Mortality Projections associated with the ’00’ series tables

London 12/9/2005

Edinburgh 14/9/2005

Agenda

- Introduction
- P-spline model
- Lee-Carter model
- Illustrative Results
- Use of Stochastic Methods in practice
- Conclusions/Future Work

  Lunch

- Software workshop

Introduction

- Mortality Projections
  - Historical perspective
  - Need for stochastic approach
- Overview of recent work
- Feedback received on Working Paper 15
- What Happens Next?

Mortality Projections

- Projection factors first produced for a(55)
- 92 series - single projection factor
- Cohort effect - range of 3 projections
- Need for stochastic approach

Overview of recent work

- Working Paper 1
  - An interim basis for adjusting the “92” Series mortality projections for cohort effects
  - Offered a range of projections
- Working Paper 3
  - Initial exposure of various projection methodologies
  - Consultation document to guide future work
- Working Paper 11
  - Summary of responses to WP3
  - “green light” to continue work
- Working Paper 15
  - Proposed 2 methods: P-spline and Lee-Carter

Feedback on WP15

- Broad support for the introduction of stochastic methodologies
- Both P-spline and Lee-Carter wanted - no clear ‘winner’
- Support for CMI issuing illustrative software
- Software must allow actuaries to make appropriate adjustments
- Respondents clearly felt that it was inappropriate for the CMI to prescribe a method or basis…
- … but there was an equally clear demand for some guidance
- Recognised possibility of other models

What Happens Next

- Timescale for consultation & approval
  - Proposed Base Tables + full dataset released 28 Sept
  - Consultation till 31 October
  - FIMC adopt Base Tables Q1 2006?
- Status of CMI projections work
  - Peer reviewed, not approved
  - Exposing work to the profession will allow full review and issues to surface
- Future work
  - To be decided as feedback to current proposals is received and analysed

Generating Mortality Scenarios

Iain Currie

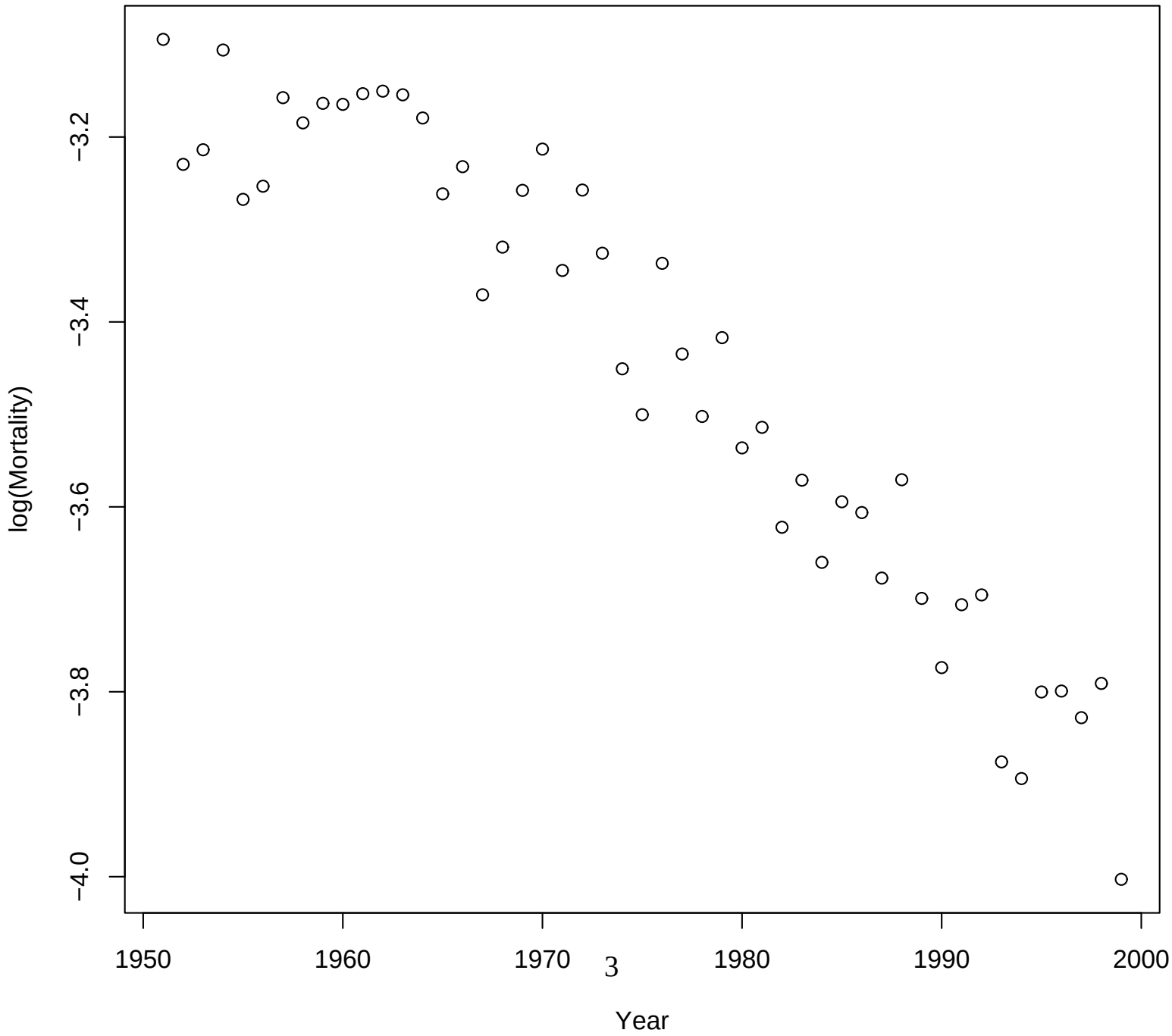
James Kirkby

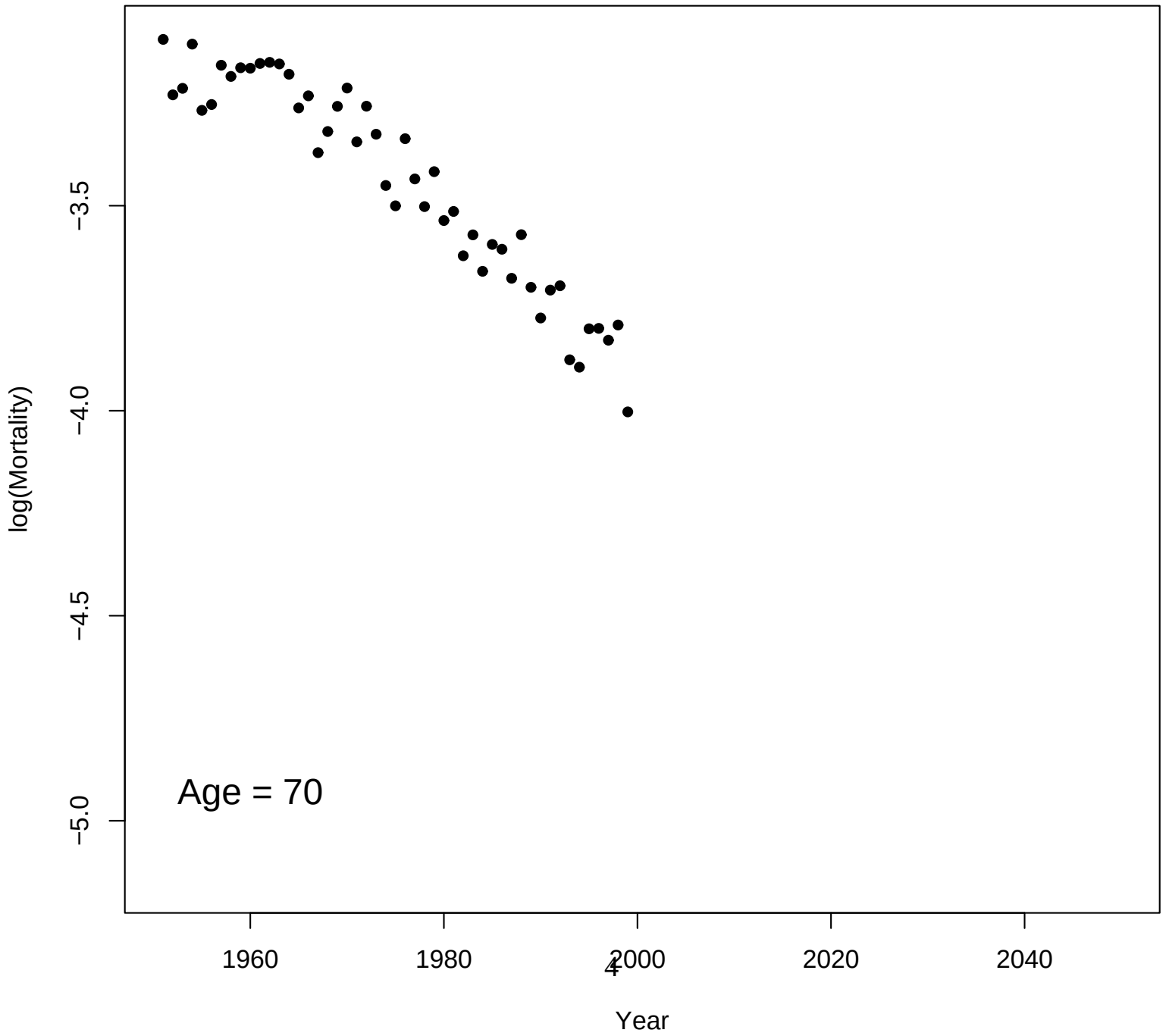
Heriot Watt University, Scotland

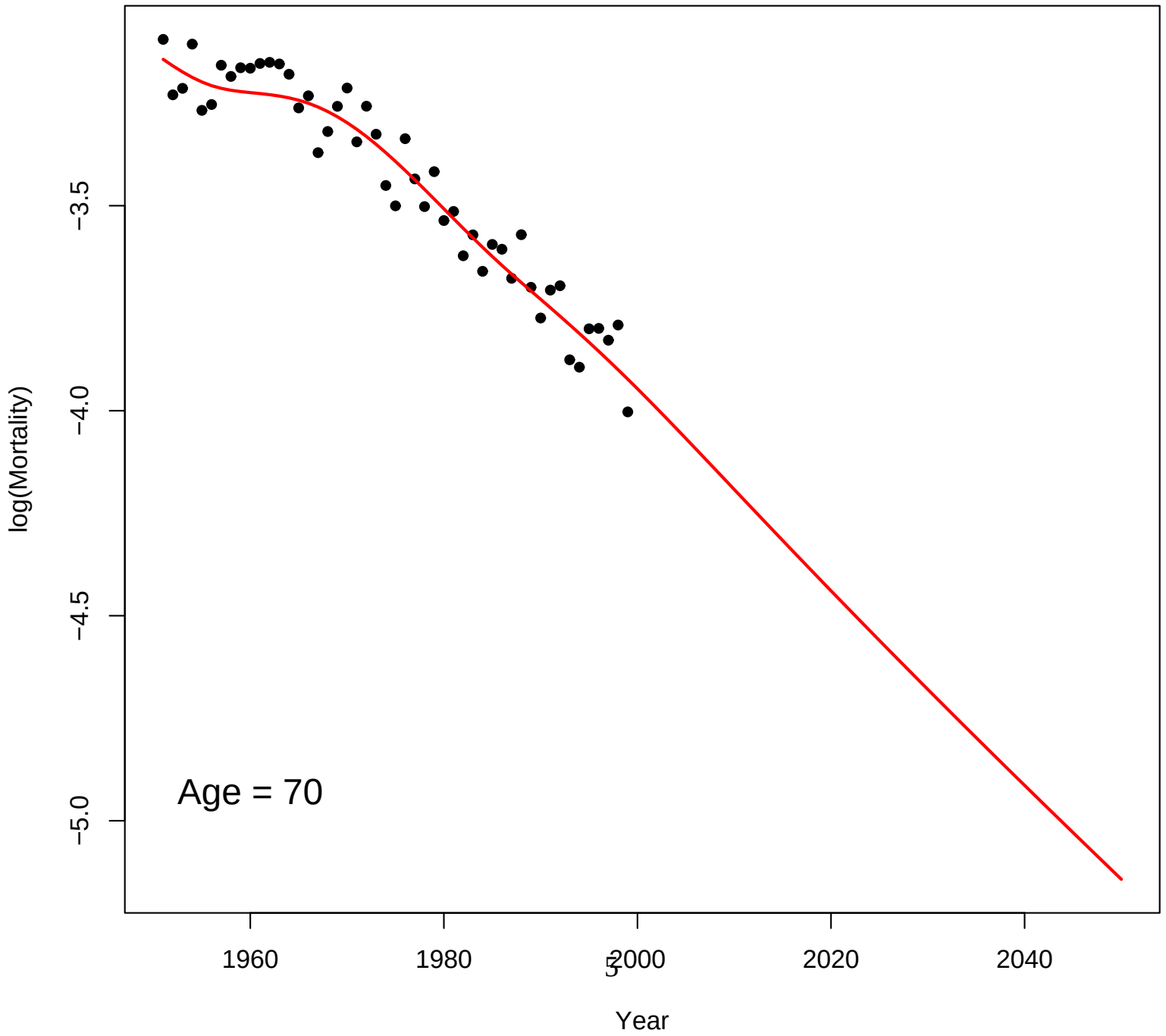
September 2005

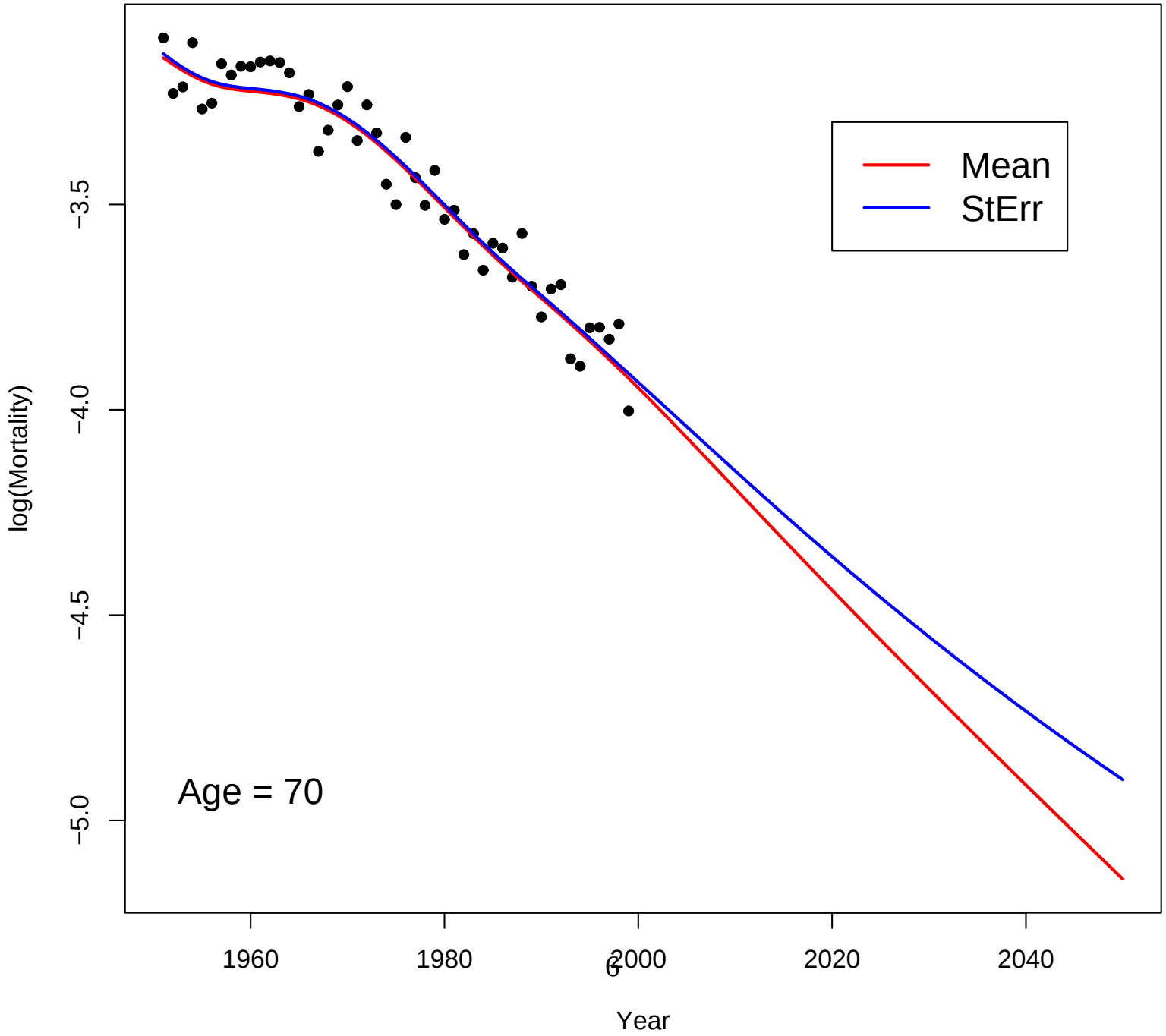
Plan of talk

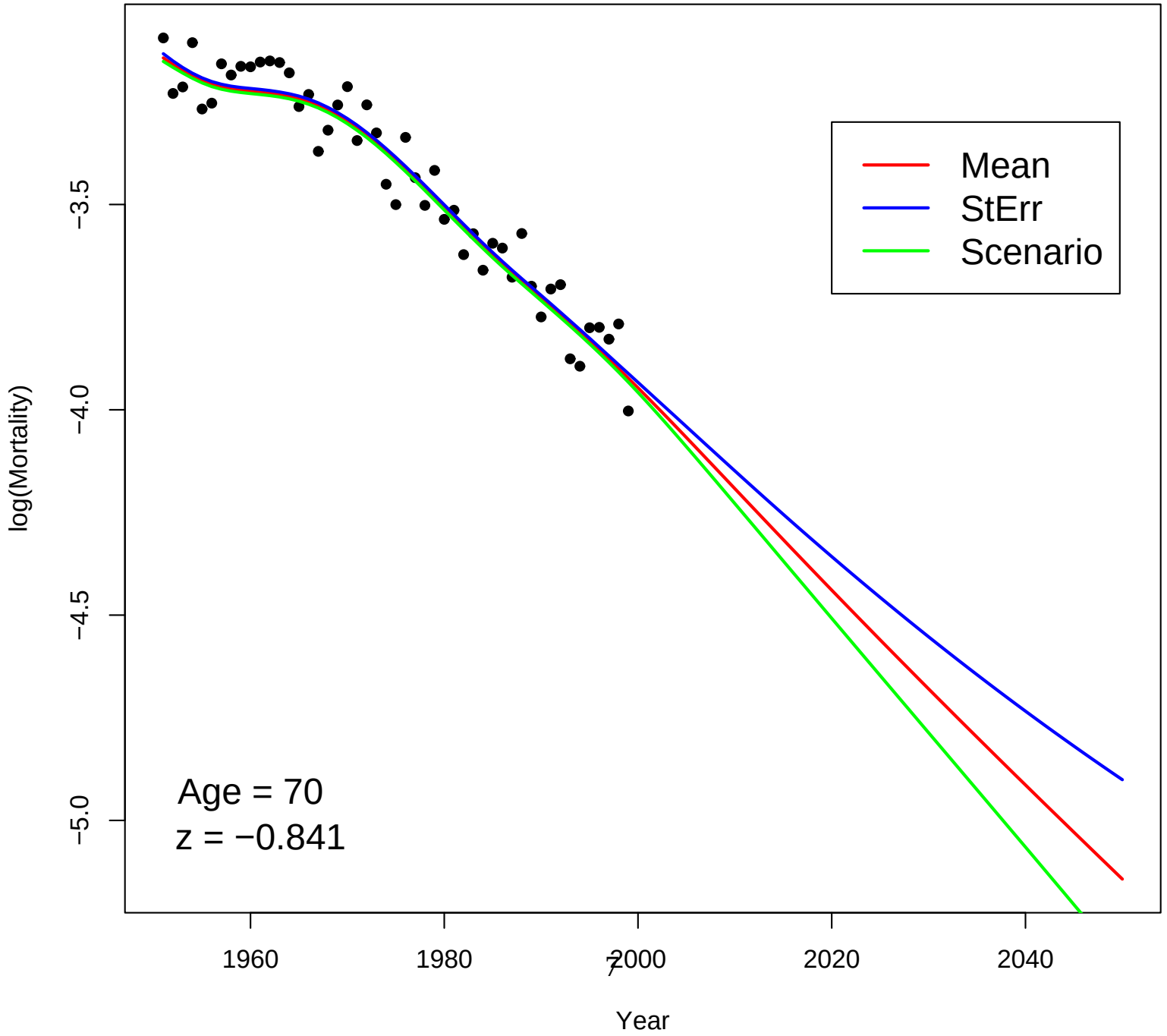
- The P-spline method
- The Lee-Carter method

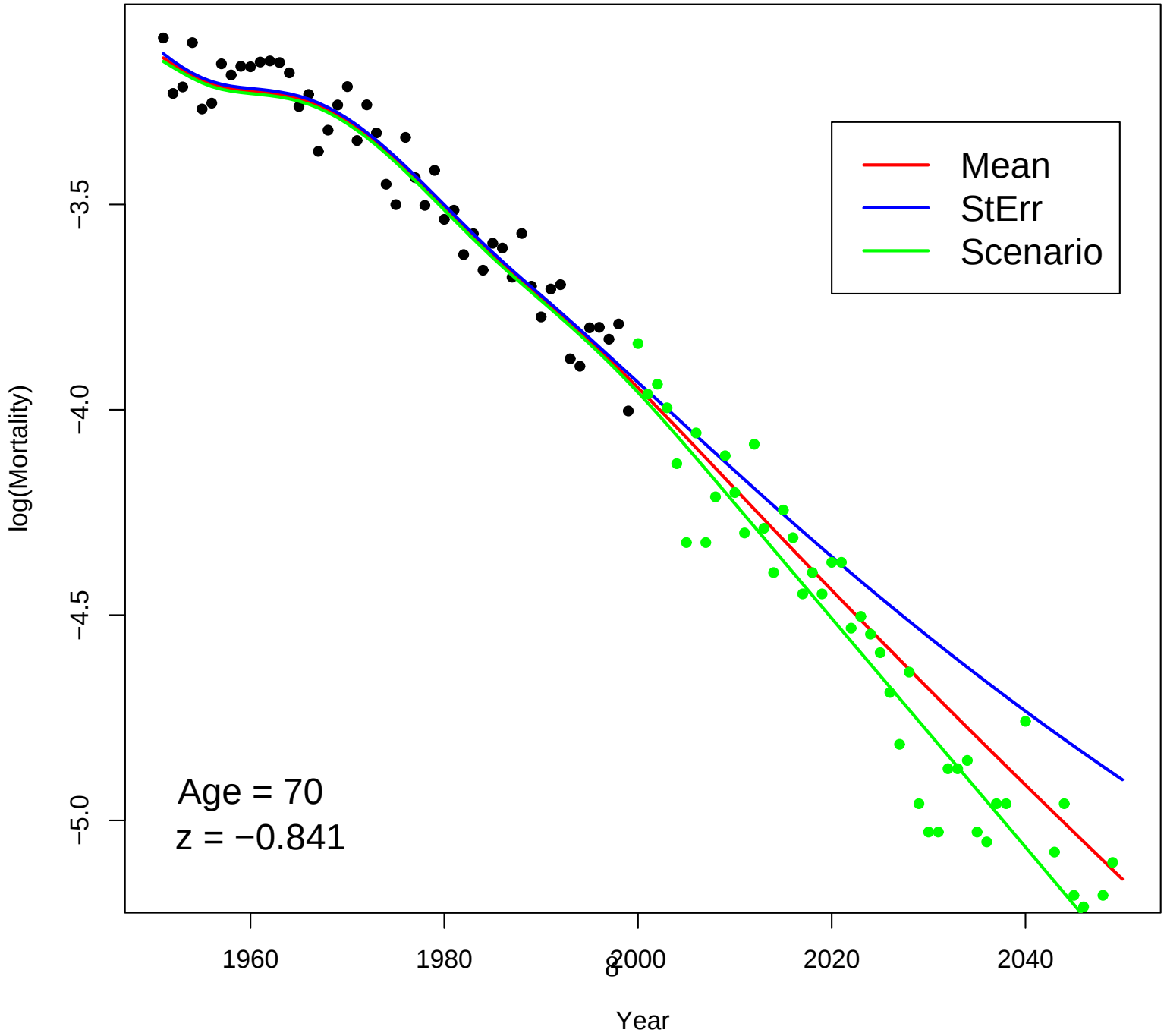

log(Mortality)
−3.2
−3.4
−3.6
−3.8
−4.0
1950
1960
1970
1980
1990
2000
Year


log(Mortality)
−3.5
−4.0
−4.5
−5.0
Age = 70
1960
1980
2000
2020
2040
Year


log(Mortality)
−3.5
−4.0
−4.5
−5.0
Age = 70
1960
1980
2000
2020
2040
Year


log(Mortality)
−3.5
−4.0
−4.5
−5.0
1960
1980
2000
2020
2040
Year
Mean
StErr
Age = 70


log(Mortality)
−3.5
−4.0
−4.5
−5.0
1960
1980
2000
2020
2040
Year
Mean
StErr
Scenario
Age = 70
z = −0.841


log(Mortality)
−3.5
−4.0
−4.5
−5.0
1960
1980
2000
2020
2040
Year
Mean
StErr
Scenario
Age = 70
z = −0.841

Scenario generating in 1-d

Data

$\boldsymbol{d} = \boldsymbol{d}_{n_y \times 1}$, deaths

$\boldsymbol{e} = \boldsymbol{e}_{n_y \times 1}$, exposures

Model

$\boldsymbol{d} = \mathcal{P}(\boldsymbol{e} * \boldsymbol{\lambda})$

$\boldsymbol{\lambda} = \boldsymbol{\lambda}_{n_y \times 1}, \qquad \log \boldsymbol{\lambda} = \boldsymbol{B}_y \boldsymbol{\theta} = \boldsymbol{m}$

Estimates

Mean curve: $\hat{\boldsymbol{m}} = \boldsymbol{B}_y \hat{\boldsymbol{\theta}}$

Standard error curve: $\hat{\boldsymbol{s}} = SE(\hat{\boldsymbol{m}})$

Deterministic scenario

Mean curve: $\hat{\boldsymbol{m}}_i = \hat{\boldsymbol{m}} + z_i \hat{\boldsymbol{s}}, \ i = 1, 2, 3, \ldots$

where $z_i \sim \mathcal{N}(0,\, 1)$.

Stochastic scenario

$\boldsymbol{d}_i = \mathcal{P}(\boldsymbol{e} * \exp(\hat{\boldsymbol{m}}_i)), \ i = 1, 2, 3, \ldots$

Scenario generating in 2-d

Data

$\boldsymbol{D} = \boldsymbol{D}_{n_a \times n_y}$, deaths

$\boldsymbol{E} = \boldsymbol{E}_{n_a \times n_y}$, exposures

Model: 2-d P-splines: $\boldsymbol{B} = \boldsymbol{B}_y \otimes \boldsymbol{B}_a$

$\boldsymbol{D} = \mathcal{P}(\boldsymbol{E} * \boldsymbol{\Lambda})$

$\boldsymbol{\Lambda} = \boldsymbol{\Lambda}_{n_a \times n_y}, \qquad \log \boldsymbol{\Lambda} = \boldsymbol{B}_a \boldsymbol{\Theta} \boldsymbol{B}_y' = \boldsymbol{M}$

Estimates

Mean sheet: $\hat{\boldsymbol{M}} = \boldsymbol{B}_a \hat{\boldsymbol{\Theta}} \boldsymbol{B}_y'$

Standard error sheet: $\hat{\boldsymbol{S}} = SE(\hat{\boldsymbol{M}})$

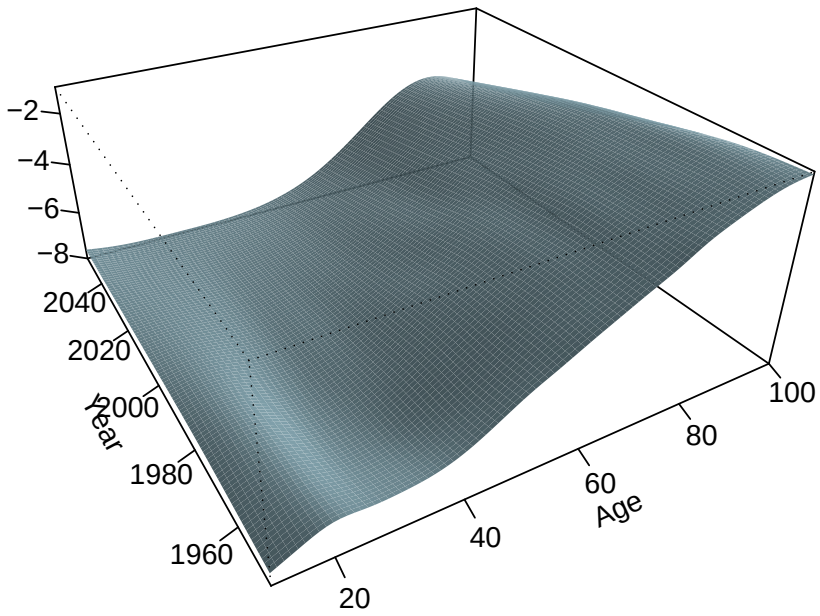
Deterministic scenario

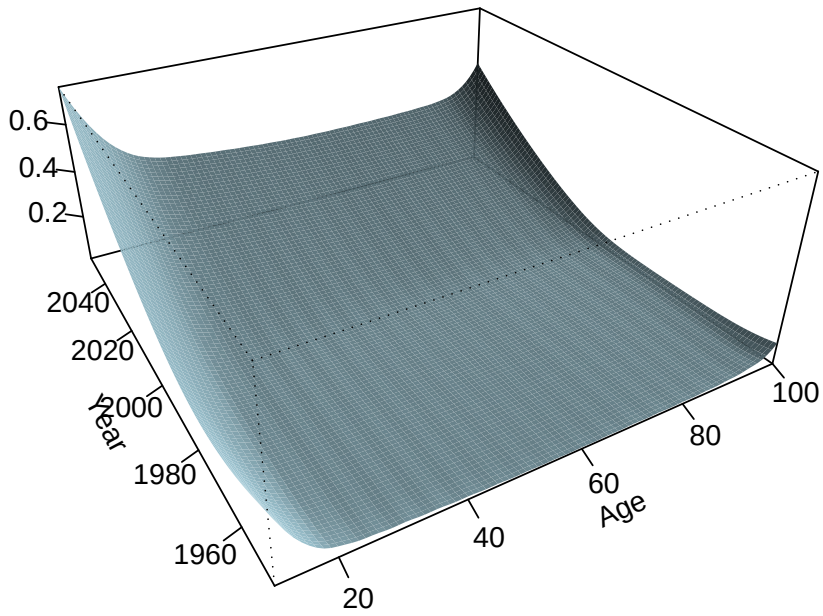
Mean sheet: $\hat{\boldsymbol{M}}_i = \hat{\boldsymbol{M}} + z_i \hat{\boldsymbol{S}}, \ i = 1, 2, 3, \ldots$

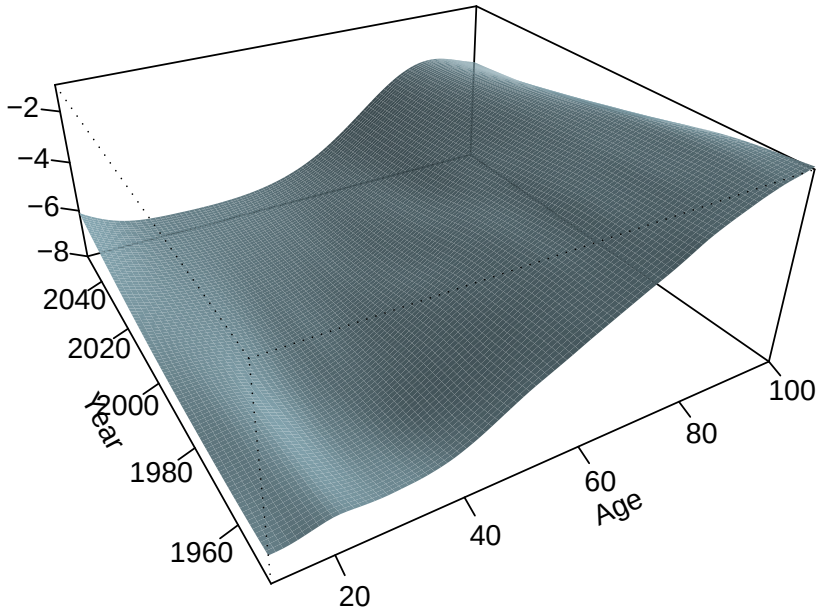
where $z_i \sim \mathcal{N}(0, 1)$.

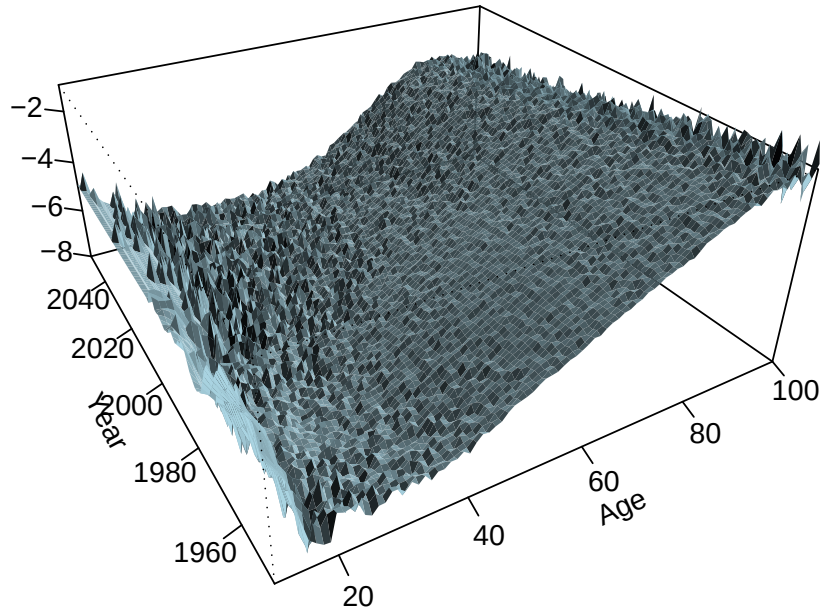
Stochastic scenario

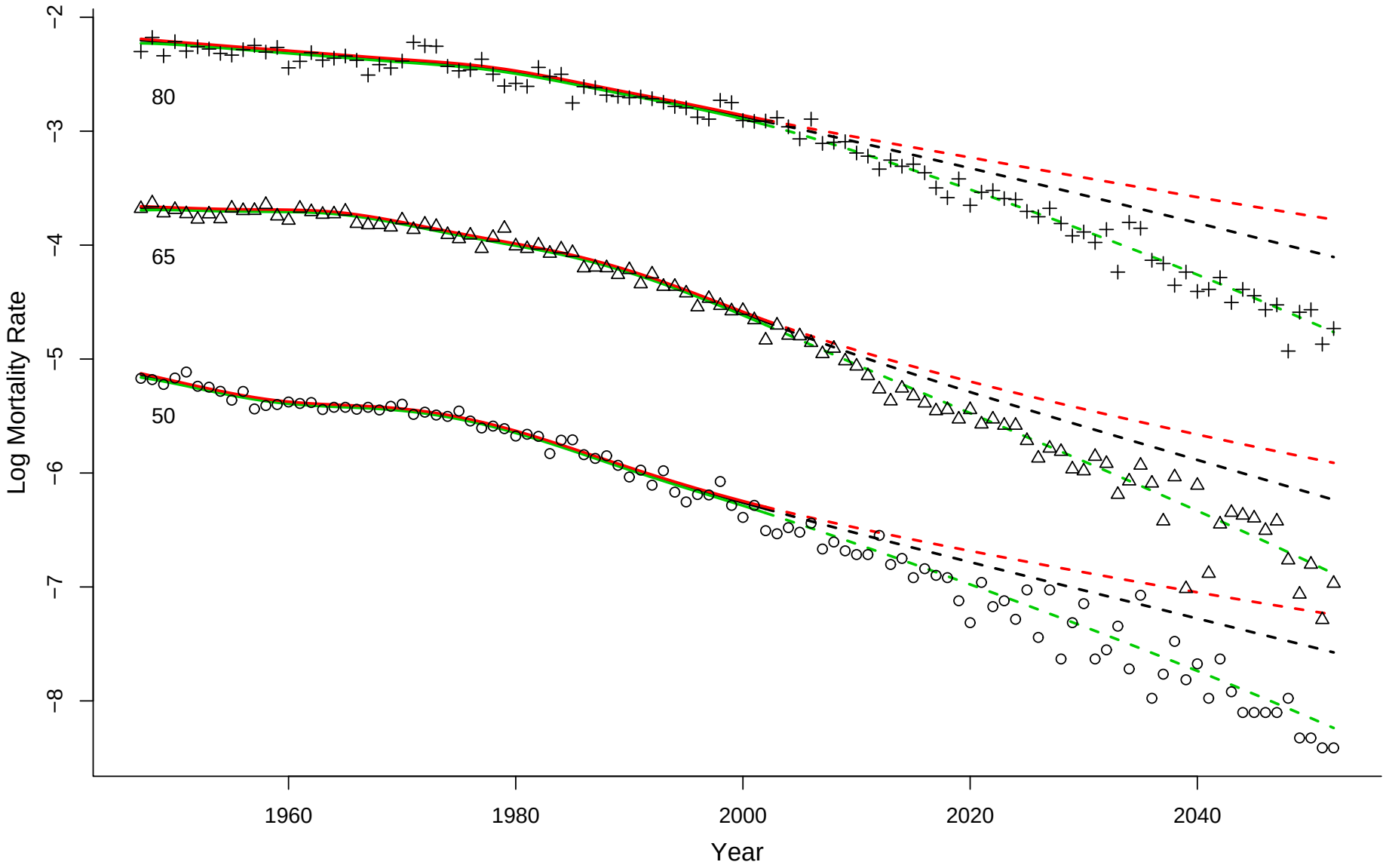
$\boldsymbol{D}_i = \mathcal{P}(\boldsymbol{E} * \exp(\hat{\boldsymbol{M}}_i)), \ i = 1, 2, 3, \ldots$


−2
−4
−6
−8
2040
2020
2000
1980
1960
Year
20
40
60
80
100
Age


0.6
0.4
0.2
2040
2020
2000
1980
1960
Year
20
40
60
80
100
Age


−2
−4
−6
−8
2040
2020
2000
1980
1960
Year
20
40
60
80
100
Age


−2
−4
−6
−8
2040
2020
2000
1980
1960
Year
20
40
60
80
100
Age


Log Mortality Rate
−2
−3
−4
−5
−6
−7
−8
80
65
50
1960
1980
2000
2020
2040
Year

Lee-Carter model

Data

$$\boldsymbol{D} = \boldsymbol{D}_{n_a \times n_y}, \quad \text{deaths}$$

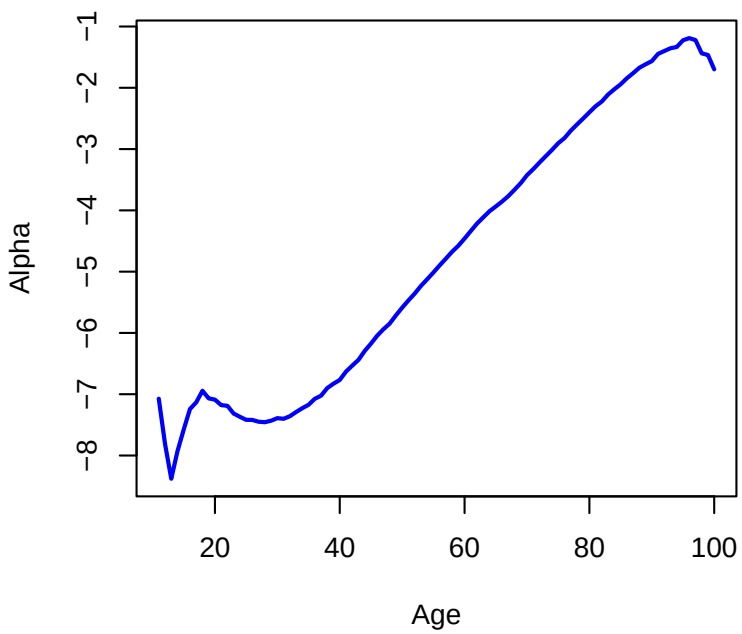
$$\boldsymbol{E} = \boldsymbol{E}_{n_a \times n_y}, \quad \text{exposures}$$

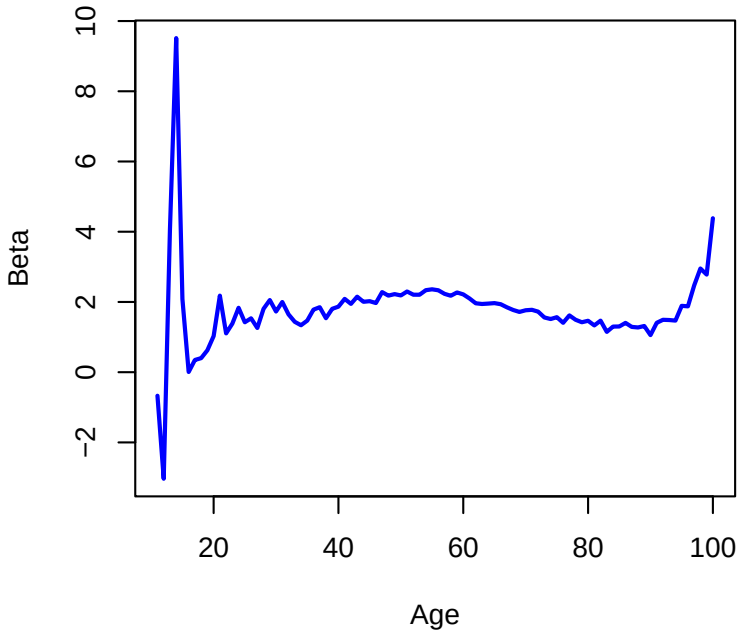
Model: Lee-Carter

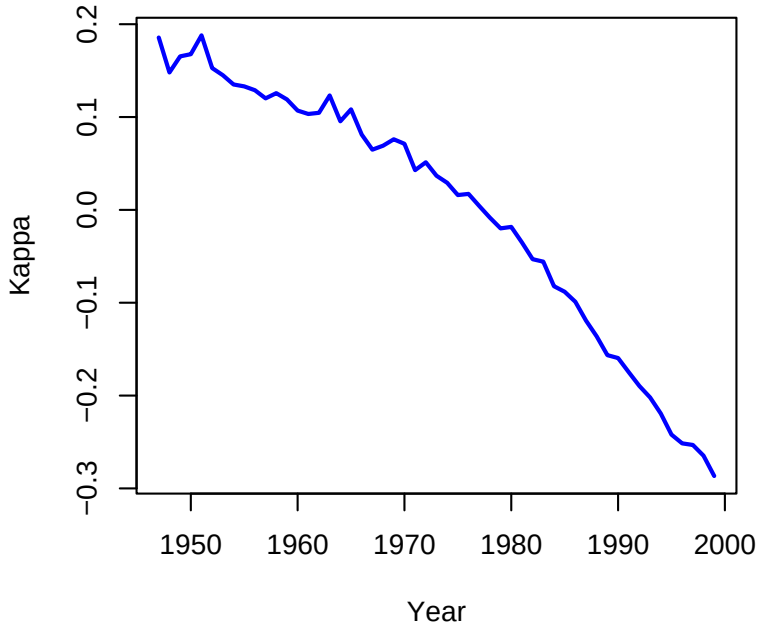
$$\boldsymbol{D} = \mathcal{P}(\boldsymbol{E} * \boldsymbol{\Lambda})$$

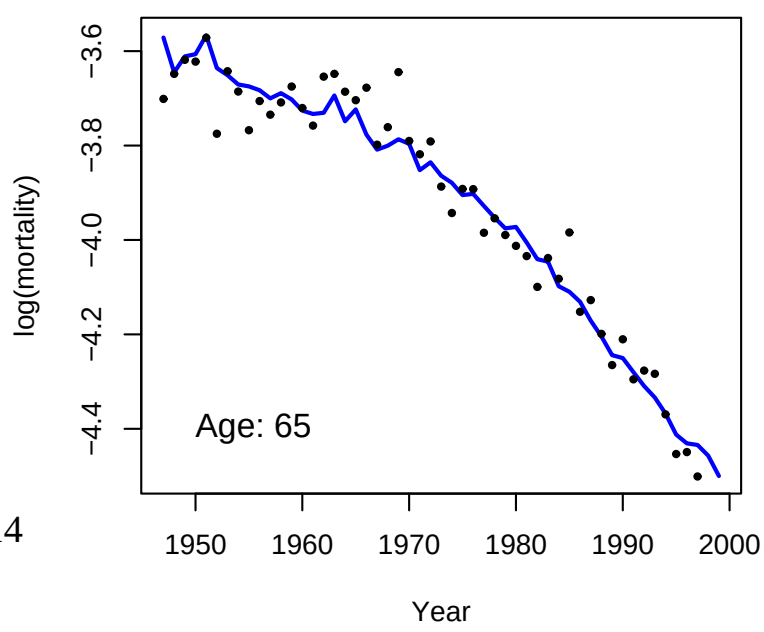
$$\log \boldsymbol{\Lambda} = \boldsymbol{\alpha}\mathbf{1}' + \boldsymbol{\beta}\boldsymbol{\kappa}' = \boldsymbol{M}$$

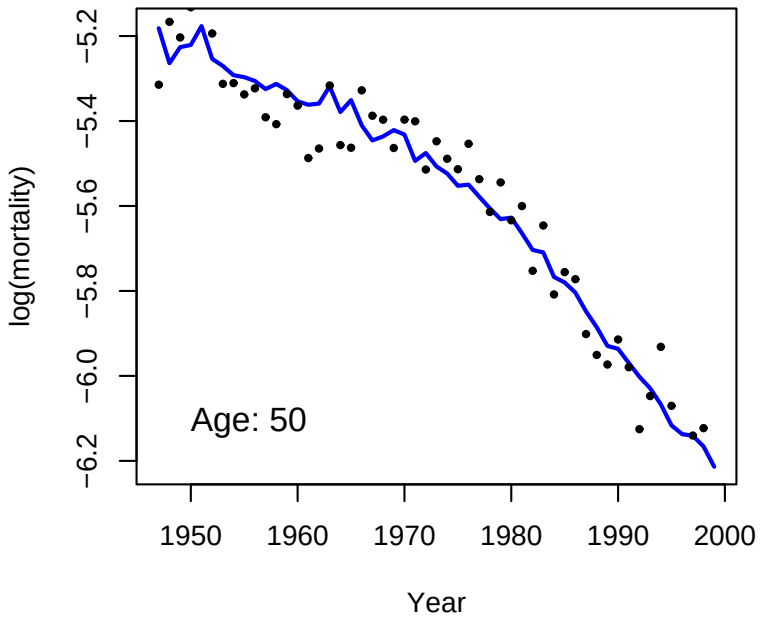
or $\log \lambda_{ij} = \alpha_i + \beta_i \kappa_j$

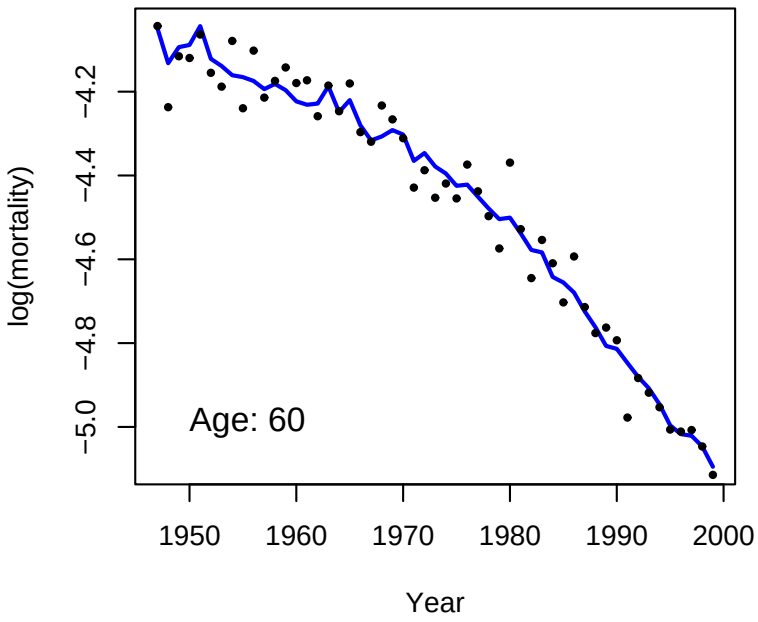

Alpha
−1
−2
−3
−4
−5
−6
−7
−8
20
40
60
80
100
Age

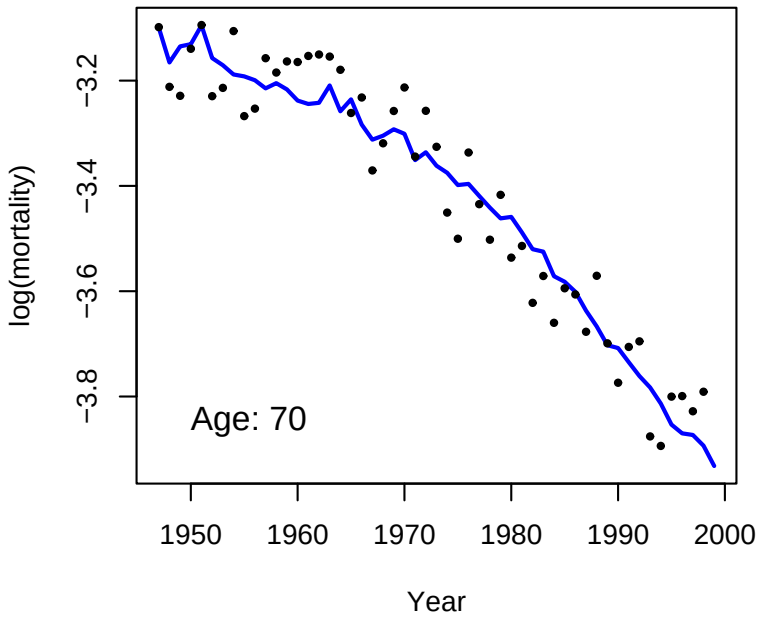

Beta
10
8
6
4
2
0
−2
20
40
60
80
100
Age

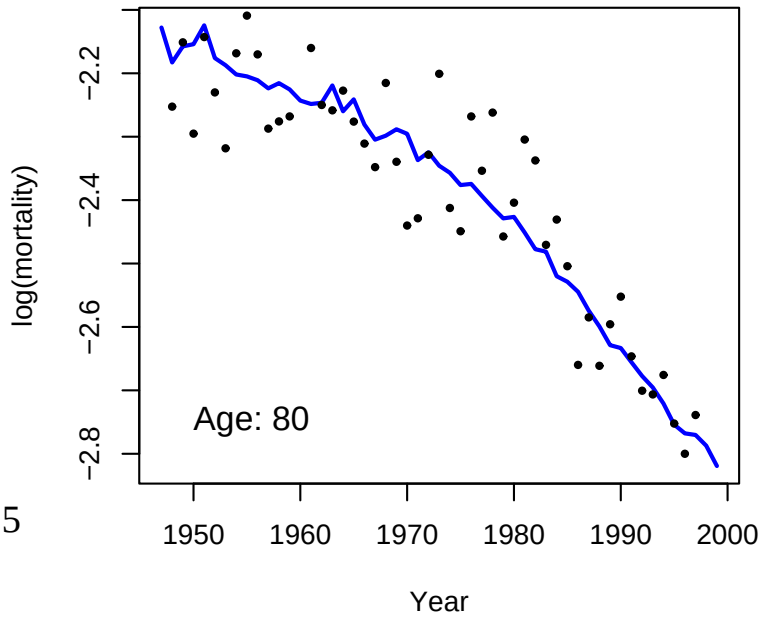

Kappa
0.2
0.1
0.0
−0.1
−0.2
−0.3
1950
1960
1970
1980
1990
2000
Year

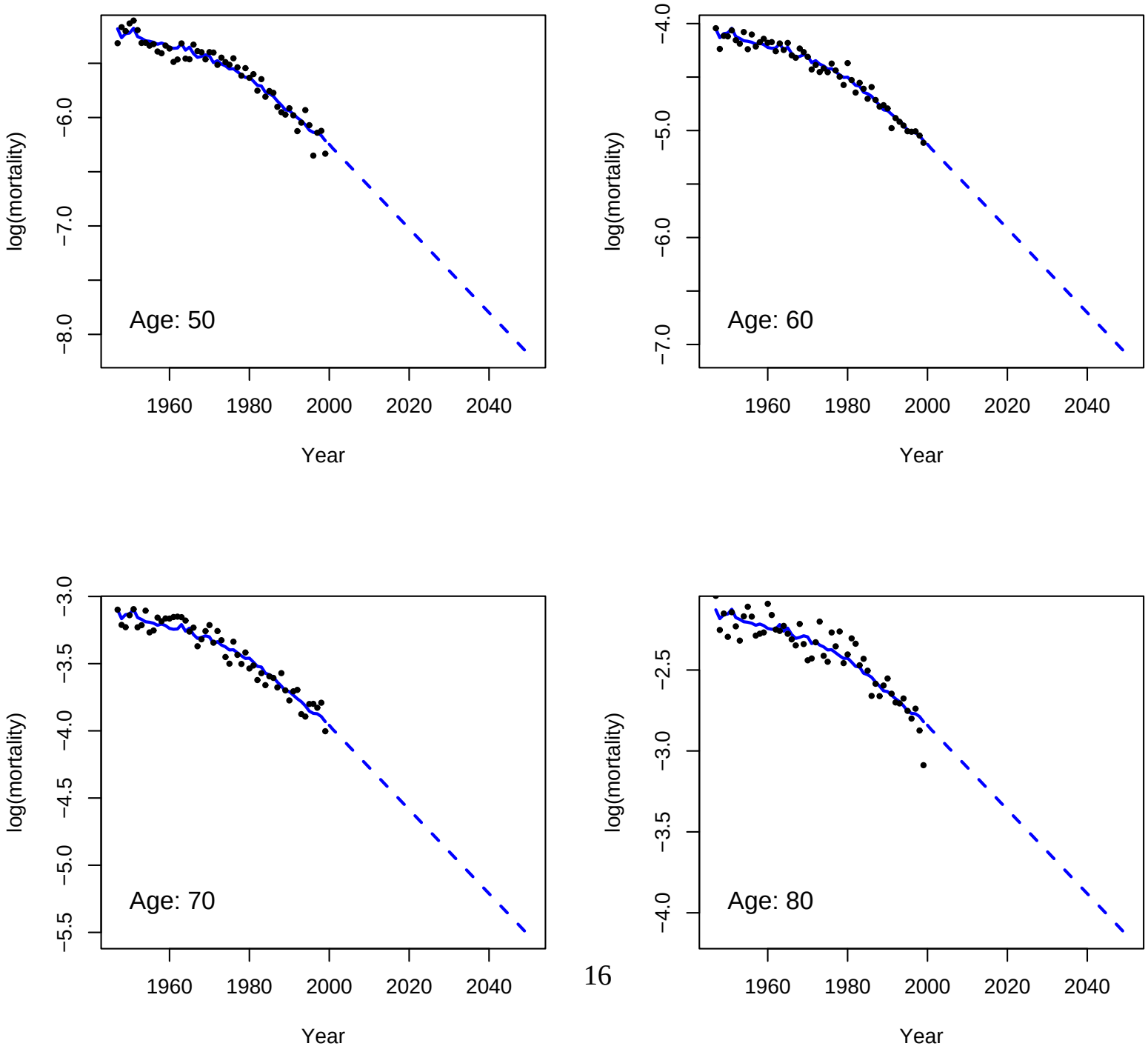

log(mortality)
−3.6
−3.8
−4.0
−4.2
−4.4
Age: 65
1950
1960
1970
1980
1990
2000
Year

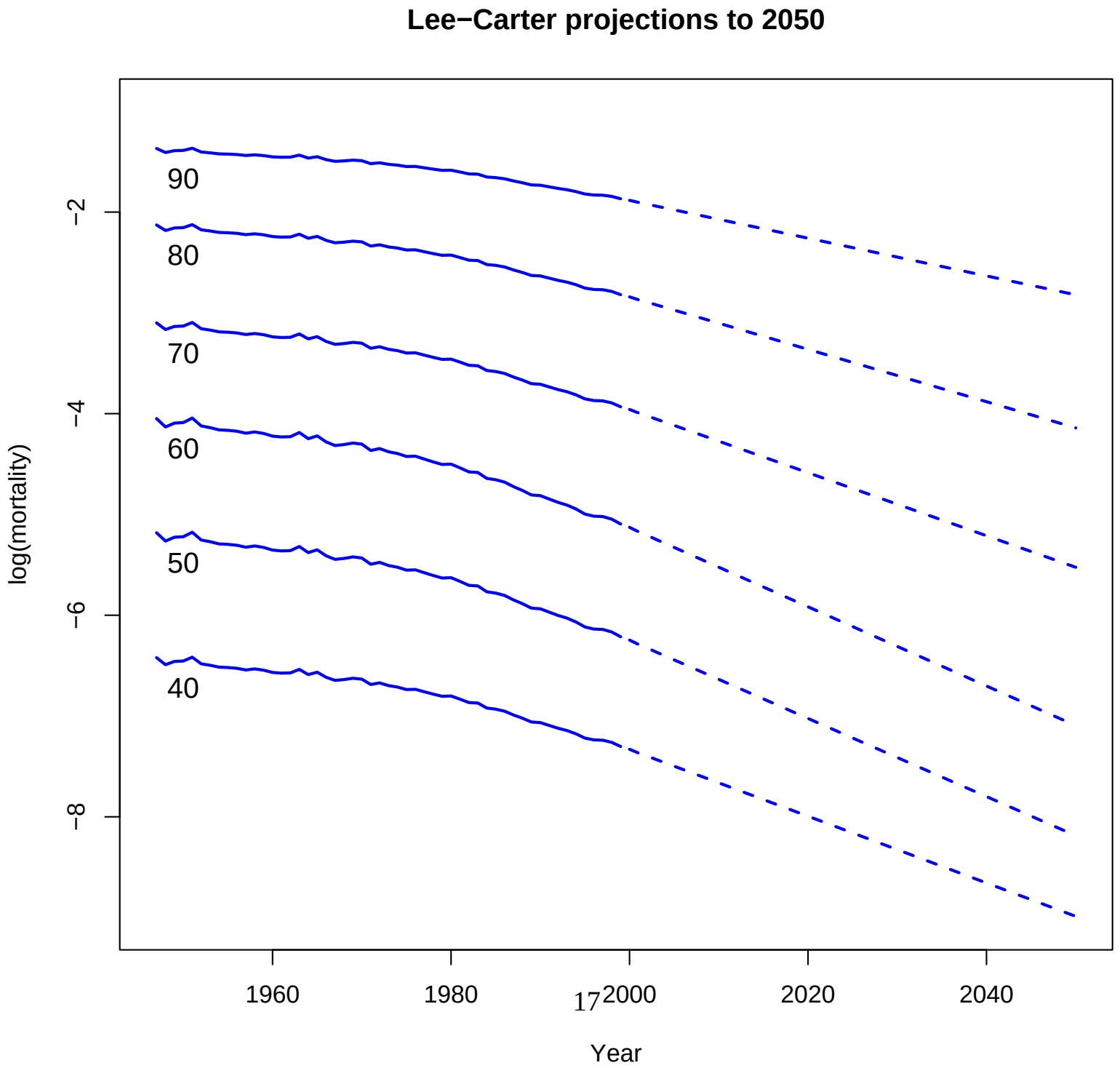

log(mortality)
−5.2
−5.4
−5.6
−5.8
−6.0
−6.2
Age: 50
1950
1960
1970
1980
1990
2000
Year


log(mortality)
−4.2
−4.4
−4.6
−4.8
−5.0
Age: 60
1950
1960
1970
1980
1990
2000
Year


log(mortality)
−3.2
−3.4
−3.6
−3.8
Age: 70
1950
1960
1970
1980
1990
2000
Year


log(mortality)
−2.2
−2.4
−2.6
−2.8
Age: 80
1950
1960
1970
1980
1990
2000
Year


log(mortality)
−6.0
−7.0
−8.0
Age: 50
1960
1980
2000
2020
2040
Year
log(mortality)
−4.0
−5.0
−6.0
−7.0
Age: 60
1960
1980
2000
2020
2040
Year
log(mortality)
−3.0
−3.5
−4.0
−4.5
−5.0
−5.5
Age: 70
1960
1980
2000
2020
2040
Year
log(mortality)
−2.5
−3.0
−3.5
−4.0
Age: 80
1960
1980
2000
2020
2040
Year


Lee−Carter projections to 2050
log(mortality)
−2
−4
−6
−8
90
80
70
60
50
40
1960
1980
2000
2020
2040
Year

Bootstrapping I

Pearson residuals $\boldsymbol{R}_p$

$$\boldsymbol{R}_p = \frac{\boldsymbol{D} - \hat{\boldsymbol{D}}}{\sqrt{\hat{\boldsymbol{D}}}}$$

Permute row one of $\boldsymbol{R}_p$ - and solve for new data!!

Repeat for all other rows using the same permutation.

Refit bootstrap data and forecast.

Bootstrapping II

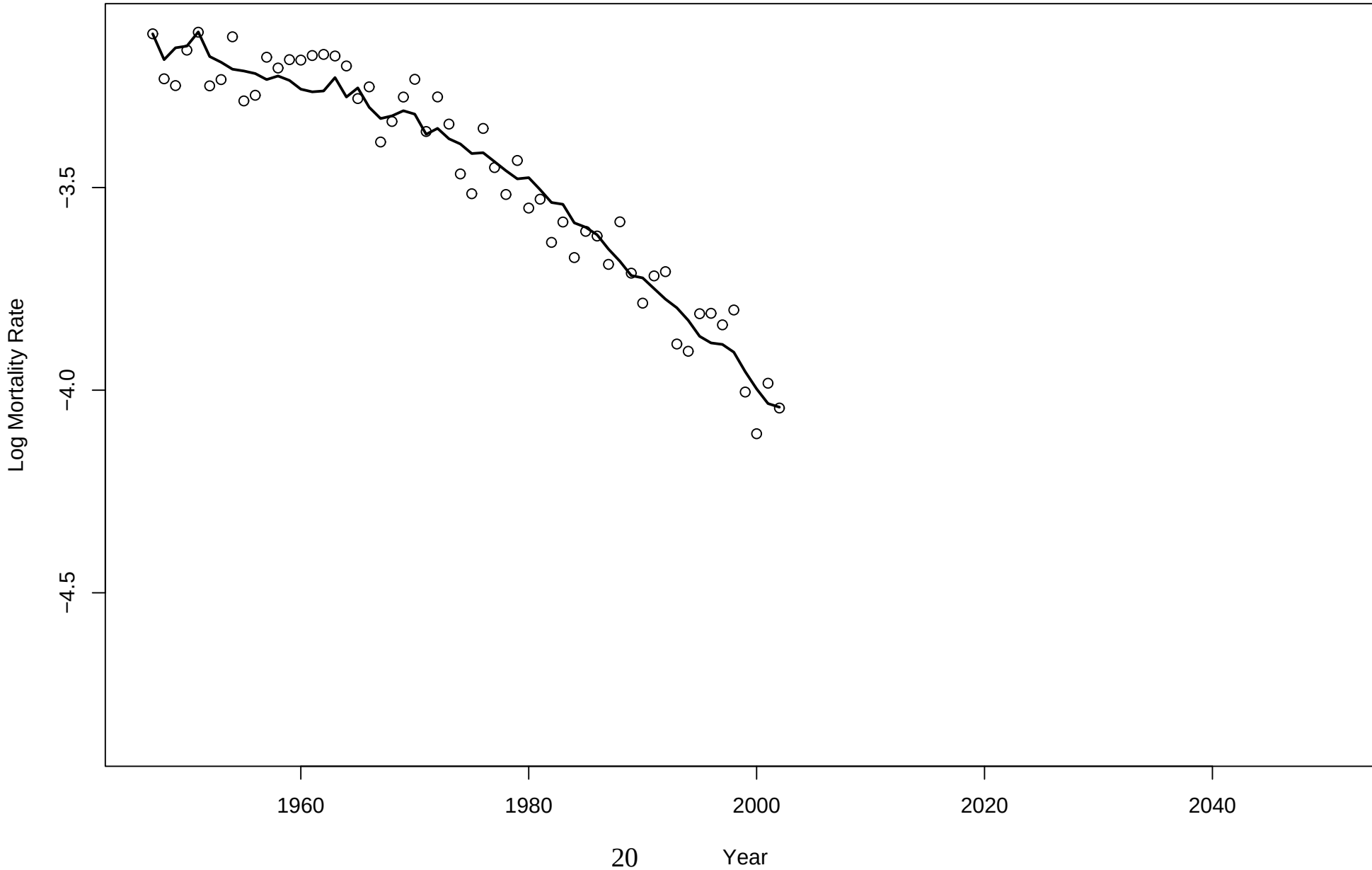
Deviance residuals $\boldsymbol{R}_d$

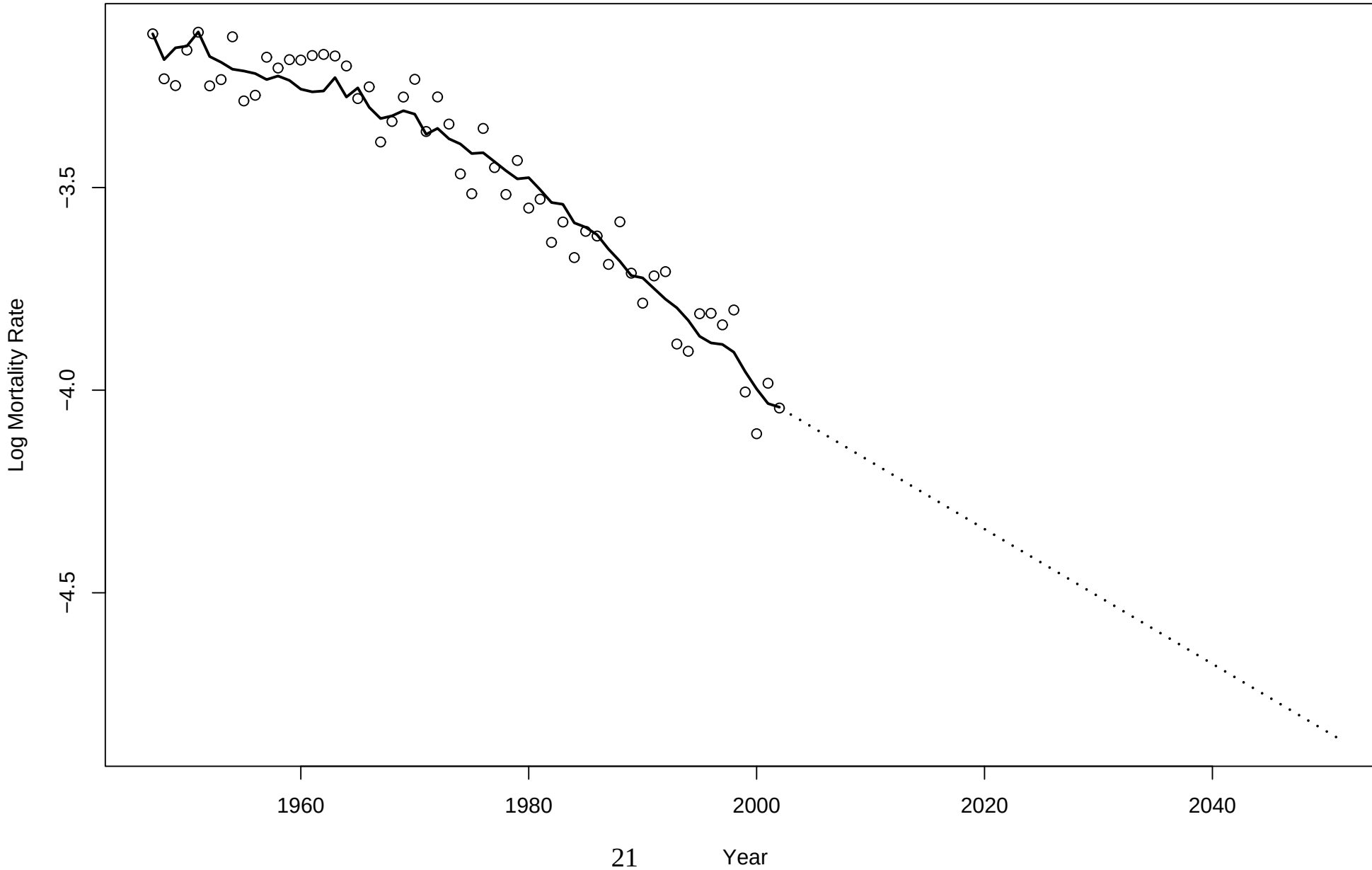
$$\boldsymbol{R}_d = \text{sign}(\boldsymbol{D}_{x,t} - \hat{\boldsymbol{D}}_{x,t})\sqrt{2[\boldsymbol{D}_{x,t}\log\frac{\boldsymbol{D}_{x,t}}{\hat{\boldsymbol{D}}_{x,t}} - (\boldsymbol{D}_{x,t} - \hat{\boldsymbol{D}}_{x,t})]}$$

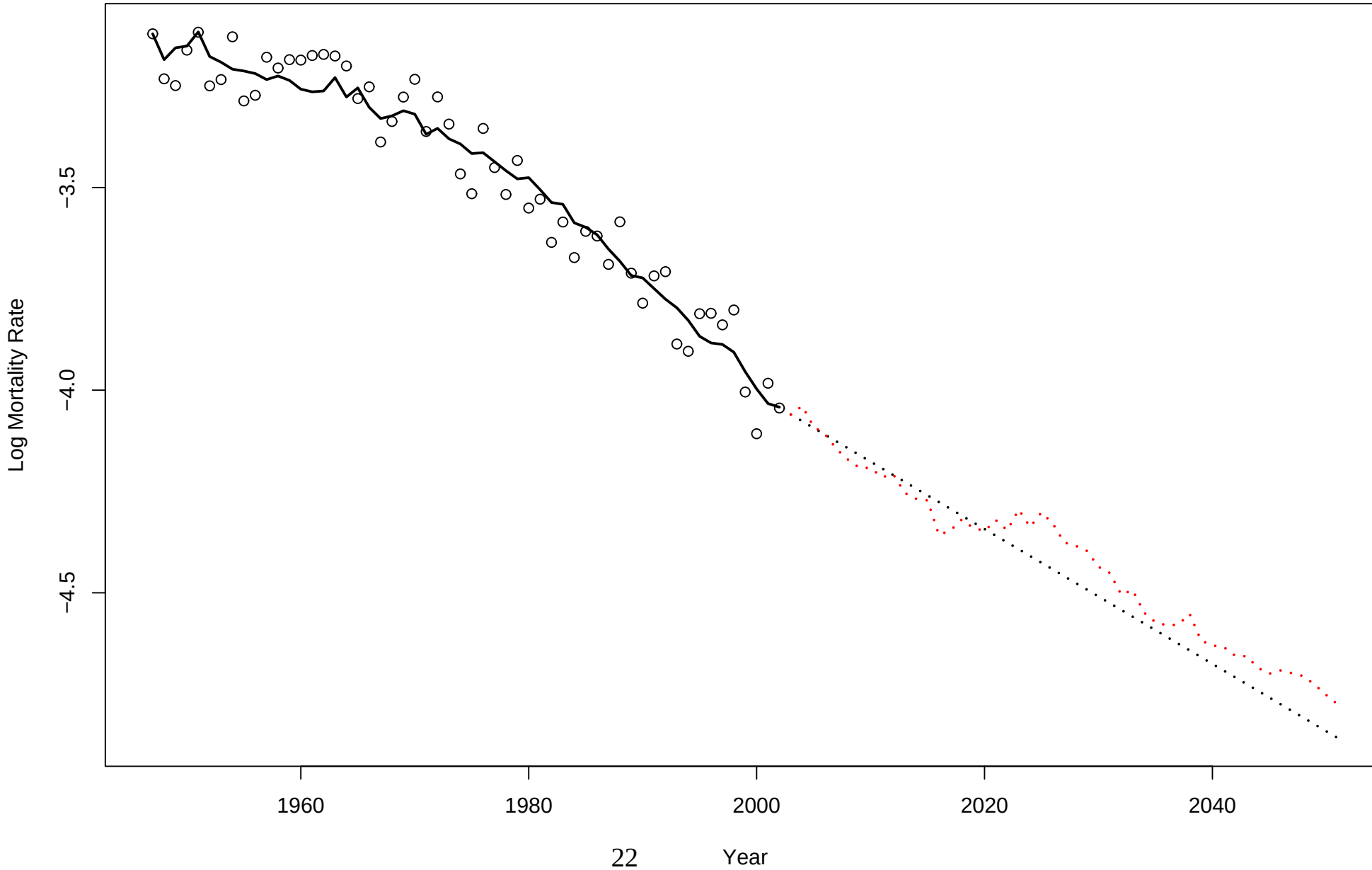
Permute row one of $\boldsymbol{R}_d$ - and solve for new data!!

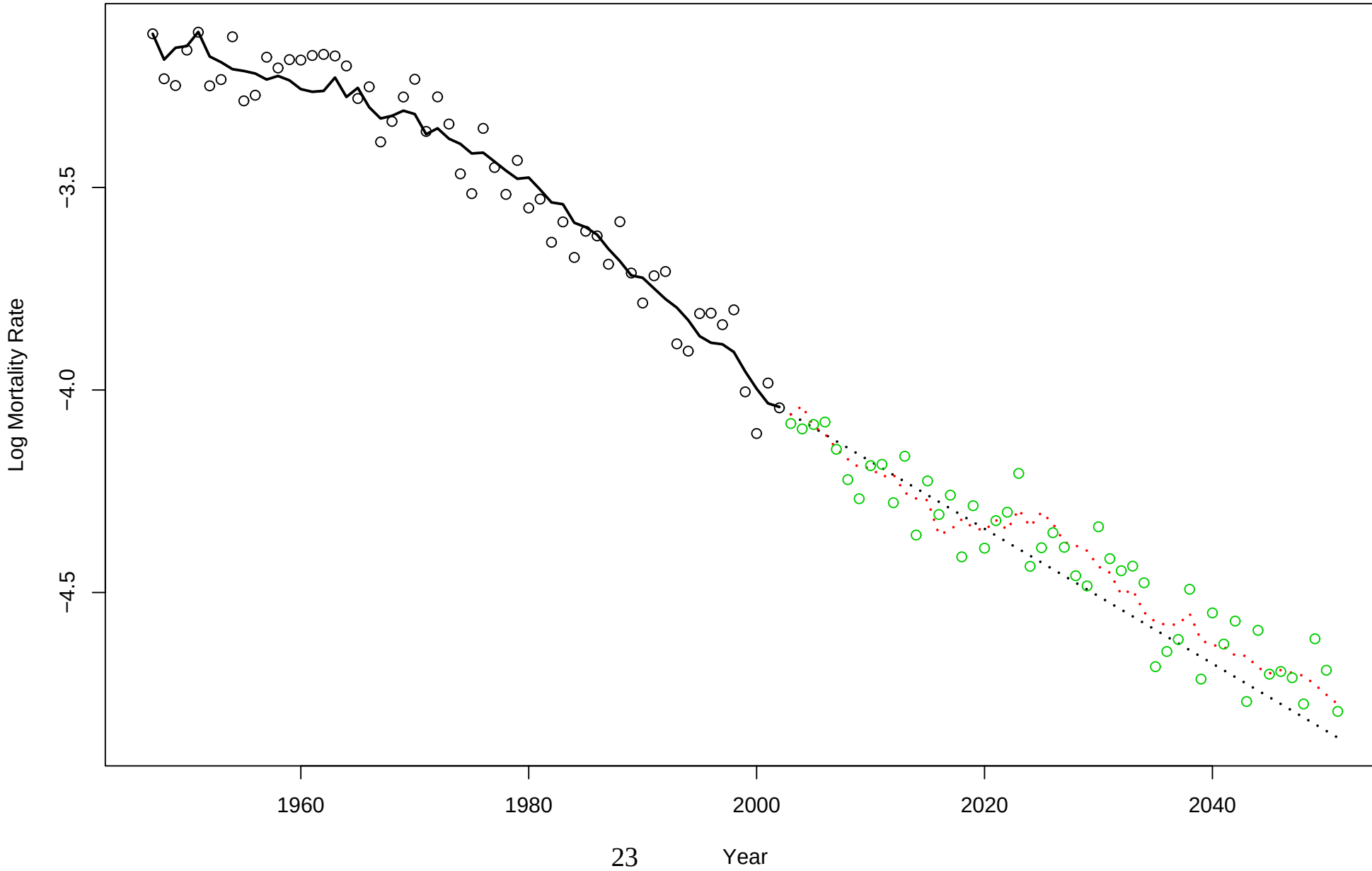
Repeat for all other rows using the same permutation.

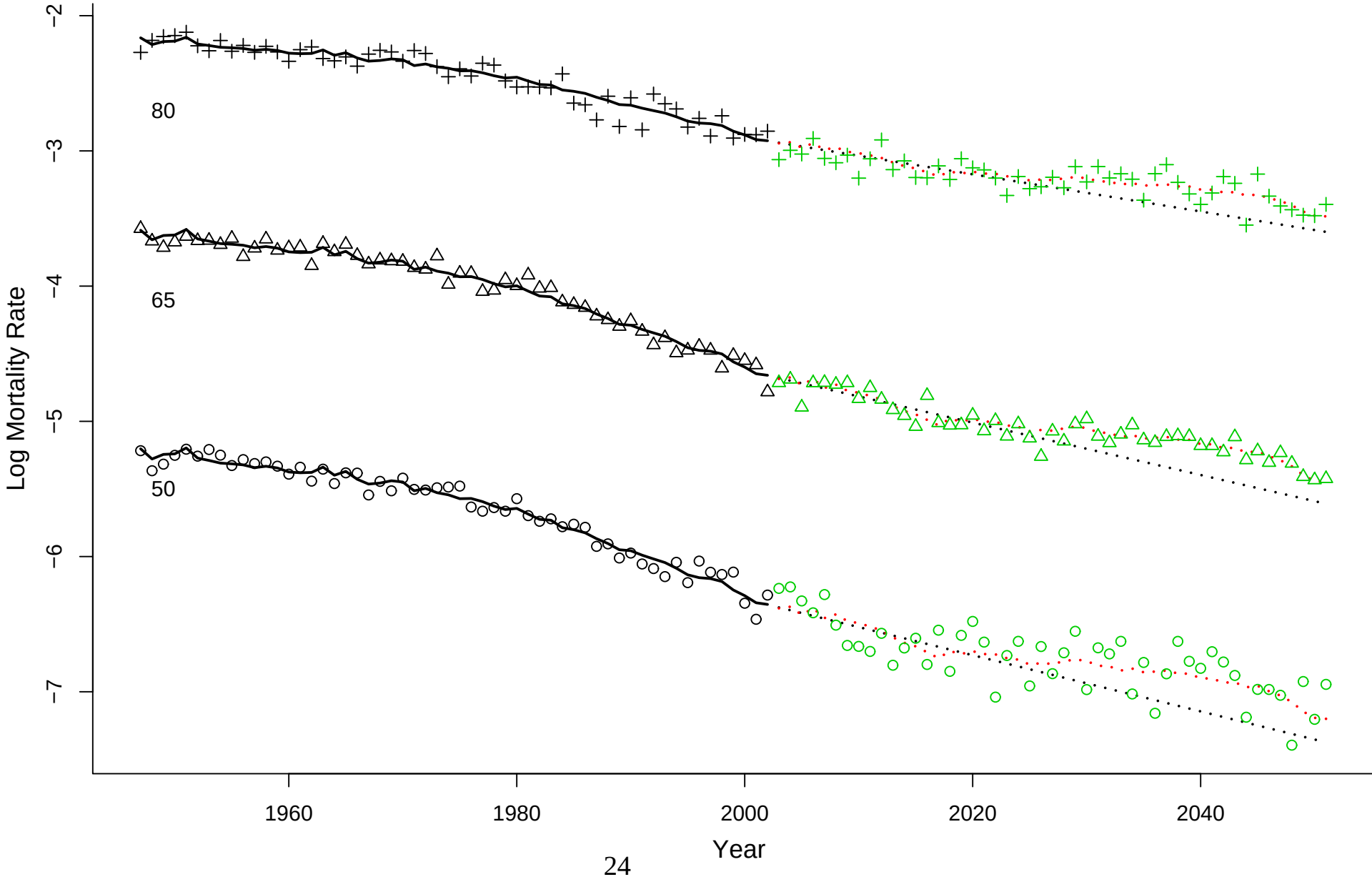
Refit bootstrap data and forecast.


Log Mortality Rate
−3.5
−4.0
−4.5
1960
1980
2000
2020
2040
Year


Log Mortality Rate
−3.5
−4.0
−4.5
1960
1980
2000
2020
2040
Year


Log Mortality Rate
−3.5
−4.0
−4.5
1960
1980
2000
2020
2040
Year


Log Mortality Rate
−3.5
−4.0
−4.5
1960
1980
2000
2020
2040
Year


Log Mortality Rate
−2
−3
−4
−5
−6
−7
80
65
50
1960
1980
2000
2020
2040
Year

References

Lee-Carter models

Lee & Carter (1992) J American Statistical Association, 87, 659-675.

Brouhns, Denuit & Vermunt (2002) Insurance: Mathematics & Economics, 31, 373-393.

Penalized spline models

Eilers & Marx (1996) Statistical Science, 11, 758-783.

Currie, Durban & Eilers (2004) Statistical Modelling, 4, 279-298.

Richards, Kirkby, and Currie, (2005) BAJ, to appear

Web sites

`http://www.ma.hw.ac.uk/~iain/research/papers.html`

`http://www.ma.hw.ac.uk/~iain/workshop/workshop.html`

Agenda

- Introduction
- P-spline model
- Lee-Carter model
- **Illustrative Results**
- Choosing a mortality basis for reporting purposes
- Conclusions/Future Work

Lunch

- Software workshop

Illustrative Results

Rajeev Shah

Illustrative Results

- Using the output from Stochastic models
- Model differences:
  - P-spline (period penalty) v P-spline (cohort penalty) v Lee–Carter v 92 Series
- Effect of using different datasets for P-spline
- Effect of using different parameters for P-spline
- Progress of projections 1984-1992 for P-spline
- Progress of annuity values 1984-1992

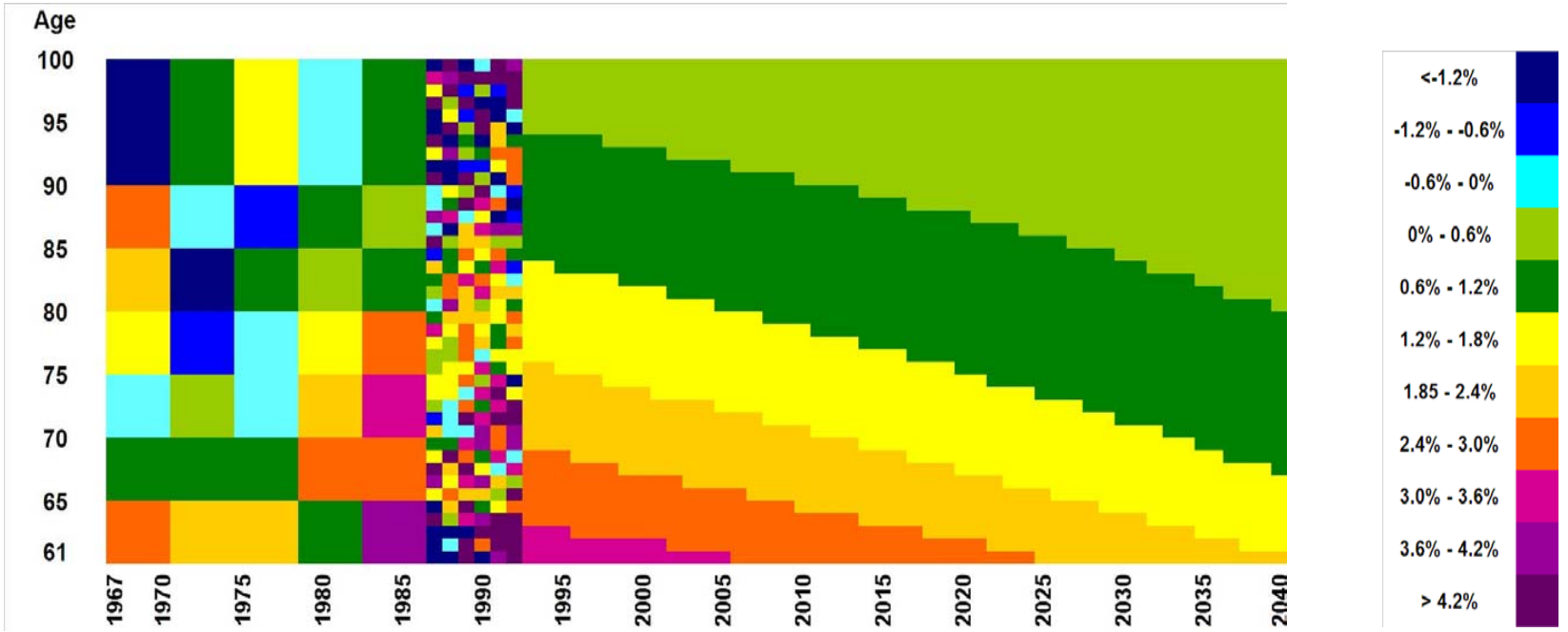
Using the output from Stochastic models

- Models generate μ_x (for data at age nearest)
- Estimate q_x as $1 - \exp[- (\mu_x + \mu_{x+1}) /2]$
- Calculate Improvement Factors = $q_{x,t} / q_{x,(t-1)}$
- Decide on start data
  - Adjust Base Table for recent improvements and office experience
- Apply Improvement Factors to start data to get projected $q'_{x,t}$
- Calculate annuity values

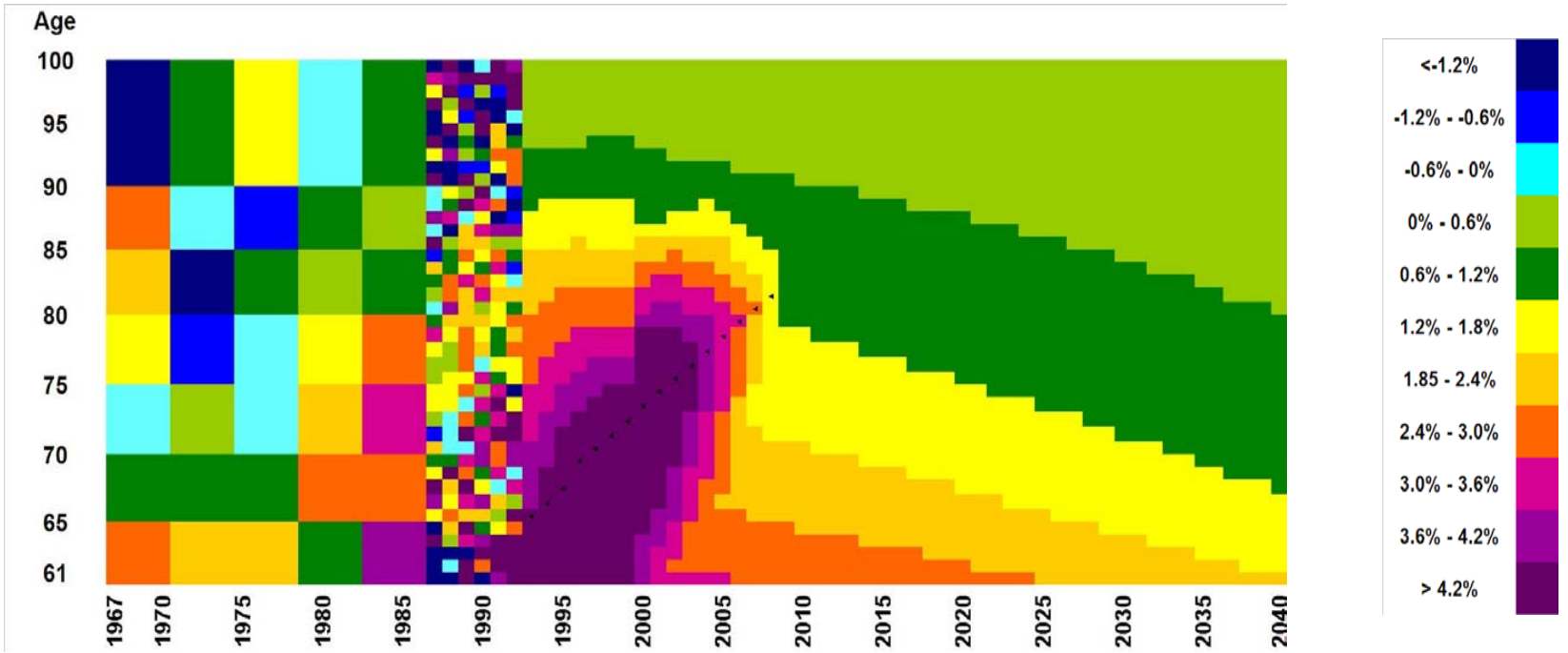
Model differences:

P-spline (period penalty) v P-spline (cohort penalty) v Lee–Carter v 92 Series

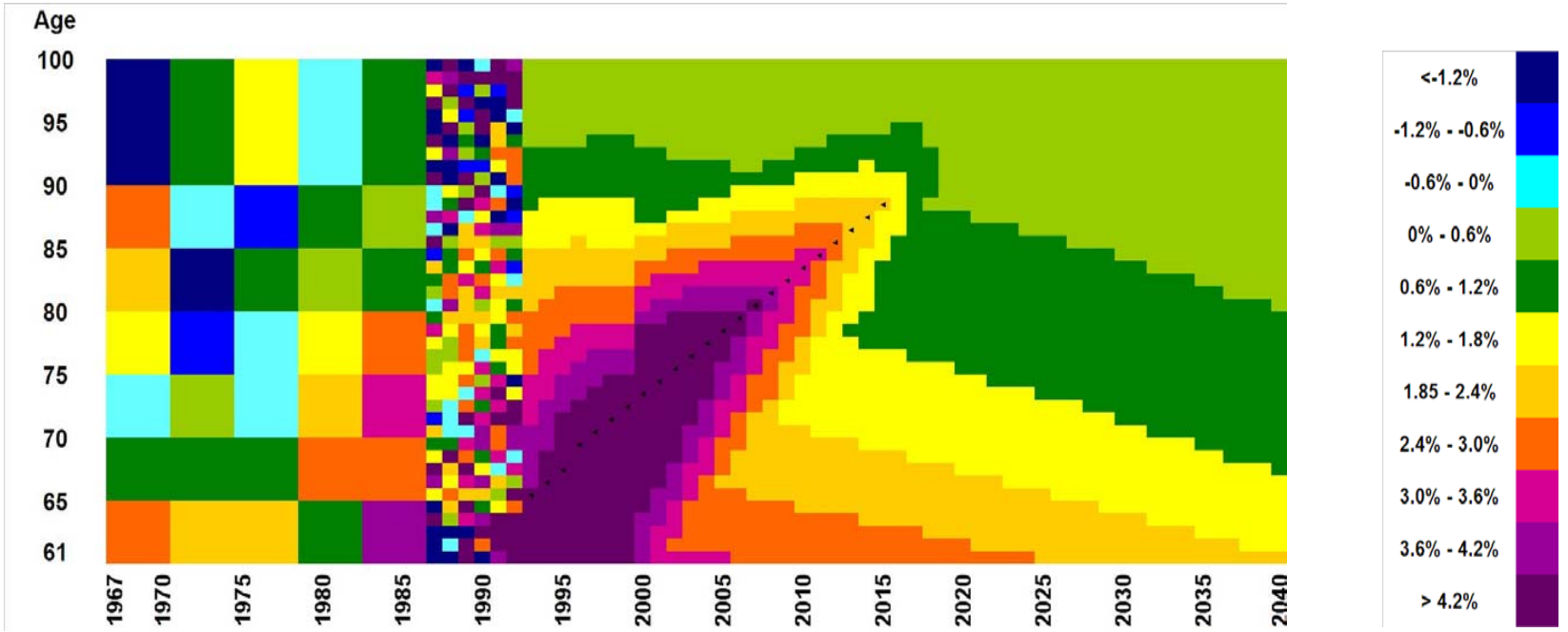
92 Series projections



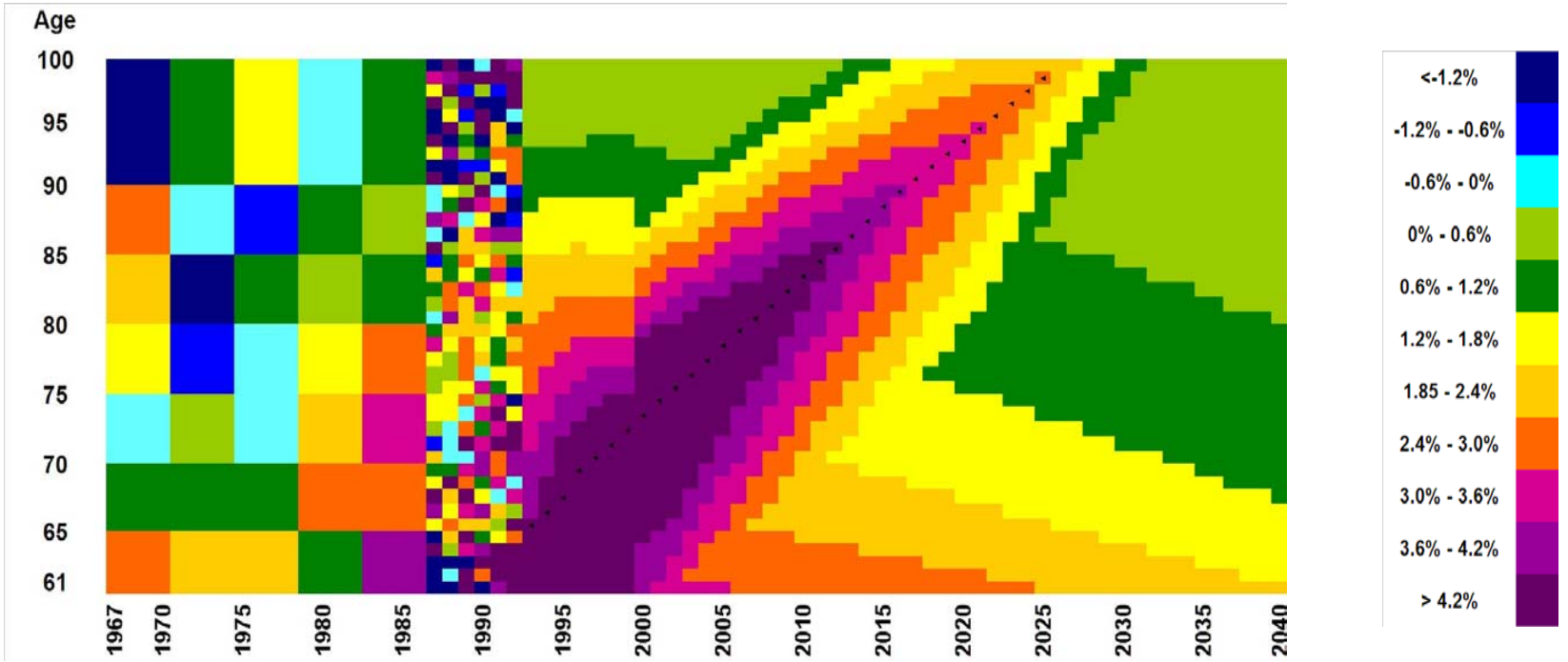
92 Series SC projections



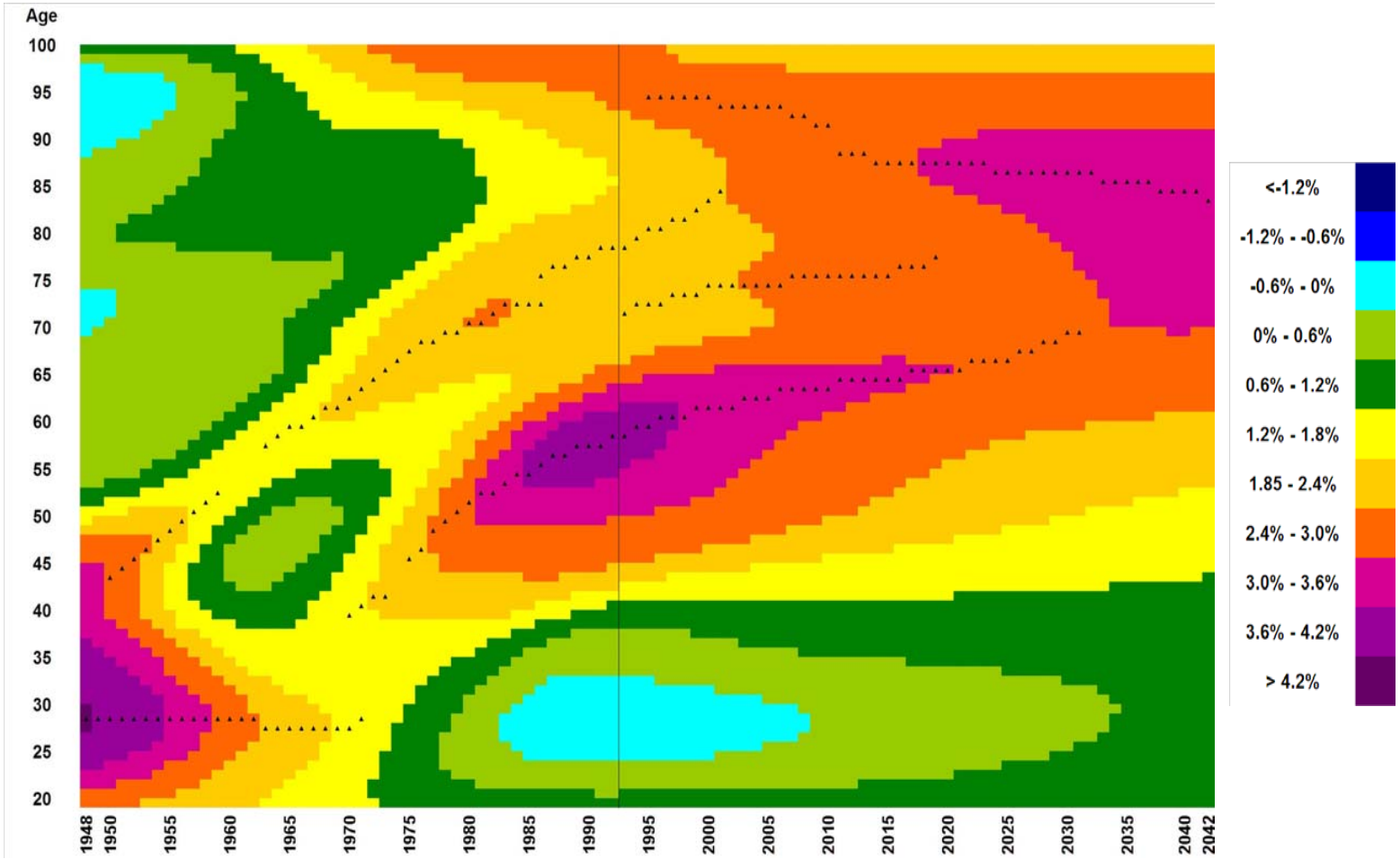
92 Series MC projections



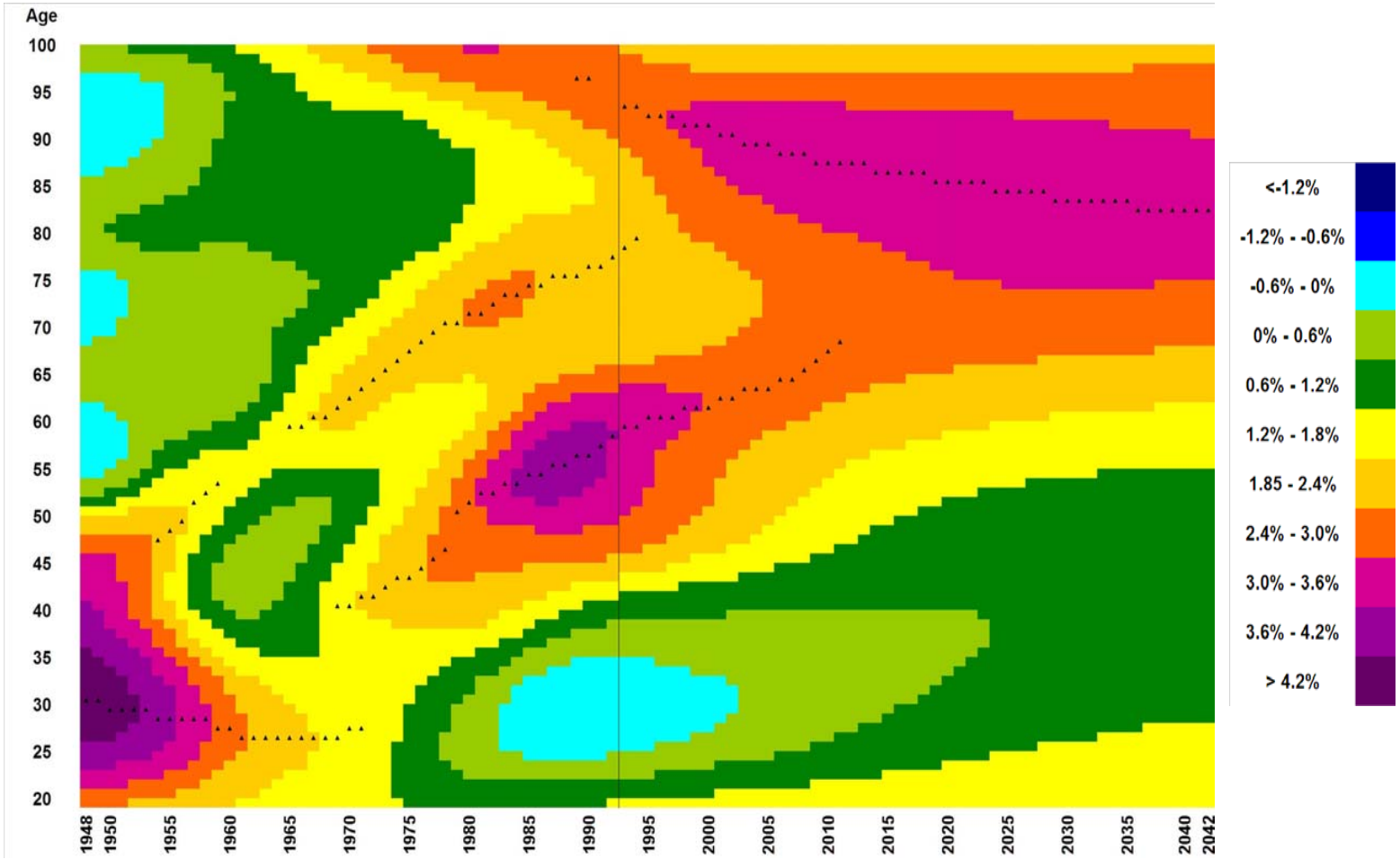
92 Series LC projections



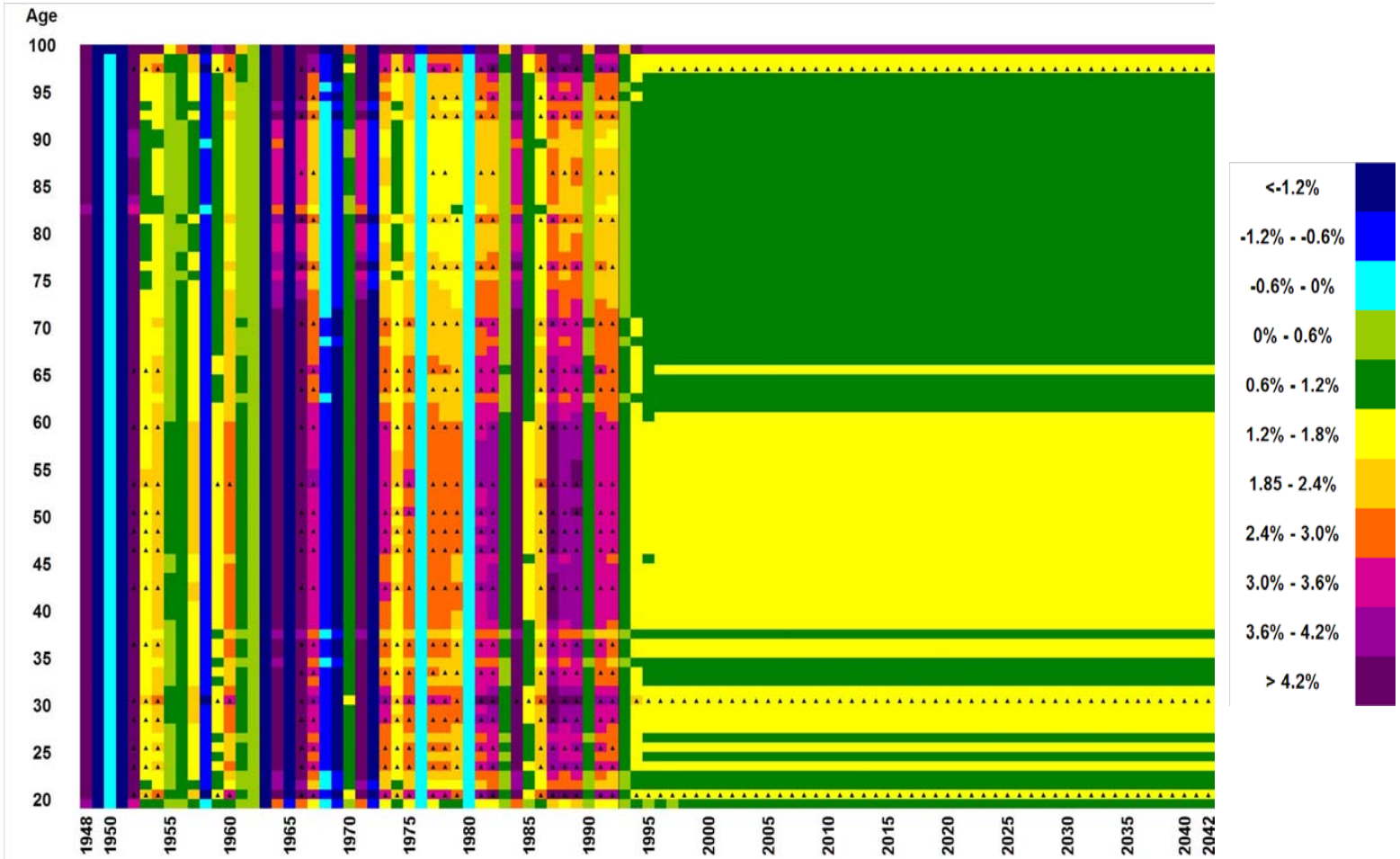
P-spline Cohort Penalty



P-spline – Period Penalty



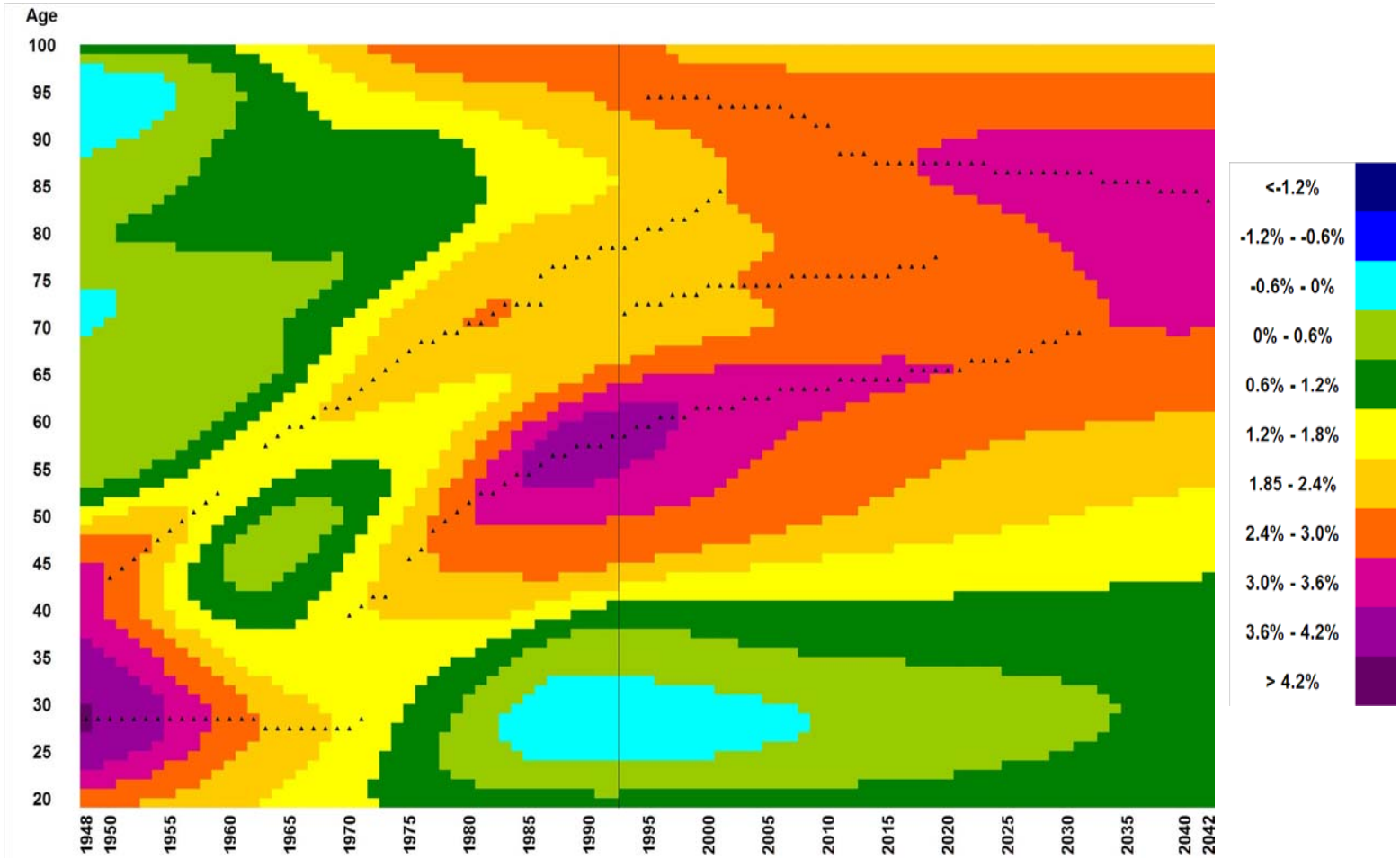
Lee-Carter


Age
100
95
90
85
80
75
70
65
60
55
50
45
40
35
30
25
20
1948
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015
2020
2025
2030
2035
2040
2042
<-1.2%
-1.2% - -0.6%
-0.6% - 0%
0% - 0.6%
0.6% - 1.2%
1.2% - 1.8%
1.85 - 2.4%
2.4% - 3.0%
3.0% - 3.6%
3.6% - 4.2%
> 4.2%

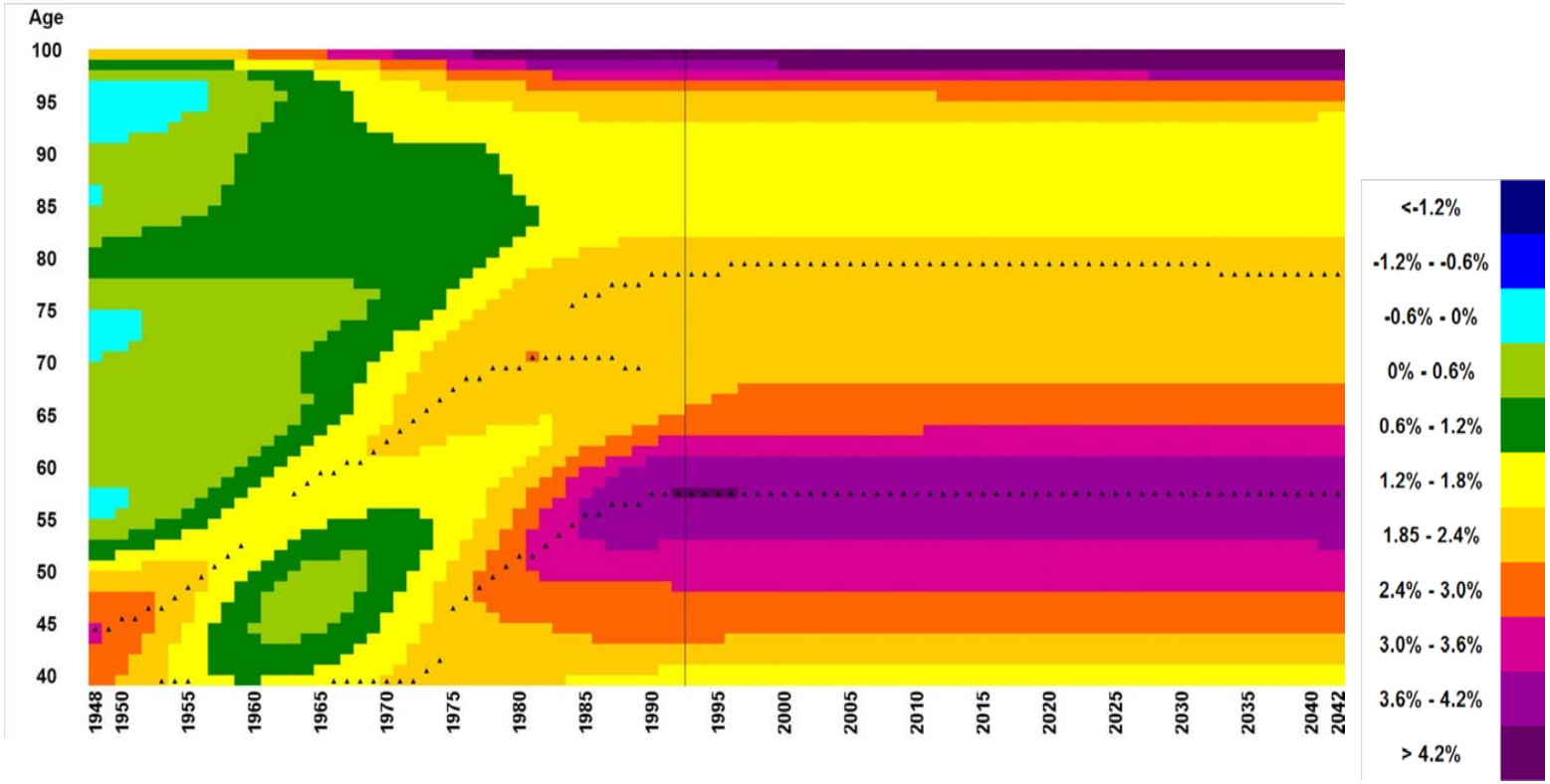
Effect of using different datasets

- Uses P-spline model
- Assured Lives Data to 1992

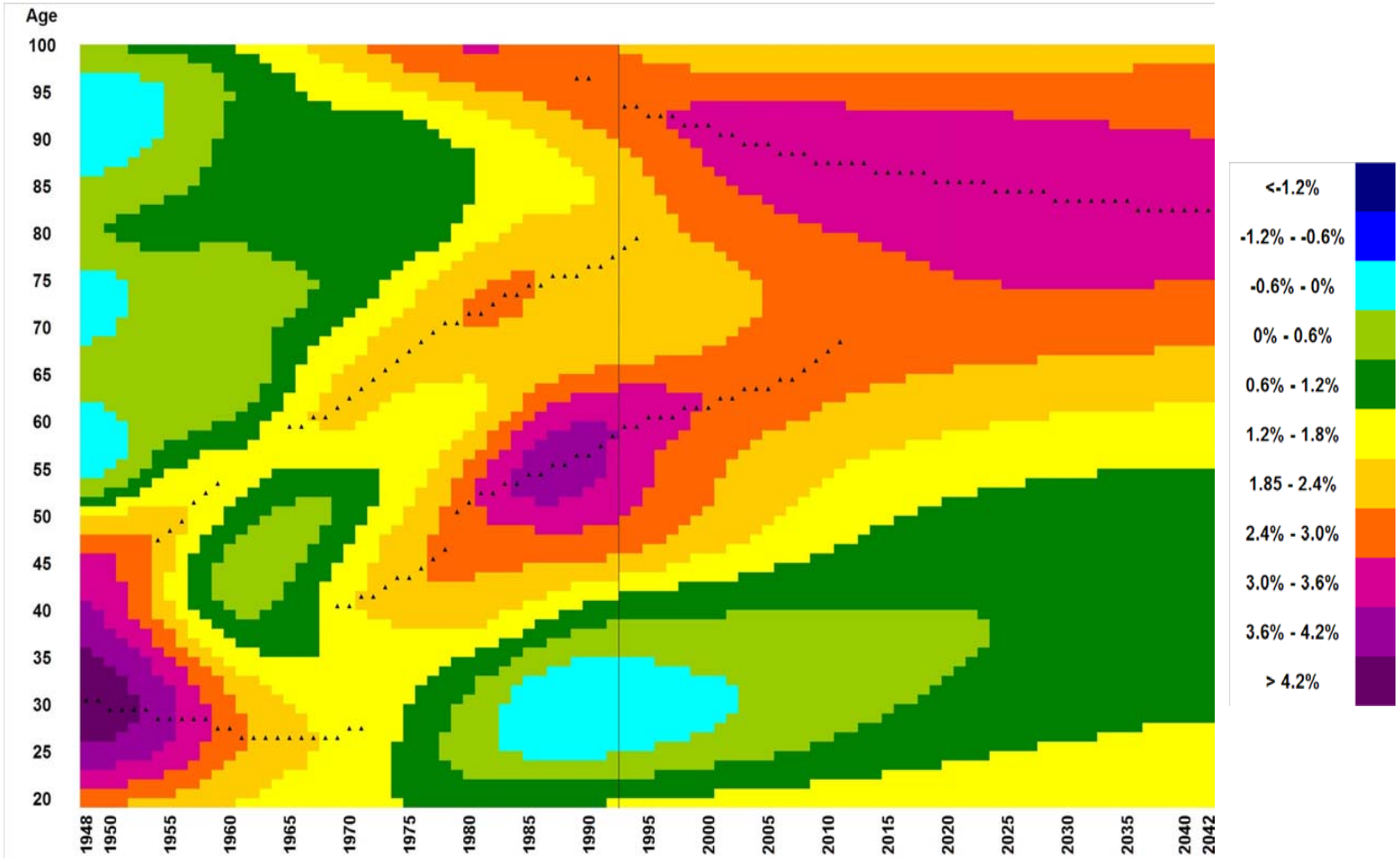
Ages 20-100 – Cohort Penalty



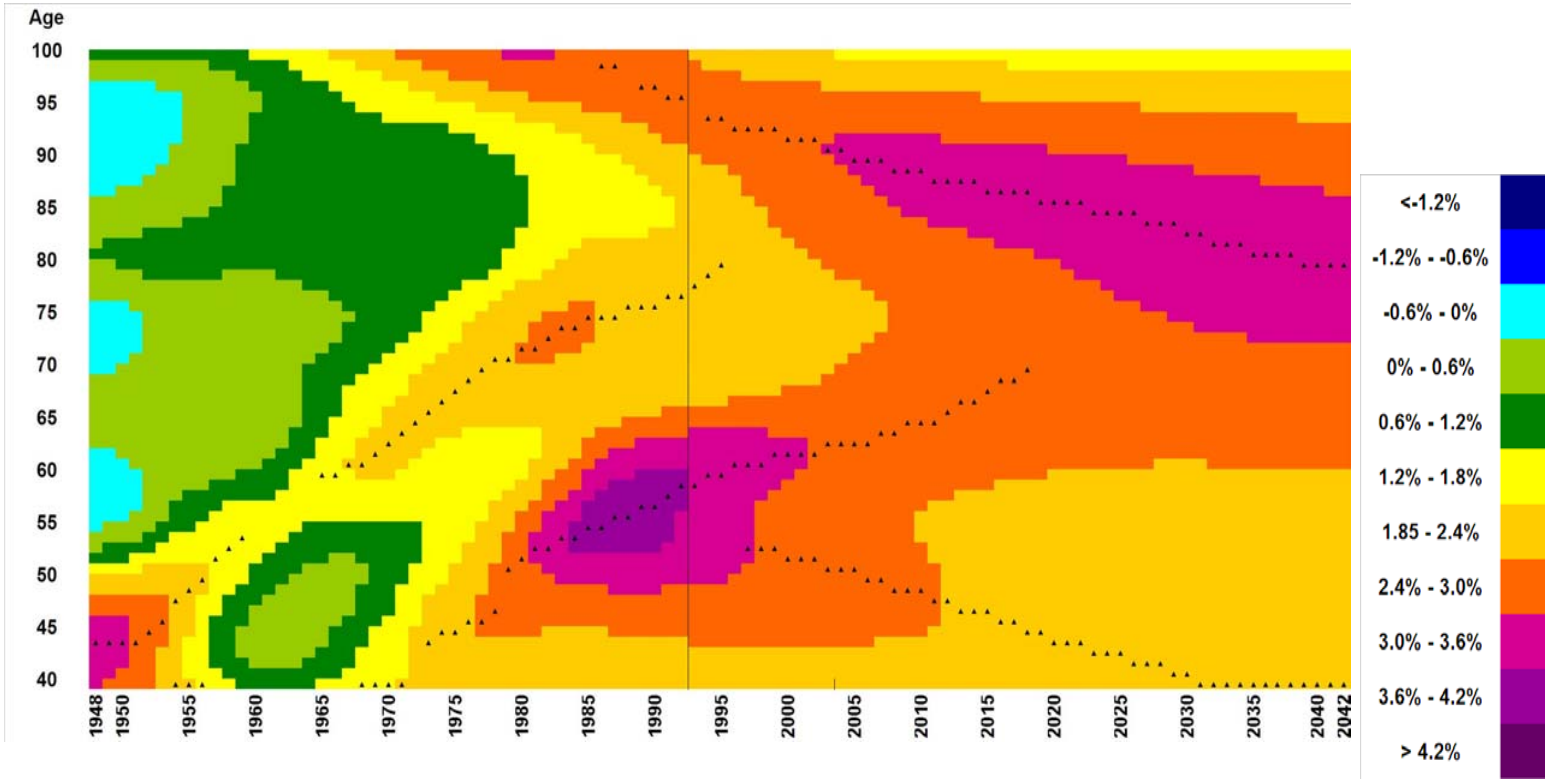
Ages 40-100 – Cohort Penalty



Ages 20-100 – Period Penalty



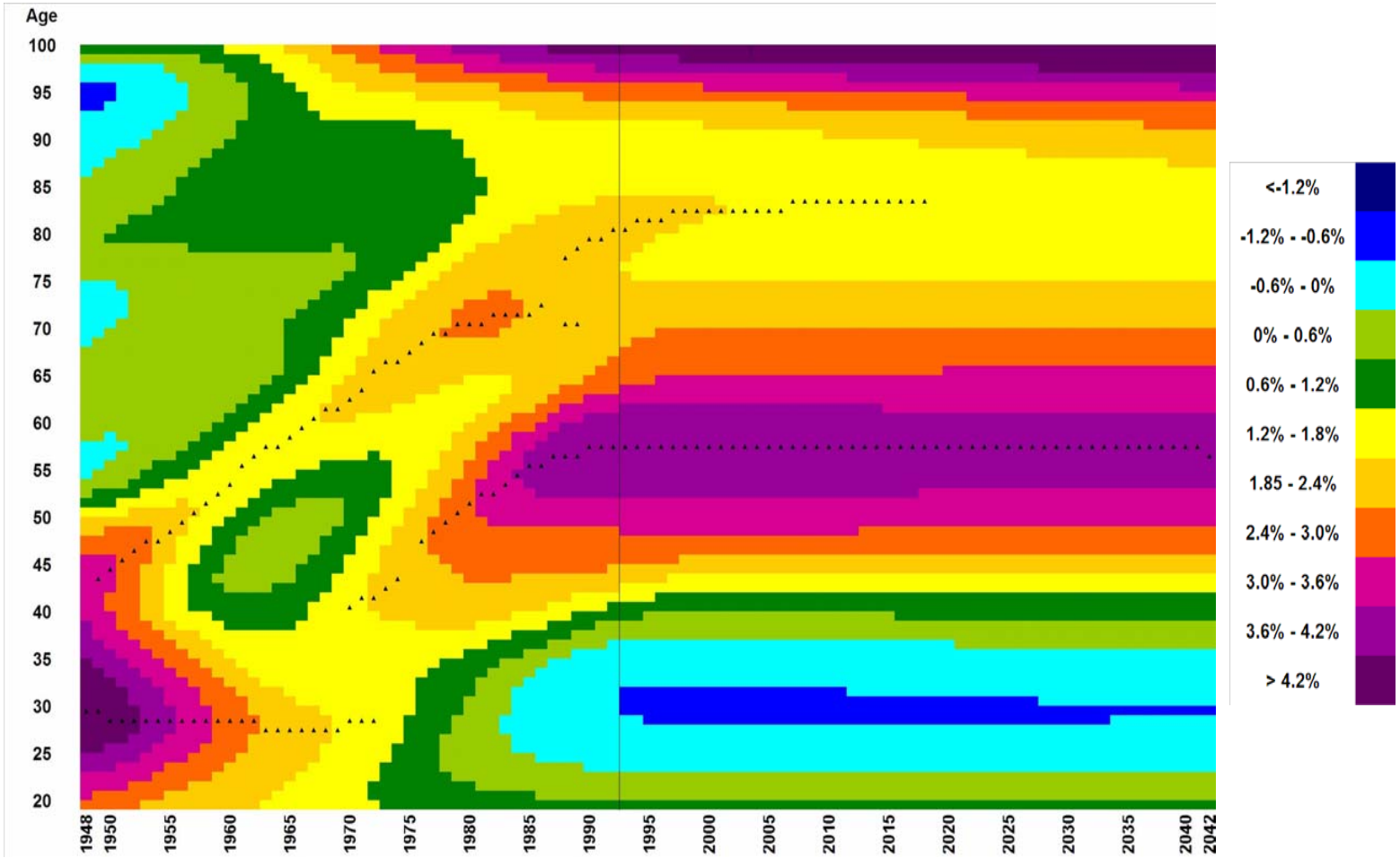
Ages 40-100 – Period Penalty



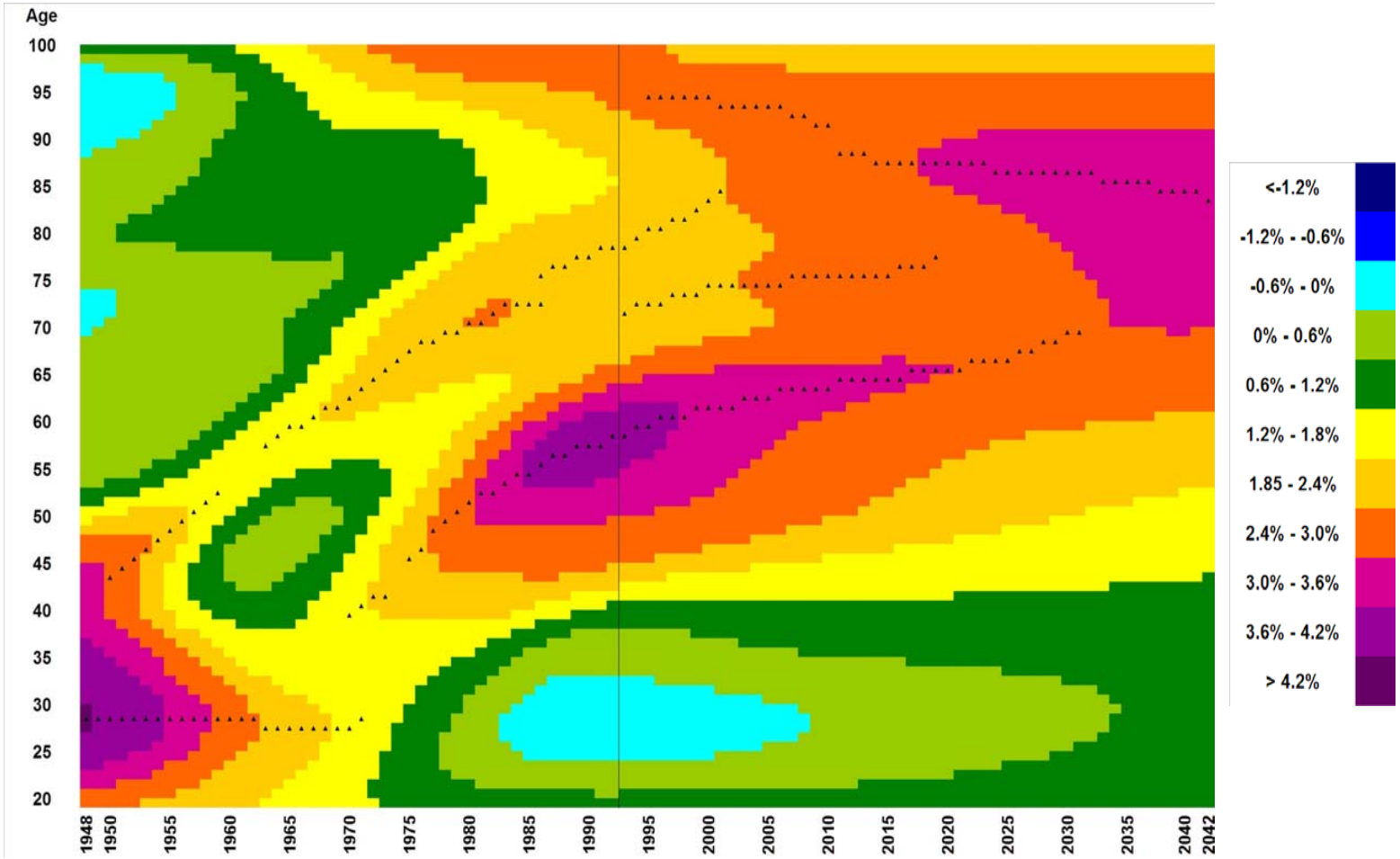
Effect of using different parameters

- Uses P-spline model
- Assured Lives Data to 1992

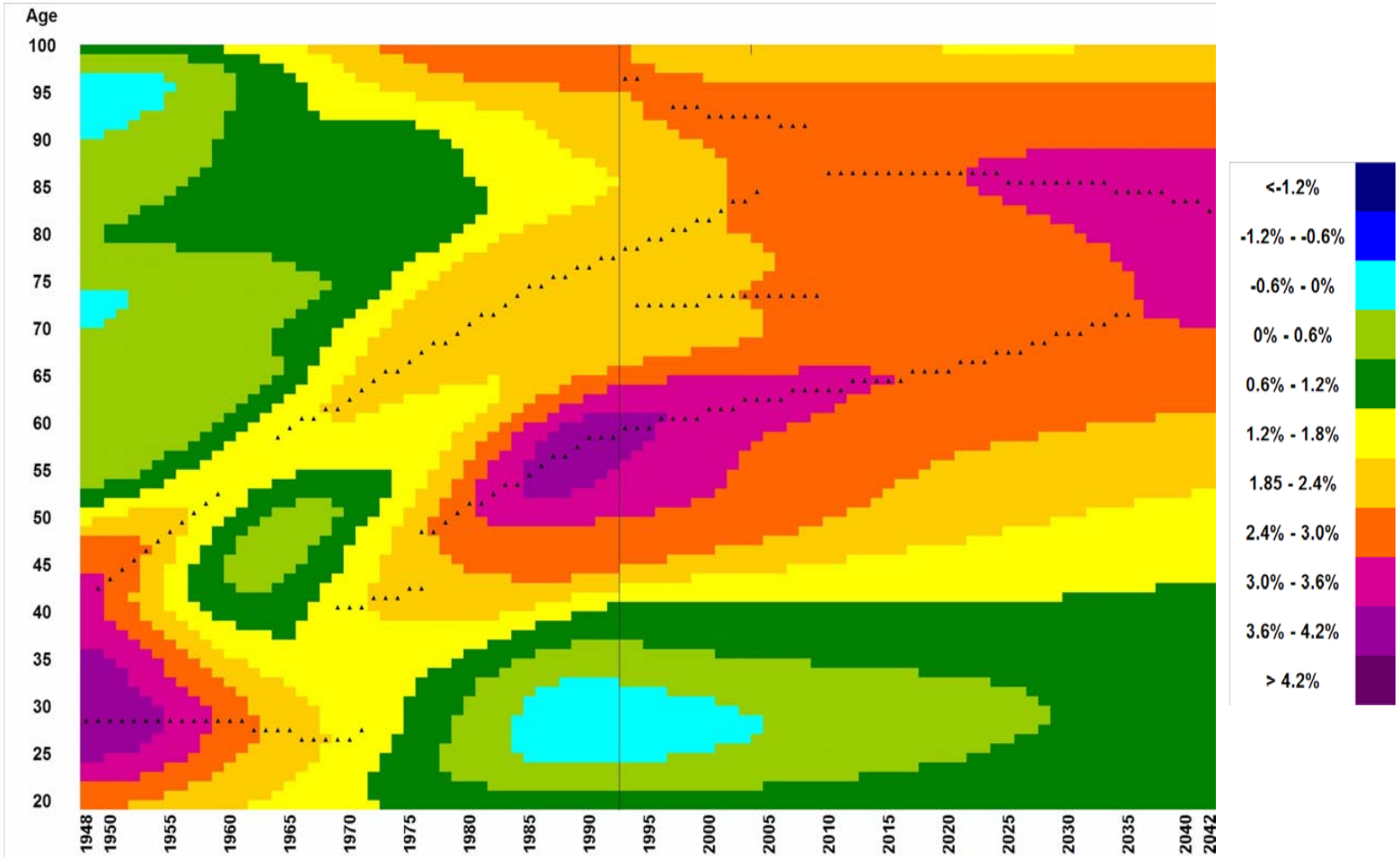
Cohort with knots every 6 years



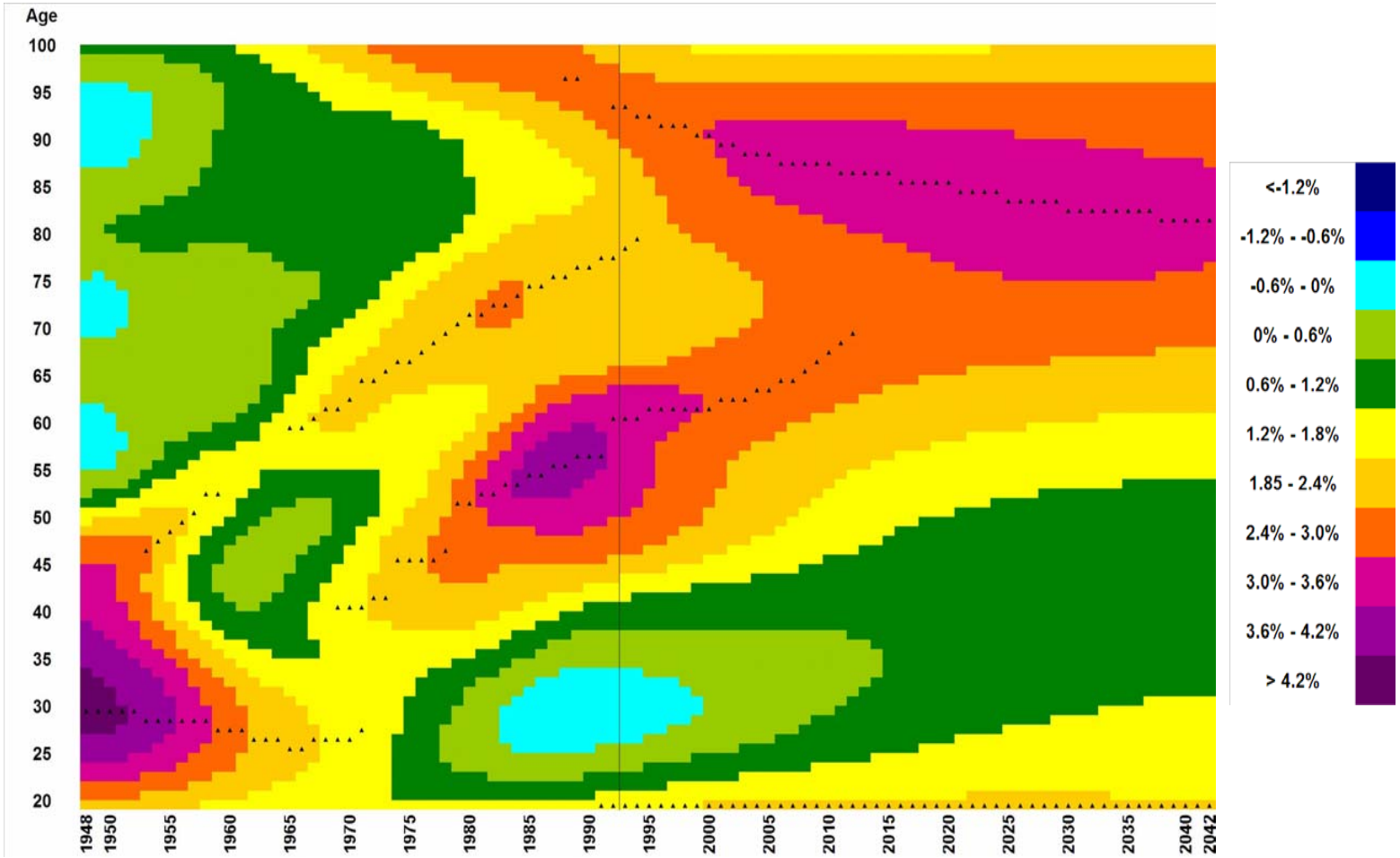
Cohort with knots every 5 years



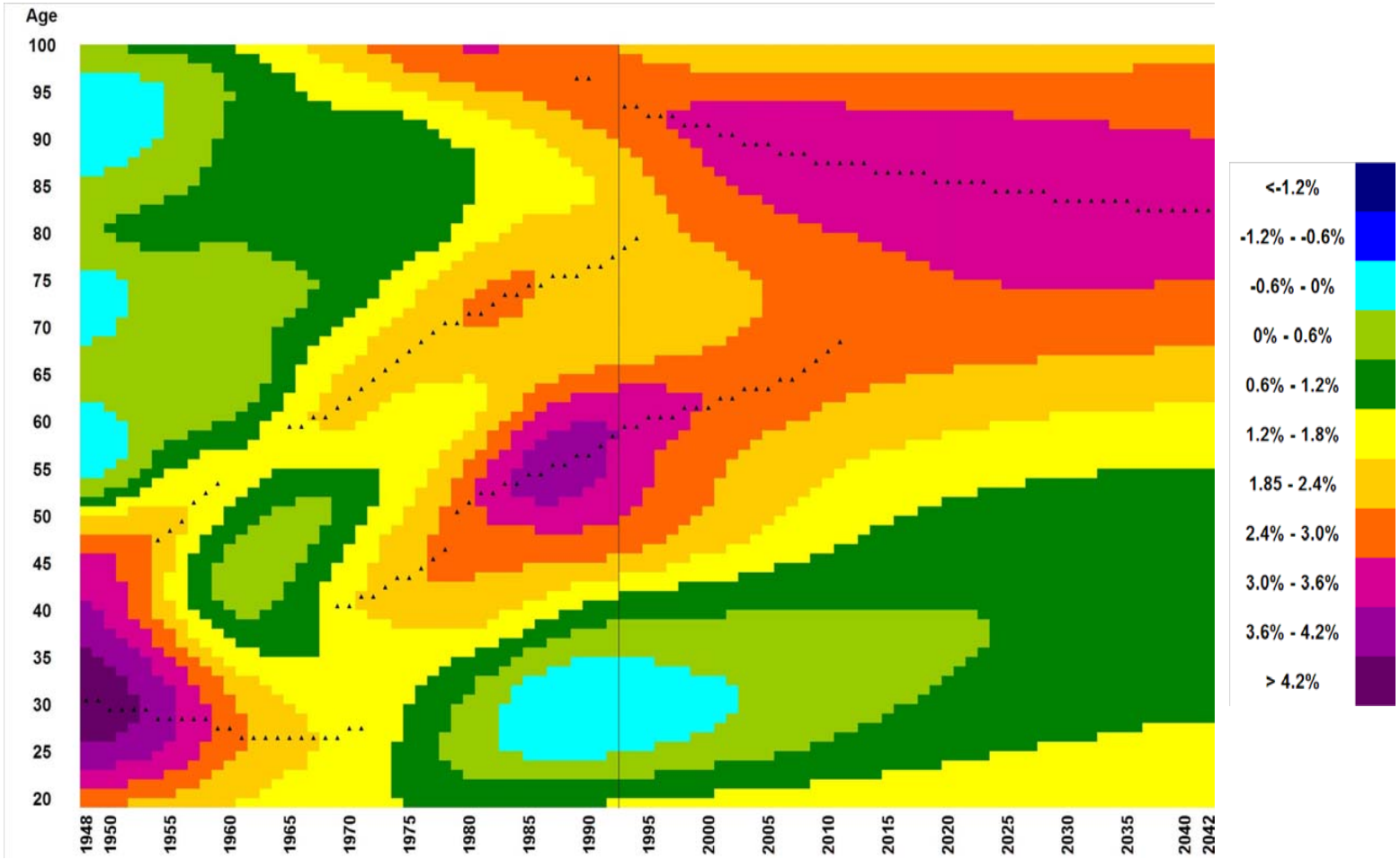
Cohort with knots every 4 years



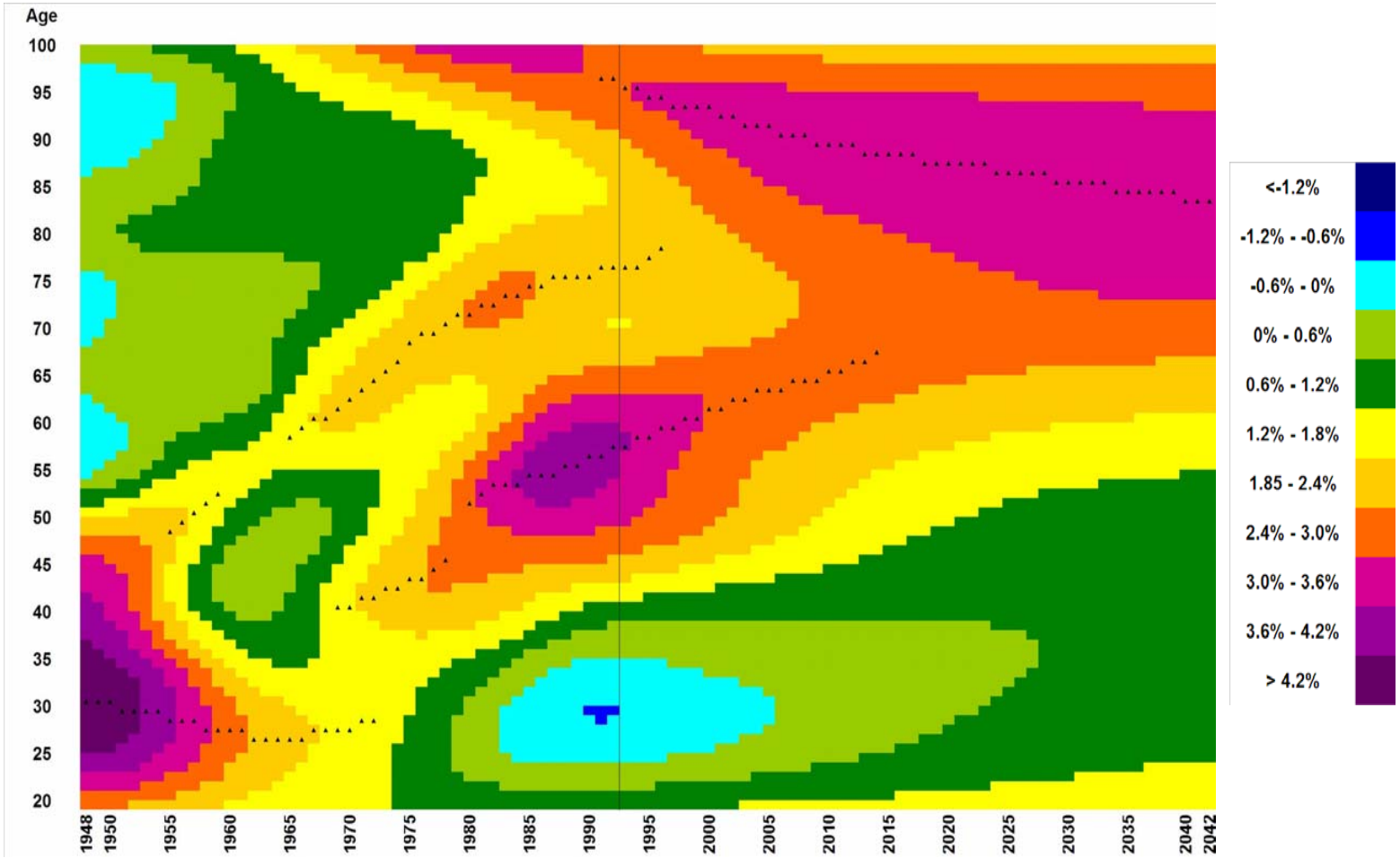
Period with knots every 3 years



Period with knots every 4 years



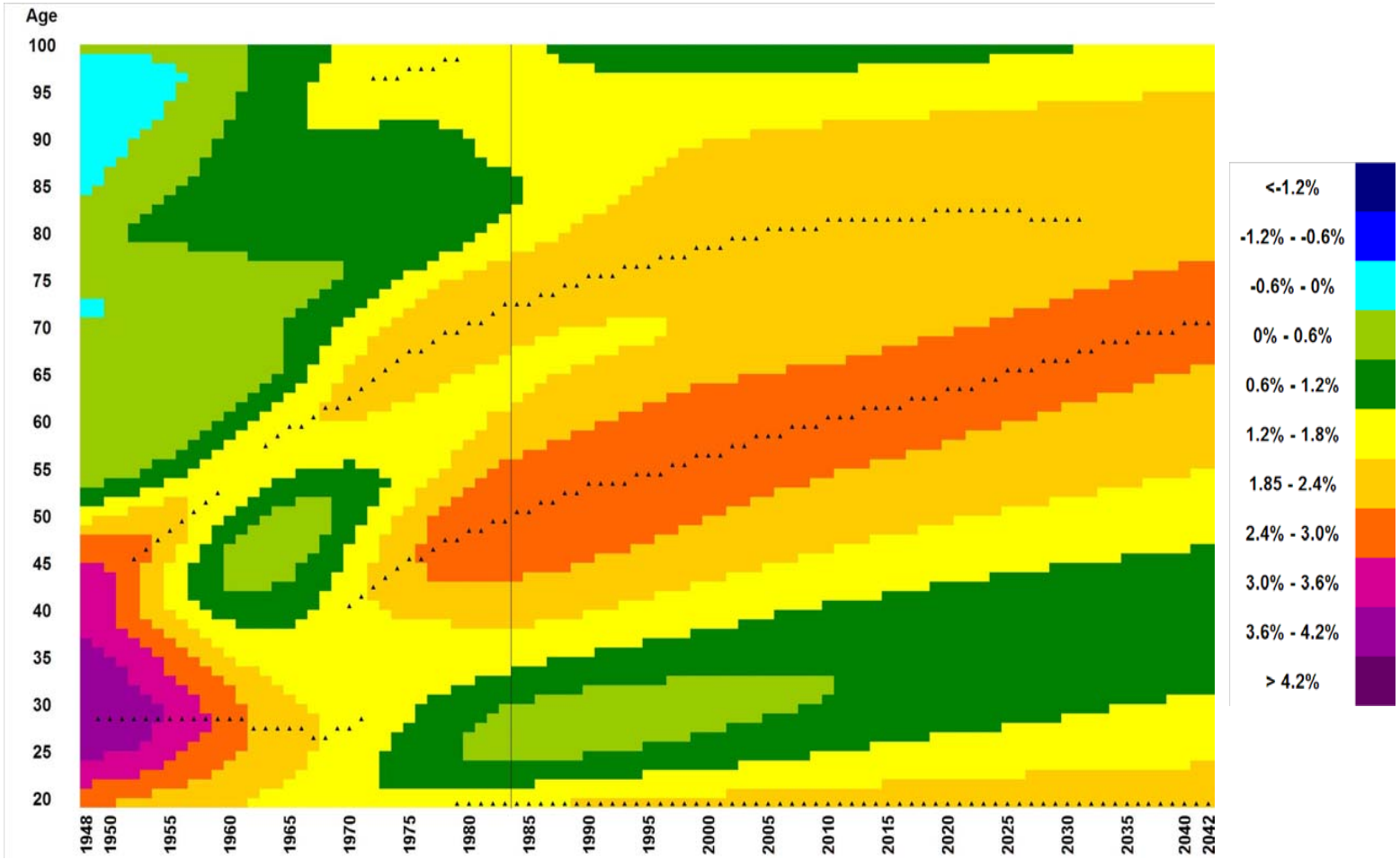
Period with knots every 5 years



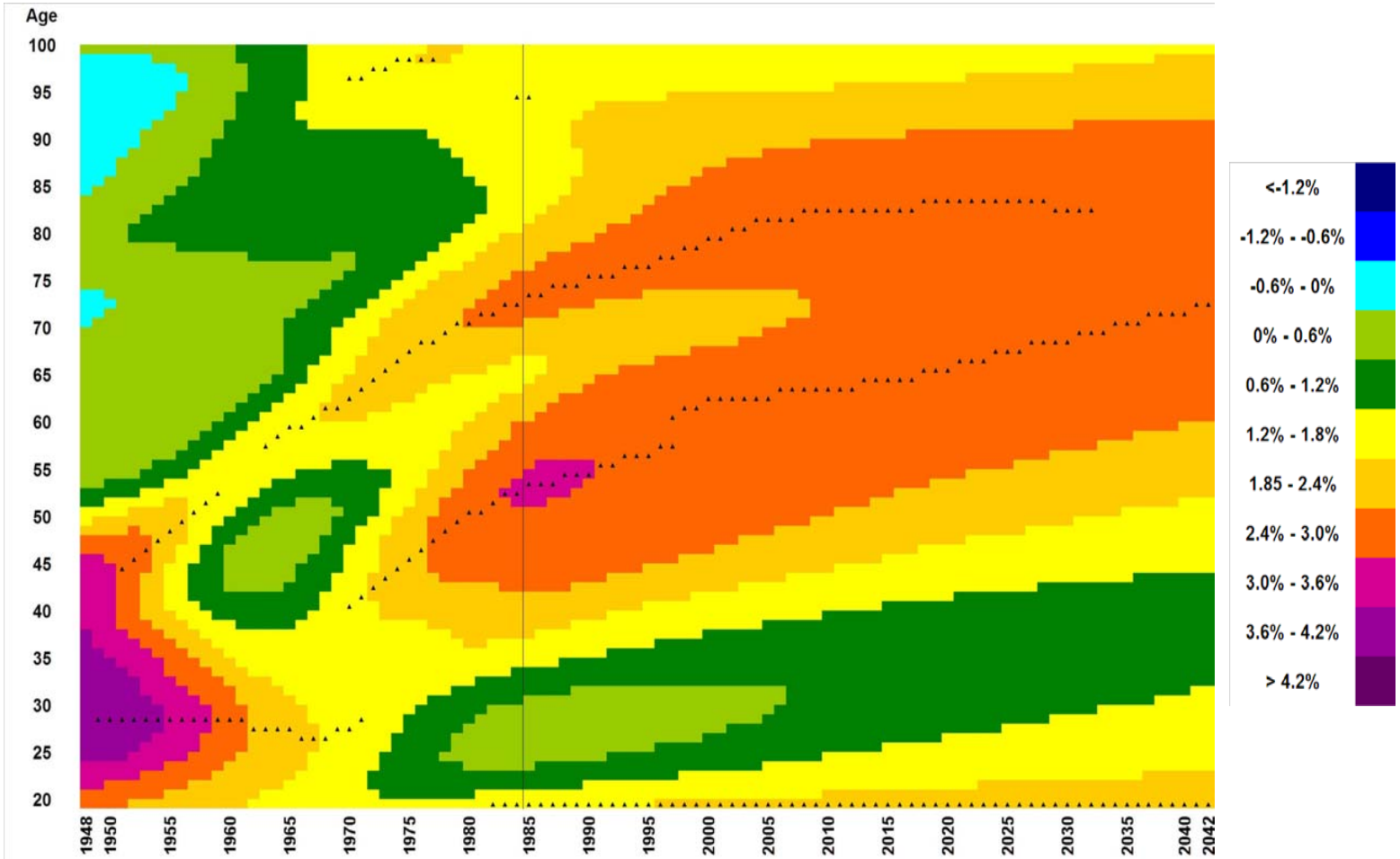
Progress of projections 1983-1992

- Uses P-spline model
- Cohort penalties

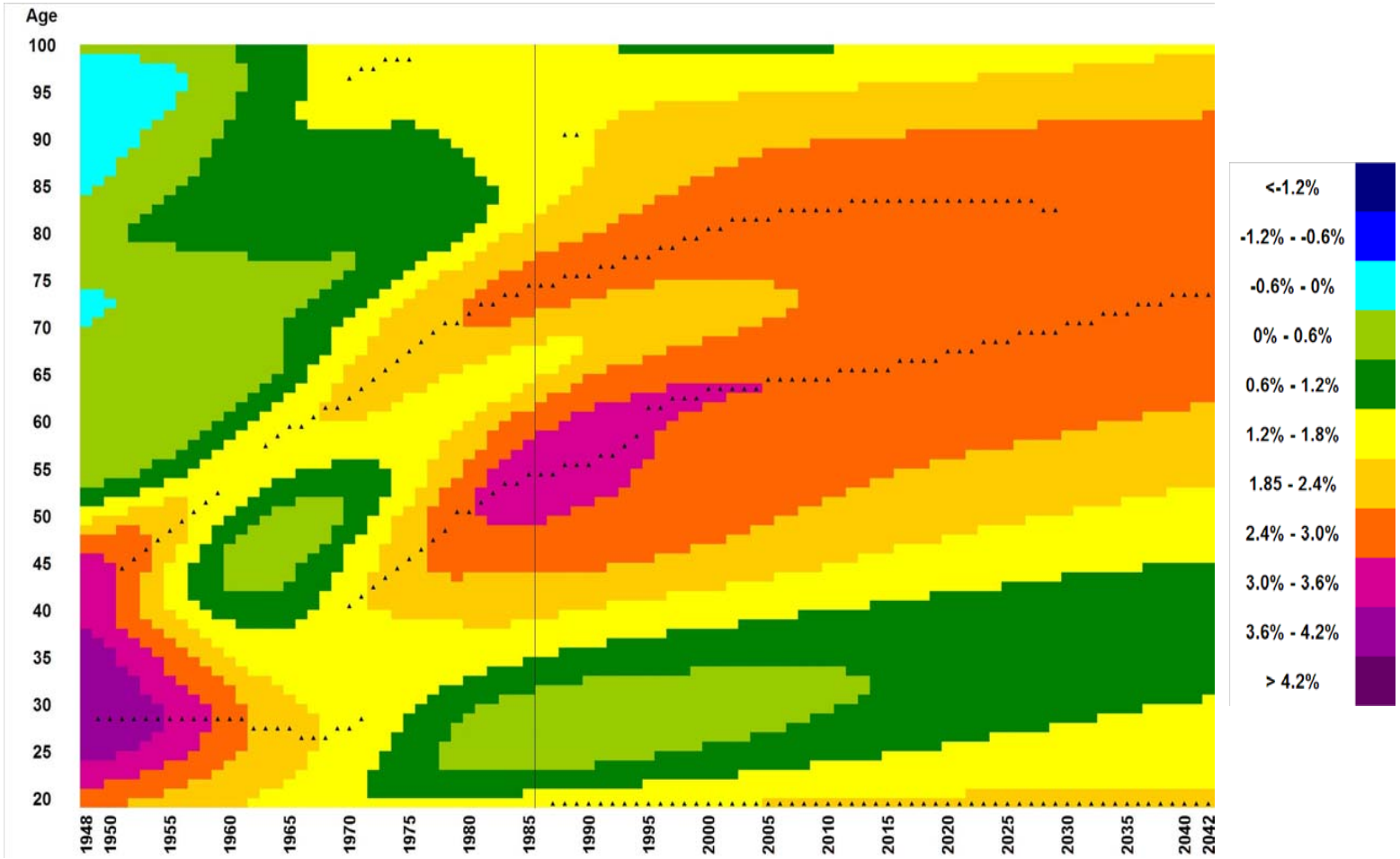
Data to 1983 – Cohort Penalty



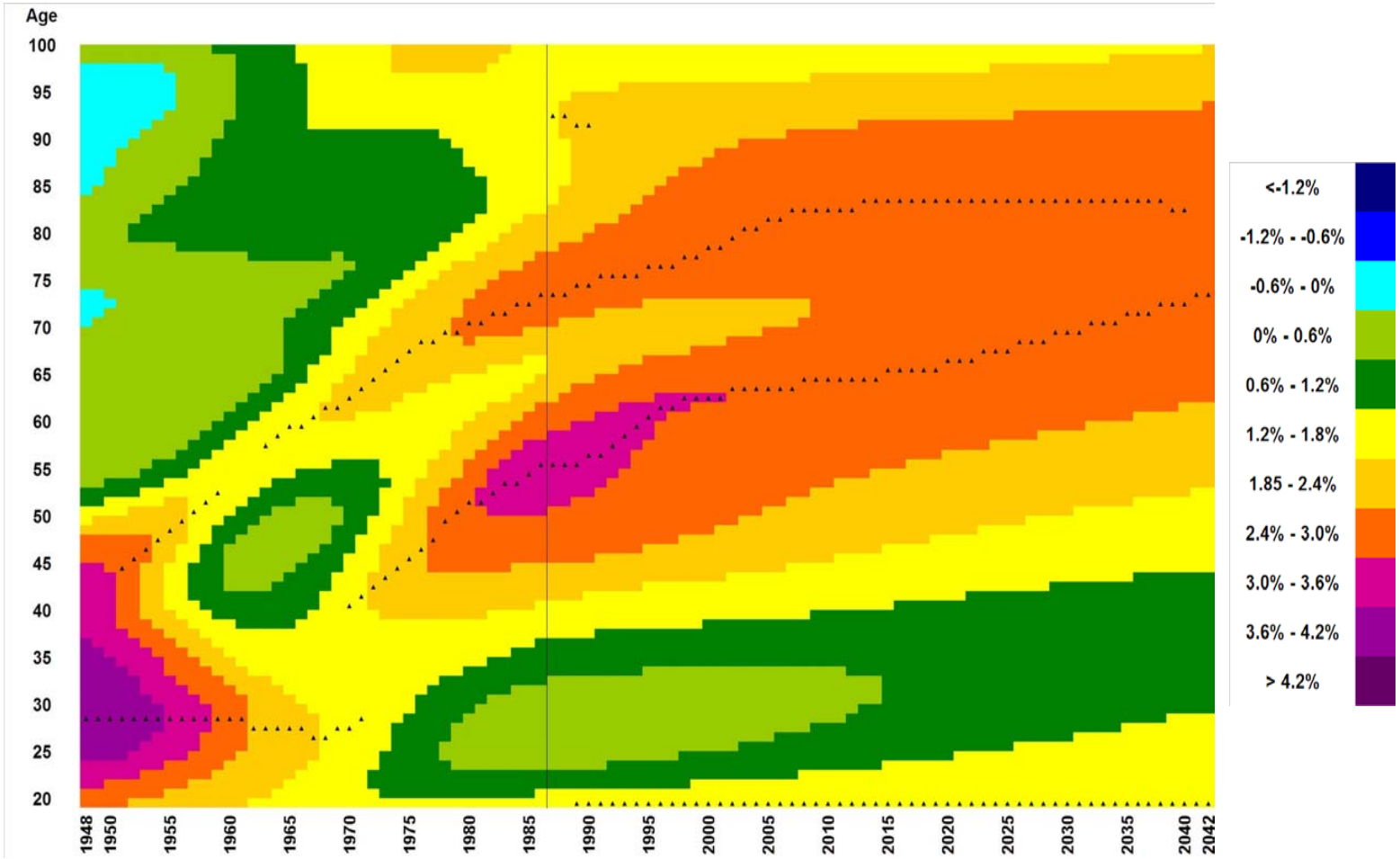
Data to 1984 – Cohort Penalty



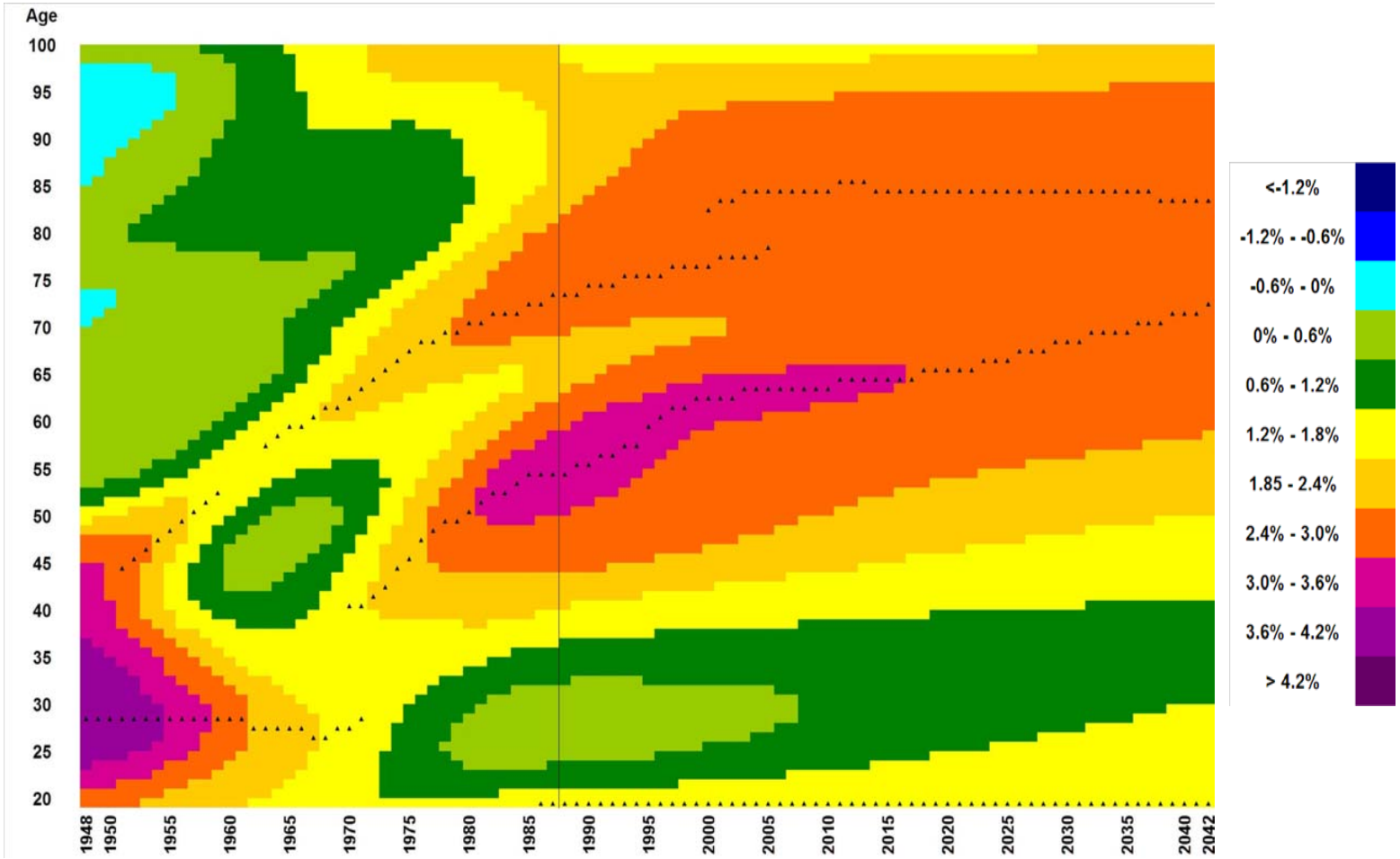
Data to 1985 – Cohort Penalty



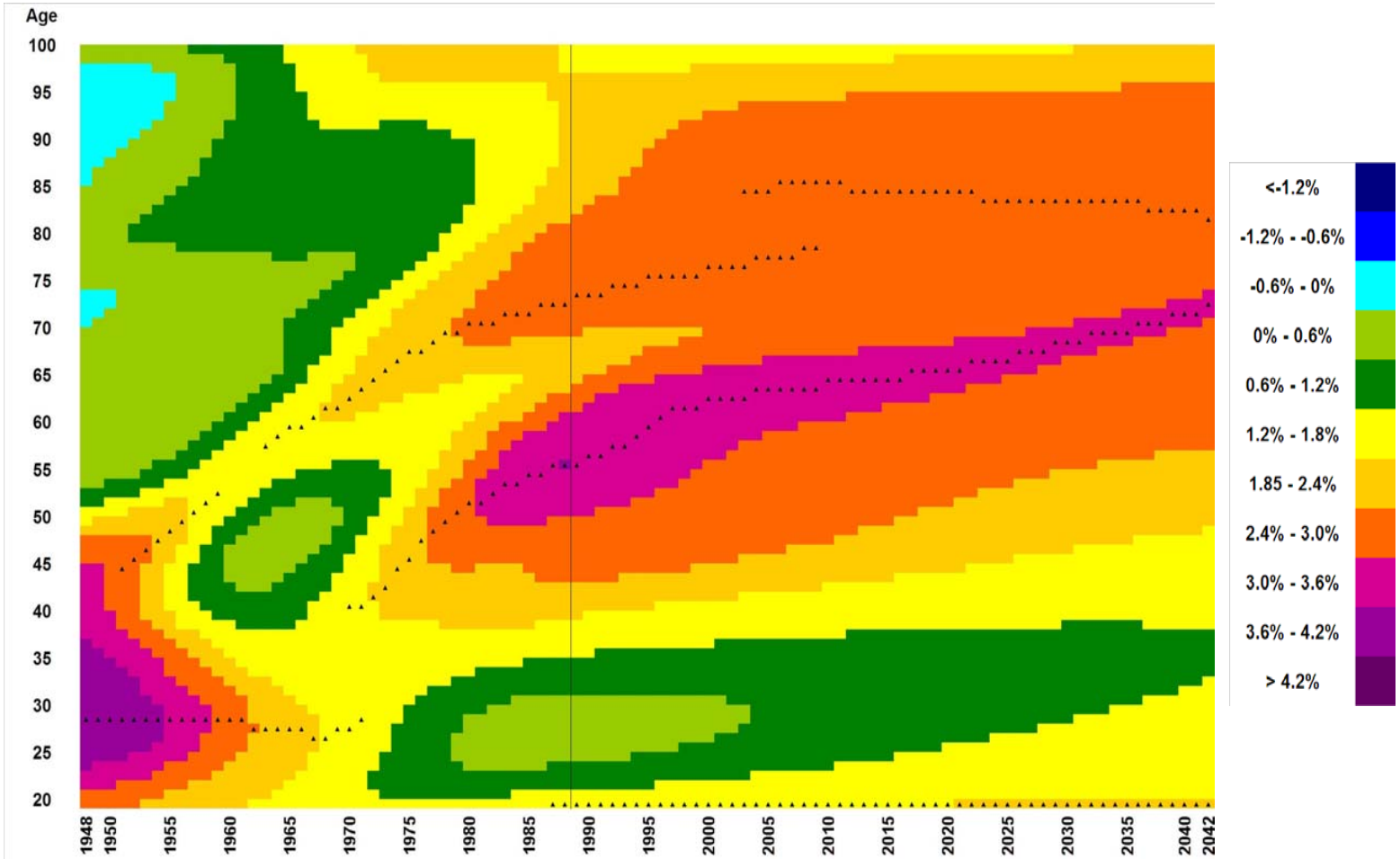
Data to 1986 – Cohort Penalty



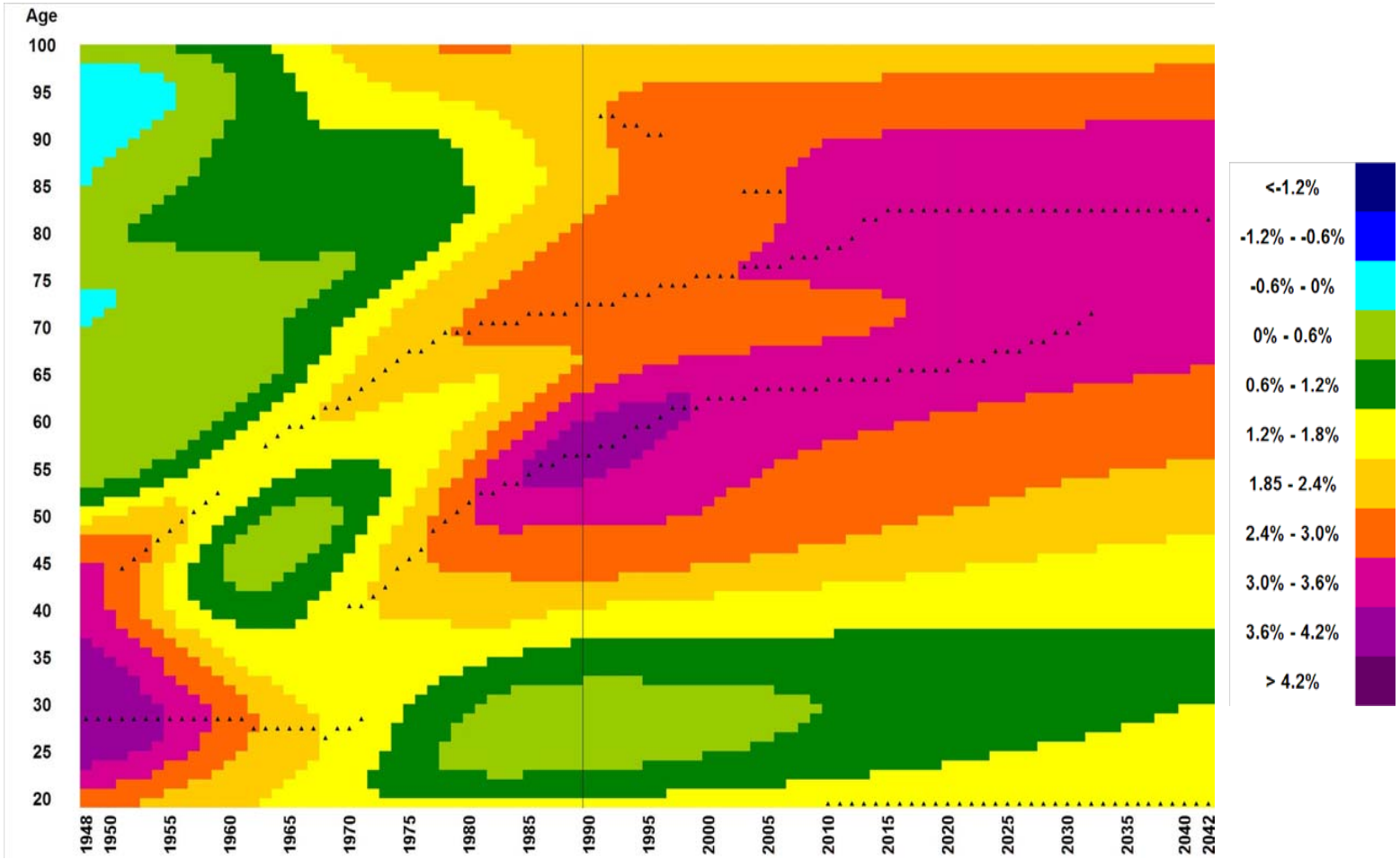
Data to 1987 – Cohort Penalty



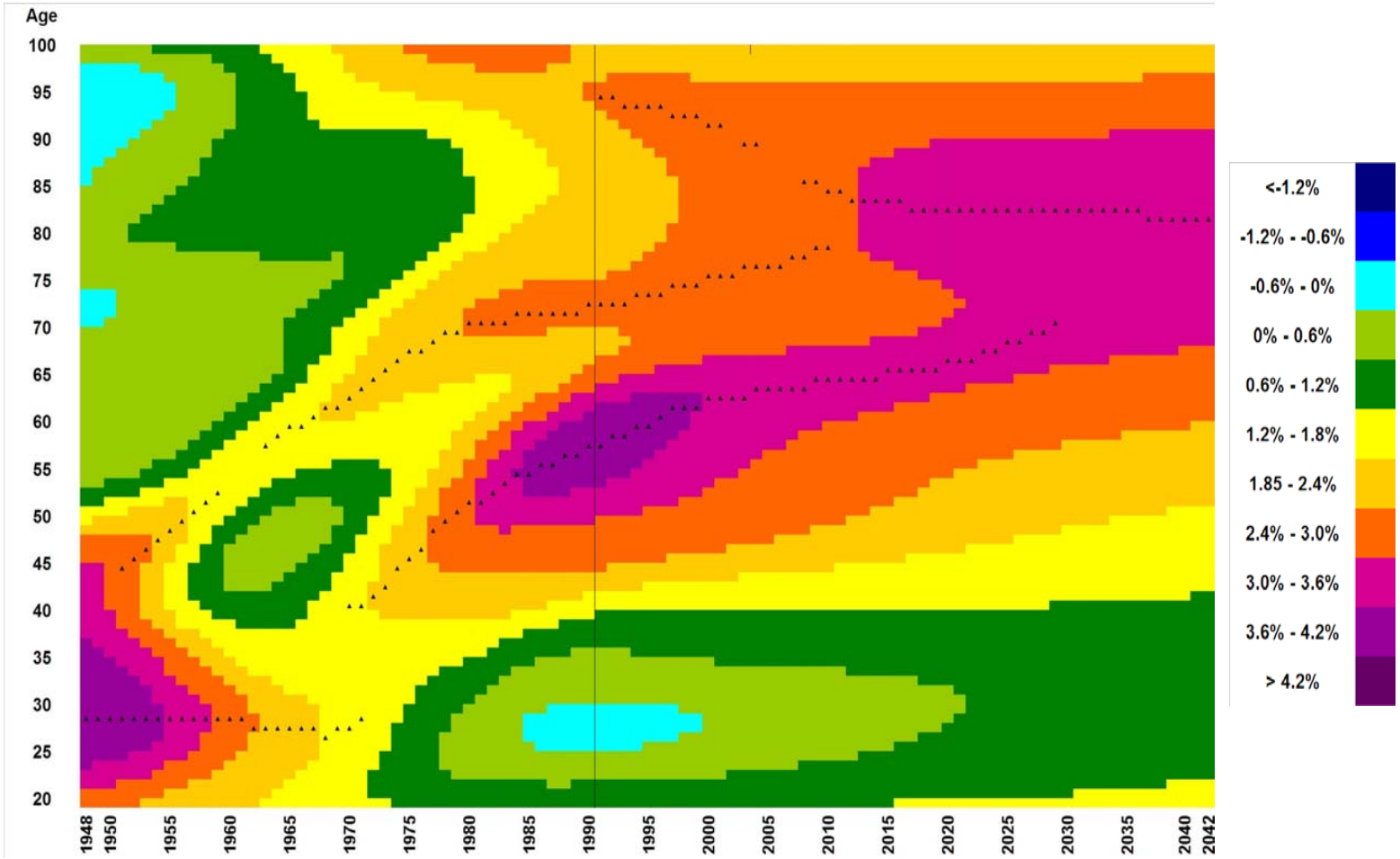
Data to 1988 – Cohort Penalty



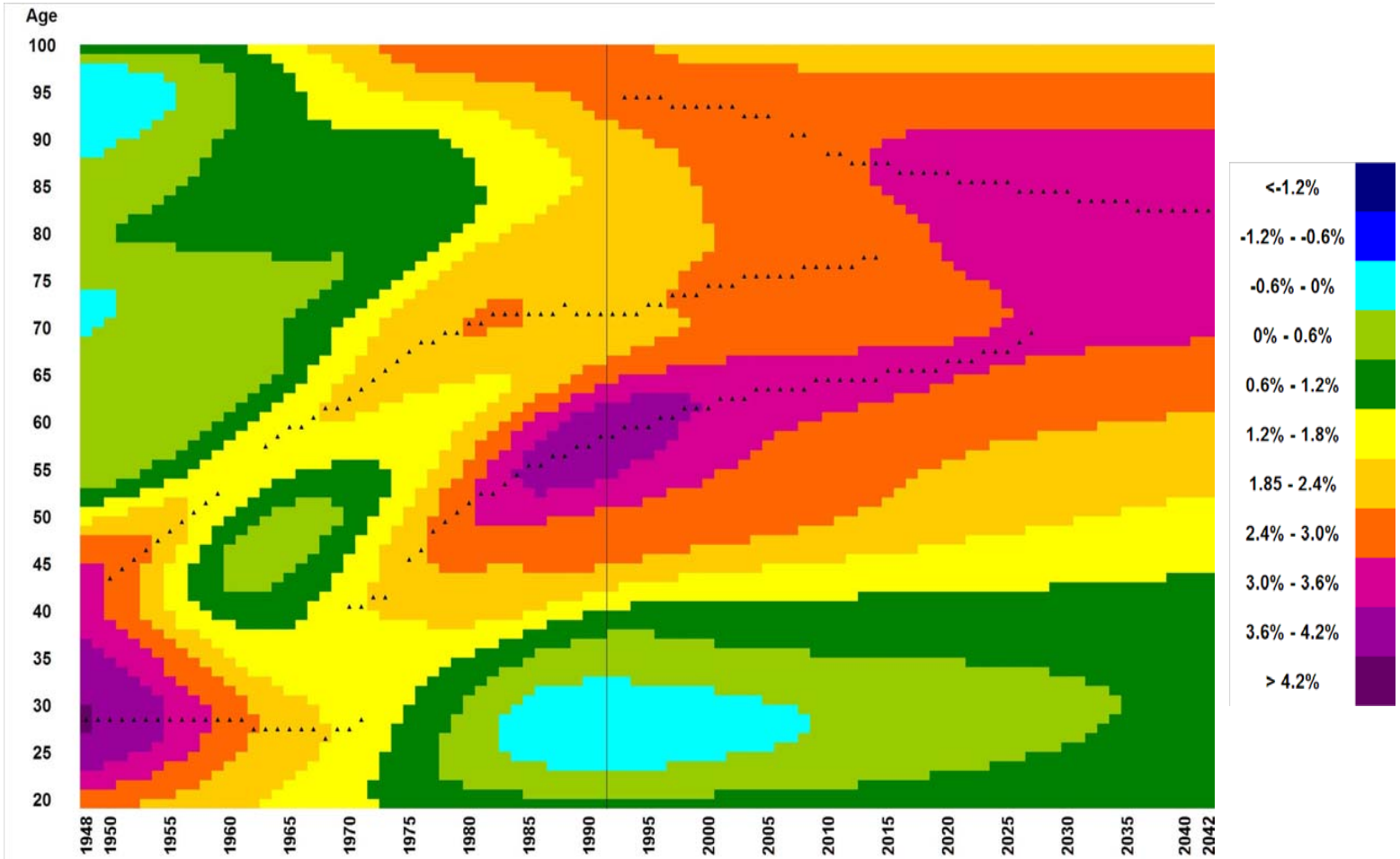
Data to 1989 – Cohort Penalty



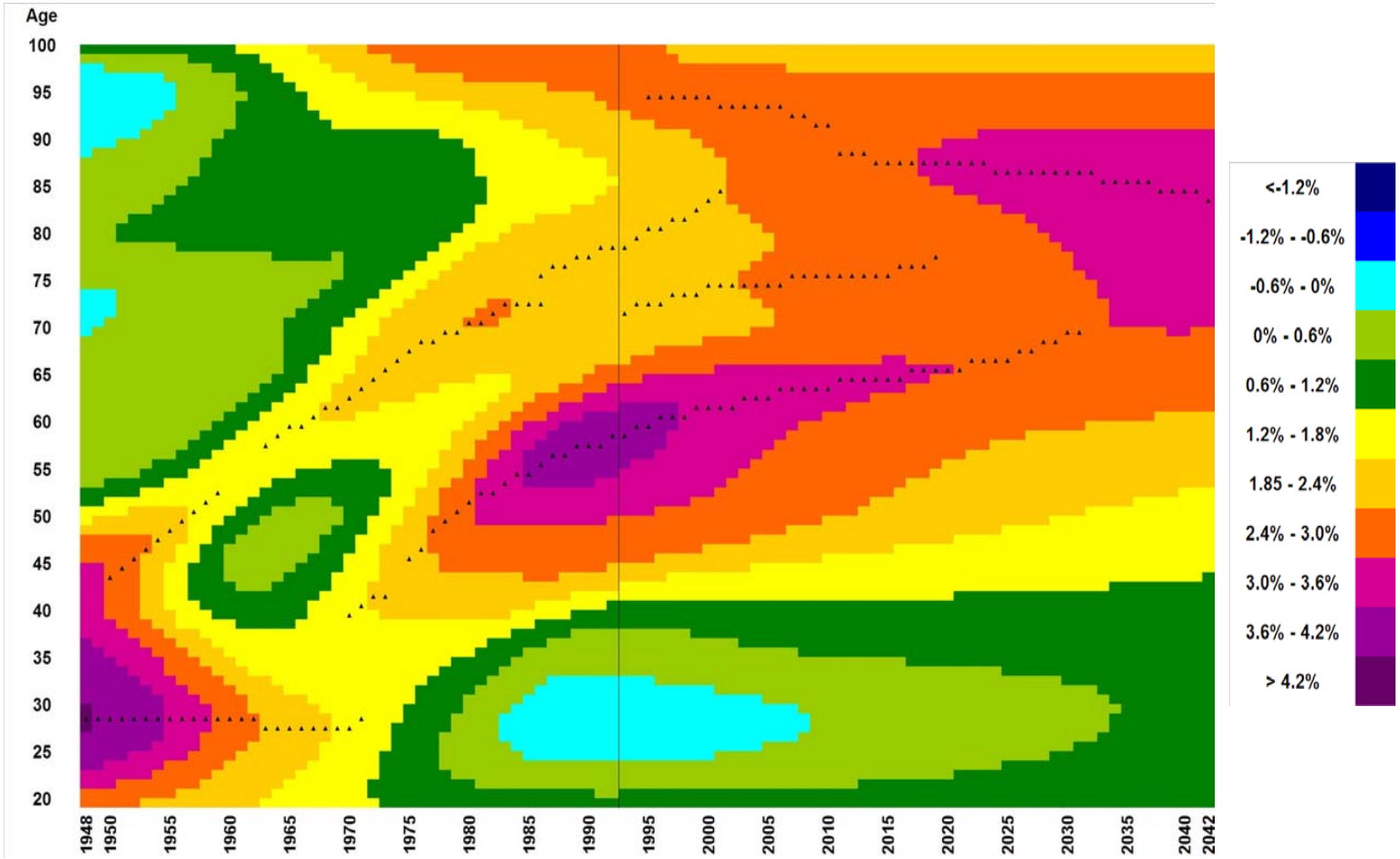
Data to 1990 – Cohort Penalty



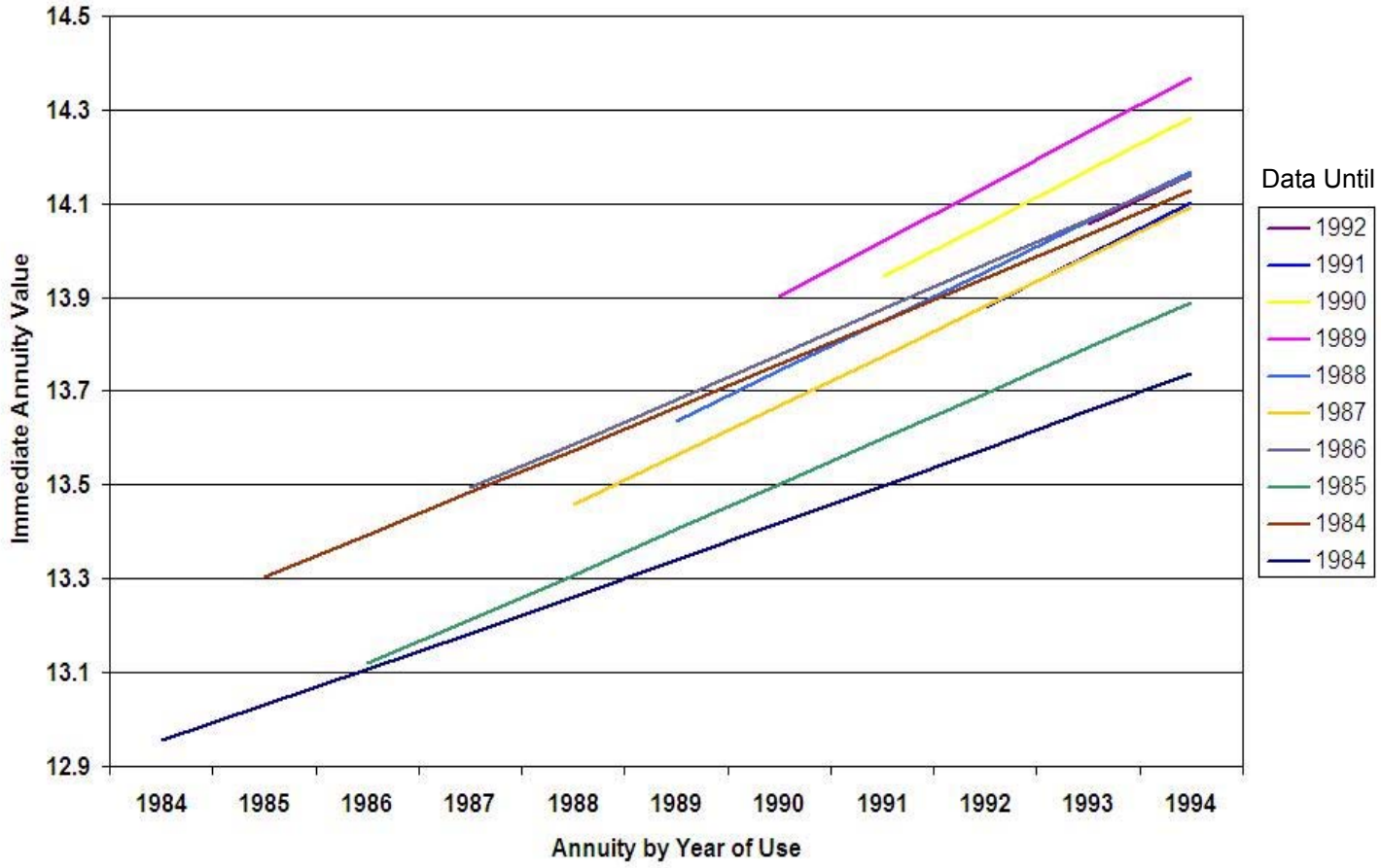
Data to 1991 – Cohort Penalty



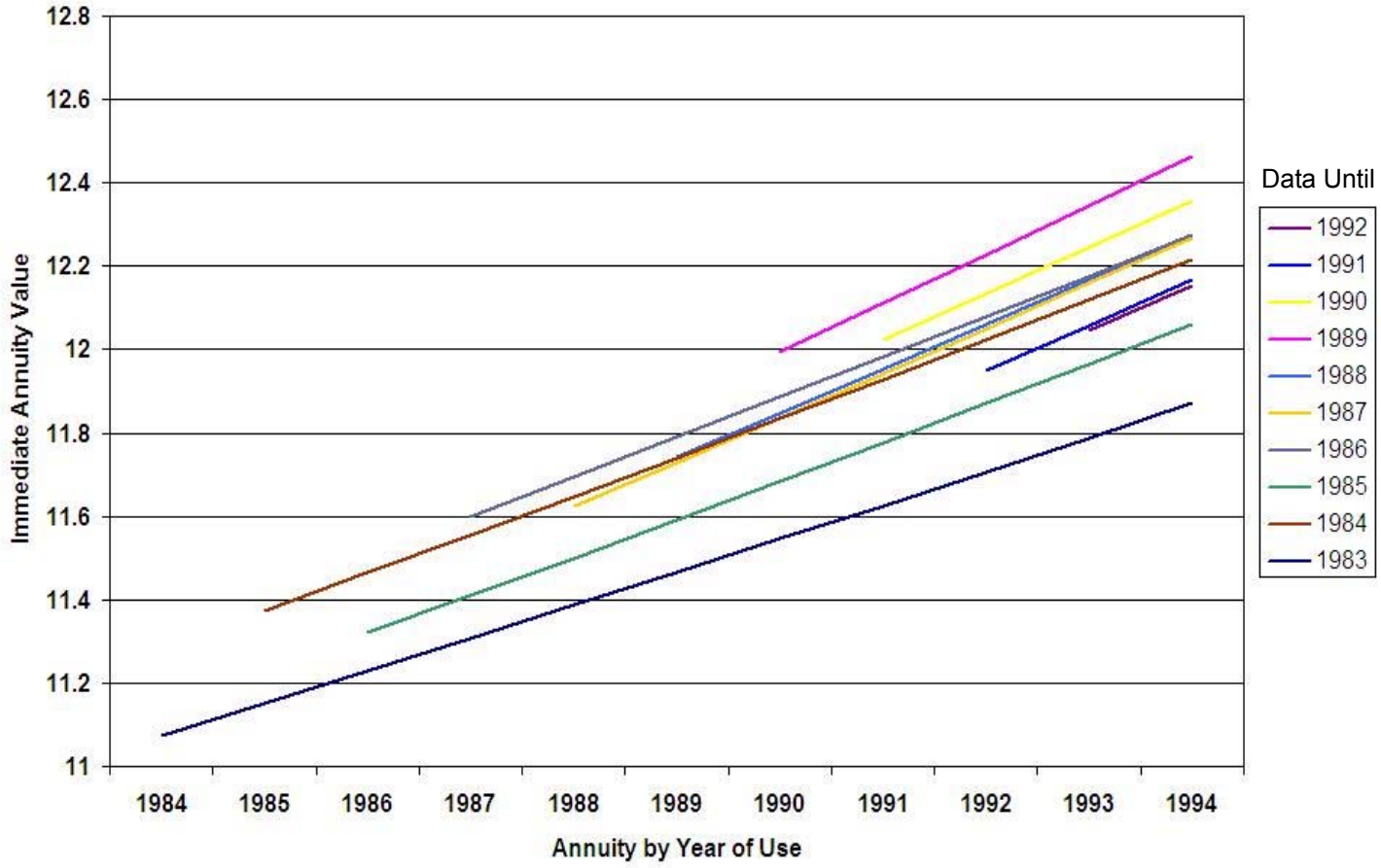
Data to 1992 – Cohort Penalty



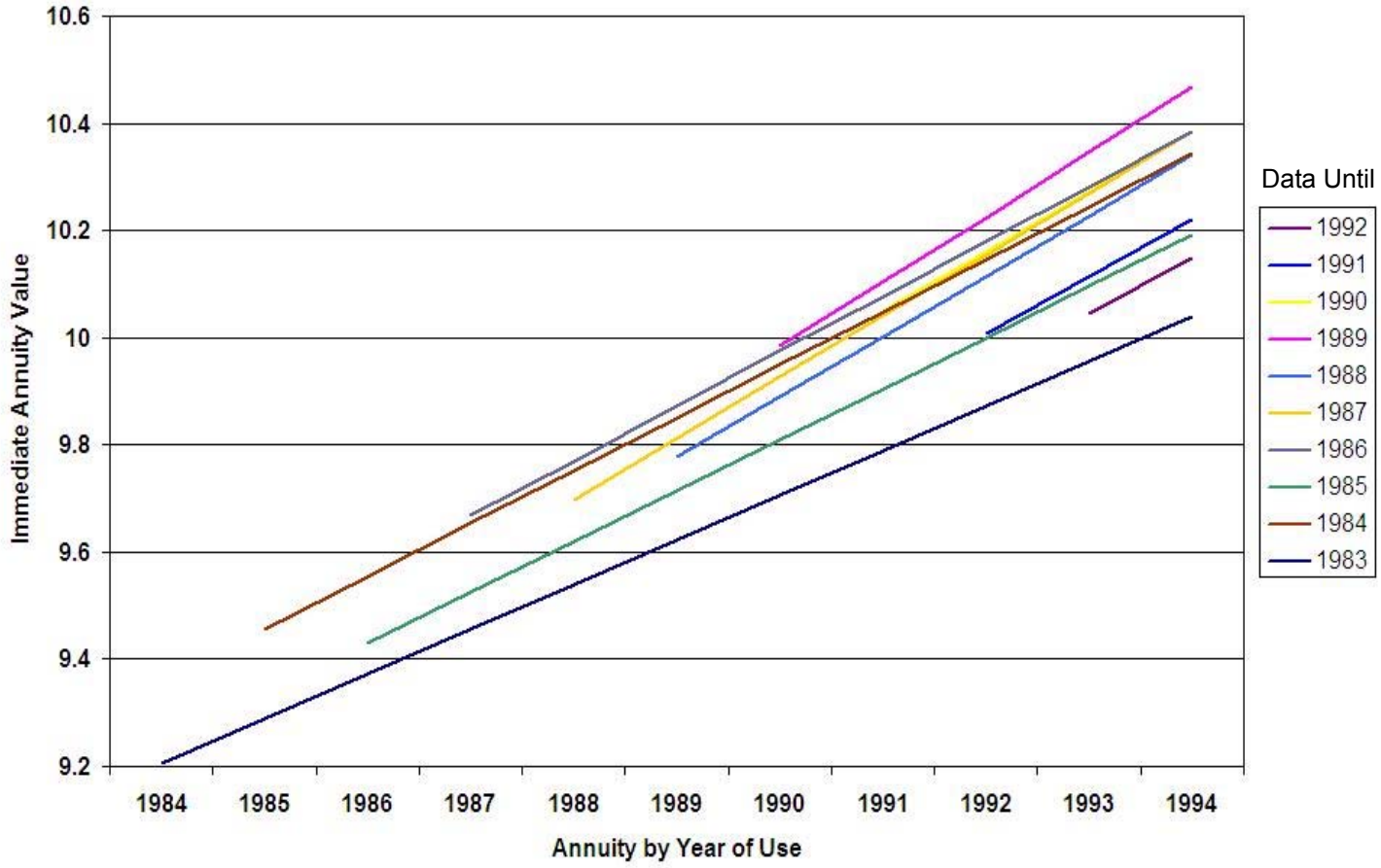
Cohort Penalties – Age 60 – 4.5%



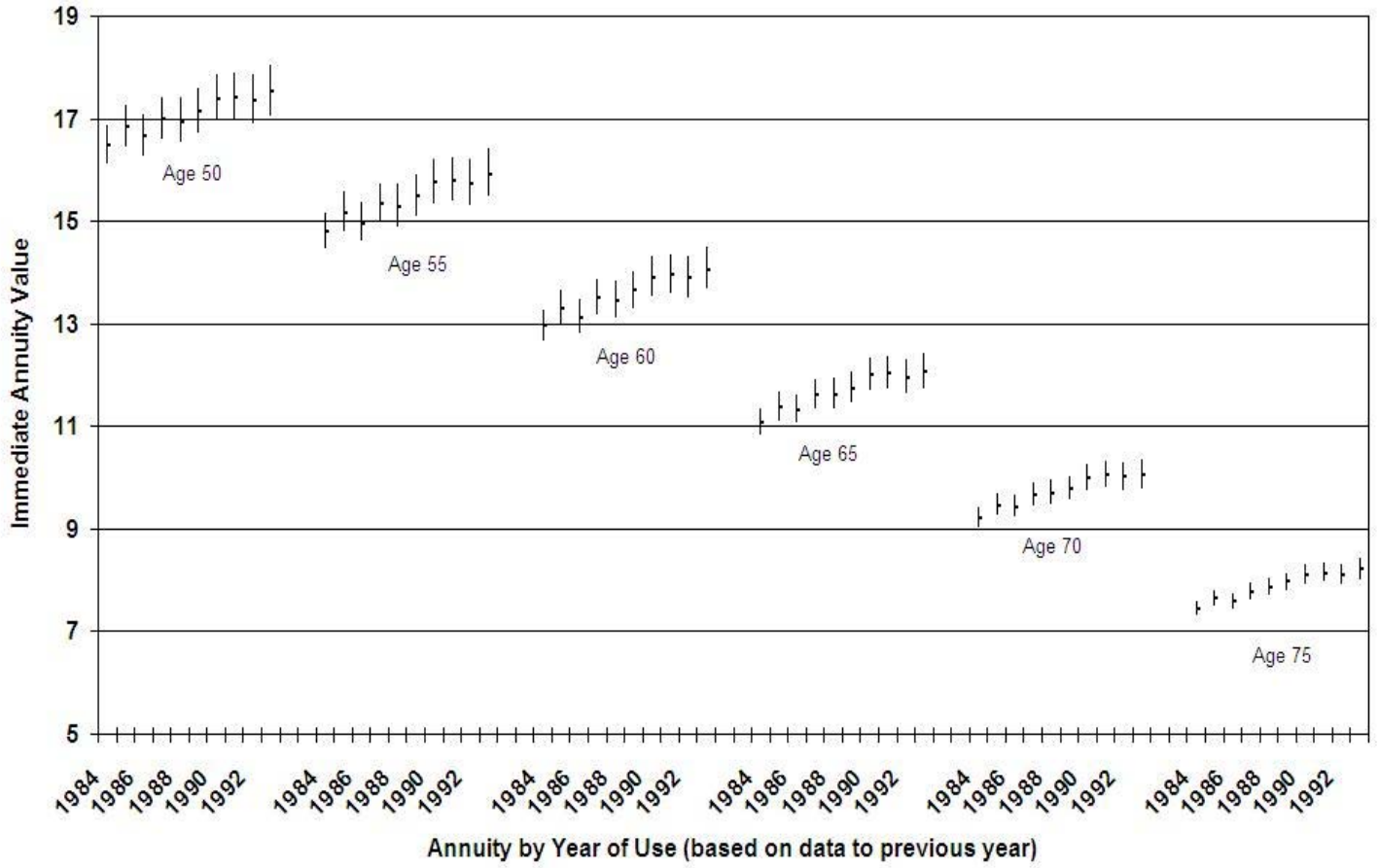
Cohort Penalties – Age 65 – 4.5%



Cohort Penalties – Age 70 – 4.5%



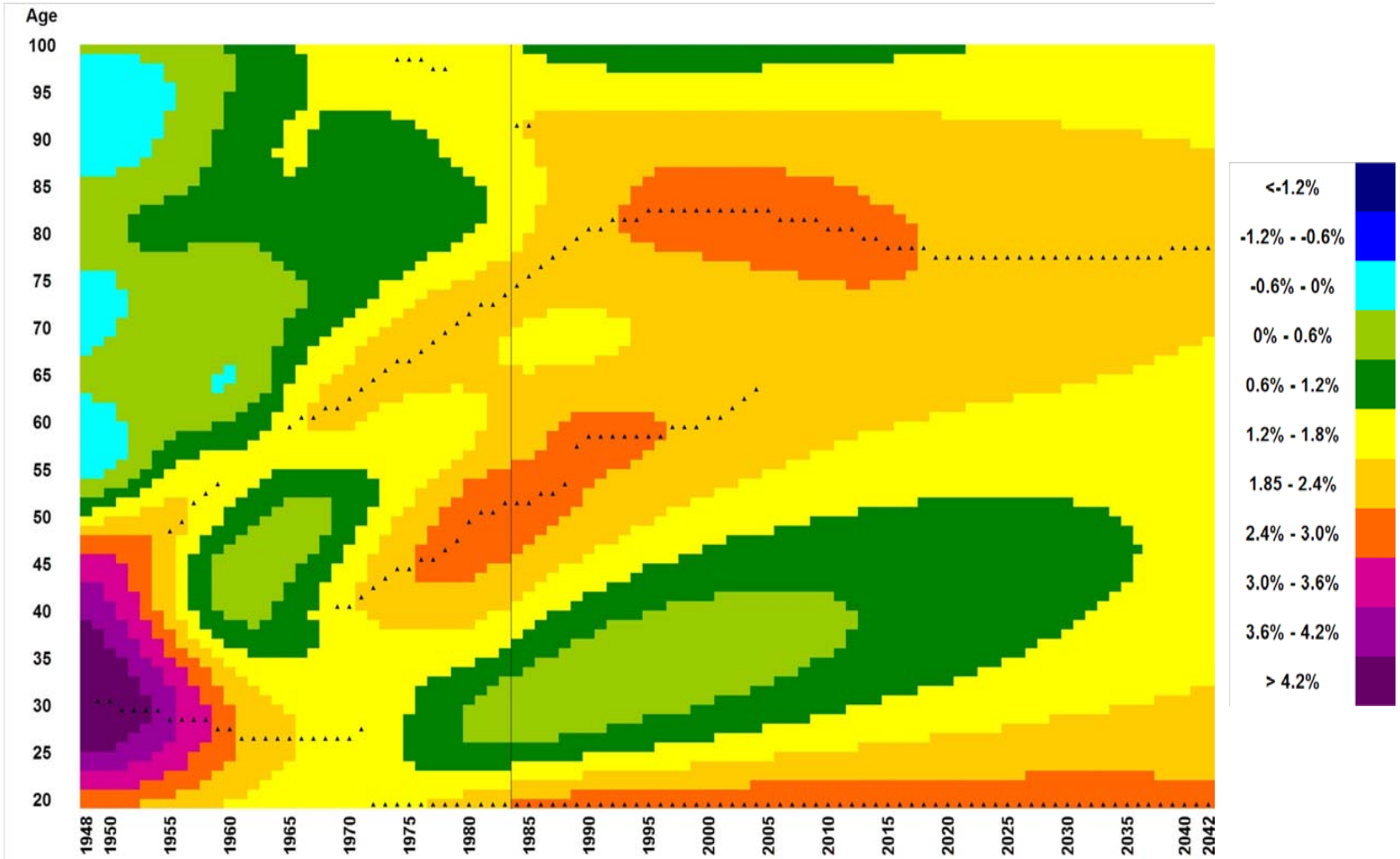
Cohort Penalties



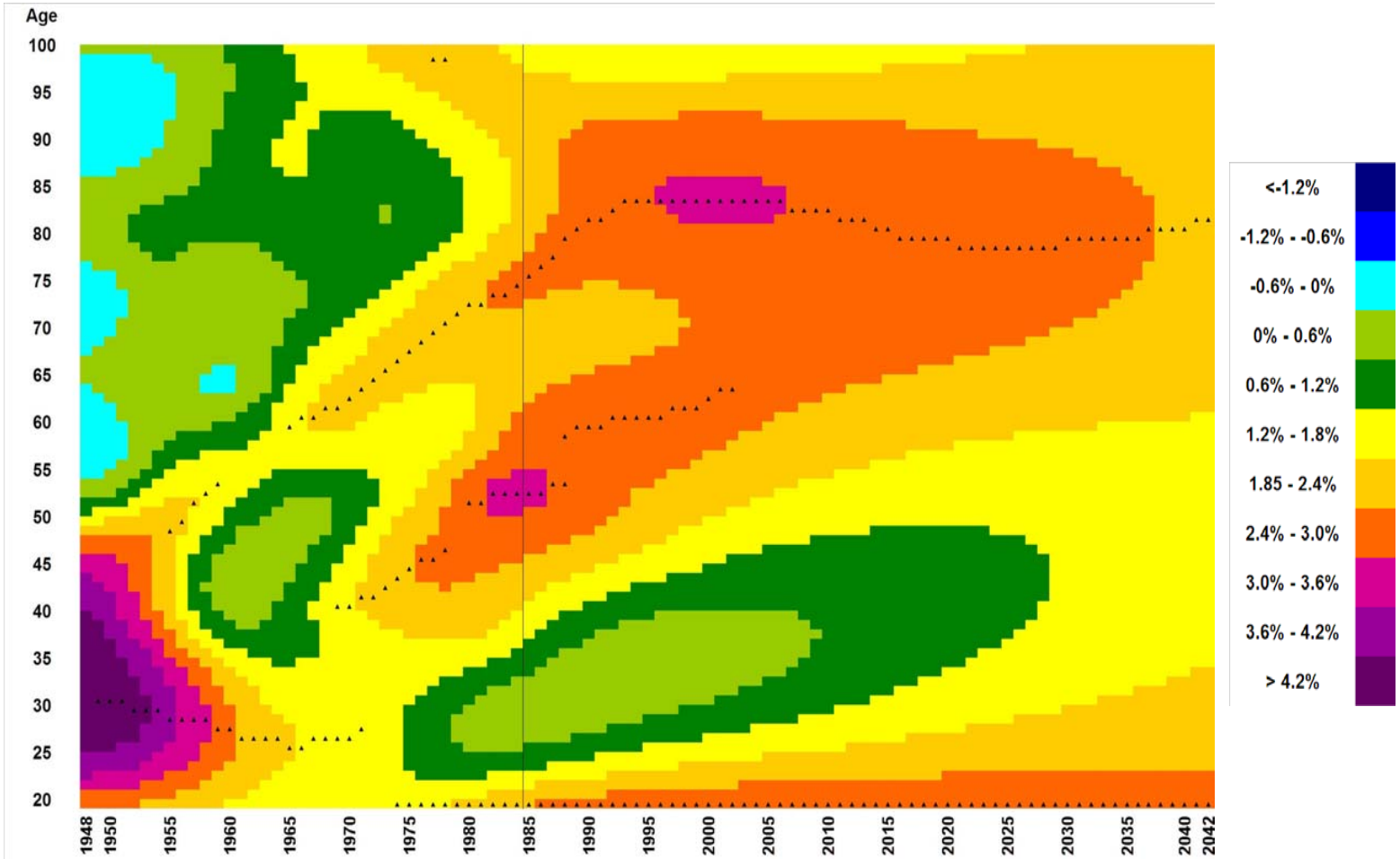
Progress of projections 1983-1992

- Uses P-spline model
- Period penalties

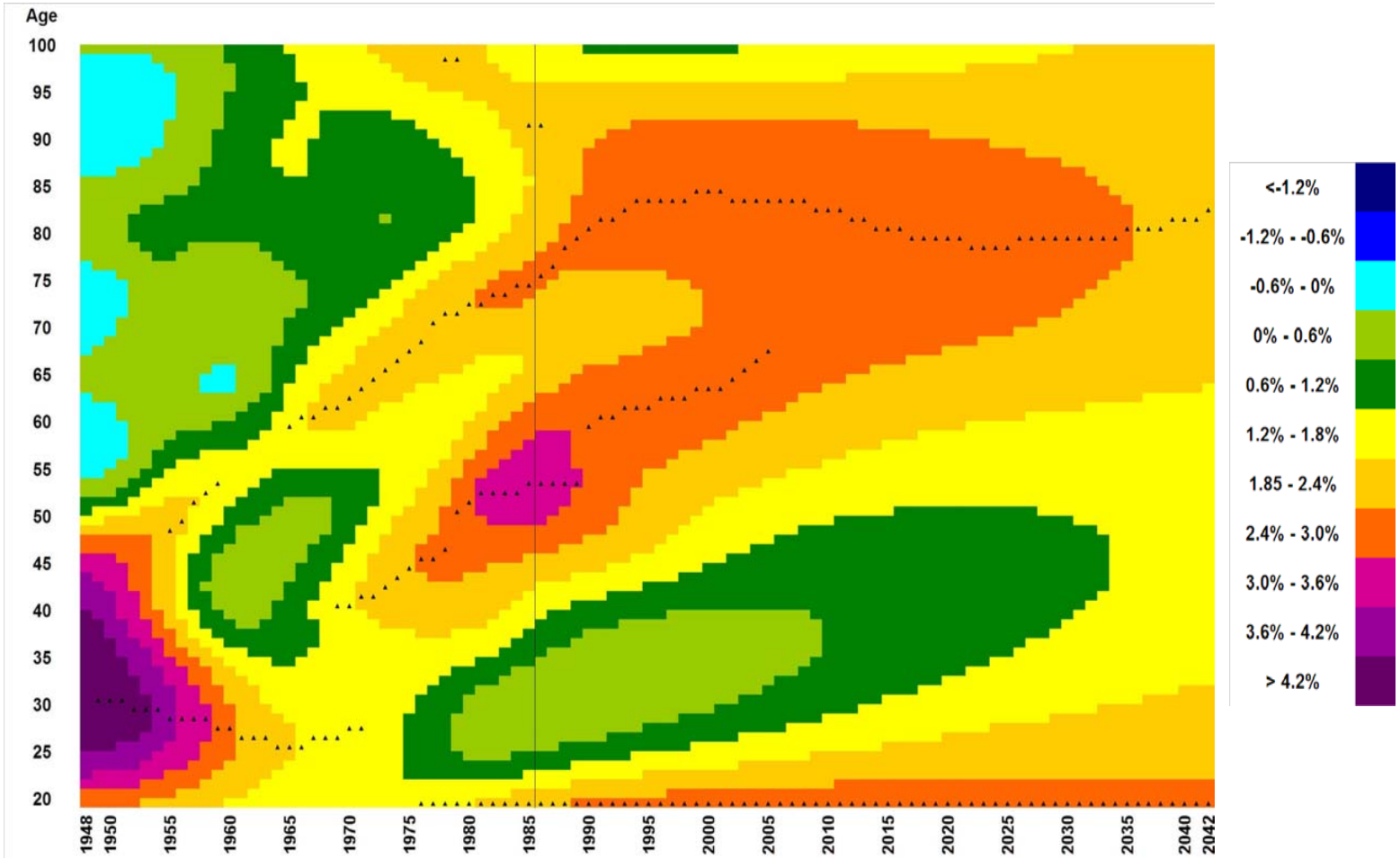
Data to 1983 – Period Penalty



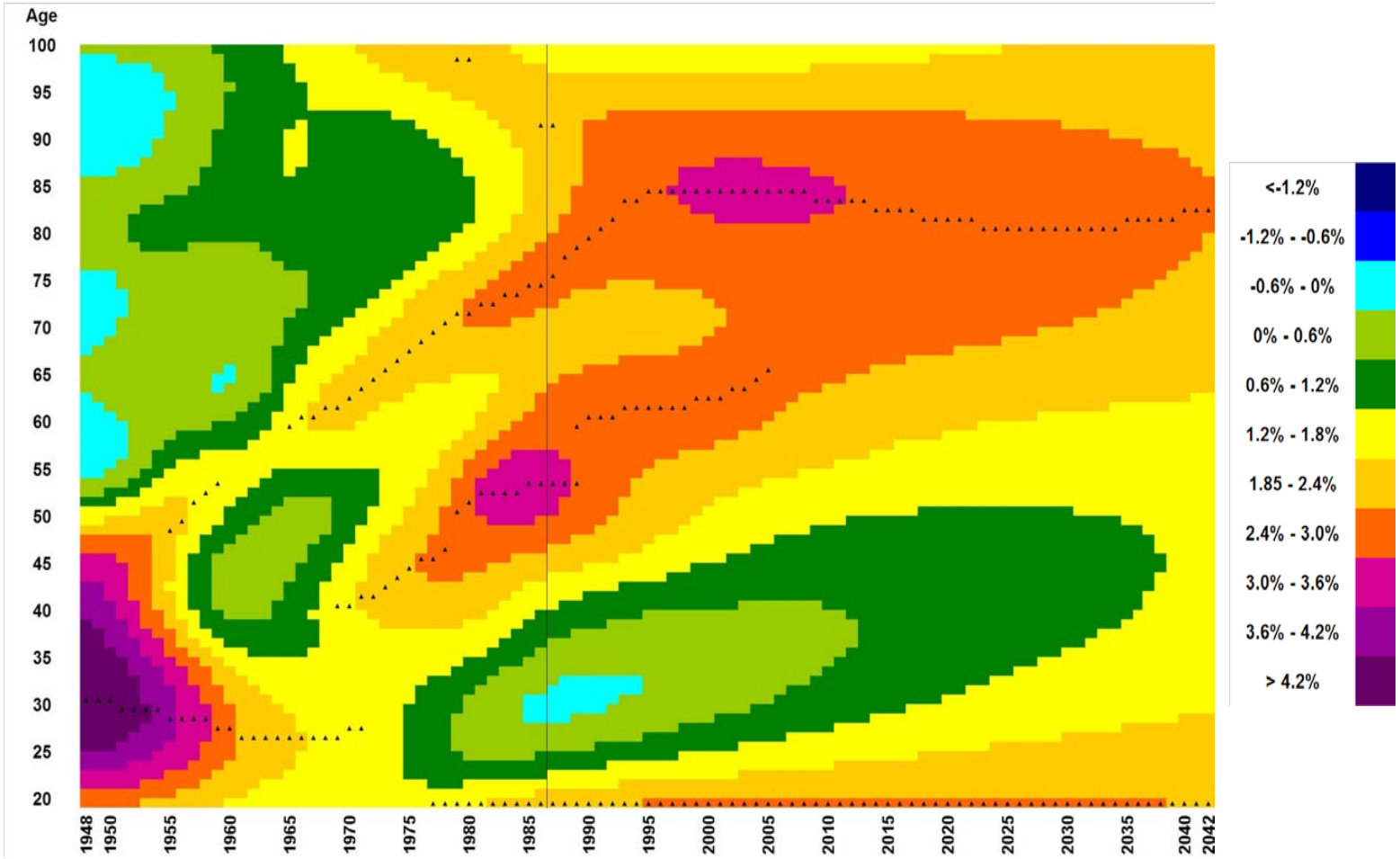
Data to 1984 – Period Penalty



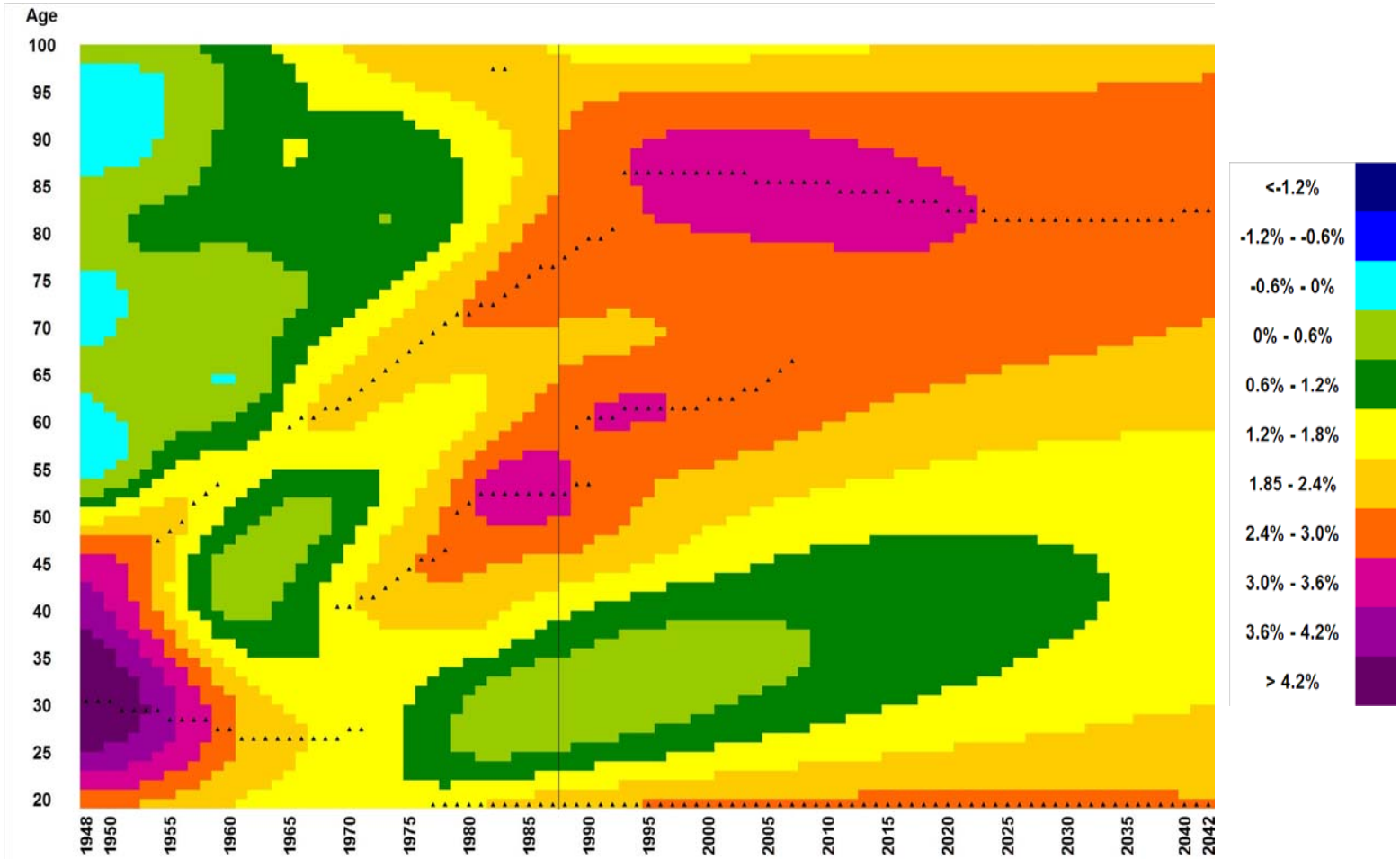
Data to 1985 – Period Penalty



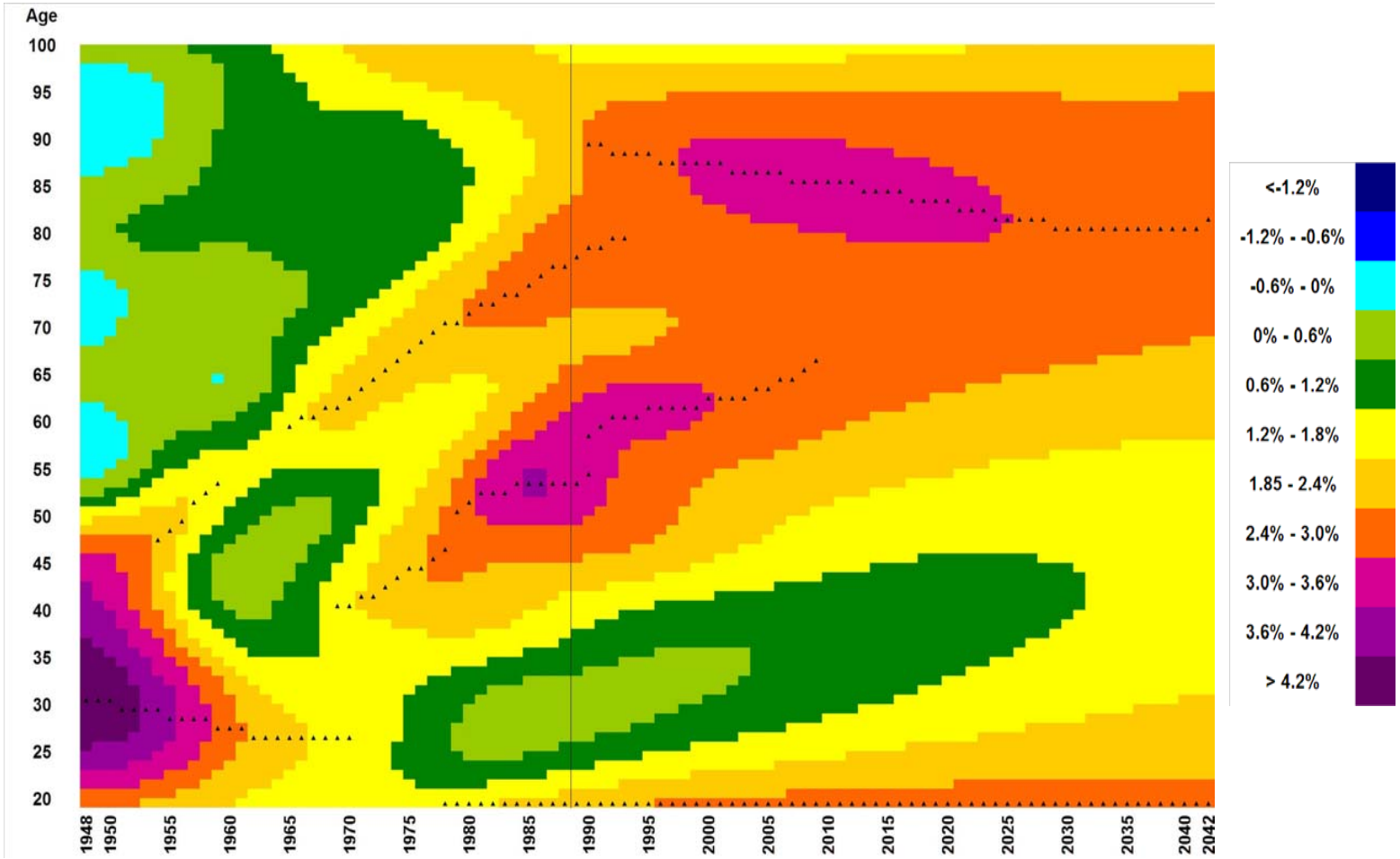
Data to 1986 – Period Penalty



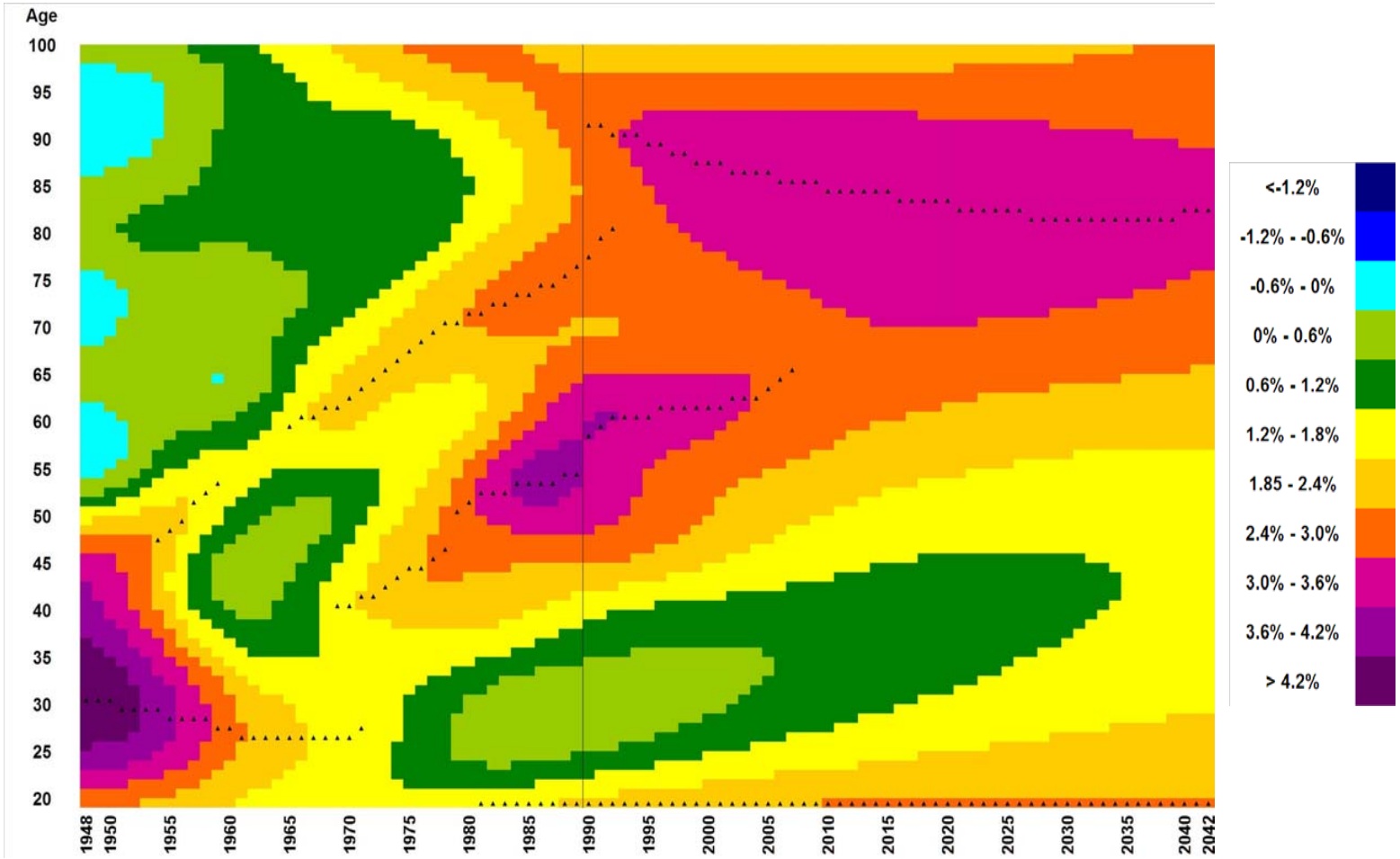
Data to 1987 – Period Penalty



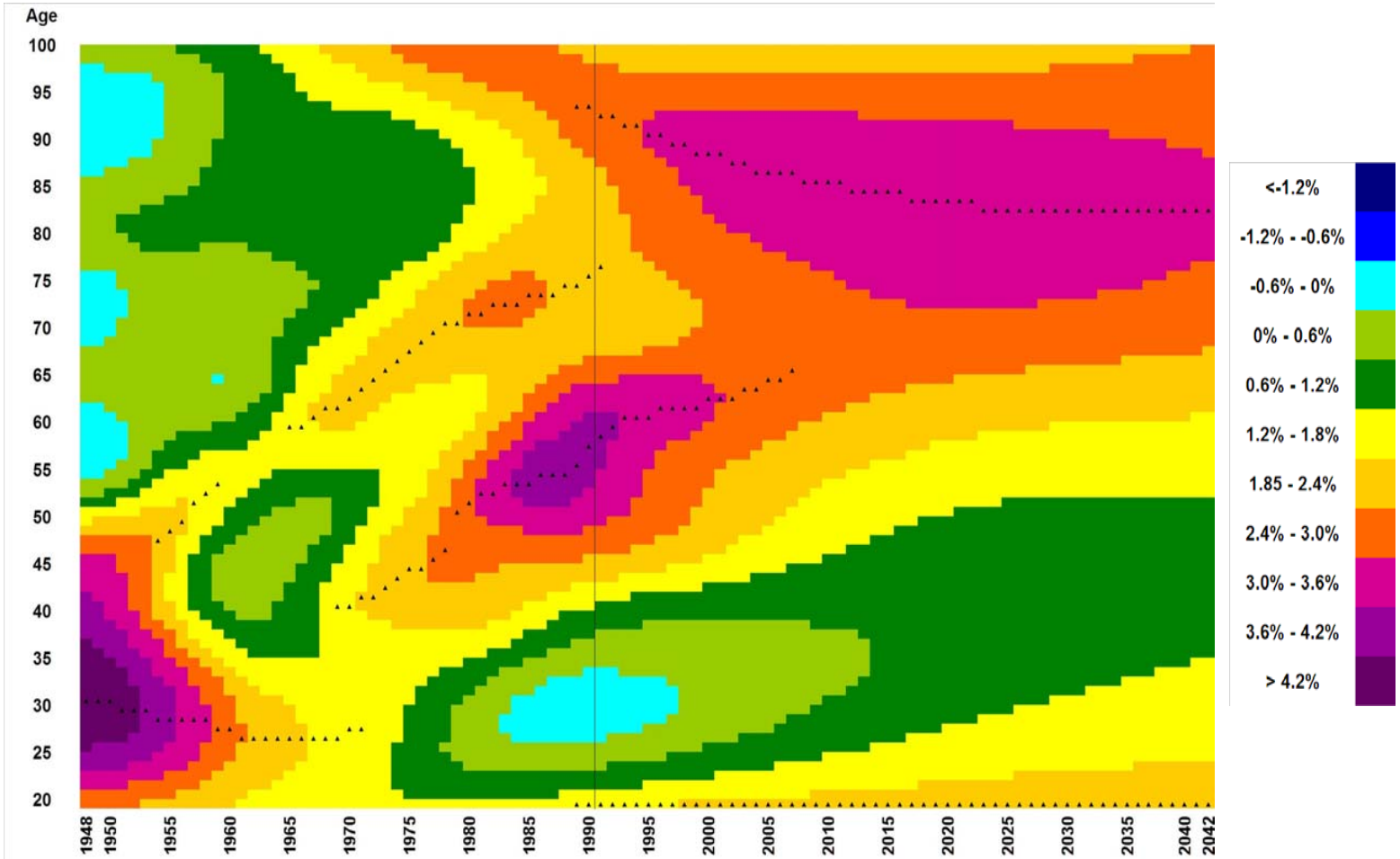
Data to 1988 – Period Penalty



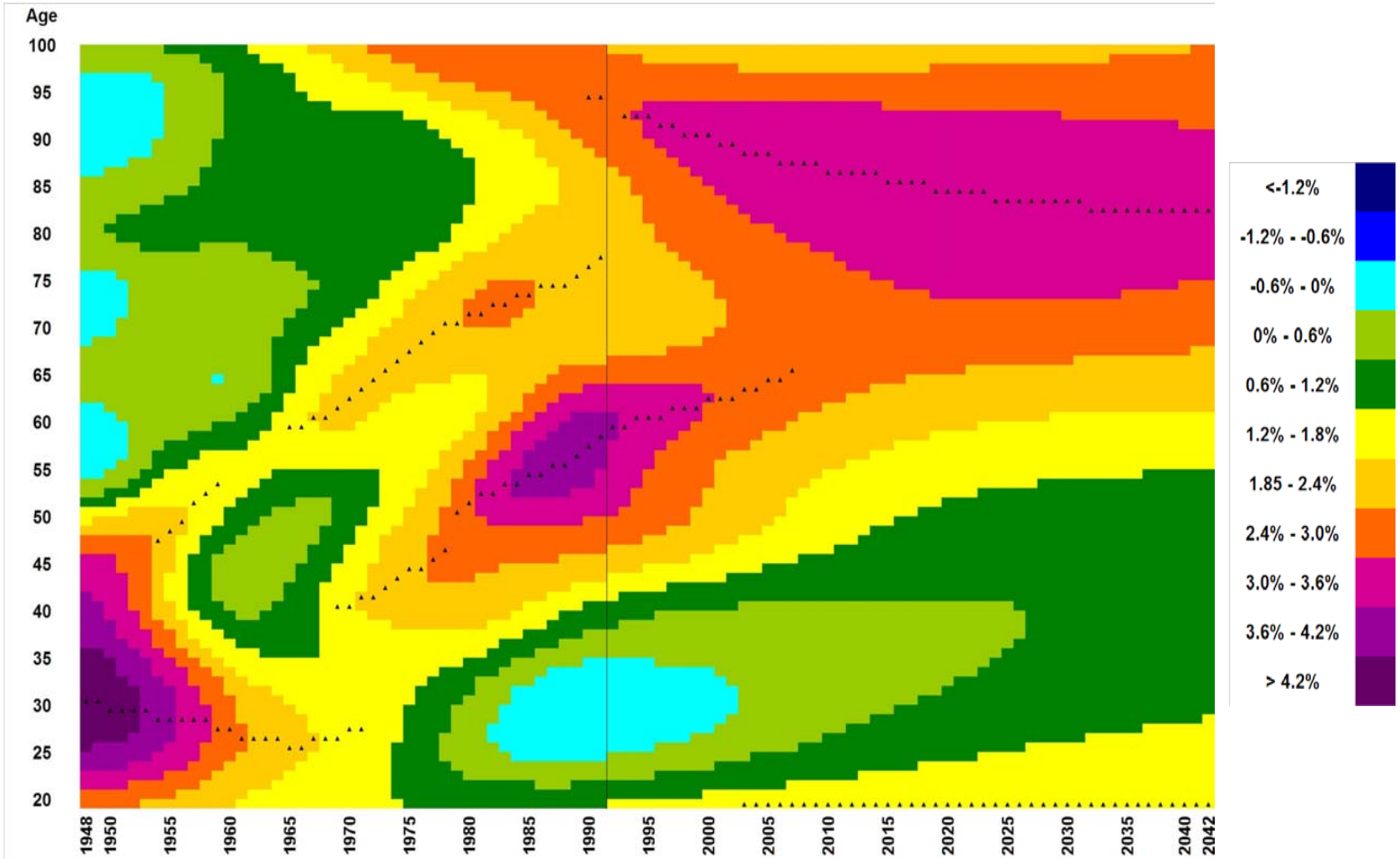
Data to 1989 – Period Penalty



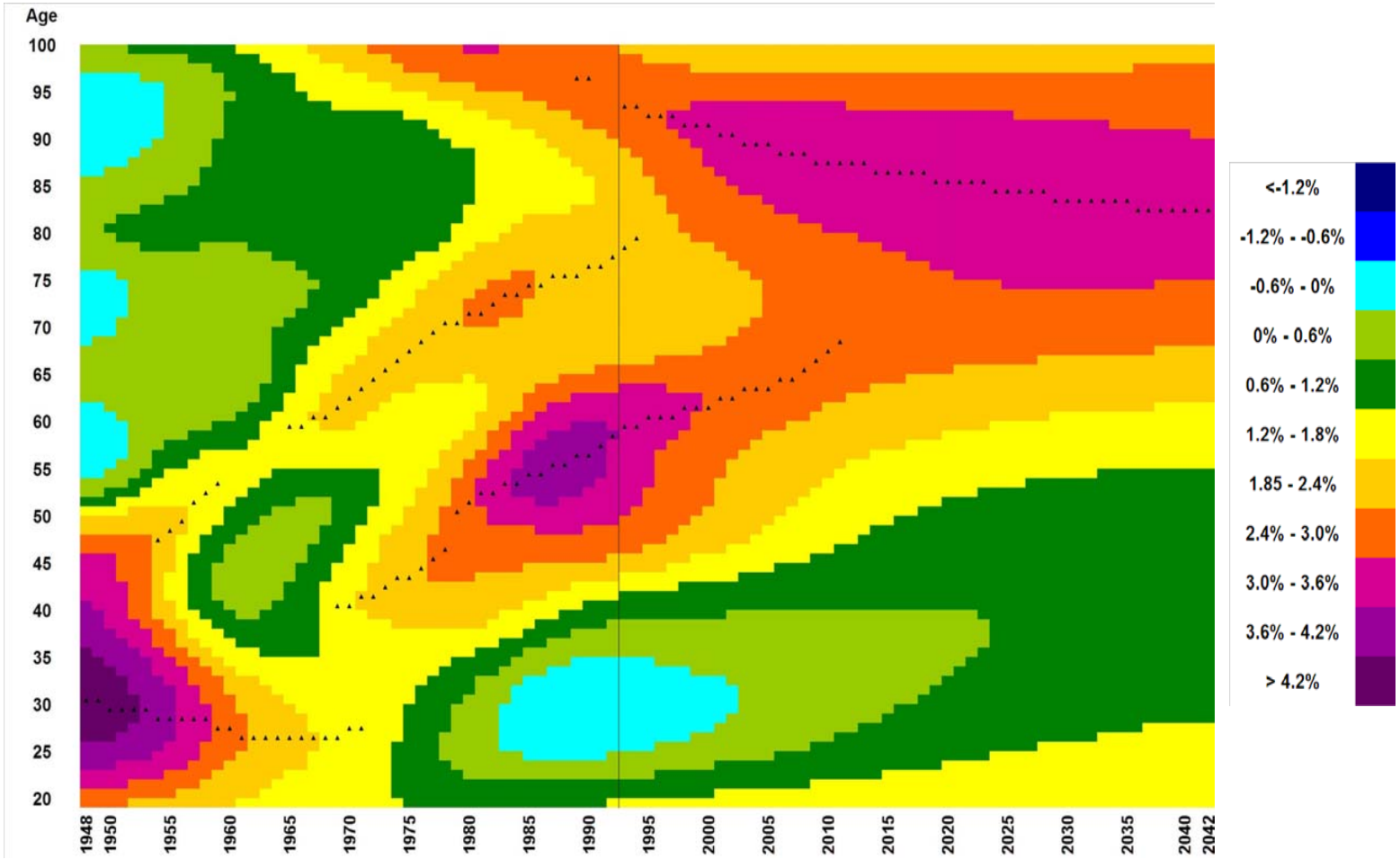
Data to 1990 – Period Penalty



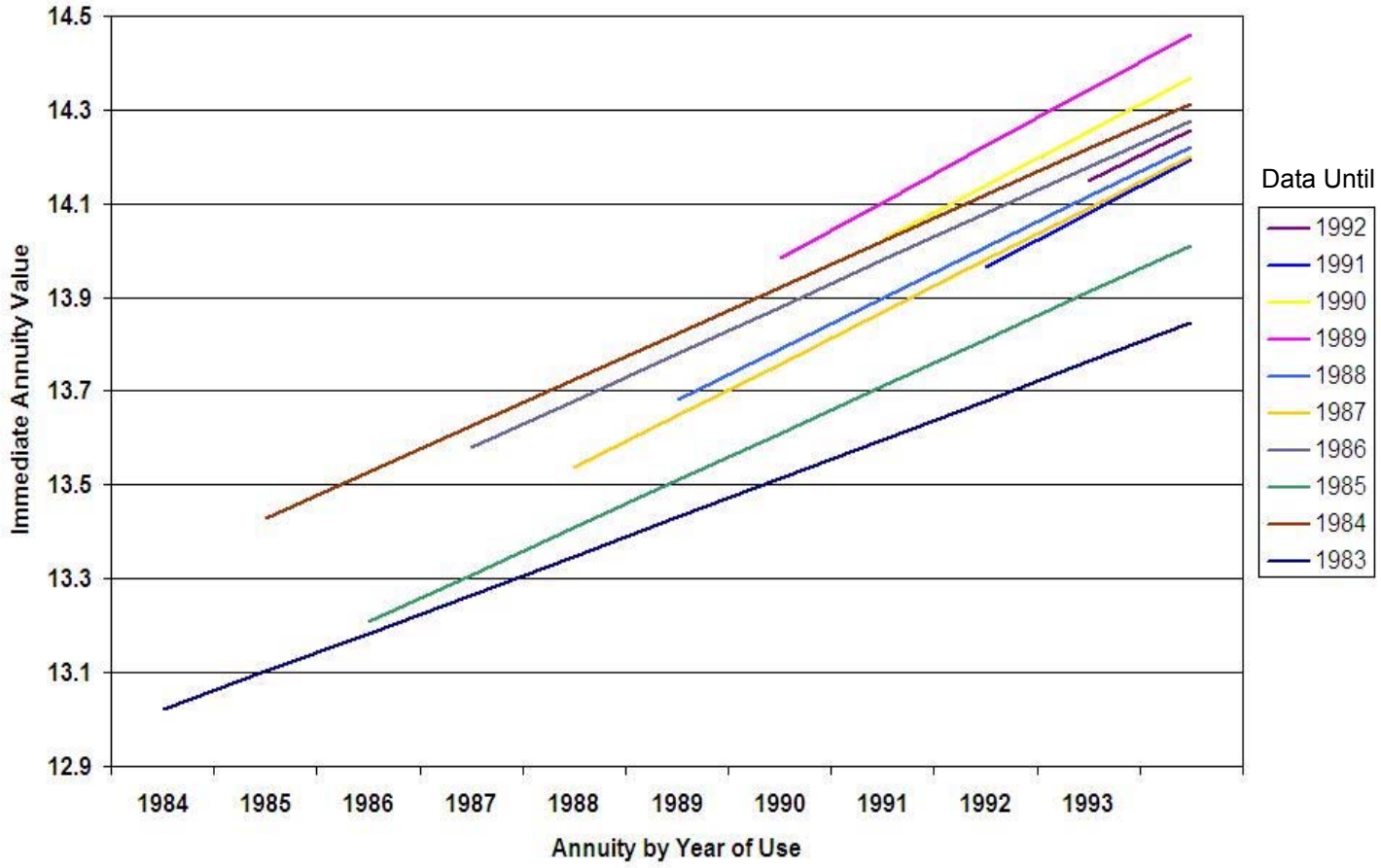
Data to 1991 – Period Penalty



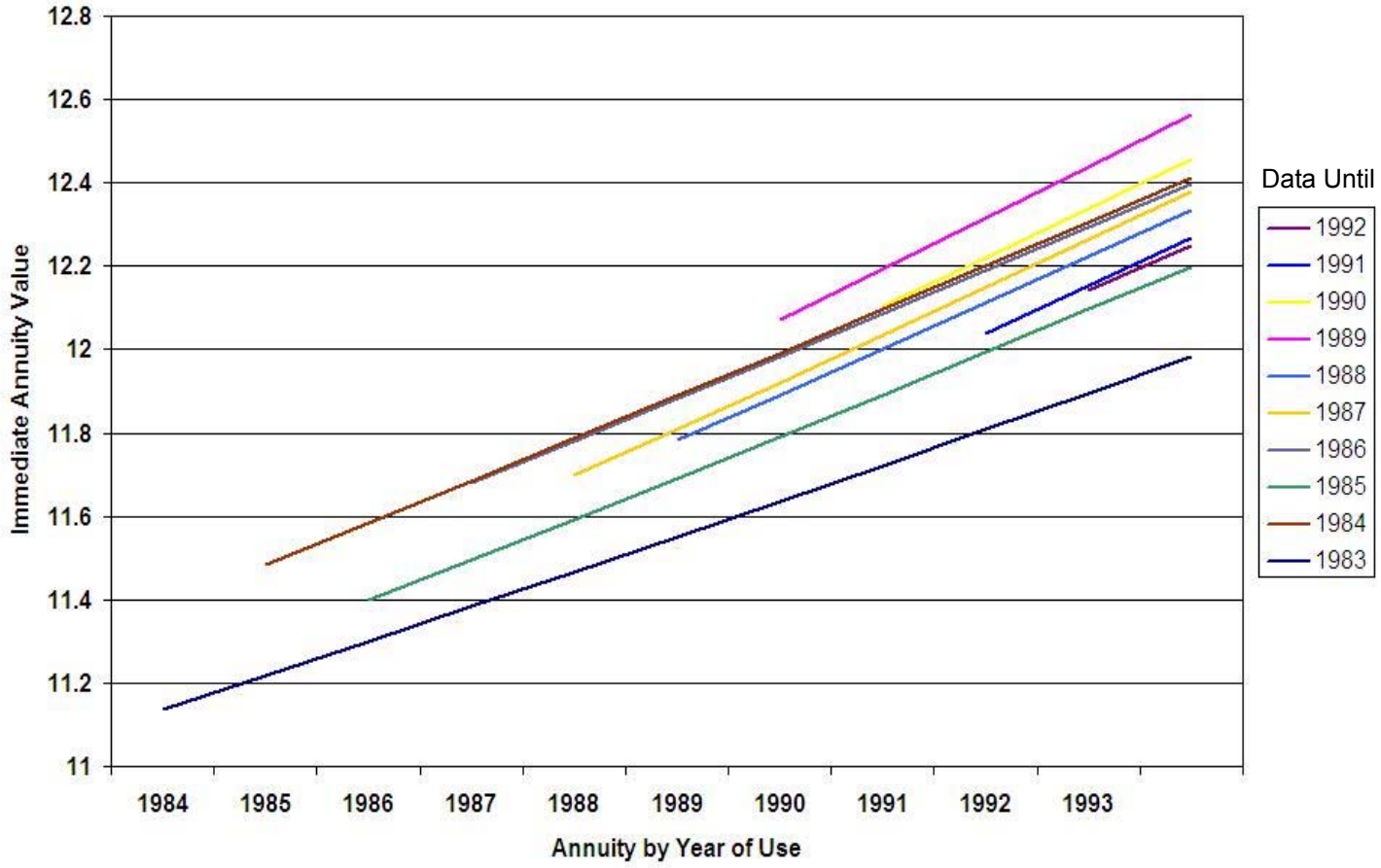
Data to 1992 – Period Penalty



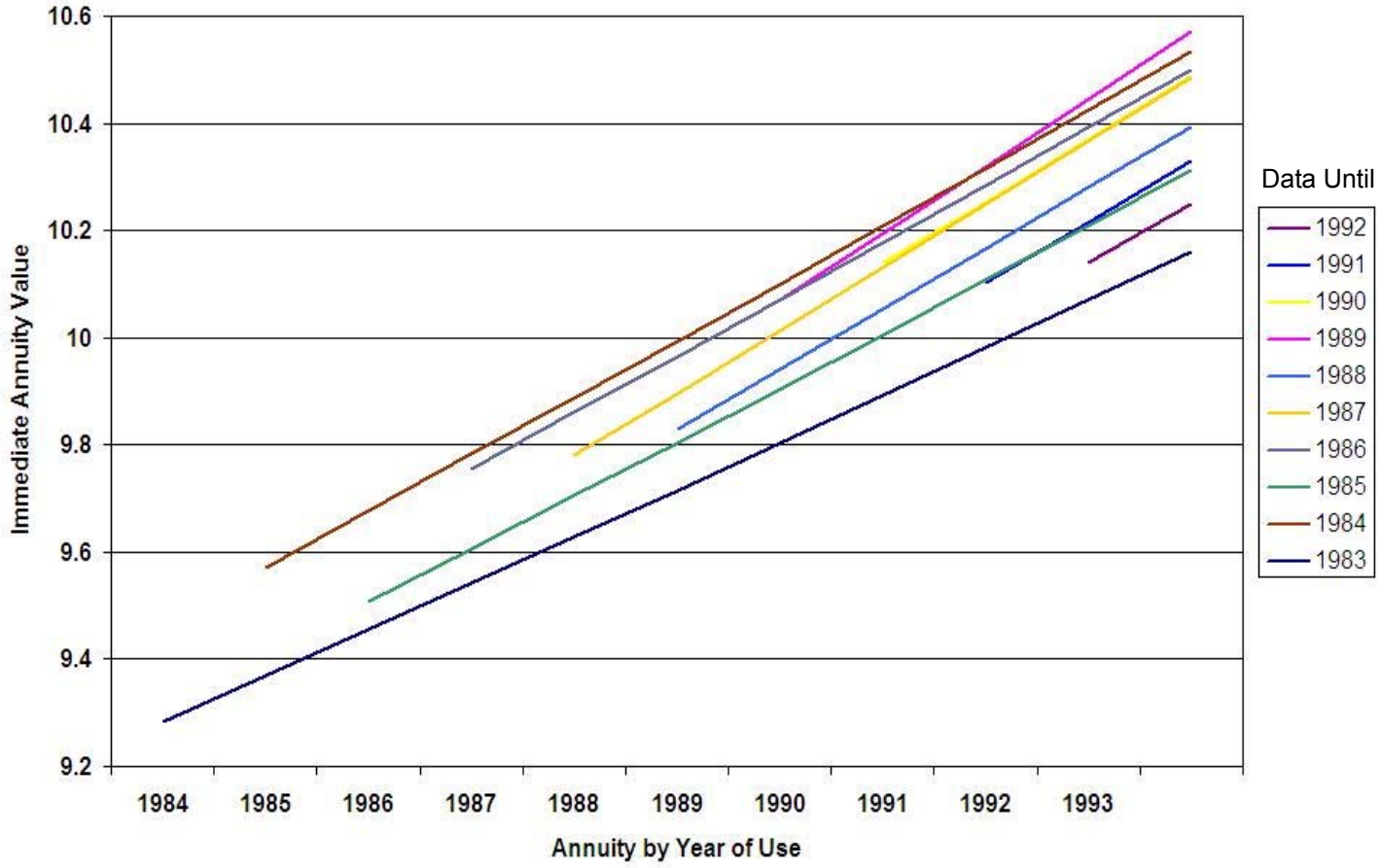
Period Penalties – Age 60 – 4.5%



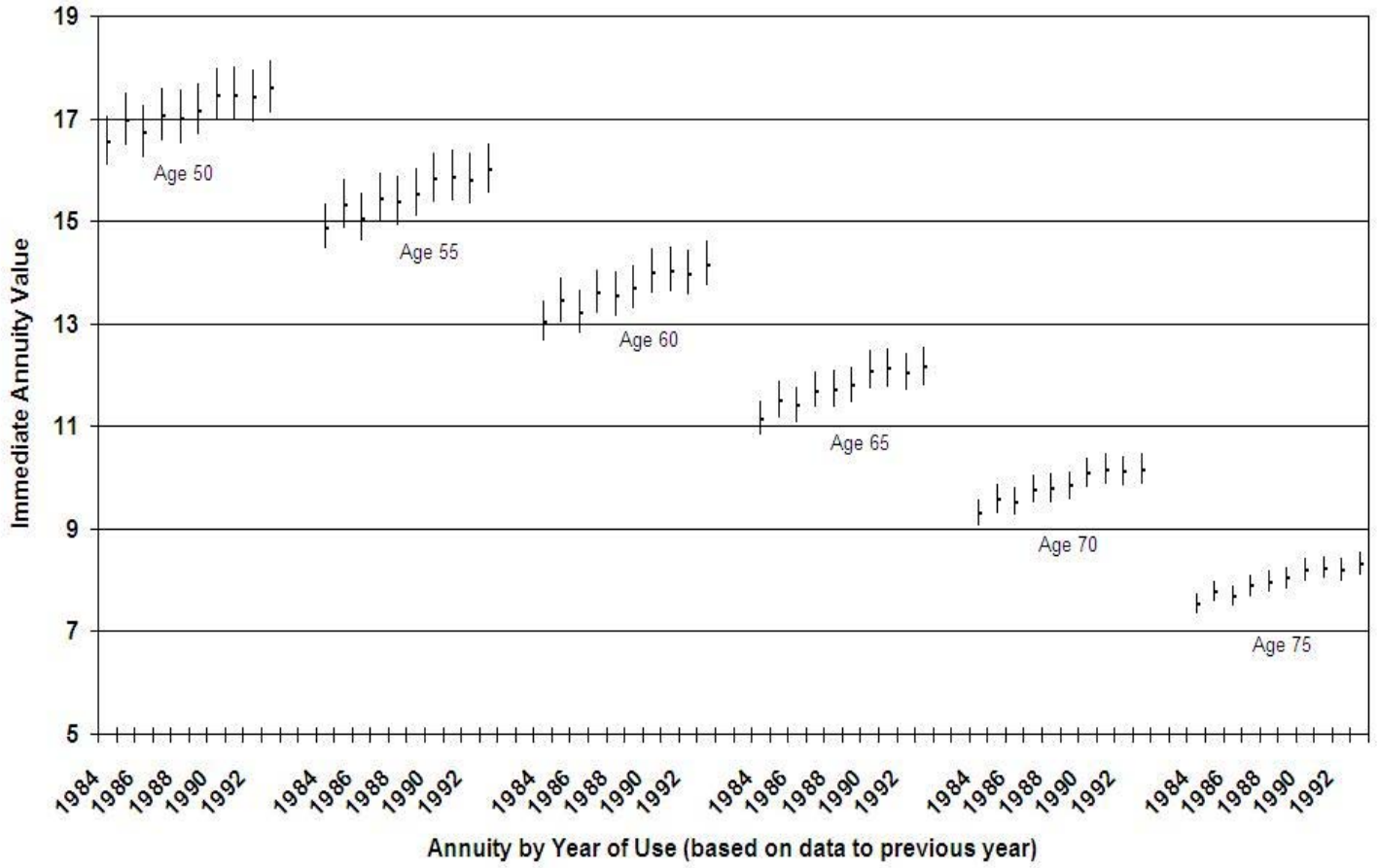
Period Penalties – Age 65 – 4.5%



Period Penalties – Age 70 – 4.5%



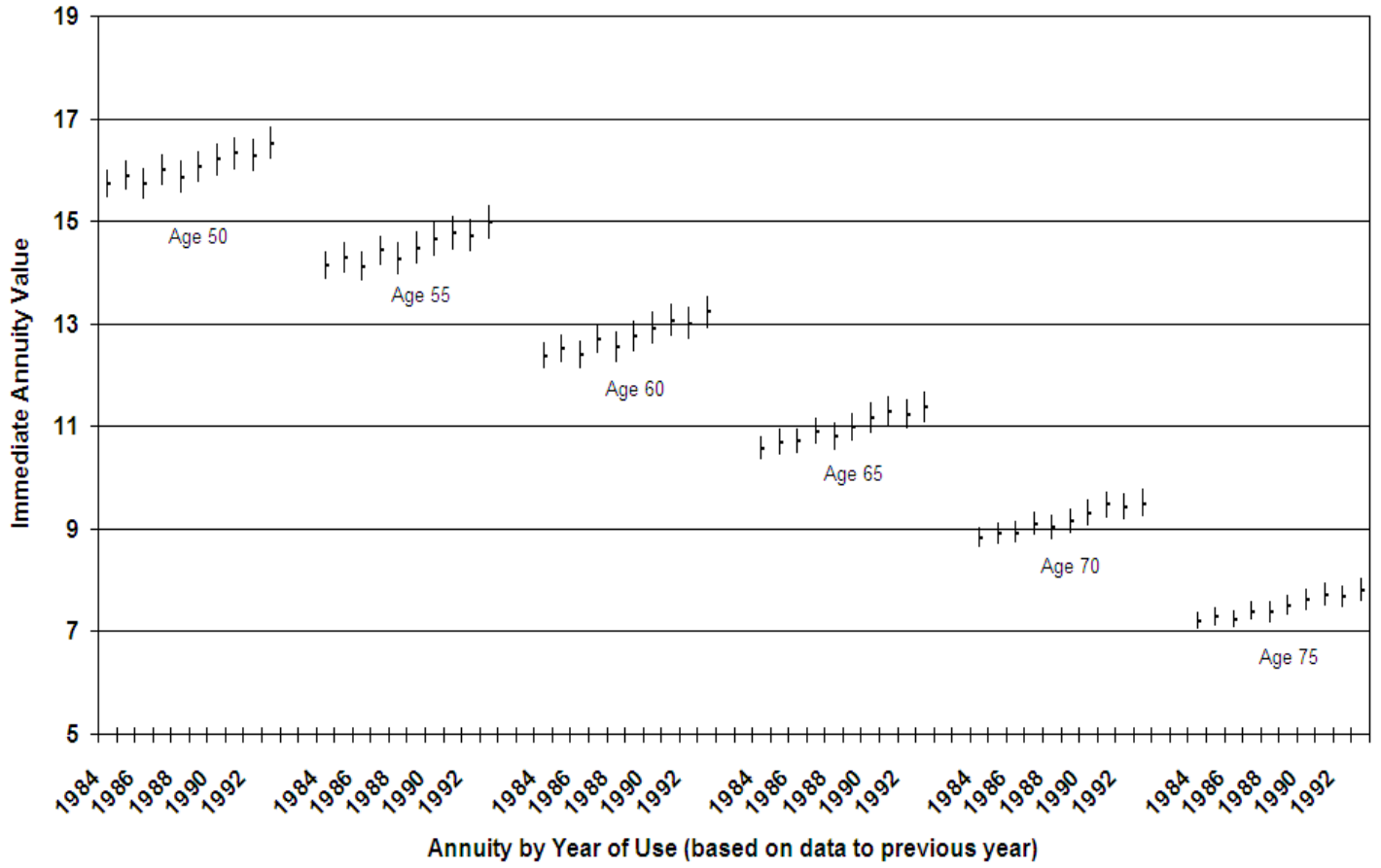
Period Penalties



Progress of projections 1983-1992

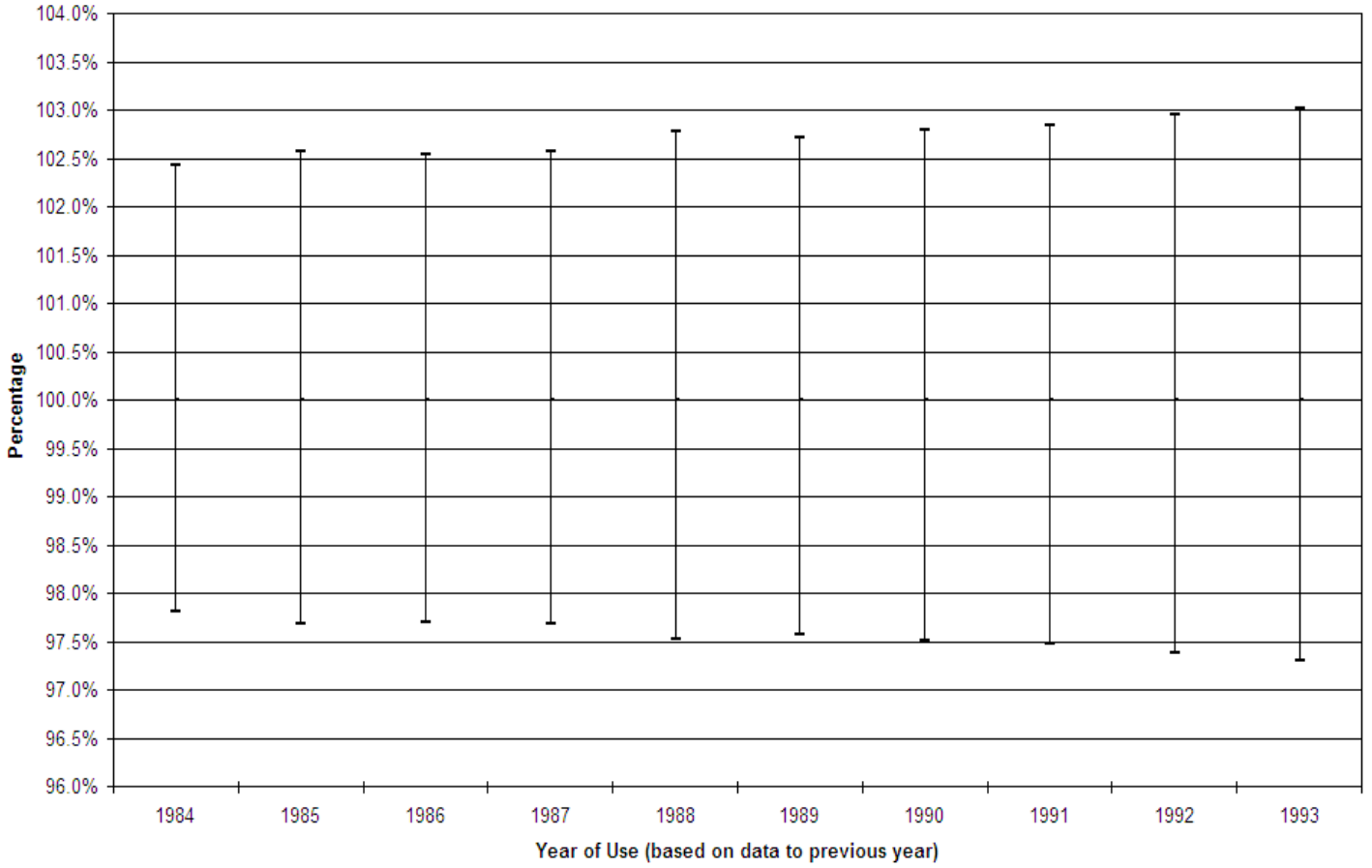
- Uses Lee-Carter model
- Period penalties

Lee-Carter


Immediate Annuity Value
19
17
15
13
11
9
7
5
Age 50
Age 55
Age 60
Age 65
Age 70
Age 75
1984
1986
1988
1990
1992
Annuity by Year of Use (based on data to previous year)

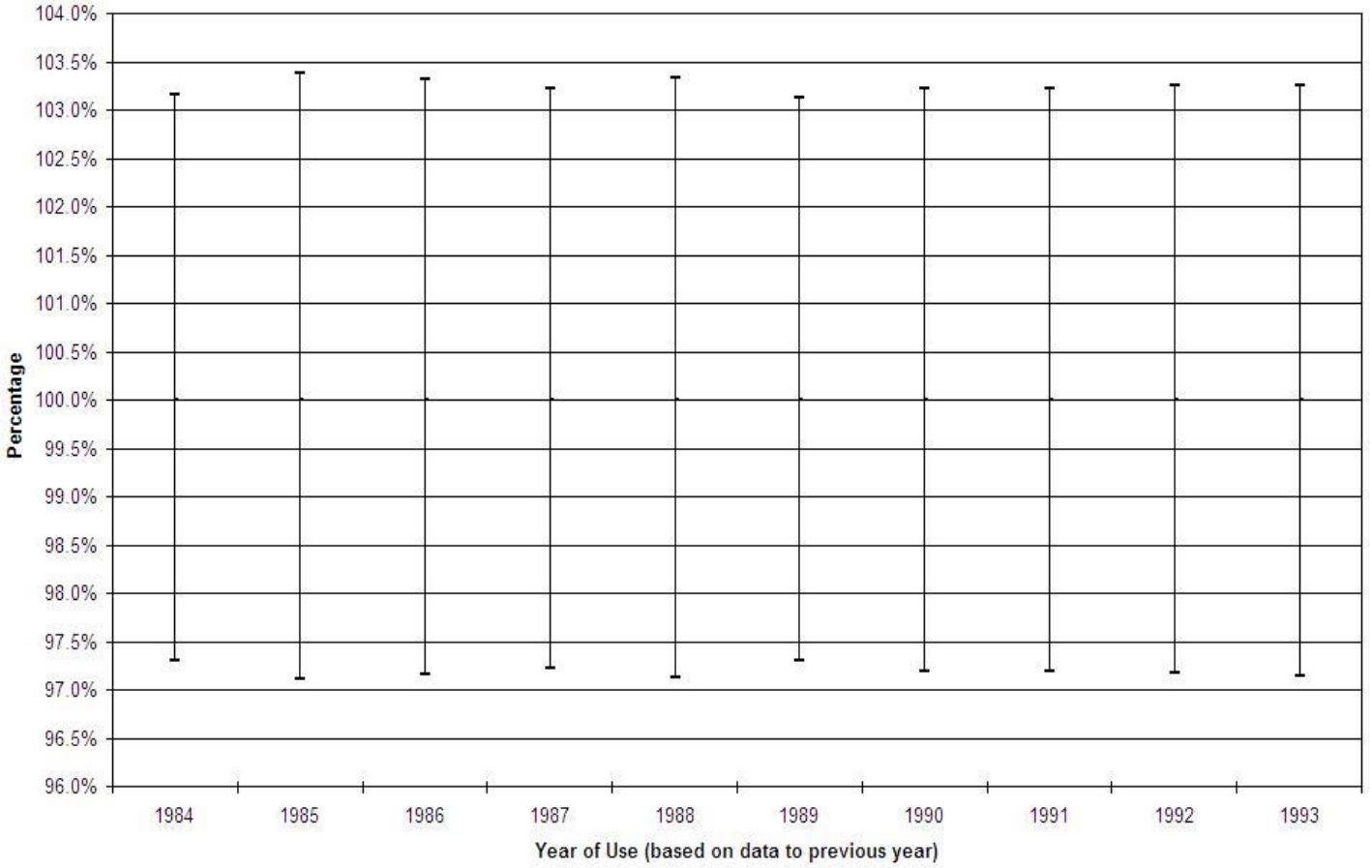
Portfolio Risk Capital – Cohort Penalties

(95% Confidence Interval)



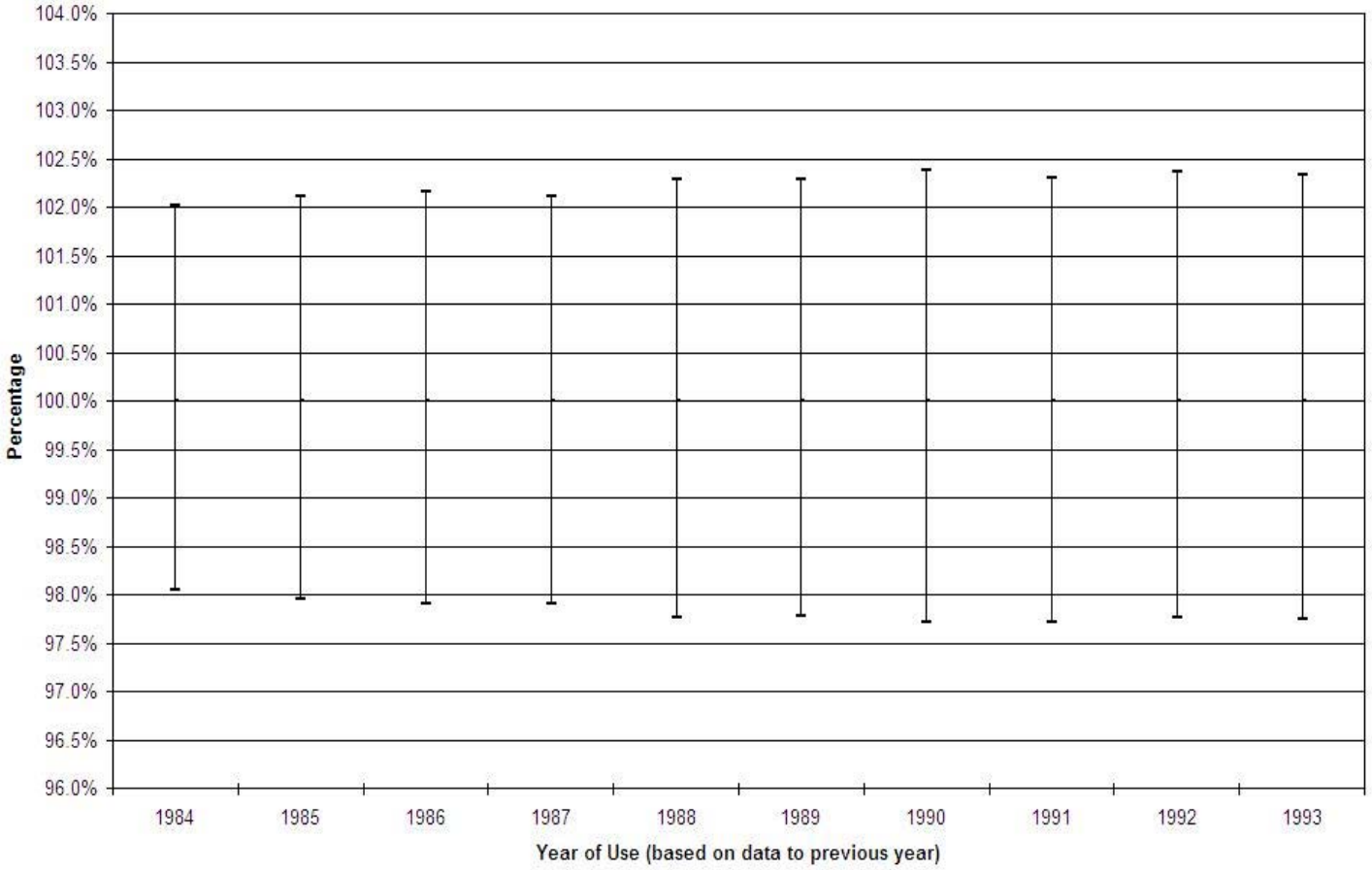
Portfolio Risk Capital – Period Penalties

(95% Confidence Interval)



Portfolio Risk Capital – Lee Carter

(95% Confidence Interval)



Agenda

- Introduction
- P-spline model
- Lee-Carter model
- Illustrative Results
- Choosing a mortality basis for reporting purposes
- Conclusions/Future Work

Lunch

- Software workshop

Discussion Forum – Choosing a mortality basis for reporting purposes

Stephen Richards
and Keith Miller

Choosing a basis for reporting

- Choice of base table and base mortality
- Choice of projection basis for future mortality
  - Stochastic v deterministic
  - Probabilistic v non-probabilistic
  - Model selection: P-spline v Lee-Carter v other
  - Penalty options: age-period v age-cohort
  - Dataset and parameters
- How do we enable the Board to make a decision?