and interest in 1716 (*Uytreening der Kanssen in het Speelen* – 'Calculation of chances in games'⁴¹) that included work on lotteries and tontines. In his work of 1753 (*Vervog van de beschryving der staartsterren*) on astronomy with an emphasis on comets, together with geography, there is an early map of New Guinea delineated as far as it was then known. There is a final section of more than 200 pages on the population of Holland, widows' and orphans' funds and other actuarial matters.

There is a French translation of Struyck's actuarial work⁴² published in Amsterdam in 1912. The translation omits from the 1753 work some text, mainly a large chunk on Dutch villages. Anders Hald, in his *History of Probability and Statistics before 1750*,⁴³ has a good note on Struyck (pages 394-395), who he describes as slightly improving the results of the results of four pioneers (Montmort, J and N Bernoulli and De Moivre). A more detailed look at Struyck including his actuarial contribution and biographical information would be interesting.

In 1897 a Dutch Insurance Company published the *Bouwstoffen voor de Geschiedenis van der Levensverzekeringen en Lijfrenten in Nederland*⁴⁴ ('Fundamentals of the History of Life Insurance and Annuities in the Netherlands'). This contains a wealth of biographical information on Dutch early actuarial pioneers, insurance companies and other matters. There was a French translation⁴⁵ dated in 1898. The company ceased trading in 1922.

Germany

There is a lot of early demography in Germany and the most important demographer is Johan Peter Süßmilch. His main work *Die Göttliche Ordnung* ('God's Divine law') had four editions and additional to the first edition published in 1756. The 1741 and the 1765⁴⁶ editions

have been the subject of works published by INED⁴⁷. Other German writers in the field include Marperger and Justi.

Johan Nicolaus Tetens wrote an extensive two-volume work on life contingencies⁴⁸ in 1785-86. It came about because he had given advice to the Calenberg Widows' Fund that was in difficulties. The whole of the second volume is devoted to widows' fund problems and includes the valuation of widows' funds. This work has hardly been researched by English language authors and may be worth further investigation. Incidentally, Tetens's birthplace was in a part of Schlegswig-Holstein that was Danish at the time. For that and other reasons he is sometimes regarded as Danish rather than German.

Widows' funds first appeared in Germany at the beginning of the seventeenth century. They did not appear in Great Britain until right at the end of the seventeenth century. When I search the internet for early widows' funds items, I often find eighteenth-century German widows' funds. There is little British material; I find nothing French or Dutch. Why?

Peter Koch published a work on insurance pioneers and their ideas (*Pioniere des Versicherungsgedankens*⁴⁹. Wiesbaden, 1968). It consists of short biographies. In addition to the internationally known pioneers, Graunt, Halley etc., it has much information on important German actuarial and insurance figures.

General Comment

Quite a lot of actuarial history consists of examining formulae in books to find the earliest mention and then the further development. I wonder if we should sometimes move away from that. When a person writes a book it is usually for a reason. Perhaps we should look for the reason including the social, political economic, religious and other influences at work. A summary of the content of a work including these influences together with biographical material as well as the mathematical content may make for more varied and interesting reading.

Religion

There seems to be a religious interest in the early actuarial science and insurance area. I've already mentioned the revocation of the edict of Nantes, which affected De Moivre and Duvillard. You noticed that the Italian work on De Moivre was edited by two religious men. Johann Peter Süßmilch in Germany in 1741 wrote a major demographic work titled 'God's Divine Law' that ran to four editions. God's law was the mortality table.

In the United Kingdom, William Derham wrote *Physico-Theology* a religious work that included actuarial matter such the proportion of deaths to births from the first edition of 1713 onwards. There were many editions⁵⁰. It was *Physico-Theology* and Nicholas's Struyck's 1740 work that stimulated Süßmilch to write in 1741.

1647 saw the publication of "A generall Bill of Mortality, of the Clergie of London, which have beene defunct by reason of the Contagious breath of the Sectaries of that City, from the yeere 1641 to this present yeere 1647, with the severall Casualties of the same"⁵¹ during the English Civil Wars.

In life assurance there was a religious divide particularly in France lasting almost until the end of the nineteenth century. Protestants were in favour of life assurance. Catholics were against as life assurance put a price on a life and the life of a man was above price

Translation

Just about any foreign language work I have mentioned could be considered for translation. They would all be valuable for specific areas of research. There are, however, some works which give a broad picture of a wide area of actuarial history.

Foremost is perhaps Heinrich Braun's *Geschichte der Lebensversicherung und der Lebensversicherungstechnik*⁵² ('History of Life Assurance and Life Assurance Mathematics'), which was published in 1925 and reprinted with an index in 1963. It is absolutely packed with information and very little is out of date.

Hans Schmitt-Lermann's 159-page *Der Versicherungsgedanke im deutschen Geistesleben des Barock und der Aufklärung*⁵³ ('Insurance Thought in German Scientific and Cultural Life in the Baroque and Enlightenment Periods') published in 1954 is a little more specialised but well worthwhile. More general and encompassing the whole insurance area is Ludwig Arps's *Auf sicheren Pfeilen – Deutsche Versicherungswirtschaft vor 1914*⁵⁴ ('On Secure Foundations – German Insurance before 1914'), a work of nearly 700 pages published in 1965.

A couple of years ago I heard that Peter Koch, whom I mentioned earlier, had been asked to write a history of German insurance and actuarial science. It may be worth waiting a year or two for this to be published, as it may be a good item for translation.

In the French language there is P J Richard's *Histoire des Institutions d'Assurance en France*⁵⁵, 1956 ('History of the Insurance Companies in France') that covers all forms of insurance. Also of some value is George Hamon's *Histoire Générale de l'Assurance en France et à l'Étranger*⁵⁶, 1897 ('History of Insurance in France and Overseas'), a work of nearly 800 pages.

Puzzling Items

In 1823 Johan Heinrich Meyer wrote a General Introduction to Annuities and Reversions (*Allgemeine Anleitung zur Berechnung der Leibrenten und Anwartschaften*⁵⁷). The first Germany-wide life insurance company in Gotha owned volume 1 and Anders Hald owned a copy of both volumes. I have looked at both volumes. It seems to be a good early work on life contingencies, but I can find no reference to it in any actuarial literature in any language. It is, however, mentioned in a journal of astronomical news (*Astronomische Nachrichten*, volume 4, pages 493-500.)

*Arbre de Vie*⁵⁸ ('Tree of Life')

This French language item, [not illustrated here as copyright is unclear] was sent to me by a book dealer, who was unable to identify it. It seems to be eighteenth-century from the paper and from the engraved design. The engraver seems to be Italian. The numbers in the centre are apparently probabilities of life or perhaps numbers from a life table, but not in a form I recognise. I referred a copy to INED two years ago and they passed a copy to the University of Lyon. The *Arbre de Vie* was issued by the Institute for Political and Moral Arithmetic, but

this organisation seems to have disappeared without trace, possibly during the French revolution. The upward sloping diagonals have numbers from one to one hundred and may be ages. It is not clear what the numbers of downward sloping lines are. As yet it remains a mystery. A copy of the original full size document (missing about a quarter) can be provided for research if anyone wishes to shed light on its origin.

Trevor Sibbett
June 2007

References

- ¹ Heinz Leo Müller-Lutz; Karl-Heinz Rehnert; [Peter Koch] (1995), *Beiträge zur Geschichte des deutschen Versicherungswesens: Aus Anlass des 60. Geburtstages von Peter Koch*. 1995. [Festschrift for Peter Koch] (Not in Institute of Actuaries Library)
- ² Netherlands. Staten Generael der Vereenighde Nederlanden (1665). *Publicatie. De Staten Generael der Vereenighde Nederlanden : Allen den genen die desen sullen sien ofte hoozen lesen....* [poster]. - In 's Graven-Hage: By Hillebrandt van Wouw, 1665. [Library RKN: 43069]
- ³ Emmanuel Etienne Du Villard (1787). *Recherches sur les rentes, les emprunts et les remboursements*. - Paris: 1787. [Library RKN: 4364]
- ⁴ Compagnie Royale d'Assurances (1788). *Prospectus de l'établissement des assurances sur la vie*. - Paris : Lottin , 1788. [Library RKN: 15185]
- ⁵ *Ibid.*, page 53.
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