Stochastic mortality models for actuarial work

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Overview

1. Deterministic model risk
2. Stochastic model risk
3. Advanced ages
4. Conclusions
1. Deterministic model risk
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• How do you know your model is correct?
• What are the consequences if it is not?
1. Deterministic model risk — CMI 2010

- Deterministic
- Defaults to projecting *decelerating* mortality improvements...
1. Deterministic model risk — CMI 2010

Source: Improvement rates labelled “actual” in CMI 2010.
### 1. Deterministic model risk — CMI 2010

<table>
<thead>
<tr>
<th>Age</th>
<th>Mortality improvement rate (per cent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td></td>
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<tr>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>3.5</td>
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</tr>
</tbody>
</table>

- **Actual improvements to 2007 (smoothed)**
- **CMI 2010 with long-term target of 1%**
- **CMI 2010 with long-term target of 2%**

Source: Smoothed actual mortality-improvement rates for males born in 1946, together with projected rates according to CMI 2010 model using default parameters and a long-term target of 1% or 2% improvement per annum.
2. Stochastic model risk
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- Model risk most obviously applies to deterministic models...
- ...but applies to *all* models, including stochastic ones
2. Stochastic model risk

2. Stochastic model risk

Source: Richards and Currie (2009). ONS population data, fitted and forecast rates for males at age 65 under two Lee-Carter variants: DDE model (time series) and Currie-Richards model (smooth).
2. Stochastic model risk

<table>
<thead>
<tr>
<th>Model</th>
<th>Best-estimate reserve</th>
<th>99.5% reserve</th>
<th>Capital required</th>
</tr>
</thead>
<tbody>
<tr>
<td>DDE</td>
<td>13.92</td>
<td>14.42</td>
<td>3.6%</td>
</tr>
<tr>
<td>CBD5</td>
<td>13.96</td>
<td>15.04</td>
<td>8.0%</td>
</tr>
</tbody>
</table>

Source: Richards (2011). Best-estimate and stressed annuity values for a male aged 65 following population mortality in England and Wales. Continuous temporary annuities to age 105, valued at 3% per annum. DDE model is that of Delwarde, Denuit and Eilers (2007), while CBD5 model is that of Cairns, Blake and Dowd (2006), as modified by Currie (2010) to allow for a non-linear effect by age. The DDE model projects a single constant drift term, while the CBD5 model projects a bivariate drift term.
2. Stochastic model risk

- DDE model says a reserve of 14.42 suffices with 99.5% probability
- CBD5 model says the same reserve suffices with 87.3% probability
2. Model risk

- How do you know your model is correct?
- You don’t
- Must use a *variety* of models to explore model risk
3. Advanced ages
3. Advanced ages

- ONS data only goes up to age 105
- Usable CMI data stops even earlier...
3. Advanced ages

Source: CMI assured-lives data. Logarithm of crude force of mortality for CMI data at ages 50–100 aggregated over the years 2001–2005. The data above age 95 are unreliable and cannot be included in any projection model. However, actuarial calculations for annuities and pensions typically require mortality rates up age 120, so some form of extrapolation is required from age 95 to age 120.
3. Advanced ages

• How to get rates *and* projections at advanced ages?
• 2D P-spline models can extrapolate by age as well as project in time
3. Advanced ages — 2D P-spline models

Source: CMI assured-lives data fitted to 2D age-period P-spline model. Logarithm of crude force of mortality for ONS data for males in 2007 at ages 50–104, together with the fitted and extrapolated rates to age 120 under the 2D age-period model (Currie, Durban and Eilers, 2004). As with a projection in time, the uncertainty over the extrapolated rates increases the greater the distance from the actual data, i.e. there is an expanding “funnel of doubt”. In this case the funnel is rather narrow because of the strength of the age signal.
3. Advanced ages — 2D P-spline models

Source: Longevitas Ltd. Fitted and projected logarithm of force of mortality for ONS data for males. Mortality rates are simultaneously extrapolated to age 120 and projected to 2050 using the two-dimensional P-spline model of Currie, Durban and Eilers (2004).
3. Advanced ages — model risk

- Model risk demands we use more than one model
- Need other models capable of extrapolation by age
- Cairns-Blake-Dowd (2006) model can also extrapolate by age
3. Advanced ages — CBD 5

\[ \log \mu_{x,y} = \kappa_{0,y} + \kappa_{1,y} S(x) \]

- \( \kappa_0 \) and \( \kappa_1 \) form a bivariate time series for projecting future rates
- \( S(x) \) is a smooth function of age

Source: Longevitas Ltd. Parameters for the Cairns-Blake-Dowd model number 5, as modified by Currie (2010). The function $S(x)$ is smoothed by splines with a five-year knot spacing, which enables age extrapolation as in the 2D P-spline models.
3. Advanced ages — CBD 5

Source: Logarithm of force of mortality for ONS data for males. Mortality rates are extrapolated to age 120 using the S(x) function, and projected to 2050 using the bivariate time series for $\kappa_0$ and $\kappa_1$. The fitted rates have ridges due to period effects.
4. Conclusions
4. Conclusions

- CMI 2010 model defaults to decelerating mortality improvements
- Model risk is very substantial for mortality projections
- Don’t rely on a single projection model or methodology
- Stochastic projection models can project by age as well as time
References I


Currie, I. D. 2010 On a model of Cairns, Blake and Dowd, Working paper available on request

Currie, I. D. 2011 Forecasting mortality at high ages, Longevitas blog

References II


Richards, S. J. and Currie, I. D. 2009 Longevity risk and annuity pricing with the Lee-Carter model, British Actuarial Journal (to appear)

Richards, S. J. and Currie, I. D. 2011 Extrapolating Mortality Projections by Age, submitted article

Richards, S. J. 2011 Applying the brakes, Longevitas blog

Richards, S. J. 2011 Model risk, Longevitas blog