## STORM RATING IN THE NINETIES



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The chart below shows the estimated storm losses for UR Household Buildings policies from 1920 to 1990. The extreme right hand side of this chart illustratea vividly the sharp increase in storm losses at the end of this period primarily from the october 1987, January 1990 and the February 1990 storms.


Householders, insurers and reinsurers are still counting the cost of these storms. The reingurers responge has been to witharaw capacity and implement fivefold increases in catastrophe excess of loss rates, Insurers analysed these losses in considerable detail but there is little evidence that any district rates have been adjusted as a result of any of these investigations.

This report outlines the causes of storms and considers the evidence for increases in their frequency and severity. A study of UK stora events irom 1920 to 1990 , conducted by the University of East Anglia climatic Research Unit for Commercial Onion, is then used to estimate the likely insurance losses from these storms. An index of relative storm exposure is calculated at postcode area level, the rating implications discussed and broad storm risk areas plotted on a map. The data is then used to identify appropriate distributions for the frequency and severity of 0 K storms and a simulation model is used to investigate the annual storm loss distribution and to calculate risk rates on line for various layers of storm excess of loss reinsurance. These results are compared with current market rates and the effectiveness of traditional reinsurance for syoothing storm losses is discussed.

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### 1.1 Barckground

The UK had a relatively "calm" period for a number of decades until the storm of october 1987 caused widespread damage to a corner of the south-east corner of England. This storm was said at the time to have been a one in two hundred year event based on the frequency of such high winds in the areas aftected.

This assessment was soon to be questioned by the events of early 1990 when a very stormy period in January caused higher insurance losses from a wider area of southern England and parts of Europe. A further, less damaging, storm followed in February 1990.

The reinsurance market was impacted by a series of other major losses during this period, especially from Hugo and Piper Alpha. These losses had a spectacular effect on the LWX market and resulted in a significant loss of capacity in this market. In turn this reduced catastrophe excess of loss capacity with rates, for the 1991 renewal eeason, increasing by up to five times theix pre-1990 levels.

Although a significant increase in reinsurance rates is normal after heavy losses, there is in addition a popular view in certain quarters that the frequency and severity of storms has increased, and will continue to increase, due to effects of emissions into the atmosphere and the resultant global warming. These beliefs are often attributed to predictions from weather forecasters and are sometimes validated by comparisons of the frequency of such events over the recent past.

The storms of 1987 and 1990 were analysed by insurers in great detall down to postcode level. These storms produced different patterns of loss. The 1987 storm affected a narrow land mass, and caused proportionately higher damage to this affected area, than the Jenuary 1990 storm which affected a much wider area and cost the insurance industry more in total.

The gross cost of the 1987 and 1990 storns accounted for about $50 \%$ of the household buildings insurance premiums for each of these years.

The results were somewhat inconcluaive, especially as the recent storms were considered unrepresentative both in terms of the areas affected and also in their severity. There is no evidence that any differential rating for storm, at postcode area level, has been incorporated into the rating of this business or that any concluaions on the likely annual cost of storms were derived from any of these studies.

The Working Party was set up to begin to investigate sone of the many questions that naturally arise from the events described above. At the time, october 1991, it was not clear of how much progress, if any, could be achieved in this endeavour.

## Terna of riforence

The Working party adopted the following terms of reference for its study:
To inventigate the frequency and sovarity of
zuropean torme and particularly any trends in
these and to evalunte the finmncial offect of
these ontantrophes, on insurars and their
reinaurera, by use of an appropriate
statintical model of the annual insurance cost
of atornt.

The availability of data on UK storms, and the time needed to analyse this data, meant that the actual statistical analysis was restricted to UK Household Buildings covers only.

The intention, initially, was to consider European storms as this is a better description of the storms which affect both the UK and large parts of North Nest Europe.

Some work on European storms is being carried out by two of our European friends, Robert Alting von Geusau and Charles Levi. It was hoped that this work could be incorporated into this report but distance and time (not TaD) have made this impossible. A report covering the European aspects is being prepared by our colleagues and should be available at the November conference.

As changes in the frequency and severity of UK storms will be reflected in changes in frequency and severity of European storms we have retained the original terms of reference.

This paper is based primarily on a report on "Storm Severity over Britain" written for Commercial Union by Dr J.P. Palutikof and A.R. Skellern [11] of the climatic Research Unit of the School of Environmental Sciences of the University of East Anglia. This report, which will be referred to as the "UEA" report from now on, examined the frequency and severity of wind storms from 1920 to 1990 in order to place the events of 1987 and 1990 in their long term context. The researchers identified 47 storms, classed as severe, during this period. Each of these storms is described in the report and the maximum gust speeds and duration, as well as the size of area affected, are given. The report also has a map for each storn showing the areas where gust speeds exceeded 60 and 80 knots.

The Commeroial Union (CU) agreed to release this report to the Working Party and also supplied a sample of its own exposure and loss data from the 1987 and both the 1990 storms for analysis. The CO has recently made copies of the UEA report available to other interested parties and the authors hope to publish work related to this study in the near future.

The working Party also obtained data from the ABI Household Risks Statistics Panel on the January 1990 and October 1987 storms [1]. These reports were confidential to the contributing panel members and we are particularly grateful to the Panel, and to Brian Hudson the ABI chief statistician, for agreeing to release these to the working Party.

### 1.4 Report structure and contents

The report is in six sections, including this Introduction (Section 1). There are, in addition, four appendices containing background details.

Section 2 deals with weather systeas and the causes of storms and briefly reviews the historic occurences of storms and changes over time. Finally, the section deals with the theories for increases in the frequency and severity of these events and discusses the effects of global warming.

Section 3 reviews the damaging effects of storms and discusses the difficulties in comparing the cost of historic storms. The availability of (UK) storm damage data is considered and the approach ueed in translating the information to loss estimates is outlined.

Section 4 presents the results of the "analysis" of UK storms from 1920 to 1990. The seasonality of these storms, their frequency and severity distributions are considered. The increase in frequency over the last ten years of this period is reviewed in the context of the underlying annual frequency distribution. Finally, an index of storm risk damage, at posteode area level is estimated, with broad "storm risk" bands depicted on a map and the rating implications are discussed.

Section 5 describes how statistical models may be used to assess storm exposures and a simple model of UK Storm Losses is identified, and its parameters estimated, from the storm loss estimates of the previous section. A simple simulation model, based on the identified frequency and severity distributions is then described.

Section 6 reviews the results obtained from the simulation model and discusses how these may be used to assist the direct writer in establishing the element of the storm peril rate to be incorporated into the overall rate for Household Buildings. The reinsurance rating implications are then considered a chart of risk rates on line is presented for a range of frequency and severity assumptions. Finally, the effectiveness of current reinsurance programmes in smoothing catastrophe losses, is discussed and a possible alternative outlined.

### 1.4 Limitations of the Analysis

The report considers only storm risk and any values derived are for this peril only. Catastrophe reinsurance also covers other perils than just storm, such as freeze and flood losses. Although these perils are unlikely to significantly increase risk rates for the higher reinsurance layers, they will contribute to the lower layers of exposure. The simulation program can be adapted easily to incorporate additional perils.

The results contained in this paper are contingent on the validity of the many assumptions and simplifications that were made in order to derive them.

In particular, building regulations and construction standards differ across the country and differences in these will have some effect on the losses to be expected from a given level of wind speed. This potentially key factor has not been considered in this study and the percentage damage caused depended only on the estimated gust speed.

The types of housing, their size, density and general proximity to trees may be further relevant factors. The damage model was based on the 1990 and 1987 storms where the damage was caused mainly in southern England. Further distortions could arise from applying these damage factors, which are based on proportions of sums Insured, to areas of the North where average sums Insured are generally lower.

The translation of the storm gust speed maps into estimated speeds at postcode area level was fairly subjective and it may be possible to use actual wind speed data for the identified storms. This should be the first step to be undertaken in revisiting any of this work.

The ABI sample data were used to derive, and calibrate, the loss severity formula. With two data sets, and two parameters to estimate, a good fit to the aggregate losses was guaranteed. when tested against the only other samples available, the model overprojected losses by over $10 \%$. The reasons for this possible bias were not investigated due to lack of time. The reasons mentioned above may be relevant as well as differences in sums insured and claims procedures.

The organisations that have contributed information for this study, that is the UEA, the $C U$ and the ABI, have not seen any part of this report prior to its submission for publication and they have not been asked to approve the methods used in interpreting their data or to endorse any of the assumptions or results. The responsibility rests totally with the working party.

[^0]This section outlines what constitutes "weather" and how storns, particularly winter storms, arise, fistoric variations in weather, and storminess, over the last four centuries are then outlined before the key questions of increased storm activity and global warming are considered.

### 2.1.Wepather and the causes of storms.

Temperature.
Solar radiation reaching the earth neats the surface, which in turn heats the atmosphere from below. Overall, the earth is in energy balance, with energy received from the sun equating to energy used to heat the atmosphere or lost into space; if this were not so, discernable changes in overall average temperature would arise. However, within the earth's energy system, variations in temperature both on the surface and in the atmosphere clearly arise, and it is the uneven distribution of temperature in the earth's atwosphere which gives rise to many of the phenomena we refer to as "weather".

These variations in temperature result largely from the decrease in the angle of incidence of the sun's rays with increasing latitude, so that the energy received per unit area is less. Atwospheric temperature will thus be highest at the equator and lowest at the poles. There is of course also a measonal effect, depending on which hemisphere is tilted towards the sun.

Another important cause of temperature differences at the earth's surface is the disposition of land and sea, as continents and oceans respond differently to incoming solar radiation. The proportion of radiation reflected into the atmosphere varies with latitude, and this variation is greater over water than over land, with the greatest proportion reflected being in the bighest latitudes. Also, once water is warm it contains a great deal of heat energy and so takes longer to cool down. The result is that temperature variations are much smaller over the oceans than over land. The larger proportion of ocean in the southern hemisphere means that summers are cooler but winters milder there.

It can be shown that the maintenance of the earth's normal temperature distribution requires a transfer of heat from low latitudes to high latitudes, and up to $80 \%$ of this transfer is achieved by the circulation of the atmosphere.

The detail of this circulation is complicated, but essentially large scale convection currents carry warm air polewards and cold air towards the equator. At the same time, water vapour produced by evaporation in the tropics is carried polewards and condenses in colder regions releasing latent heat.

Ocean currents also make an important contribution to the transfer of heat from the tropics to the poles; in the northern hemisphere the principal curxents are the Gulf Stream and the North Atlantic Drift.
winds.
In general the earth's atmosphere rotates with the earth, but there exists much local variation, and the relative motion of the air is experienced as wind. This movement of the air results principally from variations in air temperature. The major wind belts are the Trade Winds, the Westerlies and the Polar Winds and arise mainly from the difference in heating between the equatorial regions and the poles and the large scale convection cells that are set up as a result. They are also influenced by the disposition of the continents and the differential heating of the atmosphere over land and sea.

The unequal heating of the earth's atmosphere gives rise to variations in air pressure. Naps showing the distribution of pressure over the earth's surface join places with equal pressure with lines known as isobars. As pressure is constant along an isobar it follows that the direction of most rapid pressure change is perpendicular to the isobars. The spacing of the isobars indicates the rate of change of pressure, known as the pressure gradient, and the closer the isobars, the more rapid the change so the stronger the winds.

Models for atmospheric_circulation.
For the earth the polar regions represent heat sinks and the equatorial zone a heat source. It might then be thought that a convection cell would be established in each hemisphere, with the air sinking at the poles, spreading across the earth's surface, rising at the equator and returning to the poles at altitude. However,
this model takes no account of the earth's rotation and so bears littl由 relation to reality.

The three main wind belts in each hemisphere can be explained by the existence of the "coriolis force" which results from the earth's rotation on its axis, and which causes motion of the air mass relative to the earth balow. For example, as air moves towards the equator, the earth's rotational speed is increasing and the ground beneath is moving faster than the air above, so that relative to the earth the air moves westwards. Similarly, air moving towards the poles will travel faster than the ground below and so move relatively eastwards. The result is the production of cyclones (with an area of high pressure in the centre) and anticyclones (with an area of low pressure in the centre). In the northern hemisphere, cyclones exhibit anticlockwise rotation and anticyclones clockwise rotation, and these directions are reversed in the southern hemisphere.

## Extratropical_(Ninter)Storms

The Shorter Oxford Dictionary defines a storm as a


Meteorologically, storms may be of various types depending on theix origin and the geographical regions in which they occur. However, since we shall confine our attention to northern Europe, and the British Isles in particular, the type of most interest is the "extratropical" or winter storm, as tropical cyclones do not arise in this region and the tornado risk is small.

Extratropical (inter) storms originate from substantial cold air masses moving from polar areas to more moderate climates. The difference in temperature between such polar air masses and the surface of the water in the more moderate zones is particularly great in autumn and winter since the air at the poles cools more rapidiy than the air and sea in lower latitudes. There can be a sharp change in temperature over a relatively short distance at a latitude of about 50-60 degrees.

This boundary between the heavy cold air and the lighter warm air is known as the "polar front" and is in constant motion as the cold air moves southwards, pushing the warm air northwards. An approximately circular eddy of the two air masses is produced, with an area of low barometric pressure in the middle. A system of this type may build up increasing momentum for several days if the imbalance of the air masses is maintained or increased by the influx of more cold and warm air masses. Strong air currents are generated around the core of the growing low pressure area, which may measure up to 3000 km in diameter.

The intensity of the air currents depends on the difference in barometric pressure between the sub-tropical high pressure area and the polar low pressure area. The low pressure eddies develop almost exclusively over the sea, where friction forces are low, and normally lose momentum rapidly as they travel over land. The wind velocity in winter storms may reach $200 \mathrm{~km} / \mathrm{h}$ at sea and along the coast. Wind damage starts to occur at about $60 \mathrm{~km} / \mathrm{h}$.

### 2.2 The measurement of storms and their severity

## History of measurement

Storm events have been observed and recorded for centuries, with records going back almost as far as written history. However, reliable instrument-based recordings are only available for relatively recent events, i.e. those occurring within the last two centuries or so since instrumental measurements of adequate quality and density have only been available since measuring networks were instituted in the 19 th century.

Assessments of historical storms rely on non-meteorological judgements as well as such instrument measures as may be available. Using a combination of meteorological measures and other historical evidence, Lamb has been able to take a view of the strength of major storms occurring since 1509 in his book "Historic Storms of the North Sea, British Isles and Northwest Europe".

## Meteorological measurement

Many of the measures used to assess the severity of storms are in fact measures of the damage caused rather than measures of the storm phenomenon per se, and these
measures of damage will be considered in more detail later. In order to make a meteorological assessment of a storm, the following factors are considered:

```
wind speed, direction and energy spectrum
temperature
atmospheric pressure
duration
area affected
```

Wind speed, direction and energy spectrull
In modern times, surface wind speeds have been capable of direct measurement using anemometers, but the earliest such readings date from the 1880 s . Prior to that date, approximate wind speeds may sometimes be deduced from recordings of barometric pressures, since the wind speed is related to the pressure gradient. However, since around 1700 winds have been categorised according to the appearance of the sea or the effect on shipping.

Apart from wind velocity, the direction of wind is also an important factor which may be quantified by suitable measurements. Substantial changes in the direction of wind or wind attacks from a direction adverse to the roots of a tree or foundations of a building may cause much greater damage than would normally be the case.

Due to the turbulent nature of the wind, the kinetic energy inherent in wind is spread out over a wide range of varying periods and frequencies. This spread or distribution is referred to as the energy spectrum of the wind and is crucial to the extent of damage suffered by buildings or components particularly susceptible to vibration.

## Temperature

Siting of the thermometers used was a difficult problem for early observers and the best consistent practices did not emerge until the late nineteenth century. Before that, the instruments were commonly positioned in unheated north rooms or on an open north wall, sheltered from rain and direct sun. A great variety of instrument scales were in use, and conversion of historical observations to Celsius can be difficult and imprecise.

## Barometric pressures

Barometer readings become increasingly reliable from the mid eighteenth century onwards, and some usable earlier
observations were made by leading scientists of the day and by the staff of the observatory of Paris. However, certain early instruments gave trouble as the fluid (usually mercury) stuck to the sides of the glass. The unit of measurement was generally the length of the mercury column, but this can be converted to millibars and corrected for the approximate height of the instrument above sea level and standard gravity.

## Duration

The duration of wind impact is an essential parameter for determining the loss potential of a storm, since damage often only results after a large number of wind attacks (so-called load cycles) causing fatigue and, eventually, destruction of the material.

## Area_affected

In assessing the overall severity of a storm, the size of the area affected is also of interest. The damage caused will also depend on whether the storm passes mostly over the sea, over open country, or over densely populated areas. Although storms lose intensity as they pass from over sea to over land, severe storms may cause widespread damage before dying down.

## Accuracy and consistency of metsourements

In attempting to analyse historical data, allowance therefore has to be made for the nature of the instruments used, the approximate nature of any corrections which may have to be applied to convert readings to modern scales, and the probable incompleteness of series of measurements. For more recent events, the data available are likely to be more plentiful and reliable. However, some degree of standardisation may still be appropriate to enable valid comparisons to be made. For example, the windstorm risk increases with height above the ground, so comparative data should be adjusted to standard elevation of 10 metres.

In order to assess local variations in windstorm severity, a great density of observations will be required and these will rarely be available in the right place at the right time.

We are naturally interested in the most extreme and severe events, but measuring instruments may be less reliable or fail completely under peak loads.

Lamb defines a severity index for assessing the relative strength of storms, which takes the form

$$
(V \max )^{\wedge}{ }^{3} A \max * D
$$

where $\nabla$ max is the greatest surface wind speed, $\lambda$ max is the greatest area affected by damaging winds and D is the overall duration of occurrence of damaging winds (or alternatively the duration in some place of particular interest). The cube of the wind speed is used as being proportional to the power of the wind and this relationship is generally found to hold with regard to the relative damage caused to properties.

This crude index is not appropriate for measuring relative property damage from storms as it takes no account of insurance density in the area affected or the range of wind speeds experienced over this area.

A damage equation based on this formula is developed and calibrated in Appendix 2 and then used to obtain likely loss estimates for the catalogued UK storms based on estimates of wind speed in the various postcode areas.
2. 3 Variations in weather since the sixteenth century

It is clear from a study of the major storms which have been recorded from the middle of the sixteenth century up to the present that there have been variations in storminess over this time. These more general variations in storm intensity seem to have occurred at various times over the past 400 years, and several particular periods or cycles may be distinguished:-

## The Little Ice Age

The so-called Little Ice Age is normally regarded as covering the sixteenth and seventeenth centuries, and was marked by generally cooler temperatures, cold winters and periods of significant storminess. The eighteenth and early nineteenth centuries may be considered a period of recovery after the previous two cold centuries, sharing some of their characteristics although possibly to a reduced extent.

It appears from Prof Lamb's storm analysis [9] that the Little Ice Age saw a greater intensity and frequency of storms, with impressive climaxes in the periods 1570 to 1620, 1690 to 1705 and 1790 to 1800. The increased intensity of storms in this colder period may be
attributed to the source of potential energy generated by the enhanced thermal gradient between the colder ocean surface in the seas about Iceland and the ocean south of $50-55 \mathrm{~N}$ and the Bay of Biscay.

## The recovery

The period from the eighteenth century to about 1950 was a further period of recovery when overall temperatures increased and storminess was at a generally lower level. The exceptions to this are mainly to be found towards the end of each century, and seem to be part of the pattern described below.

## 1950 to the present

Since 1950 there appears to have been an increase in the frequency of great storms and very deep low pressure centres. This apparent increase in storminess is supported by meteorological observations from various sources on and around the North Sea, and is sometimes alleged to be part of the greenhouse effect. As we shall see in our statistical analysis, the case is far from proven for the British Isles and there is a wide range of opinion among meteorologists themselves; it may simply be the "fin de siecle" phenomenon observed each century.

## cycles_in storminess

Prof Lamb's study of the last 400 years indicates an increase in storminess towards the end of each century the 1580 s and 1590s, the 1690 s , the 1790 s, the 1880 s and 1890s, and the 1980s (and 1990s?). It is thought that this roughly 100 -year cyclic variation in storminess superimposed on the major trend may be associated with a variation in solar activity. These solar variations are considered to be related to 200-year and 400 -year osciliations in atmospheric radiocarbon, which seem to be firmly established and are registered in dating records which go back 7000 years or more. It appears that there is also a cyclical variation of around 100 years' length in ocean surface temperatures world-wide and in their differences from air temperatures over land regions. The ocean temperature oscillation lags about 20 years behind the over land changes because of the inertia of large masses of water.

Weather forecasts are predictably unreliable. Yet predictions of significant long term increases in worla temperatures, and their cataclysmic consequences, are easily accepted despite the efforts of the scientists to explein the limitations of these predictions.

Weather predictions are based on models of varying sophistication limited only by the speed of the latest generation of super-computers. Recent research, and hardware developments, have led to more complex models of atmospheric circulation which take account of jet streams and other activity in the higher layers of the atmosphere as well as the coriolis forces.

Current weather models may be very complex and can consume immense computing power, but they are still quite limited in what they model. The atmosphere is modelled, for example, but not the biosphere or geosphere. Modelling the cloud cover, and its heat reflecting properties, is still to be incorporated, and the ocean layer is restricted to 50 metres. The models tend to be global and local implications, for the UK for instance, are inferenced from the giobal results. The results, especially long term predictions, are prone to uknown but potentially very large errors.

It is important to remain objective when considering theories for increases in storm frequency and severity. To quote from Prof Stephen Hawkings' "A Brief History of Time":
" a theory is just a model of the universe, or a restricted part of it, and a set of rules that relate quantities in the model to observations that we make. It exists only in our minds and does not have any other reality (whatever that may mean). A theory is a good theory if it satisfies two requirements: It must accurately describe a large class of observations on the basis of a model that contains only a few arbitrary elements, and it must make definite predictions about the results of future observations... Any physical theory is always provisional, in the sense that it is only a hypothesis: you can never prove it."

Returning to the theories of increases in storminess, to what extent have their predictions fallen in ine with the weather? How far can they predict the oscillations that make tracking temperature changes and other symptons of change so difficult? what corrections do they make for earth's own contribution, volcanoes for example?

## Cycles and oscillations

An earlier section outlined the evidence of cycles in the weather. If the only measurement of weather is over the period from trough to peak, a very misleading conclusion will energe. Even if there is indubitable evidence of an increase, say, in temperature, how are we to say whether it will carry on increasing or whether the peak is about to be reached? A number of approaches seem to be chartist or correlational in nature without an understanding of cause and effect.

We have also highlighted the limitations in trying to ascertain weather from its effect. What does "world temperature" actually mean? If a measurement in central London had been taken with the same equipment on the same site over five hundred years, it would undoubtediy have revealed an increase sinply because of the local warming effects of urban density. So, how reliable are these measures on a global scale? One researcher has suggested that they need weather stations every few metres.

## cloud cover

From the sun's perspective, its energies are either absorbed or reflected by the earth. If there is global warming, more water can be held as vapour, and thus, cloud cover could increase. As it does so, more heat would be deflected into space and a new equilibrium emerge. Work in this area depends on sattelites and is in its infancy. To the best of our knowledge these effects are not adequately incorporated into the theories or in the current genexation of weather models. It remaing a hypothesis as arafted here.

## Global Warming

The world is a giant greenhouse. Radiation from the sun is able to penetrate the atmosphere. Some bounces off the surface and a proportion escapes back into space. The amount which escapes compared to that which is trapped governs the global temperature. In relative terms the amount of energy released into the atmosphere through activity on the earth is very small compared to the radiated solar heat.

Certain gasses are thought to be better insulators than othere and these include carbon dioxide and fluorocarbons. Since the amount of these gases released into the atmosphere has been increasing, it is predicted that the greenhouse effect will become more pronounced and so the earth will heat up.

Without a greenhouse effect at all, life would not be possible as the Earth would be too cold. And without weather to distribute the sun's energy around the globe, the tropics would overheat and the temperate regions be worse than arctic. 80, ve need to live in a greenhouse with a weather gystem to have life as we knov it.

## Will Britain Become Kindier?

The Meteorological office was quoted by the Sunday Times on 10 November 1991 as predicting a 14 rise in temperature by 2025 but that storms driven to Britain across the Atiantic could weaken as a result. We have here to distinguish between what occurs in the tropics and our temperate or "extratropical" zone. Hurricanes or oyclones are expected to incresse in frequency and intenaity if the earth warms up beokuse they take their energy primarily fron the warm seas. South America might even start to experience them if warming offsets the effect of cold antarctic currents.

Extratropical storms, on the other hand, depend for their intensity on the difference in temperature between cold polar air masses and the hot tropical air. Current global varming theory predicts that the warming will be uneven with the poles soeing most change. Thus, the tamperature difference between the sir masses should lessen and Britain will be calmar, not windior. The lesser contributor towards intensity of our storms is the amount of water vapour in the air. This ia expected to increase but the net effect can clearly not be known although it might appear logical to expect the improvement in the most significant driver to "win".

Are Storms on the Move?
Another issue is not only whether storms are getting more frequent or intense but also whether they have moved. The 1987 and 1990 storms were of an intensity some 5 times more common over Scotland than over Southern England. The preferred track of Atlantic storms is between Iceland and Scotland-see Appendix 1; since damage is caused to the south of such tracks, it is usually scotland that bears the brunt. Hence trees are more regularly culled by nature and building regulations more stringent there.

Dr David Bennetts of the Met office did not regard the 1987 or 1990 storms as anything other than natural events. We are prone to short memories; even in the literature we must beware instant theorising.

In Munich Re's "Windstorm" [10], there is a 1 million year temperature chart (p 106) which illustrates how exceptionally warm the last 10,000 years have been. The volatility of temperature is graphically shown even if we might be somewhat circumspect in our views of putting a thermometer to a planet - under the tongue, in the armpit or where...? On page 7 of the same book is a history of major windstorm disasters. This, though, is restricted to the last 30 years and shows 8 in the 1960's, 13 in the $1970^{\prime} s$ and 29 in the $1980^{\prime} \mathrm{s}$.

We noted earlier that Prof Lamb has observed 20/25 year periods of usually stormy weather occurring towards the end of each century and associates this with a cyclical pattern of disturbance on the face of the sun. It is precisely over this period that the Munich Re's figures have been compiled; arguably they are simply not looking back far enough to find the real trends but are looking only at a segment of the cycle.

## Sea Levels

Rises in sea levels are an area of much concern and not only because of the insurance implications. If sea levels rise, as polar icecaps melt, then wore regions will become exposed to the combined effects of high water and wind. Since polar lcesheets hold about $2 \%$ of earth's water, the potential is frightening.

Yet even here, there is no unanimity. Dr Andrew Solow of the Woods Hole Oceanographic Institute in Massachusetts claims that global warming could actually increase snowfall at the North and South poles, thickening the ice rather than melting it.

## Is Global Waraing a Useful Theory?

How does the global warming theory measure up to Hawking's razor? At first glance, the . $5 \%$ increase in global temperature since 1900 accords with that predicted by the extra greenhouse gases pumped into the atmosphere. This rise is, though, within the earth'g natural variation. Further, we should beware a global temperature.

The theory predicts that temperatures in the polar regions will rise geveral times faster than the average. Yet it has been in the tropics that the most apectacular increases have been observed. If we have to be guarded about the essential tenet of the theory - its ability to model temparature - how much more sceptical do we need
to be about all the other predictions?
And we can quote from the UEA report [11] to the CU:


#### Abstract

"predictions of future elimate in a bigh greenhouse gas vorld are based on modelling experimente. The models are not yet able to reproduce the features of the atmospheric circulation with sufficient aceuracy for us to rely on their estimates of future conditions."


The_sun
All the energy we use comes from the sun - a postage stamp-sized piece could power 50060 -watt light bulbs. So, relatively small variations in the energy given off by the Sun could have a significant effect on the earth's temperature and may be linked to the late century storms mentioned earlier.

The theoretical background is based on the effect on the Earth's weather of variations in the sun's magnetic field; these variations are said to vary according to sun-spot activity. Models have been developed based on sun-spot activity, and whilst still to be accepted, are claimed by their developer to have superior predictive properties than more accepted models. What is clear is that we should look more widely at the causes of our weather and include in our studies the sun, volcanic activity and ocean currents, while not dismissing the greenhouse theory.

## Man Exacerbated Problem

Insurers' losses from windstorms over the past 30 years would appear to have increased faster than can be explained by inflation. The superficial response has been to claim this as evidence for deteriorating weather and, thereby, as support for climatic change. There are other contributory factors, however.

The following are taken from "Windstorm" [10]:

```
* increasing population density makes the effect
of any one storm greater in the aggregate than
was the case historically;
* increases in concentration of values as
standards of living rise;
```

* settlement of high risk zones eg coastal areas

```
in the tropics - hotels on palm-tringed
paradisiacal beaches;
* industrialisation in dangerous regions eg oil rigs in the Gulf of Mexico and the North Sea;
* there is incrased insurance coverage because greater affluence results in higher density and broadened policy wordings;
```

Most of the above observations only become insurance problems when the storn risk is not correctly assessed in the rating calculation. The concern should be to ensure the required anount of premium is charged. Premiums in more exposed areas need to be higher than those in less exposed areas. All of the above points explain exactly why the cost of storm losses should rise at a higher rate than just inflation.

The skewness of the storm loss distribution, compounded with expected increases resulting from the factors above, will result in some very large increases in the absolute cost of storm damage from time to time.

It is clear that if we aliow global varging to take the blane for the apparant deterioration in atorm losede we will loge the opportsinity to teckle very real iesues vhich are much closer to being within our eontrol.

### 3.1 The damaging effects of storms

It has already been noted that storms are often measured by the degree of damage caused rather than by meteorological quantities. This is perhaps not surprising since storms would be of little lasting significance if it were not for their, possibly, devastating effects. Aspects of damage which may be taken into account include the following:-

```
destruction to property
changes to the coastline
changes to the landscape
damage to trees
loss of human life (on land)
shipping and lives lost at sea
insurance losses
```

Wind acts on its surroundings not only by way of pressure and suction forces, but also through particles or objects carried with the wind - rain, snow, hall, sand and water spray, or even branches torn off trees and pieces of roofs and masonry. In addition, strong winds or windstorm may increase the risk of fires breaking out and spreading.

The most dangerous side-effect of storms is their influence on bodies of water, building up large waves which break against the shore or banks of a lake. This makes the level of water rise - storm surge - and the resulting waves and strong currents may cause severe damage as a result of flooding and erosion.

In the case of historical events, our knowledge will rely at least as much on accounts of the effects of the storm and the damage caused as on such meteorological observations as may be available. However, it may be difficult to make objective comparisons of the effects of historical storm events and to assess their equivalent in modern terms, since

> - historical accounts will be subjective and possibly prone to exaggeration

- the information available may be far from complete
-ifestyles, and in particular the ownership of high value capital goods, have changed significantly over time
- damage to property depends on the construction and strength of physical structures, which may change over time
- population densities change over time, and it is likely that there has been a trend towards development in more exposed areas


## Factors affecting the deqree of danage caused

The degree of damage caused will depend not only on the meteorological severity of the storm but also on the susceptibility of the objects or structures with which it comes in contact.

The great storms of 1987 and 1990 highlighted the susceptibility of trees to storm damage, and uprooted trees or broken branches which then damaged buildings and vehicles.

The standard of construction of buildings will determine the degree of damage sustained, and trends towards larger, lighter buildings may result in higher levels of damage because vibrations and resonance are more readily set up. In general, buildings are constructed to withstand the wind strengths expected to occur in the locality.

New buildings must be constructed in accordance with the local Building Regulations, which differ from place to place. Regulations generally require properties to be capable of withstanding the maximum wind speed to be expected on average once in 50 years as a mean 10 minute value and a peak gust.

It may be argued that if Building Regulations were more rigorous and more uniform across the country, this would reduce the level of damage caused by severe storms. However, there are a number of practical difficulties associated with establishing and maintaining such regulations:-

- sufficient historical meteorological data is often not available;
- even 50 - year values may still present an excessive residual risk;
- very complex regulations are difficult to interpret and appiy, and may not therefore be fully complied with;
- regulations will be suitable for buildings of relatively simple designs and standard shapes, but may be less appropriate for more unusual structures;
- it may take many years for new research tindings to be incorporated in the regulations;
- regulations do not make sufficient allowance for interaction effects with adjacent buildings;
- ultimately, there is no substitute for practical experience;

The damage likely to be caused by storm-driven coastal flooding can be effectively limited by adequate coastal defences such as the East Anglian walls and the Thames Barrier.

## 3,2 The UK storm data and the UEA reoprt

The storm of october 1987 (CAT 87J) was a severe blow to weather forecasters, householders, insurers and, uitimately, their reinsurers. As outiined in Section 1 these lossess were analysed in some detail by insurers, especially on domestic buildings policies where postcode level information was more readily available. There is however little evidence that any usable conclusions on the storm risk at district level could be drawn from the analysis and reflected in rates.

Overall buildings rating levels were being increased at this time, in any case, and postcode rating implemented to reflect, primarily, the risk of subsidence which had a more easily identifiable cause.

For any significant progress to be made a much longer history of storms, and their likely insurance losses, was clearly requiret. A very large amount of meteorological data is collected by a numerous weather stations across this country and elsewhere and is available on monthly basis. This data includes wind and gust speeds at each of these weather station locations. Although it is possible to derive relative storm risk bands from such data, and such maps exist and are used, for example, in local building regulations, until recently the insurance loss data necessary to complete this process for insurance rating purposes has simply not been available.

This position was however to change during 1991 partly as a direct result of work commissioned specifically to gain a better understanding of these events. This year also from the publication of a substantial book by prof. H.H. Lamb on the "Historic Storms of the North Sea, British Isles and Nortwest Europe" [9]. This very comprehensive stydy, mentioned earlier, identified storms over at least five centuries. The book includes descriptions and weather maps (pressure isobars) for a large number of these storms. It is possible, for specialists, to estimate wind speeds fron the density of these isobars.

The UEA study [11], commissioned by Commercial Union, identified a number of other storms that could be classed as severe but which were not included in the Lamb catalogue.

These missing storms affected mainly Northern England and Scotland and were included in the UEA report. The UEA report comprised 47 UK storms during the 71 year period of the study from 1920 to 1990. All these storns had measurable land areas thought to have experienced gust speeds in excess of 60 knots ( $110 \mathrm{ks} / \mathrm{hr}$ ), which is taken as sufficient to result in significant damage to property. Each storm is described and a map of the UK is drawn for each storm identifying the areas where gust speeds are thought to have exceeded 60 and 80 knots. Some extracts trom this report, including sample maps, are to be found in Appendix 1.

## 3.3 optimising the use of the available data

The first task was to establish whether the UEA maps for the 1987 and 1990 storms gave an acceptable representation of the actual damage sufferred in these storms. If some relationship can be derived to fit this data then an attempt can be made to "translate" all the UEA maps to insurance losses by applying this relationship to estimated gust speed and storm duration. (A detailed description of how this was done can be found in Appendix 2.)

A successtul outcome of this approach could yield estimates of "storm losses" for each of these nistoric storms, based on current exposures, for any given portfolio of household buildings policies and produce storm loss information down to postcode area (i.e. first 2 letters) level. (See Section 4)

The postcode information, if considered reasonable, could then be reviewed to see if any conclusions on relative storm exposure could be drawn. This would be a highly speculative approach but worth investigation. (See Section 4.3)

The UEA catalogue of storms can also be used to investigate the frequency of UK storms over the last seventy years and to consider any changes in the underlying frequency of these events. Loss estimates for all the catalogued storms could be used to examine the loss size alstribution for storn damage. (See Section 5)

The frequency and severity distributions could then be modelled and computer simulation used to investigate the implications, for insurers and reinsurers, of storm danage. Some results based on this approach are presented in Section 6.

### 4.1 Estimated UK Storm Losses 1920-1990

The UEA maps were converted to estimated gust speeds for each postcode area and a formula was then applied to estimate the damage caused. The details of the derivation of this formula, and its calibration, are described in Appendix 2. Exposures by postcode (SI) are then used to obtain a loss estimate for each postcode area from each storm.

The loss estimates for each of these storms, expressed in pence per $£ 1000$ of Sum Insured, are shown in the table below, based on the 1990 ABI sample exposure mix. The cost estimates will vary with the geographical mix of exposures and care is required in applying these results to a portfolio with a different exposure profile.

TABLE 4.1.1 : STORM CHARACTERISTICS AND LOSS ESTIMATES

|  | BFEbs |  | Fratis |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \$ | 26-Feb-90 | 8 | 111024 | 24.8 | \% ${ }^{3}$ | 29-Jul-56 | 1929606 | 12.8 |
| \% | 25-Jan-90 | 14 | 105267 | 112.2 | 25 | 21-Dec-54 | 3104445 | 4.1 |
| 3 | 16-Dec-89 | 13 | 8635 | 3.5 | \%\% | 26-Nov-54 | 566035 | 8.6 |
| \% | 13-Feb-89 | 9 | 83474 | 7.1 | 2 2\% | 03-Mar-53 | 21115547 | 18.4 |
| \$ | 09-Oct-88 | 31 | 63736 | 14.5 | \% | 17-Dec-52 | 17100333 | 16.1 |
| \% | 16-Oct-87 | 8 | 39064 | 100.9 | 30 | 30-Dac-51 | 770726 | 4.0 |
| \% | 24-Mar-86 | 12 | 35363 | 17.8 | 3 | 09-Oct-49 | 313158 | 1.4 |
| $\stackrel{*}{*}$ | 13-Jan-84 | 3 | 97454 | 7.4 | 4\% | 16-Mar-47 | 8106912 | 31.6 |
| \% | 01-Feb-83 | 21 | 49344 | 16.8 | 83\% | 18-Jan-45 | 1139886 | 7.8 |
| 1. | 23-Nov-81 | 12 | 11102 | 0.1 | 4 | 07-Apr-43 | 11181750 | 42.9 |
| M | 04-Dec-79 | 8 | 5345 | 0.5 | \%\% | 23-Nov-38 | 12121304 | 52.2 |
| $1 \%$ | 11-Dec-78 | 6 | 27205 | 3.1 | 35 | 04-Oct-38 | 448933 | 7.6 |
| 8\% | 02-Jen-76 | 5 | 210123 | 33.5 | 3\%. | 16-Jan-37 | 72467 | 0.1 |
| \% | 27-J.an-74 | 15 | 18093 | 4.7 | 3:3 | 26-Oct-36 | 11122126 | 18.8 |
| Ss | 12-Jan-74 | 11 | 123360 | 12.9 | \%\% | 18-Oct-35 | 15102388 | 24.1 |
| \% | 02-Mar-73 | 41 | 10691 | 5.9 | 40 | 16-Sop-35 | 1150166 | 25.1 |
| 1\% | 12-Nov-72 | 9 | 20149 | 4.1 | 4 | 05-Dec-29 | $20 \quad 18000$ | 7.6 |
| 18. | 14-Jan-68 | 15 | 31662 | 48.1 | 4 | 23-Nov-28 | 18117603 | 33.3 |
| 14 | 06-Mar-67 | 12 | 47288 | 1.7 | 4. | 16-Nov-28 | 951400 | 11.7 |
| 20 | 16-May-62 | 9 | 45232 | 1.9 | $4 \%$ | 06-Jan-28 | 71960 | 10.0 |
| 21 | 16-Feb-62 | 16 | 107734 | 14.2 | 45 | 28-Oct-27 | 1370315 | 20.0 |
| 23 | 11-Jan-62 | 16 | 115136 | 29.8 | 46 | 28-Jan-27 | 1185941 | 9.2 |
| 2 | 16-Sep-61 | 20 | 75661 | 6.4 | $4 \%$ | 26-Jan-20 | 951991 | 1.0 |
| 24 | 04-Nov-57 | 4 | 39064 | 10.1 |  |  |  |  |

The average cost of these storms is 18.1 p per 81000 of SI. The annual cost, over the 71 year period covered, is 12.0 p per $£ 1000$ of $S I$.

These estimates are likely to be very approximate as some very broad assumptions have had to be made to derive them. They identify, much as expected, the January 1990 storm (Cat 90 A ) as the most severe, with the October 1987 storm (Cat 87J) as the second most costly storm of the last seventy years.

Two gtorms, 10 and 37 in the catalogue, are estimated to have cost relatively little and this is also clear from their gust apeed maps which show only a small, remote, area affected by gusts in excess of 60 knots. For the purposes of the subseruent analysis we will remove these two torgis from our litt. The average cost of the remaining 45 storms is then $18,9 p$ per E1000 of SI. The annual cost is unaffected by this change.

### 4.2 Seasonality of UK storm

Chart 4.2.1, overleaf, shows the frequency by month of the 45 storms in the revised list and identifies January as the "stormiest" month with 11 storms during the period. The months of october and December each had seven storms, November had six and Narch five. As expected the chart shows the higher incidence of storms during the first and fourth quarters, with 20 storms in each, and a relatively "calm" period during the intervening six months.

This chart does not allow for differences in seasonal severity of storms. Chart 4.2 .2 is based on the storm cost estimates given earlier, with the cost of each storm expressed as a percentage of the overall (71 year) cost. Each bar consists of the sum of these percentages for the storms that occured during the month and each storm may be identified by a different hatch pattern. The January 1990 storm is the first storm in the January bar and the October 87 storm is the second storm in the october bar.

The charts show that the month of January has been the stormiest, and costliest, month of the period from 1920 to 1990 included in the UEA study even after excluding the costliest storm of this period, the January 1990 storm.



## 4. 3 Postcode aret analysie gnd spatial map

Table 4.3.1 below shows the estimated relative storm losses, for each postcode area, over the period 1920-1990. Two valuea are shown against each postcode. The second value (ALL) is the crude estimate calculated by adding all the loss estimates for the postcode before normalising. The first column (ADJ) is adjusted by removing half the estimated losses for both the October 1987 and January 1990 storms.

TABLE 4.3.1 : STORM RISK INDEX BY POSTCODE AREA

| $\begin{aligned} & \text { PCOST } \\ & \text { CODE } \\ & \hline \end{aligned}$ | INDEX |  | $\begin{aligned} & \text { POST } \\ & \text { CODE } \end{aligned}$ | INDEX |  | $\left[\begin{array}{l} \text { POST } \\ \text { CODE } \end{array}\right.$ | INDEX |  | POST | INDEX |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | ADJ | ALL |  | ADI | ALl |  | ADJ | ALL |  | ADi | Als |
| G | 278 | 241 | OL | 142 | 123 | NR | 87 | 88 | NW | 65 | 76 |
| KY | 278 | 241 | $L$ | 140 | 122 | DT | 86 | 89 | PE | 62 | 59 |
| ML | 275 | 239 | CT | 137 | 175 | DA | 83 | 99 | SN | 62 | 61 |
| TD | 269 | 234 | BN | 137 | 177 | CM | 83 | 95 | RG | 61 | 65 |
| FK | 265 | 230 | cw | 135 | 121 | BH | 83 | 91 | TF | 61 | 53 |
| EH | 262 | 228 | TS | 131 | 114 | BS | 81 | 75 | WD | 59 | 65 |
| KA | 250 | 217 | B8 | 130 | 113 | P | 80 | 87 | NN | 58 | 57 |
| PH | 195 | 169 | LS | 129 | 114 | CB | 80 | 81 | EC | 58 | 70 |
| Iv | 183 | 159 | DH | 129 | 112 | RH | 79 | 97 | wc | 58 | 70 |
| DD | 182 | 158 | CH | 125 | 109 | 日R | 79 | 93 | LE | 57 | 55 |
| DC | 179 | 156 | BL | 125 | 108 | Lb | 77 | 70 | KT | 56 | 68 |
| PL | 179 | 179 | TN | 121 | 161 | LU | 76 | 77 | B | 55 | 49 |
| wN | 168 | 159 | SR | 119 | 103 | MX | 76 | 75 | SE | 55 | 68 |
| WA | 166 | 148 | Yo | 119 | 105 | RM | 75 | 89 | He | 55 | 63 |
| DL | 166 | 144 | Br | 116 | 101 | so | 74 | 82 | OX | 54 | 54 |
| PA | 165 | 144 | AB | 116 | 101 | IG | 74 | 89 | H2P. | 54 | 58 |
| TR | 165 | 179 | LN | 114 | 103 | 10 | 73 | 93 | UB | 54 | 64 |
| LA | 161 | 140 | L. | 113 | 100 | SG | 73 | 75 | EN. | 54 | 63 |
| s | 160 | 143 | EX | 111 | 111 | SM | 72 | 85 | sw | 54 | 66 |
| CA | 157 | 136 | Hu | 109 | 98 | SP | 72 | 74 | w | 53 | 66 |
| NE | 156 | 135 | DN | 109 | 98 | Al. | 71 | 76 | cv | 51 | 48 |
| M | 151 | 132 | EW | 109 | 95 | CR | 71 | 86 | HR | 51 | 46 |
| TQ | 151 | 160 | SA | 103 | 96 | DE | 70 | 63 | SL. | 51 | 59 |
| HD | 150 | 134 | ss | 102 | 118 | BA | 70 | 68 | DY | 50 | 44 |
| SR | 149 | 131 | Co | 102 | 111 | GU | 69 | 86 | TW | 49 | 58 |
| FY | 148 | 133 | TA | 98 | 96 | SY | 68 | 60 | Ws | 48 | 43 |
| WF | 146 | 130 | PO | 97 | 119 | ST | 67 | 60 | WR | 47 | 42 |
| HX | 146 | 131 | CF | 97 | 88 | E | 66 | 77 | WV | 44 | 39 |
| BD | 145 | 128 | ME | 96 | 120 | NP | 65 | 59 | G. | 41 | 39 |
| PR | 144 | 129 | NG | 87 | 77 | N | 65 | 76 |  |  |  |
| HH | 143 | 126 |  |  |  |  |  |  | ALL | 100 | 100 |

The index values are based on some fairly broad assumptions and need careful interpretation. The following map shows an approximate spatial distribution based on the adjusted (for the 19875 and 1990A storms) index values. The four bands used are: 0-70, 71-130, 131-180 and over 180. A map, with the index values shown for most postcode areas, can be found in Appendix 3.


### 4.4.Rating implications at postcode area level

Table 4.3.1 shows a very wide variation in estimated storm experience across the UK with the lowest, adjusted, risk index at around $40 \%$ of the average risk index and the highest at just under $280 \%$ of the average level. Assuming that these index values are accepted, the risk differences indicated would, under normal circumstances, lead to differences in rates.

The map shows well defined areas of high and low risk from storm and these areas correspond, reasonably well, with expectations and the general wind factors derived from meteorological data. The main difference is that the figures derived above indicate the storm risk relativities by district in terms of possible insurance losses per unit of sum insured, all other factors being equal, and may thus be of some use to insurers. However care is required before these relativities can be reflected in postcode area rating.

The estimated annual cost of these storms is around 12p per $£ 1000$ of SI and would thus roughly account for $10 \%$ of annual losses on a typical uK domestic buildings portfolio. Using the adjusted index values tabulated above, and assuming these can be applied in the rate calculation, they indicate a storm risk rate range from around 5 p to over 30 p per $£ 1000$ of SI. Even if these values could be adjusted to allow for other factors, such as construction, size, type and so on, the index differences are unlikely to become insignificant and would, probably, still warrant some recognition in the rating to reflect the storm element.

Assuming, for the sake of illustration, that the storm component in the current rate is 20 p , or $10 \%$ of, say, a £2 per $£ 1000$ SI gross rate, then adjusting this using the extreme values above will result in a range of rates from around $£ 1,90 \mathrm{p}$ to over $£ 2.30 \mathrm{p}$, representing reductions of $5 \%$ and increases, in the extreme, of over 15\%.

It is unlikely that, in the short term, the established market will adjust its rating of this business to account for any such differences in the storm risk. A new insurer operating directly, however, and more able to target its market may perceive this as a marketing opportunity.

### 5.1 Statistical model泉 and their uses

Statistical models are an attempt to translate a real process from its initial context into a simplified statistical aepiction in order to gain a better understanding of the real, and much more complex, process.

The frequency and severity of US hurricanes has led to fairly complex models of hurricane damage being developed. These nodels can be very flexible and are extensively used, in both deterministic and a stochastic modes, to estimate likely insurance losses before they occur, whilst they are occurring and after they have occurred. They estimate losses at a particular location from either a projected storm path, in their stochastic mode, or an actual storm path when deterministic. The damage model may take account of actual policy details such as size, and type of construction. They are also used to measure and control exposures and to assist in rating.

The interested reader should review the paper by K.M. Clarke [2] and the extensive description in D.G Friedman [4]. The development of such models reguires a significant amount of insurance and weather data, as well as effort and cost. Needless to say there is no attempt to develop such a model for uK storms in this paper. At least two European reinsurers are believed to be developing models of this (US) type, using wind speed data collected by the weather stations dotted across Europe.

A much simpler model, based on estimating aggregate losses fros a frequency and a severity distribution, will be identified and then used to simulate outcomes of storm damage for further analysis. Simple models of this type estimate losses for a (market) portfolio rather than estimating losses at policy level which are then aggregated. With care, different portfolios can be evaluated by selecting appropriate model parameters.

A first step in determining how the parameters of a particular portfolio may differ from the assumed market mix will be a comparison of the portiolio storm exposure profile relative to this market mix. This can be estimated by calculating the portfolio weighted storm index value, using a set of storm index values for each postcode area, such as one of the sets shown in

Table 4.3.1, and comparing this with the assumed market value of 100. Values above 100 would indicate a portfolio likely to suffer relatively higher storm losses, per $£ 1000$ SI, as this portfolio appears to have a bigger proportion of its exposure in the higher storm risk areas.

These simple models are very easy to develop, require only a handful of parameters and a pocket calculator with a random number key. This method requires patience, however, and a PC spreadsheet package makes it much more acceptable and time efficient.

The first task then is to identify an appropriate trequency distribution for UK storms.

### 5.2. The frequency distribution of UK stoxms

The frequency distribution is usually represented by one of the standard, discrete, statistical distributions. The binomial, the negative binomial or poisson are the obvious choices. Alternatively unit random numbers can be used to select the number of events from any made-up distribution by simply using the cumulative probability values. The frequency analysis is usually fairly straightforward and the results relatively insensitive to the choice of model but can be very sensttive to the choice of model parameters.

The following table shows the annual frequency distribution of the 45 storms identified during the 71 year period 1920-1999.

Table 5.2.1 Annual frequency of UK Storms 1920-1990

| No | freq | Years |  |
| :---: | :---: | :---: | :---: |
| 0 | 37 |  |  |
| 1 | 25 |  |  |
| 2 | 7 | $27,35,38,54,74,89,90$ |  |
| 3 | 2 | 28,62 |  |
| 4 | 0 |  |  |

The sample mean and variance of this frequency distribution are . 634 and .607 respectively.

Both the poisson and binomial distributions can approximate this frequency distribution reasonably well.

As the mean of the sample exceeds the variance the binomial distribution would be the usual first choice. Fitting the binomial requires the estimation of two parameters, $N$ and $p$, where $N$ is the number of trials and $p$ is the probability of a success from each of the $N$ independent trials. The maximum sample frequency is 3 and this would be the lowest choice for N. care is needed in choosing a suitable value for $N$ as it has an obvious interpretation as the theoretical maximum number of storms during a year. various combinations of estimates of $N$ and $p$ can be tried subject to the condition that $N \times p=.634$, the sample mean.

The poisson distribution requires only a single parameter and this may be estimated by the sample mean of .634. The poisson has the advantage that it carries no implicit maximum bound for the number of storms in any one year.

The table below shows the observed frequencies against a range of fitted frequencies based both on the poisson and binomial distributions. In the case of the binomial various values of N are used.

Table 5.2.2: Annual Frequency Comparison


For the purposes of this investigation any of the above distributions can be chosen without loss of generality.

Selecting the binomial distribution requires a choice for $N$ and as mentioned earlier this implicitly defines a theoretical maximum number of storms in a year. The poisson distribution does not have a theoretical upper bound and to avoid unnecessary complications has chosen to represent the annual frequency of UK storms.

## Investigating changes in frequency

The association of a theoretical distribution to the annual frequency of storms can be used to explore changes in the frequency of these events and consider their statistical significance.

Using the poisson distribution, and assuming independence, the distribution of storms in a thirty year period will also be poisson with mean thirty times the annual mean, or 19 events. The theoretical standard deviation for this period is then 4.4 events.

There were 16 events during the period from 1931 to 1960 and 22 during the period from 1961 to 1990. Both of these are within one standard deviation of the expected value.

The same argument applied to ten year period, yields an expected 6.3 events with a standard deviaton of 2.5 . The nine storms identified during the last ten years studied, from 1981 to 1990, is a little over one standard deviation above the mean, hardly proof of a significant increase in frequency.

This statistical representation of the annual storm frequency implies that significant increases and decreases in frequency will occur from time to tine especially when relatively short periods, say of five to ten years are considered. This is simply a consequence of the high coefficients of variation which are $40 \%$ for the ten year period and 56\% for the five year period.

Looking at the seven, 10-year periods covered by the study, the statistical model has a mean of 6.34 events with a standard deviation of 2.52. The range for one standard deviation either side of the mean is then 3.82 to 8.86. The actual observed frequencies were:

| Yr | $81-90$ | $71-80$ | $61-70$ | $51-60$ | $41-50$ | $31-40$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No | 9 | 7 | 6 | 7 | 4 | 5 |

The above discussion should suffice to demonstrate that statistically, at least, the case for increases in the frequency of storms is far from proven. More importantly either the poisson or binomial representations provide a reasonable fit to the annual number of UK storms during the period from 1920 to 1990.

Identifying an appropriate loss size distribution is much more subjective and difficult. The usual choice, assuming a suitable theoretical distribution can be found, is between the exponential, the log-normal, the Weibull or the pareto distributions. The pareto is often preferred (by reinsurers) as it has a heavier tail and so tends to be more conservative, or safer, in estimating the impact of large losses. This is particularly relevant in the evaluation of the higher reinsurance layers.

It is worth recaliing, before any results are derived, that the storm loss figures to be used are themselves estimates and the sample size is small. The sample values for the 45 events range from .5 to 112 and have a mean of 18.9 and a standard deviation of 22.7. Two of these losses, the storms of October 87 and January 90 , are at or above 100 on this scale and the next highest loss is estimated at only 52.2p per $£ 1000$ SI.

Both the log-normal and Weibull distributions fit the sample data well enough to be acceptable models, at the usual confidence levels, using the chi-square test.

The log-normal distribution, fitted to the 45 storm loss estimates shown in Table 4.1.1, has a mean of 20.7 and a standard deviation of 34.7 . The probability of exceeding 100p per $£ 1000$ SI is $2.6 \%$. The same probability for the Weibull distribution fitted to this data is $0.7 \%$ which is clearly too low compared to the sample value of 4.4\% or 2 events out of 45 .

For reinsurance considerations, and more particularly for estimating the risk rates for higher levels of reinsurance, choosing either of these distributions may be considered unsatisfactory, especially by reinsurers, as their tails are "light" compared to the preferred distribution for such purposes which is the Pareto.

The Pareto distribution does not fit the data particularly well across the complete range of losses. The Mean Residual Life (MRL) plot is sometimes used to identify a likely loss distribution.

The MRL function of $X, \operatorname{MRL}(X)$, is defined by:

$$
\operatorname{MRL}(X)=\text { Average (ail values exceeding } x \text { ) - } x
$$

It can be shown that in the case of the Pareto distribution the MRH function is linear. More details of the use of this function can be found on page 108 of Hogg \& Gluckman [8].

Chart 5.3.1 below shows the MRL function for the UK storm loss estimates. The linear trend line is fitted to the losses over 14 p and indicates that a pareto distribution may provide a reasonable fit to losses in excess of this amount. The mean residual values of losses below this value do not easily fit around this linear trend.


The observations above were used to it a composite distribution to these losses with a Pareto tail. This distribution was obtained empirically by successively eliminating the lower cost claims until the remaining claims matched claims from a Pareto distribution.

For example taking claims in excess of 10p resulted in an estimate of the Pareto parameter of 1.1071. This was then tested against the sample of 24 losses, exceeding this limit, by comparing the number of events that are expected to exceed a given size of loss. The process was repeated with various other starting values and results compared with the actual sample values. The distribution was truncated at 500p per $£ 1000$ SI.

The following table shows a range of values obtained in this way using start values of 10p and 14 p and varying the Pareto parameters.

Table 5.3.2 Comparison of actual and estimated number

| Pareto Pareto | Scale <br> Shape | $\begin{array}{r} 10 \\ 1.11 \end{array}$ | $\begin{array}{r} 10 \\ 1.15 \end{array}$ | $\begin{array}{r} 10 \\ 1.05 \end{array}$ | $\begin{array}{r} 14 \\ 1.4 \end{array}$ | $\begin{array}{r} 14 \\ 1.26 \end{array}$ | 14 1.15 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Value | Actual | <----Estimated no exceeding value ----> |  |  |  |  |  |
| 10 | 24 | 24.00 | 24.00 | 24.00 | n/a | n/a | n/a |
| 14 | 20 | 16.42 | 16.21 | 16.74 | 20.00 | 20.00 | 20.00 |
| 20 | 13 | 10.95 | 10.67 | 11.38 | 12.09 | 12.68 | 13.16 |
| 25 | 10 | 8.48 | 8.19 | 8.92 | 8.81 | 9.52 | 10.10 |
| 30 | 8 | 6.87 | 6.59 | 7.30 | 6.79 | 7.52 | 8.13 |
| 40 | 5 | 4.90 | 4.66 | 5.29 | 4.50 | 5.16 | 5.75 |
| 50 | 3 | 3.76 | 3.54 | 4.10 | 3.25 | 3.84 | 4.37 |
| 75 | 2 | 2.28 | 2.12 | 2.54 | 1.79 | 2.22 | 2.62 |
| 100 | 2 | 1.57 | 1.45 | 1.77 | 1.15 | 1.47 | 1.79 |
| 200 | . 00 | . 56 | . 50 | . 65 | . 35 | . 49 | . 62 |

This practical approach was adopted primarily due to lack of time and also as it illustrates more clearly what the underlying assumptions mean in terms of loss likelihood.

Looking at the results, tabulated above, an acceptable fit is obtained by selecting the lower limit at 14p. The pareto shape parameter estimate based on the sample of 20 losses that exceed this new limit is 1.401 .

The Pareto shape parameter was varied to minimise the chi-squared values calculated from the actual and expected values shown in a table similar to Table 5.3.1 above. This process iaentified a shape parameter of 1.26. This choice fits the actual data well and estimates 1.78 losses in excess of 100 p , during the 71 year period, compared to the 2 in the sample. This set of assumptions also indicates a loss in excess of 200 p every 115 years - based on .62 losses in the 71 year base used for the table.

The losses below 14 p have an average of 6.2 p , that is just below the range mid-value. The distribution looks reasonably uniform in this range and, as the impact on results from a more accurate empirical fit is minimal, a uniform rectangular loss distribution is assumed for these losses.

## 5.4: A. Model of UK Storm Losses 1920-1990

The storm loss size distribution for UK Household Buildings policies, is then selected to be a composite distibution with a uniform first segment up to a loss of 14 p per $£ 1000$ SI. The second segment, above 14 p , is a truncated Pareto alstribution with a scale parameter of 14 and shape parameter 2.15 and an upper limit of 500p.

The select frequency distribution is, as before, a poisson with parameter 0.634 and the relative frequencies of claims in the two loss size segments are 0.5556 (25/45) in the lower band and $0.4444(20 / 45)$ in the higher band.

There are clearly limitations to any results derived from the above model. Perhaps the most serious is the assumption of independence of events. The historic analysis contains periods of particularly stormy conditions. None of the UK storms identified for the period from 1920 to 1990 are within the usual 72 hour limit that separates catastrophe events for reinsurance purposes. This is not conclusive, however, and such considerations will be an area of argument, and counter argument, especially between insurers and their reinsurers.

Results derived from this distribution are described in Section 6 and the impact of varying some of the parameters, and the loss size distribution, are discussed.

### 5.5 The increage in Storm losses. An explanation?

Chart 5.5.1 below shows the estimated storm losses for UK household policies from 1920 to 1990 and shows the significant increase in these losses at the end of this period.


Chart 5.5.2 shows the US Catastrophe Losses for the period from 1961 to 1989, expressed as a percentage of Property Premiums. The figures are extracted from the recent report by Conning \& Company [3].


These two charts exhibit some similarities. The high losses at during the 1987 to 1990 period in the first, UR, chart are matched by the events of 1964 and 1965 on the uS chart, which includes hurricane Betsy in 1965. Hurricane Hugo contributes to the high 1989 losses. We expect 1992 to at least match the 1989 level following the recent losses from hurricane Andrew.

This concentration of extreme losses to short periods, as experienced in the UK for the 1987 to 1990 period, is clearly not a new, or unique, feature as the US data clearly demonstrate. Can the events of 1987 and 1990 be considered sufficient evidence of a significant increase in the frequency and/or severity of UK storms?

The pattern of lostes shown in both the charts above is more eanily explained by the stmple statietical medel described in section 5.4 and ariaes, primarily, from the ekewnegs of the lose sise diletribution and the relative infrequency of these eventa.

Chart 5.5 .3 below has been plotted from a 71 year period simulation using the "select" set of assumptions by the prograill described in Appendix 4.


The reader is invited to study the chart above and compare this with the two earlier charts before forming an opinion on this key issue.

## Introduction

The previous section described an approach for selecting appropriate frequency and severity distributions for UK storms, and identified a set of parameters based on the estimated UK storm losses for the period from 1920 to 1990. The two distributions are combined in a simulation program which is described in Appendix 4. The rest of this section reviews the results obtained by using versions of this program and discusses some of the implications for both direct insurers and reinsurers.

Three sets of assumptions are chosen to indicate a likely range of values. Firstly the two distributions identified, and fitted, in the previous section are chosen as the "select" assumptions as they provide a reasonable fit to the actual data. Secondly the log-normal distribution, with parameters estimated from the data, is used to represent a less severe loss distribution, whilst retaining the "select" poisson basis. Finally the frequency of storms over the period from 1981 to 1990, together with the conditional probability of a large loss (i.e. >14p) also derived from the storms of this period, are used to define a very severe set of frequency (and severity) assumptions.

To recall, the "select" assumptions are then a poisson frequency of .634 (45/71), with a composite loss severity distribution with a uniform first segment up to a loss of 14 p . The second segment is a Pareto with a scale parameter of 14 and a shape parameter of 1.26 , truncated at 500p. The conditional probability of a loss below 14p is . 556 or $25 / 45$.

The more extreme distribution has a poisson frequency of 0.9 , based on 9 storms in the period 1981-1990, with a conditional probability of a loss below 14p of .333, based on 3 losses below 14p out of the 9 occurring.

The log-normal distribution has a mean of 20.7 and a standard deviation of $\mathbf{3 4 . 7}$. The underlying normal distribution has a mean of 2.363 and a standard deviation of 1.156 .

## 6. 1 The Annual Storm Loss Distribution.

The first use of the simulation program is to derive the annual storm loss size distribution for the three sets of assumptions described above.

The model was run for 50,000 periods for each set of assumptions and the results are shown, in graphical form, on Chart 6.1.1 below. The chart shows "return periods" defined here as the inverse of the probability of exceeding the given loss amount. The thicker line shows the results for the "select" set of assumptions.


The annual average losses from these simulations were 14.0, 27.4 and 13.3 respectively, compared to the base sample average of 12.0 . The log-normal distribution estimate is nearer this sample mean loss and the extreme frequency assumptions, as is to be expected, result in an average annual loss which is more than twice the actual average value.

The standard deviations of these annual estimates are 35.1 , 52.3 and 31.9 respectively. The simulated values are allowed a bigger range than the sample range, with the Pareto truncated at 500 and the log-normal unbounded. Truncating the Pareto at alternative limits does not appear to affect the results to any degree especially at the range of values likely to be of interest to practitioners.

The chart indicates that storm losses comparable to those suffered in 1987, caused by cat 87J, at around 100 p per $£ 1000$ SI have a return period 40 years. Losses of twice this severity have a return period of about 130 years under the same "select" assumptions. The comparable figures under the severe assumptions are about 15 and 50 years respectively. These periods illustrate the extremity of these severe assumptions.

### 6.2 Rating implications for Household Buildincs

The previous chart is redrawn below with a restricted range of loss and with only the "select" line shown.


This chart may now be used to consider the storm risk rate to be included in the overall risk rate for household buildings. If, for the sake of argument, management decide that they need to cover their storm losses nine out of ten years then the required risk rate to be charged oan be read off from Chart 6.2 .1 as 35p. This is two and a half times the underlying annual risk rate obtained from the simulation.
clearly the rate actually charged to cover the risk of storm will need to reflect the actual cost to the insurer, given his retained risk, the cost of his capital and the cost of his reinsurance protections. only some of the reinsurance aspects are considered below.

## 6. 3 Reinsurance consjderations

As the program simulates individual losses it can be used to evaluate any reinsurance programme.

Chart 6.2.1 above can be used, with appropriate care, to provide answers to questions likely to arise in the selection of appropriate priority limits for storm stop Loss reinsurance, assuming such a product is on offer.

For example to cover annual storm losses likely to occur once in ten years requires a lower limit of 35p, as identified above. The amount of cover available can then be used to identify the implicit upper limit of this cover in terms of return periods.

The remainder of this section will consider the more traditional Catastrophe Excess of Loss reinsurance with the usual single reinstatement at cost proportional to amount only.

The program uses 10 layers of loss and accumulates results for each of these layers. The priority points were set set at $0,2,5,5,10,25,50,100,150,200$, 300 and 500. Recall that 100 equates to an October 1987 size loss or approximately half the premium from a household buildings acount. The computer program priority limits can be changed very easily if required for evaluating a specific reinsurance programe.

The spreadsheet accumulates various statistics for each layer. The number of losses in the layer, the cost of these losses, the amount reinstated and the amount in excess of the cover are accumulated. The sum of squares is also calculated and is available for estimating standard deviations of all these statistics.

For illuatration purposes only Risk Rates on Line, or RROL, are consiáered.

We define the RROL as the reinsurers expected loss, net of the pro-rata reinstatement premium, expressed as a percentage of the layer size. So RROL = Expected net loss to layer / ( Layer + Expected amount reinstated).

Taking the first layer of 2.5p XS Op, for illustration, a typical result may look as follows:
Average simulated gross cost of layer $=1.6793$
Average simulated net cost of layer $=1.5830$
Average amount reinstated $=1.2246$
The RROL is then $1.5830 /(2.5+1.2246) * 100 \%=42.5 \%$
Note that the difference between the gross and net layer costs arise from the uncovered portions of third or subsequent claims. The amount reinstated accumulates for the year up to a maximum of the layer amount.

The chart below shows the RROL's for the three scenarios described above from the same simulations of 50,000 periods each.


200


### 6.4 Reinstatements and thein impact

Under the storm frequency assumptions derived from the experience over the last seventy or so years the probability of three or more storms in any one year is fairly small. Combined with the skewness of the loss gize distribution this makes the initial value of two of more reinstatements of little significance.

Table 6.4.1 below shows an extract from results of the simulation using the extreme frequency assumptions.

Table 6.4.1 Simulation Results: High Freq Assumptions

| Lower <br> Himit | Upper <br> Limit | Layer Ampunt | Full Cost | Reduced* Cost | Impact Reins |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 2.5 | 2.5 | 2.181 | 2.004 | $8.12 \%$ |
| 2.5 | 5 | 2.5 | 2.050 | 1.899 | $7.37 \%$ |
| 5 | 10 | 5 | 3.698 | 3.468 | 6.22 名 |
| 10 | 25 | 15 | 7.109 | 6.924 | 2.60\% |
| 25 | 50 | 25 | 4.545 | 4.531 | $0.29 \%$ |
| 50 | 100 | 50 | 3.661 | 3.660 | 0.03\% |
| 100 | 150 | 50 | 1.712 | 1.712 | $0 \%$ |
| 150 | 200 | 50 | . 978 | . 978 | $0 \%$ |
| 200 | 300 | 100 | . 988 | . 988 | $0 \%$ |
| 300 | 500 | 200 | .503 | .503 | $0 \%$ |
| Tote |  |  | 27.424 | 26.667 | 2.76\% |

* Reduced by limiting losses to twice layer anount

The table shows that only $2.76 \%$ of the losses would not be covered by the assumed single reinstatenent available. Most of this shortall is in the lower layers. The layer of 25 xs 25 , for instance, starting roughly at a quarter of the cost of cat 87 J is high enough for the initial value of a second reinstatement hardly to register.

These calculations consider the value of more reinstatements at the inception of the cover and are not to be confused with the conditionel velue of additional reinstatements, sought by some insurers, after an early (and sometimes late) penetration of their catastrophe covers.

### 6.5. Risk_rates versus market rates

The model derived Risk Rates on Line are not directly comparable with actual market reinsurance rates as these will be for wider cover (i.e. more perils) and will usually also cover commercial properties as well as household buildings and will additionally include safety, expense and profit margins.

One recent example for catastrophe cover for a uk household account will be used to illustrate the current differences between the estimated RROL for storm and the market rate for UK catastrophe cover.

The cover in question, expressed in per $£ 1000$ SI, is for 60p Excess of 30 p and has an indicated market cost of around $30 \%$ rate on line or 18 p per f 1000 SI .

The estimated Risk Rate on Line for storm using the "select" set of assumptions is approximately $5 \%$ or $3 p$ per $£ 1000$ SI. Under the extreme assumptions these storm risk rates double.

The comparison may be distorted by the inclusion, in the current market rating of these covers, of a "pay-back" element. However, a premium which appears to be as much as six times the risk premium level must question the cost effectiveness of traditional catastrophe excess of loss reinsurance.

## 6. 6 Effectiveness of catastrophe reinsurance

Catastrophe reinsurance pricing is clearly judgemental although this paper has set out an approach which provides a theoretical framework where both actual experience and simulated futures can be examined. In the recent past, this cover has been relatively cheap in the UK but the consequence is the demand for huge increases in order to achieve "payback" following the recent storms. The period in which reinsurers seek to recover their outlay is often as little as 4 or 5 years, a fraction of the shortest return period we have postulated. This is compounded by reinstatement premiuns and aggregate deductibles which severely limit the value of the cover at present.

The volatility of catastrophe prices significantly erodes the stability of results which insurers seek through their reinsurance programmes. Further, these covers are generally placed on an accident year basis, leaving insurers without the ability to incorporate the new premiums into their own rates until the following renewal.

It is scarcely surprising, therefore, that many of the old gentlemen's agreements to renew and offer payback are being formalised through spread loss contracts. However, this very formalisation can be their undoing since off-balance sheet liabilities or assets are then created which the DTI requires to be recognised in its returns.

If the traditional reinsurance market is unable to offer a stable enviroment, insurers may elect to move towards greater retentions, aided by the establishment of a catastrophe reserve. There are tax implications here, of course, and the outcome of the ABI's lobbying is awaited with interest. We have indicated a possible rating approach where the aim is to cover losses in nine years out of ten; reinsurance plus the reserve would be geared for the exceptional tenth year (which could be 1992 1) and the simulation progran used to investigate possible rules for operating a storm catastrophe equalisation reserve.

Although this is considered outside the scope of this paper and has not been investigated further the results of such an investigation are likely to show that the cost of the traditional forms of reinsurance are disproportionate and their "smoothing" inefficient.

### 6.7. Managing the impact of catastronbe Lossess

One concern is whether reinsurance is the right medium for protection against catastrophe. In this arena, reinsurers are in danger of becoming concentrators rather than spreaders of risk because of the rarity and scale of the losses. Indesd, time is the only spreading mechanism they can offer and that period is all too brief. So, perhapa we should stand back and take a fresh view of the problem.

Who else takes risks about the future? Futures exchanges are a potential source of capacity with the bonus that financial and weather-related catastrophes are generally independent. This is closer to the risk spreading concept of insurance than traditional reinsurance companies can offer. While abortive attempts have been made by some exchanges before, in Chicago they seem to be making real progress. Recent press coverage indicates, that in the us at least, such hedging may become available at a cost of around $5 \%$ of the amount at risk. It would appear that this minimum cost will then determine the priority limits of the cover, rather than be determined by the required cover. The ovaluation of these contracts may well be the next application of our work.

## gTORM RATING IN THE KINETIES

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## A1.1 Characteristics of Storms in the Storm Catalogue <br> Dates, duration in hours, maximum wind speeds, area affected by winds exceeding 60 knots and the unadjusted severity index

## A1.2 Sample Storm Damage. Maps

Brief descriptions and Maps for the 7 storms identified as the most costly in Table 4.1.1 of the main report.

## Al. 3 sample Storn Tracks

Each of the Maps in A3.2 also plots the position of the low pressure centres, or depression tracks, that these storms "hung on". These are meteorological maps and do not show the position of the maximum wind speeds or the maximum damage.

For these storms damage occurs to the right of the storm track and maximum damage can occur at locations which are very far from this actual low pressure track. A review of the Map for Storm 2, the October 1987 storm, illustrates this point.


APP1．1：Characteristics of storm in storn catalogue

| Storm No． | Drate | Durailon | ｜idex．wiñ | Area | Severity |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | ［ H 官． | ［Kn．］ | ［ $\mathrm{Km} \mathrm{m}^{2}$ ］ | fricex |
| 1 | 26．2．90 | 突 | 100 | 111024 | 88 |
| 2 | 25．1．90 | 14 | 100 | 105267 | 1474 |
| 3 | 16－17．12．89 | 13 | 34 | 8635 | 45 |
| 4 | 13，2．89 | 9 | 128 | 83474 | 1503 |
| 5 | 9．102．88 | 31 | 96 | 83735 | 1740 |
| 6 | 15．10．87 | $\theta$ | 100 | 39044 | 313 |
| 7 | 24．3．08 | 12 | 76 | 353831 | 108 |
| 宜 | 13．1．84 | 3 | 118 | $974{ }^{\text {c }}$ | 400 |
| 9 | \＄2．83 | 21 | 106 | 40344 | 1203 |
| 10. | 23－25．11．81 | 12 | 80 | 11108 | 60． |
| 11 | 48．12．79 | 8 | 96 | 5345 | $3{ }^{3}$ |
| 12 | 11－12．1．78 | 6 | 74 | 27245 | 6 |
| 13 | 21，7雨 | 5 | 116 | 210120 | 186 |
| 14 | 27．28．1．74 | 15 | 7 | 10096 | 134 |
| 15 | 121.74 | 11 | 114 | 120990 | 3010 |
| 18 | 23．4．73 | 41 | \％ | 10901 | 124 |
| 17 | 12．13．14．72 | 9 | 68 | 20149 | 57 |
| 18 | 14－15．1．85 | 15 | 116 | 31002 | 741 |
| 19 | 6．3．67 | 12 | 79 | 4720 | 20 |
| 20 | 16．5．0\％ | 6 | 浐 | $48 \times 23$ | 20\％ |
| 21 | 78－172．20 | 18 | 12 | 10yrad | Smo |
| 22 | 11．1．8\％ | 18 | 9 | 11513． | 1348 |
| 24 | 16－17．気耍 | 20 | 8 | 78089 | 117 |
| 24 | 4.11 .57 | 4 | 80 | 8009 | 0 |
| 25 | 20．738 | 19 | 80 | 20908 | \％ |
| 28 | 21－23．12．54 | 3 | 90 | 109448 | 2 m |
| 27 | 20－30．11．54 | 5 | 100 | 00093 | 308 |
| 2 | 31－12．83 | 21 | 0 | 118947 | 1074 |
| 23 | 17.12 .8 | 17 | \％ | 109330 | 180 |
| 30 | 30.12 .51 | 7 | 9 | 70ras | 411 |
| 31 | 0.10 .2 .49 | 3 | 7 | 1915： | 14 |
| 32 | 76．3．47 | 8 | 8 | 10.912 | 808 |
| 33 | 10．1．45 | 11 | 0 | 3900：9 | 413 |
| 34 | 7.4 .43 | 11 | 0 | tet750 | 1034 |
| 35 | 25－24．11．38 | 12 | 84 | 121304 | 1200 |
| 33 | 4．10．3\％ | 4 | 9 | 4 | 143 |
| 37 | 15－221．37 | 7 | 73 | 2407 | 7 |
| 势 | 25－27．10．36 | 11 | 留 | 122128 | 1031 |
| 39 | 18－18．10．35 | 15 | 0 |  | 1047 |
| 40 | 16－17．935 | 11 | 15 | 50196 | 332 |
| 41 | 5－7．12．23 | 20 | 钽 | 16000 | 319 |
| 42 | 22－25．11．28 | 10 | 0 | 117609 | 1759 |
| 43 | 16．17．1928 | 0 | 70 | 31400 | 159 |
| 4 | 67.128 | 0 | 73 | 71900 | 224 |
| 45 | 28－29．10．27 | 13 | 0 | 70315 | 等3， |
| 46 | 28．1．27 | 11 | 94 | 85941 | 785 |
| 47 | 26．27．1．20 | 9 | 63 | 51991 | 117 |

## Stora No. 2 : 25 January 1990

Unusual features of this storm include:
i) explosive deepening;
ii) the associated vorticity pattern was characterized by marked descent to the rear of the trough and ascent ahead of it (as in the October 1987 stoxm);

1ii) the low was embedded in a strong baroclinc zone which resulted in it moving very rapidily (at around 50 knots) and deepening rapidly as it crossed the UK. As in the october 1987 storm, the rapid speed of movement was an important addition to the surface wind.

25th January 1990


A small intense low formed olose to the British Isles on 14 October and changed the pattern of air flow bringing moist warm air towards Britaln from the SW. At the same time, in the mid-Atlantic, there was $a$ deepening of cold ain from the north.

The polar front at the surface in the early hours of 15 October lay at latitudes $38^{\circ}-45^{\circ} \mathrm{N}$, associated with a strong upper-air jet strean, with the exit zone lying over the south of England. Incipient wave disturbances began to form along the frontal zone, and ultimately one developed into a cyclone centre over the Bay of Biseay, with pressures as low as 952 mb . This system crossed England from SW to NE in the early hours of 16 october.

One remarkable feature of this storm is that the strongest winds did not occur until dfter the passage of the cold front. The onset of southerly winds in the warm sector brought unusually high temperatures, with a rise of $7.9^{\circ} \mathrm{C}$ in one hour observed in parts of the $S E$ of England. Pressure increases of 8 mb in one hour were measured in the same region.

16th October 1987


This storm affected the North sea especially the east cost of England and Scotlond. The greatest damage and loss of life wos caused by a great sea surge in Lincolnshire, East Anglia and the Thames estuary. The coincidence of strong northerly winds blowing down the North Sea with high spring tide was responsible for this flooding. The strongest pressure gradient was measured over the central North Sex at around midnight, and the gradient wind was described as 'phenomenal': in the range 100-130 knots.

This storm developed in the cold air from the Arctic in the rear of a depression which had approached the Faeroes from the WSW on 30 January and then moved SE to the German Bight. As the depression moved SE, pressure rose rapidly to the west, producing a very steep pressure gradient.

Although this storm brought tremendous damage and loss of life to the UK (around 350 lives lost) and mainland Europe, this was due largely to flooding rather than directly to the high wind speeds. The storm track of this event has been used by Mundch Reinsurance to specify their probable maximum loss scenario (Munich Re, 1990).


This storm was associated with an intense depression moving rapidly $5 E$ from Iceland. North-westerly gales were experienced over Scotland and the eastern half of England. In parts of Scotland, hourly wind speeds of greater than 50 knots were recorded.

7th April 1943


When this frontal cyclone was forming in the western Atlantic, it was already drawing in very cold air to the rear from Greenland and the Davis Strait. It deepened rapidly to cross the British Isles as a 960 mb centre on 23 January. This storm advanced 1000 nautical miles in 24 hours (a speed of 43 knots) from 22-23 January but then slowed to half this speed in its passage to Norway.
storm 35 brought widespread south-westerly and westerly gales to England, with unusually high wind speeds in inland areas. A storm surge occurred in the German Bight.

23rd-24th November 1938


This gale was part of a westerly sequence which lasted about one week. The main depression centres lay in a belt between $58^{\circ}$ and $63^{\circ} \mathrm{N}$. Vigorous secondary depressions developed from the waves on a trailing cold front ruming between the Azores and the North Sea.

Gale-force winds first affected NW Ireland, at around 1800h on 14 January, then moved to affect the whole of Scotland and northern England. Wind directions within the gale were SW to westerly. The greatest intensity lay in a narrow belt across central scotiand, especially elasgow.

14th-15th January 1968


The Capella storm was notable for the great area affected by damaging winds, not only in the UK (see accompanying map) but also across Europe, and for its prolonged duration, up to 24 hours in eastern Englend.

A low pressure area formed in the central Atlantic between $20^{\circ}$ and $50^{\circ} \mathrm{N}$ in the last few days of 1975. Meanwhile, on the eastern side of the Atlantic, a strong thermal gradient developed, between temperatures of $11^{\circ} \mathrm{C}$ over England and temperatures of $-11^{\circ} \mathrm{C}$ over southwest Iceland. A breakaway depression from the low pressure area, moving eastward across the Atlantic, encountered this zone of themmal contrest and deepened rapidiy as it crossed the British Isles. The pressure fell as low as 968 mb , over Denmark.

2 January 1976







## RPPEADIX 2

## CONVERTIMG THE URA MAP8 TO LOBEES

## A2. 1 Estimating qust speeds for postcode areas

The conversion of the UEA storm maps into insurance losses involves a number of stages. Firstly some method is needed to obtain gust speed estimates at postcode area level from these maps. Secondly a formula is required to estimate the percentage loss from a given gust speed, storm duration and exposed sum insured. Finally an actual exposure value is required for each postcode area for the portfolio being evaluated.

To obtain gust speed estimates it was assumed that the closeness of the gust speed isolines could be interpolated, somehow, to obtain an estimate of gust speed for each postcode area. This speed gradient assumption appeared to correspond reasonably well when October 87 storm was considered as this had a clearly defined, and relatively narrow, speed bands as can be seen from the map for Storm 6 in Appendix 1.

As a simple example a postcode with a centre roughly in the midale of the $60-80$ knot band would be have a value of 70 knots. Where possible the distance from the nearest 60 knot line was estimated for each postcode and related to the width of the 60-80 knot band nearest to this area to estimate the speed by linear interpolation.

This process was first applied to the three storms for which exposure and loss data were available and could be used to validate and calibrate a damage formula, With over 120 postcode areas and 47 maps this was ultimately a laborious process with an uncertain outcome.

An alternative, which should produce more reliable estimates, would be to use actual figures of maximum wind and gust speeds published, at least monthly, for all the UK Weather stations. This approach was not followed due to lack of time, and uncertainty about the availability of this information for the whole of the 71 year period concerned as well as the difficulty of obtaining exposure and claims information to correspond to the weather station areas. This may, however, be a good starting point for a subsequent study.

## A2.2 The damace or loss severity formula

The severity index formula, described in Section 2.2, provided a starting point here.

This formula, used by meteorologists to index storm severity, has the severity proportional to the area affected, duration of the storm and the cube of the maximum speed.

```
Severity = k * Duration * Max Area * (Max Speed ) * 3
```

The October 1987 produced a much wider range of percentage losses at postcode area level (nee Appendix 3 Table A3.1) and this storm was chosen to test the reasonableness of a formula of the above type. The losses from the ABI sample were used, together with the speed estimates obtained by the process described above, and the values plotted in an $X-Y$ chart.

The crosses on the following chart show the actual damage against estimated gust speed for the postcode areas that suffered damage. The thick line is a cubic curve finally adopted as the damage formula. This curve is defined by :

```
Damage = .00345 * (Speed in knotg - 47) ^ 3
```

These values were chosen to replicate the losses for the ABI samples for both storms, after an adjustment for the longer duration of the 1990 storm.


The October 1987 storm had a duration of 8 hours compared to the 14 hours of the January 1990 storm. The severity index formula indicates that damage should be proportional to the duration of the storm. In the following chart the crosses show the January 1990 actual ABI sample losses against the estimated gust speeds and the thick curve is the earlier cubic formula with a duration factor of $14 / 8$ or 1.75 to adjust for the longer duration of the 1990 storm.


Although there is wide variation around the assumed cubic line the general shape appears to capture the data reasonably well in the circumstances. As mentioned above the constants were chosen by ensuring that the overall losses projected, based on the speed estimates, matched the ABI sample losses for both storms. The fitting process was clearly not rigorous and was adopted for simplicity and to correspond to the severity formula generally used by meteorologists. Log-1inear regression can be used to derive alternative models. This would be a more formal process which could be subjected to the usual validation tests.

## A2. 3 Validation of the loss severity formula

Returning to the actual formula chosen, the validation process consisted of projecting losses based on the smaller CU sample exposures and comparing the estimated losses with the actual ones. The choice of the formula parameters ensured that the ABI losses, for both the 1990 and the 1987 storms, fitted "perfectly".

For the smaller samples tested the formula produced higher lozs estimates than the actual losses.

The CU sample for the october 1987 storm had a loss of 90.3 p per £1000 SI. The model projected 100.6 p .

The cU sample for the January 1990 storm had a loss of 94.0 p per $\{1000 \mathrm{SI}$ compared to the model estimate of 111.0p.

The estimate for the February 1990 loss (Storm 1 in the Catalogue or cat 90G) was 22.5 p against an actual loss of 14.2 p per $£ 1000 \mathrm{SI}$.

The reasons for these differences were not investigated further. These may partly arise from differences in sums insured or types of properties insured. Ideally more samples should be tested and any differences which emerge, and which cannot be explained by random fluctuations, should be investigated.

The formula used at least overestimated the losses on all three test cases. This was considered to be a more acceptable outcome than the alternative.

The actual choice of damage model, and its parameters, warrants further study. The main differences are likely to arise at the extremes of the range of gust speeds and the choice of model may introduce some bias. This bias could result in overprojecting the less gusty storms and underprojecting the more severe storms, or vice-versa. The formula, and parameters, chosen at least ensure a good fit for the two biggest losses and would appear to overproject rather than underproject storm losses.

The base exposure and loss data used, and the estimated speeds for each storm and postcode area, are given in Appendix 3.

APPENDIX A3．1 ：EXPOSURE AND LOSS（SAMPLE）DATA

|  |  |  |  |  |  | Row | $1$ | ？ |  | 8indich |  | $\begin{aligned} & \infty \\ & 0 \\ & \hline \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| corn |  | 0 | $\cdots$ | － | 0 |  |  | A 4 | 0 |  | $00$ |  |
| 盛 | 407 | 13 | 1.9 | 1.3 | 1.4 | 6.519 | 4375 | 4.4 | 7 | 31.3 | 50. | 0． |
|  | dint | 28.4 | 32.0 | H0．0． | 136．7 | 5.7 Exib |  | 23 | 24 | 4.5 | 35.4 | 年年 ${ }^{\text {a }}$ |
|  | 2mo | 2.7 | 2.0 | 364 | 36.1 |  | 4 x 212 | 3.9 | $\underline{6.1}$ | 130．0． | 14.3 | 5.7 |
|  | 16800 | 1.1 | 1.2 | 25\％ | 215.3 | 9.7 ：食碞 | 1978 | 2.0 | 21 | 30.0 | 21.2 | M．I |
|  | 5200 | 25 | 1.4 | 32.4 | 31.3 |  | 1051 | 615．1 | $4{ }^{4}$ | 2FI． 6 | \＄74．7 | 36.0 |
|  | 4t | 2.4 | 2.2 | 99.2 | 3.1 | 42 | 720 | 13 | 10.0 | 4.0 | 54.0 | 6.1 |
|  | 1385 | 4.2 | 60.1 | 107.3 | 151.0 | 4.4 | 3121 | t．e | 0.6 | 3.7 | 14 | 24 |
|  | 3005 | 2.4 | 0.2 | 31.1 | 14.7 | 29.4 | 11717 | 103.6 | 10．s） | 132.3 | 44.6 | 6.9 |
|  |  | 785．6 | 70．5 | 17.7 | 51.4 | 1.4 | 1785 | 1.3 | t．7 | \＄3 | 3.1 | 13.1 |
|  | 46 | 157.0 | 2004 | 14.4 | 1225 | 4.1 | 12711 | 30. | 3.1 | 3.7 | 3.7 | 35.0 |
|  | （6）15 | 5.3 | 4.7 | 204.5 | 28.3 | 4.6 | tas | 63 | 3.2 | F． 7 | 40.1 | 12.7 |
|  | 21904 | 4.3 | 0.8 | 1.5 | 1.2 | 12. | 7 m 1 | 2.6 | 1.4 | 101.8 | 3.1 | 10.1 |
|  | 35 | 1.2 | 2.4 | 10 | 4.0 | 17.4 | 2 cma | 130.9 | 18084 | \＄0． | 25.7 | 6.1 |
|  | E57 | 52.7 | 1460 | 4.1 | 40.5 | 14 | Hoto |  | 90.4 | 130．0 | 25.7 | 6.5 |
|  | 1359 | 3.1 | 1.0 | 124.3 | 125.2 | 14.0 | 4 HO | 2.9 | 3.0 | 2 E .4 | 14，4 | 42.3 |
|  | T\％ | 2.2 | 0.4 | 44.0 | 252 | 38.9 | 3965 | 1.3 | 11.9 | 1064 | 車 | 4．） |
|  | 15016 | 393.0 | Lis） | \％ 2 | 14.3 | 教 | 43） | 1.0 | 4.3 | HeA | 35 | 4.0 |
|  | 2015 | 157．4 | 2653 | 策， | 155.0 | ¢ 4 ： | H6木安 | H．1 | 11.3 |  | \％ 26 | M． 1 |
|  | 734 | 195.3 | 158.3 | 151．5 | 152.2 | 6.3 ：粨品 | 2146 | 4.7 | 2.5 | 1.6 | 4.5 | 0.7 |
|  | 11890 | Ton． 3 | 21．7 | 204.6 | 212.1 | 24.6 | 11658 | 74 | 4 | 40.3 | 5\％． 5 | 11.6 |
|  | 19\％5 | 3.5 | 13 | 27．） | 27.3 | 13 | 18 tas | 40.9 | 425：1 | 272.5 | 216.4 | 162 |
|  | 473 | t．0 | 21.3 | ． 3 | 17．9 | 12.4 | 40 | 1.3 | 0.9 | 19.5 | 13.2 | 354 |
|  | 76 | 390.3 | $\pm 0.7$ | tif． 7 | mbs | 11.9 | 13641 | 6.3 | 57.4 | 17．3 | 177.2 | 5. |
|  | 2597 | 0.5 | 2.15 | 20 | 25 | 0.4 | 12\％1 | 4til | JP3 | 15.0 | 14t．4 | \＄．t |
|  | 1tcy | 20 | t．0） | W． | $2 \mathrm{2}, 3$ | 21.9 | 2＊1 | 191．t | 1stol | 16E． 6 | 1\％61 | 5.6 |
|  | 4154 | 6.3 | 20. | 4.9 | t．］ | 5.4 | 1987 | 1.7 | L， 9 | 101.6 | 52.7 | 19.1 |
|  | 2465 | 2.3 | 4.3 | 13.9 | 4.3 | 2. | 12814 | 4.7 | 4.6 | 17.6 | 316.4 | 3.4 |
|  | 9744 | 1.3 | ＊．3 | 4.1 | 4.4 | 85.0 | H04t | 4385 | 164.2 | 1514． | 14.1 | 4 |
|  | 206t | 1.8 | 1.1 | 3．5 | 15\％ | 20．4 | 为 | 413 | 3.0 | H6．5 | 136.2 | 5.7 |
|  | 534 | 19.2 | 11.3 | 2404 | 165．3 | 1.4 | ＊＊5 | 1.7 | 2.3 | 19．3 | 17.1 | 年4 |
|  | \％ | 2.5 | 4． 2 | 24.3 | 11．） | 12.1 | T6S | 4 | c． | 16.4 | 131.1 | 5.1 |
|  | 3 m | 138．6 | 10.1 | 137.1 | 1531 | 7.1 | $4{ }^{4}$ | valo | 2069 | 16.0 | 121.4 | 34 |
|  | 30.7 | 123．） | 110.1 | 122.1 | 127.8 | 1.7 | 2011 | 75 | ＊） 3 | 29．4 | 20.5 | 6.1 |
|  | 13658 | 1.5 | 2.0 | 4.5 | 4.5 | 3.7 | 11232 | 0.3 | 2ss | 20.5 | 14.3 | 10.7 |
|  | \％H | 24．0 | ¢0．9 | 146． | 2085 |  | 479 | 7．${ }^{5}$ | 34.4 | 1＊6． 9 | 23.5 | 5.9 |
|  | 15014 | 75 | 1.1 | 521.8 | 26.4 | 6. | 2012 | 1.4 | 1.3 | 6.2 | 1．0 | 25.7 |
|  | 2715 | 0.7 | 0.4 | 3.7 | 2.5 | 20 | 9701 | 37.7 | 300.1 | 456.7 | 341.4 | 4.5 |
|  | 340 | 1.8 | 2.2 | 20.7 | 35.3 | 40.418 | 71301 | 4.3 | 1.9 | 3.7 | 16.9 | 12.0 |
|  | tores | 0.9 | 0.3 | 5.5 | 1.1 | \％ | 12004 | 10.2 | \％5．5 | 94.1 | 40.5 | 6.4 |
|  | 12080 | 3.4 | 2.7 | ＊1．5 | 0.9 | 70： 8 IT | 1121 | 9.6 | 30.9 | 47.5 | 23.7 | 16. |
|  | tamo | 136．3 | 130.4 | 171.7 | 1782 | 4.6 ，ta | 503 | \＄． 6 | 6.1 | 3 c | Wens | 13. |
|  | $\square$ | 68.3 | 52.7 | 129： | 130．1 | 6.2 P | 2615 | 0.4 | 0.0 | 3.4 | 3.4 | 6.7 |
|  | 2374 | 1.0 | 0.6 | 23.8 | H． 3 | 21.4 ，7\％ | $2 \mathrm{H7}$ | 0.5 | 4.0 | 43.2 | 15.5 | 4.4 |
|  | 359 | 1.4 | 0.5 | 0.5 | 0.1 | 4.51 \％ 5 | $3 \mathrm{~m} / 4$ | 769.7 | 4n． 7 | 202．1 | 20.0 | 43 |
|  | 2130 | H． 4 | 23.4 | 136． | 13.6 | 3.4 ］ | $0 \%$ | 74 | 7.5 | 278.2 | 252.2 | \＄． 2 |
|  | 6069 | 1.5 | 3.1 | 536.9 | 54.7 | 10.4 －${ }^{\text {b }}$ | 7 | 9.4 | 6.7 | 438.5 | 330.5 | 1.4 |
|  | 43st | 2.3 | 2.4 | 3 H | 15.2 | 19.4 寺． 7 | 4102 | 15 | 1.0 | 6.7 | 13.3 | 14.7 |
| $\cdots$ | 1642 | 1.5 | 4．0 | 76.3 | 44.1 | 453 倍 T\％ | $\omega 7$ | 0.5 | 17.1 | 360.7 | 176.4 | 1.3 |
| 90 | 4089 | 160． | ＋07， 1 | 100.5 | 118.1 | 3.8 \％ | $4 \mathrm{ct1}$ | 75.6 | 53.2 | 150.9 | 165.2 | 17 |
| 0 | 9\％ | 1，7 | 0.6 | 27.3 | 17.4 | 28.6 | 510\％ | 76 | 40.9 | 11.4 | 7.7 | 6.2 |
|  | 15013 | 34.4 | 2065 | 112.4 | 76 | 9.4 W | $71 \%$ | 1．4 | 1.0 | n． 0 | 20.1 | 28.0 |
|  | 320t | 0.4 | 0.7 | 2.5 | 14.4 | 0.9 （te | 4149 | 36.3 | 24.4 | 72.4 | 74.3 | ＊．4 |
| K | 4238 | 1.0 | 0.5 | 3.7 | 1. | 70 \％ | 3684 | 41.5 | 34. | 11\％．6 | 121.7 | 4.1 |
| 炜 | 14206 | 1252 | 125．3 | 144．＊ | siz． 7 | 4.4 | 476 | 2.1 | 4.5 | 11. | 24.2 | 27.2 |
| $\pm$ | 1652 | 3.1 | 1.0 | 0.4 | 1.4 | 4.15 | 387 | 1.0 | 2.8 | 35.2 | 4.2 | 30.3 |
| EY | 3752 | 0.5 | 0.4 | 1.4 | 1.0 | o．f）．Wh | 684 | 1.4 | 1.4 | 7.7 | 36 | 12.5 |
| 1 | 3006 | 3.4 | 3.4 | 40.1 | 5.5 | 36.2 \％，vt | 404 | 1.3 | 0.7 | 120） | 9.9 | 2 z |
| H | 4093 | 1．4 | 2.5 | 34.0 | 20.2 | 4.7 \％ | 3tat | 1.1 | 5.4 | 25.7 | 45.3 | 19.4 |
| Cib | 205 | 2.6 | 5.2 | 18.7 | 725 | 35.4 \％ ro | （130） | 0.1 | 0.9 | 12.4 | 10.7 | 20.6 |
| 분 | 127is | 3.3 | 4.5 | 22t | 27.3 | 42.7 I | 209 | 0.8 | 0.0 | 0. | 20 | 0.4 |
| 1 | benso | 6.4 | 4.4 | 31.7 | 25.4 | H1．7 | 10000 | Wis | \％．3 | 1117 | 101 | 1441 |
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|  | T 1 | E | 3 | 30 | 50 | 50 | 50 | 70 | 3 | \＄0 | 50 | 53 | \％ | 50 | 50 | 50 | 5 | \％ | 3 | 3 | 50 | 5 | 50 | 50 |
|  | 7 | 宜 | 5 | 4 | 0 | 51 | 5 | 73 | 38 | \％ | 50 | 60 | 70 | 6 | 50 | 50 | 50 | 70 | 5 | 50 | 等 | 5 | 50 | 50 |
|  | V | 7 | 3 | \％ | 10 | 61 | 3 | 5 | 0 | 50 | 50 | 45 | 0 | 30 | 5 | 39 | 5 | 50 | 50 | 5 | 510． | \％ | 50 | 64 |
| \％${ }^{\text {chem }}$ | 蓈 | 50 | 5 | 3 | 0 | 51 | 5 | 70 | 6 | 34 | 50 | 34 | 㐌 | 5 | 6 | 3 | 5 | 7 | 5 | 5 | 61 | 4 | 5 | 59 |
| 4095． | 7 | 7 |  | \＄0－ | 50 | 9 | 5 | 35 | 36 | 50 | 50 | 54 | 7 | 50 | S 0 | 50 | 5 | 30 | 5 | 50 | 50 | 65 | 50 | \％ |
|  | 6 | \％ | 50 | 59 | 50 | 6 | \％ | 5 | 6 | 50 | 30 | 5 | 0 | 30 | 5 | \％ | 55 | 59 | 50 | 50 | 54 | 65 | 5 | 64 |
|  | 62 | 30 | 50 | 4 | 5 | 51 | 50 | 58 | $\pi$ | 30 | 30 | 5 | ¢0 | 50 | 7 | 5 | 50 | \％ | 4 | 5 | 70 | 5 | 67 | 3 |
|  | 8 | 7 | 30 | 50. | 3 | 4 | 灰 | 5. | 30 | 37 | 50 | 50 | 70 | 5 | 5 | 50 | 50 | S0 | 50 | 5 | 37 | 4 | 50 | 70 |
|  | 7 | 5 | 55 | 30 | 70 | 51 | 50 | 号 | 0 | 50 | 5 | 5 | 75 | 3 | 4 | ¢ | 5 | 70 | 3 | 50 | 6 | 4 | 58 | 50 |
|  | 4 | 4 | 50 | 50 | 50 | 50 | 50 | 4 | 5 | 50 | 30 | 5 | 0 | 5 | 30 | 50 | 50 | 50 | 59 | 50 | 5 | 58． | 50 | 5 |
|  | 4 | 73 | 50 | 5 | 3. | 34 | \％ | 5 | 3 | 54 | 36 | 55 | 7 | 50 | 50 | 0 | 6 | 50 | 3 | 30 | 5 | \％ | 50 | 60 |
|  | － | 67 | 50 | 50 | 60 | 5 | 70 | 5 | 30 | 50 | 50 | 50 | 0 | 30 | 0 | 5 | 9 | 59 | 5 | 5 | 53 | 6 | 55 | 5 |
|  | 䊗 | 7 | 50 | 5 | 5 | 7 | 7 | 63 | 4 | 50 | 50 | 60 | 35 | 30 | 50 | 7 | 50 | \＄ | 50 | 30 | 37 | 34， | 3 | 70 |
|  | 4 | 7 | 50 | 5 | 5 | 4 | 7 | 5 | 50 | 50 | 5 | 50 | 7 | 5 | 38 | 5 | 5 | 5 | 54 | 30 | 51 | 6 | 50 | 7 |
|  | 75 | 50 | 50 | 5 | 6 | 51 | 50 | 75 | ¢ | 50 | 5 | 50 | 7 | 50 | 60 | 50 | 3 | $\%$ | 5 | 57 | 30 | 3 | 0 | 59 |
|  | \％ | 73 | 50 | 50 | 60 | 60 | \％ | 65 | 5 | 9 | 5 | 5 | 7 | 50 | 50 | 5 | 50 | 50 | 5 | 5 | 3 | 6 | \％ | 0 |
|  | 57 | 5 | 50 | 75 | 0 | 4 | 59 | 5 | 3 | 5 | 5 | 5 | 7 | 0 | 26 | 5 | 50 | 5 | 6 | 7 | 31 | 33. | 7 | 50 |
| E＝4 | 6 | 7 | 50 | 5 | 9 | 40 | 5 | 43 | 6 | 50 | 50 | 方 | 3 | 5 | 50 | 5 | 50 | 50 | 5 | 5 | S5 | 58. | 50 | 4 |
|  | 50 | 50 | 50 | \％ | 5 | 51 | 50 | 53 | 5 | 5 5 | 50 | 30 | 3 | 5 | 70 | 50 | 50 | 5 | 6 | 7 | 75 | 30 | 7 | 6 |
|  | 4 | \％ | 70 | 5 | 7 | 5 | 30 | 30 | （1） | 5 | 50 | 50 | 6 | 50 | T | 50 | 4 | 50 | 50 | 50 | 50 | 3 | 50 | 50 |
|  | 3 | 5 | 50 | 50 | 50 | 91 | 50 | \＄ 8 | 50 | 9 | 50 | 70 | 53 | 50 | 5 | 50 | 6 | 50 | 80 | 50 | 34 | 4 | 30 | $\%$ |
|  | 方 | $\cdots$ | 50 | 55 | 6 | 51 | 5 | \＄ | 9 | 50 | 50 | 5 | 73 | 50 | 60 | 50 | 50 | 7 | 5 | 34 | 82 | 8 | 5 | 30 |
|  | 5 | 㐫 | 54 | 50 | 50 | 7 | 7 | 85 | 50 | 50 | 50 | 50 | 70 | 50 | \＄0 | 50 | 5 | 50 | 30 | 50 | 56 | 3 | 50 | 64 |
| $\cdots$ | E5 | \％ | 5 | 5 | 50 | \％ | 55 | 53 | 50 | 50 | 50 | 0 | 53 | 5 | 30 | 5 | 55 | 50 | 50 | 50 | 56 | 4 | 50 | 7 |
|  | 6 | 7 | 50 | 58 | 50 | 7 | \％ | 5 | 5 | 50 | 54 | 5 | 7 | 3 | 50 | 5 | 5 | 50 | 5 | 50 | 56 | 4 | 3 |  |
| 804tis | 74. | 客 | 50 | 6 | 60 | 51 | 50 | 73 | 3 | 54 | 30 | 4 | 7 | 10 | 35 | 9 | 9 | $\cdots$ | 0 | 50 | \＄1 | 3. | 5 | $\underline{3}$ |
| 3imin | 43 | 7 | 50 | 50 | \％ | \％ | n | 85 | 50 | 9 | 9 | 5 | 75 | 30 | T． | 50 | 50 | 5 | 5 | 5 | 名 | 65 | 5 | 50 |
| \％ | 6 | 7 | 50 | 50 | 5 | 7 | 5 | 5 | 50 | 5 | 5 | 36 | 70 | 5 | 32 | \％ | 50 | 3 | 5 | 5 | 5 | 35 | 50 | \％ |
|  | ¢ | 76 | 30 | 50 | 50 | 72 | 70 | 5 | 0 | 50 | 50 | 9 | 㐌 | 70 | 30 | 5 | 50 | 50 | 50 | 30 | 5 | 4 | 50 | 5 |
|  | $\omega$ | 61 | 501 | 30 | 40 | 5 | 5 | \％ | 33 | 30 | 50 | 50 | 75 | N | $\pm$ | 36 | 5 | 60 | 5 | 5 | 4 | 6 | 54 | 50 |
|  | 6 | 7 | 50 | 50 | 50 | 7 | 30 | 65 | 50 | 30 | 50 | 3 | 7 | 50 | 50 | 5 | 50 | 50 | 50 | 50 | 37 | S | 5 | 方 |
|  | 0 | 7 | 30 | 50 | 50 | 炜 | 5 | 65］ | 50 | 50 | 50 | 50 | 70 | 50 | 56 | 54 | 50 | 50 | 50 | 50 | 5 | 4 | 50 | 70 |
|  | 5 | 71 | 50 | 30 | 35 | 30 | 30 | $\cdots$ | 5 | 50 | 50 | 5 | 7 | 50 | 50 | 5 | 55 | 50 | 5 | 50 | 51 | 5 | 50 | 5 |
|  | 6 | Fib | 5 | 50 | 50 | 76 | 35 | 5 | 30 | 50 | 50 | 5 | 70 | 30 | 30 | 3 | 5 | 5 | 50 | 50 | 54 | \％ | 51 | 4 |
|  | 4 | 76 | S | \％ | 5 | T1 | 5 | S | 5 | 5） | 56 | 3 | 45 | $\underline{4}$ | \＄91 | 泡 | 6 | 50 | 50 | 50 | 52 | 4 | 5 | 4 |
|  | \％ | 59 | 39 | 58 | 58 | 31 | 50 | 30 | 4 | 59 | 50 | 65 | 7 | 50 | 53 | 5 | 39 | 70 | 50 | 30 | 6 | 5t． | 57 | 5 |
|  | 67 | 7 | 39 | 50 | 50 | 53 | \％ | 4 | 5 | 39 | 5 | 5 | T | 30 | 50 | 60 | 6 | 3 | 80 | 50 | 50 | 5 | 50 | 34 |
|  | 4 | 62 | 50 | 50 | 5 | 5 | 90 | 6 | 5 | 9 | 50 | 5 | 70 | 5 | 35 | 5014 | 5 | 9 | 0 | 30 | 3 | 6 | 53 | 5 |
|  | 区 | 7 | 50 | 50 | 缶 | 34 | 5 | 65 | 7 | 9 | 5 | 3 | \％ | 5 | 50 | $\mathbf{5}$ | 50 | 9 | 50 | 50 | \＄7 | 5 | 50 | 7 |
|  | 34 | 6 | 30 | 9 | 7 | 50 | 50 | 55 | 0 | \＄0 | 50 | 5 | 7 | 5 | 65 | 30 | 30 | 50 | 50 | 50 | 57 | W | 31 | 5 |
|  | 5 | 7 | 50 | 50 | T | 50 | \％ | 55 | 7 | 90 | 54 | 50 | 7 | 5 | 65 | 50 | 0 | 50 | 90 | 54 | 50 | 65 | 30 | 知 |
|  | 4 | \％ | 30 | 7 | 70 | 51 | 50 | 70 | 40 | 50 | 55 | 50 | 7 | 50 | 70 | 50 | 5 | \％ | 55 | 50 | 70 | 6 | 4 | 50 |
|  | 6） | 36 | 30 | 50 | 55 | 50 | 50 | 55 | \＄0 | 50 | 5 | 50 | \％ | 50 | is | 50 | 50 | 50 | 50 | 50 | 61 | 6 | 5 5 | 50 |
|  | 68 | 0 | \％ | 50 | 9 | 107 | 50 | ${ }^{3}$ | 50 | 50 | 50 | 3 | 40 | 5 | 50 | 50 | 40 | 50 | 50 | 50 | 51 | 4 | 5 | 7 |
|  | 51 | 92 | 7 | 50 | 46 | 60 | 50 | 55 | 0 | 50 | 50 | 50 | 0 | 50 | \％ | 50 | 70 | 50 | 50 | 50 | 50 | 6 | 50 | 50 |
|  | 6 | 9 | 2 | 50 | \％ | 6 | 50 | 30 | 5 | 50 | 50 | 6 | 4 | 4 | 70 | 39 | \％ | 3 | 30 | 5 | 50 | 65 | 50 | 50 |
|  | 7 | 53 | 3 | 5 | 55 | 55 | 50 | 70 | 7 | 50 | 50 | 4 | 7 | 50 | 55 | 30 | 3 | 7 | 50 | 3 | ＋1 | 54. | 50 | 30 |
| 嵝 | 6 | 7 | 约 | 5 | 9 | 17 | 5 | 4 | 5 | \％ | 50 | 5 | 7 | 5 | 50 | \％ | 50 | 59 | 50 | 50 | 54 | 35 | 5 | 3 |
|  | 6 | 7 | \％ | \＄0 | 50 | 0 | 5 | 65 | 30 | 50 | 50 | 50 | T | 50 | 5 | 50 | 30 | 5 | 50 | 3 | 37 | 5 | 5 | 30 |
|  | 4 | $\cdots$ | 50 | 50 | 50 | W | 5 | 65 | 30 | 30 | 50 | 3 | 70 | 3 | 50 | 50 | 30 | 50 | 50 | 5 | 57 | 5 | 30 | \％ |
|  | 7 | 70 | \％ | 多 | 匂 | \＄ | 50 | 70 | 6 | 50 | 50 | 50 | \％ | 30 | 5 | 5 | 50 | 60 | 50 | 50 | 7 | 6 | 3 | 5 |
|  | 6 | 7 | 50 | \＄0 | 50 | F | 35 | 38 | 50 | 3 | 0 | 50 | 70 | 30 | 50 | 5 | 50 | 50 | 30 | 50 | 57 | 63 | 50 | 5 |
|  | 4 | 77 | 50 | 50 | 50 | 7 | 70 | 55 | \＄5 | 50 | 50 | 55 | 10 | 50 | 50 | 50 | 30 | 50 | 50 | 90 | 87 | 65 | 50 | 30 |
|  | 7 | 鲬 | 5 | 4 | 9 | 51 | 310 | 5 | 5 | 50 | 50 | 49 | 3 | 70 | 58 | 30 | 50 | 7 | 50 | 5 | \＄1 | Sil． | 50 | 50 |
|  | 74 | 13 | 3 | 53 | 4 | 31 | 5 | 70 | 46 | 54 | 50 | 50 | 20 | 50 | $\underset{ }{*}$ | 30 | 50 | 70 | 50 | 51 | 0 | 63 | 3 | 50 |
| b2w | 7 | 8 | 50 | 3 | STS | 50 | 30 | 5 | \％ | 50 | 50 | 50 | 7 | 50 | 50 | 50 | St | 9 | 50 | 50 | 结 | 4 | \＄7 | 52 |
|  | 63 | 4 | 5 | 5 | 9 | 30 | 50 | 55 | 9 | 50 | 3 | 59 | 70 | 5 | －30 | 5 | 55 | 90 | 5 | 50 | 3 | 4 | 5 | 50 |
| － | 0 | \％ | 5 | 9 | 5 | 5 | 50 | 5 | 50 | 0 | 50 | 50 | 70 | 50 | 59 | 50 | 5 | 50 | \％ | 5 | 60 | 4 | 51 | 30 |
|  | 77 | 6 | 30 | 55 | 45 | \＄1 | 50 | 70 | 5 | 5 |  | 6 | 7 | 30 | 50 | 50 | 50 | 73 | 50 | 53 | 50 | 5 | 50 | 50 |
| 3） | 57 | 50 | 30 | 50 | 5 | 51 | 0 | 5 | 5 | 50 | 50 | 3 | 70 | 50 | 50 | 50 | 50 | 50 | 50 | \％ | 50 | 5 | 50 | 50 |


|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | ， | 48 | 6］ | ， |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ftay | \％ | \％ | 5 | 50 | 9 | 70 | 5 | 5 | 50 | 7 | 50. | 50 | 40 | 7 | 73 | 3 | 50 | 50 | 50 | 50 | 5 | 70 | 6 |
| xay | 50 | 50 | H | 50 | 5 | 50 | \＄ | 70 | 50 | 5 | 70 | 5 | \％ | 50 | 35 | 6 | 50 | 30 | 33 | 50 | 50 | 50 | 5 |
| ＋ | 30 | 50 | 50 | 50 | 50 | 50 | 50 | 70 | 50 | 7 | N | 50 | 50 | 50 | 50 | so | 50 | 2 | 3 | 5 | 50 | so | 50 |
| sict | 4 | 50 | 70 | 60 | so | 50 | 5 | \％ | 50 | 60 | 7 | 50 | 50 | 50 | 5 | 75 | 50 | 6 | 0 | 50 | 3 | 30 | 50 |
| \％ | 5 | 0 | 5 | 4 | 70 | 50 | 50 | 53 | 5 | 50 | 65 | 60 | 50 | \％ | 5 | 50 | 56 | W | ¢ | 2 | 73 | 53 | 5 |
| Exa | S0， | 70 | s | \％ | 7 | 51 | 5 | 53 | 5 | 70 | 5 | \＄ | $3{ }^{3}$ | 7 | 5 | 50 | 50 | $\pi$ | 50 | 70 | 7 | 38 | So |
| $37 \%$ | 0 | 5 | 62 | 50 | S0 | 50 | 64 | 10 | 6 | So | 产 | 5 | 50 | 50 | 50 | 73 | 50 | 65 | 70 | so | 50 | 50 | 50 |
| cide | 910 | 70 | 36 | 4 | 珎 | 50 | 5 | S0 | 50 | 7 | 6 | 2 | 50. | 70 | 6 | 50 | 53 | 2 | 5 | 0 | 7 | 50 | 50 |
| \％ 5 | $\leftrightarrow$ | 50 | ${ }_{4}^{4}$ | 30 | 50 | 50 | ${ }^{3}$ | 70 | $\underline{3}$ | 5 | 0 | 50 | 称 | 50 | 50 | \％ | 4 | 6 | 6 | 50 | so | 4 | 0 |
| \％－ | 9， | 50 | 0 | 50 | $8{ }^{3}$ | 30 | 50 | $\pi$ | 50 | 7 | 70 | 50 | 5 | 5 | 50 | 7 | 5 | 5 | \％ | 50 | $0 \cdot 1$ | 30 | 5 |
|  | 6 | 5 | 7 | 5 | 5 | 50 | 30 | 坴 | 50 | $\%$ | no | 50 | 5 | 50 | 50 | 73 | 50 | 0 | 70 | 50 | 50 | 30 | 50 |
| H20： | 0 | 30 | 5 | 50 | \％ | 6 | 5 | 50 | 50 | T | 50 | So | so | 12 | 13 | 50 | 65 | 70 | 5 | 50 | 5 | 75 | 50 |
| ＊ | 50 | 70 | 5 | 30 | $\omega$ | 6 | 30 | 50 | 5 | 0 | 5 | \＄ | 50 | 7 | 3 | 50 | 5 | 70 | 5 | 4 | 70 | 10 | 90 |
| 永了 | 50 | 60 | 50 | 50 | 50 | 50 | 50 | 70 | 53 | 15 | 30 | 45 | 30 | 50 | 30 | 46 | 5 | $\mathbf{0}$ | 5 | 53 | 50 | 50 | 0 |
| Cax | 40 | 50 | 74 | 5 | 30 | 5 | 30 | \％ | 5 | \％ | 2 | 50 | 50 | 50 | $\mathbf{3}$ | 8 | 5 | 70 | 45 | 50 | 50 | 50 | 0 |
| － 4 | 7 | 60 | $\cdots$ | 4 | 5 | 30 | 50 | 4 | 50 | 70 | $\pi$ | 9 | \＄0 | 6 | 6 | 4 | 4 | 9 | 50 | 4 | 2 | 50 | 59 |
| － x 人7 | 40 | 6 | 5 | 50 | 50 | 50 | S | 7 | 5 | 4 | 7 | 6 | 50 | 50 | 35 | 45 | 53 | 50 | 5 | 50 | So | S0 | 5 |
|  | ¢ | T | 5 | 50 | S0 | 5 | 50 | 30 | 50 | 4 | $\pi$ | 6 | 30 | 5 | 5 | 4 | \＄ | 50 | \＄ | g0 | 5 | 50 | 30 |
| \％ 4 | 5 | 50 | 5 | 5 | 5 | 30 | 33 | 20 | \％ | 5 | \％ | \＄ | 5 | 50 | 30 | 5 | \＄ | 5 | 4 | 50 | 50 | 50 | 30 |
| Forstit | 64 | 50 | 氷 | 5 | 50 | 50 | 5 | 0 | 50 | 55 | 70 | 50 | 50. | 50 | 50 | 4 | 4 | \＄ | 70 | So | 50 | 50 | 0 |
|  | 50 | \％ | 62 | 56 | 55 | 50 | 50 | \％ | 50 | \％ | 0 | 50 | 50 | 3 | 50 | 0 | 5 | 35 | 6 | 6 | 5 | 5 | 5 |
| 5xay | 50 | 1 | $\underline{8}$ | 4 | 6 | 50 | 30 | 3 | 50 | \％ | 0 | 品 | 5 | 6 | 63 | 4 | 5 | $\pi$ | S0 | S | T | 5 | 5 |
| NxM00 | （1） | 50 | 56 | 50 | 50 | 30 | 53 | \％ | $\checkmark$ | 5 | 7 | 5 | 5 | 50 | so | 6 | 5 | 35 | 63 | 50 | 50 | 5 | 50 |
| ＋ | 5 | T0 | 50 | 7 | 70 | 75 | 50 | \％ | 60 | 7 | 50 | 50 | 5 | 75 | 7 | 50 | 50 | ग | 50 | 50 | 55 | 70 | 5 |
| ARNT | 59 | 50 | 5 | H | $\omega$ | 50 | 50 | 70 | 50 | 0 | 3 | 35 | 50 | 0 | 5 | 50 | 30 | ${ }^{5}$ | S5 | 6 | 6 | 50 | 50 |
| Wex | 53 | $\pi$ | 50 | 65 | \％ | 70 | 5 | 50 | 5 | 7 | $5)$ | 5 | 5 | 7 | 7 | 30 | is | ${ }_{4}$ | 50 | 50 | T | 7 | 30 |
| 3X\％ | 5 | 70 | 30 | 2 | 60 | 60 | 3 | 5 | 70 | 70 | 53 | 50 | 5 | 70 | 70 | 10 | \％ | \％ | 50 | 70 | 7 | 50 | \％ |
| 3 | 3 | \％ | 30 | 67 | 4 | 50 | 50 | \＄0 | 6 | 2 | \＄ | 4 | 50 | 0 | 20 | 50 | 50 | 0 | 50 | 0 | T | 53 | 5 |
|  | 5 | 0 | 5 | 6 | 70 | 5 | 5 | 60 | 6 | 73 | 70 | 5 | 5 | 55 | 6 | 50 | \＄0 | 6 | 50 | 2 | 79 | 50 | 50 |
|  | $\cdots$ | 50 | 7 | 30 | 50 | 50 | ${ }^{\text {A }}$ | $\cdots$ | 53 | so | \％ | 5 | so | 50 | 50 | 7 | 53 | 65 | \％ | 30 | 50 | 5 | 50 |
|  | 50 | 86. | 6 | \＄ | W | 50 | 3 | 7 | 30 | $\pi$ | 70 | 53 | F | 55 | 5 | 50 | 5 | 5 | 65 | 55 | 5 | 50 | 5 |
|  | 90 | 30 | 5 | 5 | 50 | 50 | 5 | 3 | 4 | 55 | 70 | 5 | 50 | 50 | 5 | 65 | 0 | 50 | 55 | 5 | 5 | 3 | 50 |
|  | 50 | 5 | 5 | 50 | 5 | 50 | 5 | 7 | 6 | 53 | \％ | 50 | 50 | 50 | 50 | 6 | 3 | 5 | 5 | So | 50 | 50 | 50 |
|  | 59. | $\%$ | 50 | 75 | 5 | \％ | 50 | 50 | 70 | 70 | 5 | 50 | 50 | 75 | R | 50 | 5 | 8 | 30 | 5 | 6 | 70 | 60 |
|  | W | 51 | \＄ | 50 | 5 | 50 | 5 | 70 | 3. | 35 | 7 | 5 | 5 | 50 | 35 | 6 | 5 | 5 | 5 | 5 | 30 | 8 | 5 |
| F－ | 0 | 50 | $\pm$ | S | 50 | \％ | 50 | 2 | 50 | 6 | 70 | 50 | 50 | So | 50 | 7 | 35 | \％ | 7 | 50 | （2） | 30 | 5 |
|  | 5 | 30 | 50 | 2 | 4 | 7 | 5 | 5 | 6 | 30 | S | 50 | 5 | 7 | 7 | 50 | 35 | 5 | 50 | 50 | 80 | 3 | 4 |
| 1. | 50 | $\pi$ | 2 | 67 | \％ | 50 | 50 | 5 | 50 | 3 | 4 | 70 | 5 | 70 | 4 | 50 | 60 | 70 | 30 | 70 | 75 | 55 | 50 |
| 존 | 30 | 70 | 50 | 6 | 5 | 3 | 50 | S0 | \％ | 0 | 50 | 50 | 5 | 0 | 0 | 3 | ¢ | 5 | \％ | 50 | 50 | \＃ | 6 |
|  | 5 | S0 | ＊ | \％ | 30 | 30 | 50 | 70 | 50 | 65 | 7 | 50 | So | 5 | 5 | 6 | 5 | 4. | 6 | 50 | 5 | 50 | 0 |
| $x=$ | 4 | 31 | 5 | 50 | 50 | 50 | 60 | 20 | 5 | 50 | 70 | 50 | 50 | 50 | 5 | \％ | 5 | 40 | 6 | 50 | 50 | 50 | 50 |
| 5\％ 27 | 50 | 30 | 33 | 50 | 50 | 5 | 50 | 2 | 55 | 5 | 70 | 55 | 50 | 50 | so | 8 | 0 | 50 | 55 | 9 | So | 50 | 80 |
| West | 5 | ¢ | \％ | 67 | T， | 5 | 5 | 5 | 5 | 0 | W | 5 | 50 | 7 | 0 | 5 | 50 | 70 | 30 | 7 | 75 | 50 | 50 |
| \％mat | 50 | 59 | 50 | 7 | 70 | 5 | Si | 5 | 6 | 20 | 5 | 6 | 50 | 70 | 6 | 8 | 50 | 2 | 50 | 70 | 25 | 50 | 0 |
| \％ | \％ | 50 | 5 | 5 | 30 | 30 | 50 | 70 | 55 | 5 | 7 | 50 | 5 | 50 | 5 | 4 | 50 | 50 | 3 | 50 | 50 | 50 | 30 |
| Whect | 5 | 50. | 7 | 50 | So | 50 | 50 | \％ | 50 | T0 | 0 | 50 | so | 50 | 3 | 30 | 50 | 4 | 4 | 55 | 50 | 50 | 50 |
| \％3／ | \％ | 70 | 5 | 6 | ${ }_{4}^{4}$ | 30 | S | 9 | 6 | 0 | 6 | 4 | 50 | 70 | 4 | 50 | 3 | 6 | 30 | 2 | T0 | 50 | 50 |
| 全家 | 5 | 70 | 5 | \％ | To． | 9 | 50 | 5 | 53 | 0 | 的 | 5 | 9 | $\cdots$ | 6 | 5 | \＄0 | T | 30 | 70 | 73 | 50 | 5 |
| 1200\％ | 5 | Sol | \＄ | 30 | 5 | \％ 0 | \％ | N | 9 | $\pi$ | 70 | 5 | 5 | 5 | 5 | N | 50 | 5 | 50 | 50 | 5 | 5 | 50 |
| 2x ${ }^{2}$ | 5 | 50 | 5 | 50 | 5 | \％ | 50 | 5 | 30 | 3 | 0 | 5 | 50 | 50 | so | S0 | 5 | N | 50 | 50 | \＄0 | 50 | 50 |
| 鋃絞， | 6 | 70 | 50 | 5 | g | 50 | 5 | \％ | 50 | 0 | 30 | 70 | 50 | 50 | 55 | 6 | 3 | 50 | 59 | 35 | 50 | 50 | 50 |
| 4s\％ | 31 | \％ | 3 | 2 | 7 | 7 | 50 | 5 | So | $\cdots$ | 50 | 50 | 50 | 0 | 7 | 3 | 5 | 50 | 30 | 50 | 9 | 7 | 6 |
| \％ | 5 | 7 | 5 | 5 | 5 | 75 | 9 | 50 | 35 | \％ | \％ | 53 | 50 | 71 | 7 | 5 | 62 | 4 | 5 | 59 | 5 | 0 | 60 |
| \％． | 50 | \％ | 52 | S | 50 | 30 | 35 | $\pi$ | 60 | 5 | 70 | 35 | S0 | 30 | 5 | 6 | 5 | 3 | 6 | 50 | 30 | 50 | 5 |
| 努： 2 ， | 50 | 50 | 50 | 72 | 73 | 7 | 5 | 5 | 50 | \％ | 50 | 50 | 50 | 70 | 50 | 50 | 5 | 50 | 30 | 50 | 50 | 65 | 80 |
| 6xasit | 50. | 70 | S | 3 | 50 | 3 | 30 | so | 5 | \％ | 50 | 5 | 5 | 7 | 7 | 3 | 01 | 35 | 30 | 50 | 6 | 70 | 5 |
| 8）$\times$ \％ 3 | 50 | \％ | $\%$ | $\cdots$ | 6 | 3 | 50 | 50 | 50 | $\dagger$ | 6 | 90 | S | \％ | 6 | 55 | 6 | 交 | 50 | 70 | 3 | 5 | 50 |
| －${ }^{\text {cost }}$ | 50 | 30 | ${ }_{4}$ | 67 | 7 | 5 | 5 | 50 | 5 | 7 | 30 | 6 | 5 | 2 | 29 | 50 | 5 | 7 | 50 | 70 | 73 | 65 | 50 |
|  | 50 | 5 | 7 | 50 | 5 | 50 | 30 | T | 50. | 0 | 7 | 50 | 50 | 50 | 50 | 3 | 50 | \％ | \％ | 3 | 50 | 50 | 50 |
| C，\％\％ | 50 | 50 | 5 | 5 | 4 | So | 50 | 0 | S | \％ | 70 | 55 | 50 | 5 | 52 | 3 | 50. | 5 | 5 | 6 | 6 | 5 | 5 |
|  | 50 | so | 74 | 6 | 53 | 30 | 50 | \％ | 50 | （2） | 3 | 0 | 50 | \％ | 63 | 50 | 65 | 0 | 50 | 6 | 4 | \＄ | 5 |


|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5 | 60 | 50 | 64 | 7 | 50 | 50 | 4 | 60 | 73 | $\%$ | 6 | 39 | 6 | \＄5 |  | 50 | 60 | 0 | 76 | 6 | 50 | 50 |
|  | 5 | 50 | 50 | 70 | 70 | 9 | 50 | 55 | 6 | 70 | 65 | 55 | 5 | 5 | 65 | 50 | 50 | 7 | 50 | 70 | \％ | 50 | 50 |
|  | 3 | 5 | 56 | 5 | 5 | 50 | 50 | \％ | 3 | 5 | 7 | 5 | 30 | 50 | 4 | 65 | 50 | 50 | 55 | 50 | 5 | 50 | 30 |
|  | 50 | \％ | 3 | 67 | 6 | 50 | 50 | 4 | 5 | S | 5 | 7 | 50 | 70 | 65 | 50 | 55 | 7 | 50 | 70 | 7 | 50 | 50 |
|  | 64 | 50 | 54 | 50 | 50 | 50 | 55 | 70 | 4 | 55 | 70 | － 5 | 50 | 50 | 50 | 65 | 65 | 5 | 6 | 50 | 5 | 50 | 50 |
|  | 5 | 30 | 56 | 50 | 50 | 50 | 50 | 7 | 5 | 40 | 70 | 53 | 5 | 50 | 6 | E | 50 | 5 | 53 | 5 | 5 | 50 | 50 |
|  | 5 | T0 | 5 | 4 | 6 | 70 | 50 | 50 | 65 | 7 | 50 | 50 | 55 | 75 | 7 | 50 | 35 | 65 | 50 | 55 | 6 | 75 | $\mathbf{S}$ |
|  | 50 | 50 | 52 | \＄0 | 30 | 50 | 50 | 万 | 5 | 36 | 7 | 5 | 50 | 50 | 50 | 65 | 50 | 50 | 5 | 50 | 50 | 50 | 50 |
|  | 5 | 70 | 50 | 72 | 0 | 30 | 50 | 50 | 79 | 4 | 30 | 65 | 50 | 72 | 73 | 50 | 50 | 6 | 50 | 65 | 70 | 6 | 35 |
|  | 5 | 50 | 50 | 64 | 7 | 50 | 50 | 70 | 5 | 30 | 70 | 5 | 50 | 60 | 55 | 50 | 50 | 0 | 50 | 68 | － | 50 | 59 |
|  | 50 | 5 | 5 | 56 | 50 | 50 | 50 | 20 | 50 | 6 | 20 | 5 | 50 | 50 | 30 | 4 | 50 | 30 | 55 | 65 | 55 | 5 | 50 |
|  | 50 | 59 | 50 | 50 | 50 | 50 | 59 | 70 | 50 | 7 | 20 | 50 | 30 | 50 | 50 | 55 | 30 | 6 | 65 | 30 | 50 | 50 | 50 |
|  | 30 | 0 | 50 | 56 | 5 | 5 | 5 | 0 | 5 | \％ | 2 | 2 | 50 | 55 | 5 | 5 | 50 | 50 | 50 | 6 | 5 | 50 | \＄0 |
|  | 30 | 50 | 52 | 50 | 50 | 50 | 50 | 30 | 5 | 5 | 20 | 59 | 50 | 50 | 30 | 6 | 30 | 30 | 3 | 30 | 50 | 30 | 30 |
|  | 50 | 7 | 50 | － | 70 | 5 | 3 | 35 | 5 | ${ }_{0}$ | 6 | 20 | 50 | 7 | 6 | 30 | 5 | \％ | 3 | 9 | 13 | 50 | 50 |
|  | 50 | 34 | 6 | 50 | 50 | 50 | 36 | $7 \%$ | 50 | 9 | 7 | 50 | 50 | 50 | 35 | 5 | 50 | 50 | 6 | 5 | 50 | 5 | 50 |
|  | S | 7 | 50 | 7 | 0 | 50 | 50 | 50 | 5 | 0 | 5 | \＄ | 50 | 5 | 71 | 40 | 6 | \％ | 50 | 5 | 5 | 30 | 6 |
|  | 30 | 60 | 50 | 56 | 0 | 59 | 93 | 70 | 53 | \％ | 7 | 65 | 50 | 5 | 50 | 5 | 50 | 30 | 50 | 6 | 60 | 50 | 50 |
|  | 50 | 79 | 50 | 7 | 7 | 70 | \＄0 | 50 | 5 | $\pi$ | \＄ | 50 | 50 | 75 | 73 | 30 | 5 | 30 | 39 | \＄ | 50 | 3 | 5 |
|  | 0 | 9 | 81 | 50 | 90 | 30 | 6 | \％ | 50 | 7 | 72 | 5 | 60 | 50 | 30 | 7 | 6 | \％ | 70 | 50 | 50 | 50 | 50 |
|  | 44 | 30 | 67 | 5 | 9 | 50 | 6 | 79 | 4 | 30 | 7 | 3 | 3 | 50 | 50 | 7 | 5 | 6 | 4 | 50 | 50 | 50 | 50 |
|  | 50 | 2 | 6 | 4 | 4 | 5 | 5 | 5 | 50 | 70 | 4 | 7 | 50 | 70 | 65 | 3 | 40 | 7 | 3 | 70 | 55 | 5 | 3 |
|  | 50 | 50 | 56 | 50 | 50 | 50 | 55 | 20 | 50 | 50 | 70 | 50 | 5 | 50 | 50 | 70 | 30 | 55 | 65 | 50 | 30 | 59 | 50 |
|  | 40 | 59 | 3\％ | 50 | 50 | 90 | $6{ }^{6}$ | 20 | 4 | 50 | 7 | 59 | 50 | 50 | 5 | 70 | 30 | 60 | 65 | 30 | 50 | 50 | 56 |
| 号：mow | 30 | 50 | S | 50 | 50 | 50 | 50 | 70 | 69 | 5 | 30 | $\checkmark$ | 50 | 59 | 50 | 6 | 5 | 50 | 55 | 5 | 50 | 30 | 5 |
|  | 3 | 50 | 50 | 64 | 5 | 50 | 50 | 4 | 5 | 70 | 70 | 53 | 50 | 6 | 3 | $\sim$ | 50 | 6 | 5 | 70 | 70 | 59 | 54 |
|  | 4 | 50 | 50 | 39 | 30 | 50 | 50 | 70 | 50 | 7 | T | 50 | 5 | 30 | 30 | 50 | 55 | 7 | 4 | 50 | \＄0 | 30 | 9 |
|  | 0 | 50 | 5 | 50 | 50 | \％ 0 | 50 | \％ | 6 | 3 | 7 | 50 | 50 | 50 | 50 | 5 | 50 | 50 | 50 | 50 | 59 | 50 | 50 |
|  | 50 | 50 | 5 | 30 | 50 | 30 | 5 | 7 | 0 | \％ | 70 | ＊ | 50 | 30 | 50 | 6 | 50 | 50 | 55 | 5 | 30 | 50 | 50 |
|  | 50 | 70 | 5 | 64 | 8 | 50 | 50 | 6 | 5 | \＄ | 70 | 3 | 50 | 6 | 4 | 5 | 5 | 7 | 50 | 70 | 70 | 50 | 50 |
|  | 50 | 50 | 50 | 3 | 50 | 50 | 5 | T0 | 45 | 5 | 5 | \％ | 50 | 30 | 50 | 7 | 50 | 5 | 5 | 50 | 50 | 50 | 5 |
|  | 5 | 50 | 5 | 5 | 50 | 30 | 53 | 70 | 6 | 3 | 7 | 3 | 5 | 50 | 59 | 70 | 50 | 5 | 6 | 5 | 50 | 50 | 5 |
|  | 60 | 50 | 4 | 60 | 50 | 50 | 5 | \％ | 30 | 40 | 70 | 50 | 50 | 3 | 50 | \％ | 50 | 55 | 65 | 50 | 50 | 50 | 50 |
|  | 5 | 30 | 50 | 30 | 50 | 50 | 4 | 7 | 4 | 50 | \％ | 50 | 50 | 50 | 50 | 7 | 50 | d 5 | 6 | 50 | 0 | 50 | 50 |
|  | 4 | 7 | 等 | 0 | 50 | 50 | $\pm$ | 70 | 6 | 50 | 3 | 50 | 50 | 50 | 50 | 7 | 50 | 40 | 4 | 50 | 50 | 50 | 54 |
|  | 50 | 30 | 50 | \％ | 60 | 58 | 50 | 50 | 27 | 70 | 3 | 5 | 50 | 70 | \％ | 50 | 50 | 70 | 50 | 4 | 7 | 53 | 50 |
|  | 5 | 4 | － | 50 | 50 | 50 | 5 | 7 | \＄ | 6 | 0 | d | 30 | 59 | 50 | 45 | 45 | 50 | 6 | 50 | 30 | 50 | 50 |
|  | 50 | 70 | 4 | 6 | 30 | 50 | 50 | 7 | 30 | 70 | 70 | 4 | 50 | 60 | 55 | 55 | 50 | 70 | 5 | 6 | 65 | 30 | 50 |
|  | 50 | 50 | S2 | 30 | \＄ | 50 | 50 | \％ | 40 | 58 | 70 | 50 | 50 | 50 | 50 | 65 | 50 | 50 | 50 | 50 | 5 | 5 | 50 |
|  | 5 | 50 | 50 | 0 | 5 | 50 | 30 | 70 | 30 | N | 7 | 6 | 30 | 50 | 55 | 50 | 5 | 0 | 50 | 6 | 60 | 50 | 59 |
|  | 4 | 50 | － 4 | 50 | 50 | 50 | 35 | 2 | 50 | 60 | 70 | 50 | 50 | 90 | 50 | 9 | 56 | 6 | 0 | 50 | 5 | 5 | 50 |
|  | 30 | 5 | 30 | 7 | 0 | 20 | 50 | 50 | 70 | 0 | 50 | 90 | 3 | 7 | 7 | 50 | 50 | 5 | 50 | 35 | 5 | 7 | 3 |
|  | 59 | 60 | 4 | 60 | S | 30 | 50 | 70 | 50 | 2 | 70 | 0 | ， | 60 | 5 | 56 | 50 | 70 | 55 | 65 | 0 | 59 | 50 |
|  | 4 | 50 | 56 | 50 | 50 | 50 | 64 | T0 | 4 | 5 | \％ | 3 | 50 | 54 | 50 | 5 | 8 | 60 | 65 | 30 | 50 | 50 | 50 |
| ${ }^{2}$ | 7 | 50 | 50 | 59 | 30 | 10 | 4 | 70 | 50 | \＄ | 70 | 50 | 40 | 50 | 30 | 7 | 4 | 70 | 70 | 50 | 5 | 30 | 50 |
|  | 70 | 50 | － 7 | 39 | 30 | 50 | 50 | 70 | 4 | 6 | 70 | 4 | 50 | 9 | 50 | 70 | 4 | 70 | 70 | 50 | 30 | 30 | 5 |
|  | 9 | 30 | 50 | 67 | 50 | 5 | 50 | 50 | \％ | 万 | 5 | 4 | 50 | 5 | 7 | 50 | 9 | 7 | 5 | 73 | 70 | \＄5 | 50 |
|  | 50 | 50 | 58 | 50 | 5 | 50 | 53 | 70 | 5 | 5 | 70 | 35 | 50 | 50 | 5 | 4 | 50 | 53 | 35 | 50 | 50 | 30 | 5 |
|  | 9 | 50 | 57 | 5 | 30 | \％ | 50 | 0 | 35 | 5 | 50 | 50 | 50 | 50 | \＄ 5 | 3 | 50 | 5 | 35 | 50 | 50 | 5 | 5 |
|  | 50 | 50 | 5 | 5 | 30 | 30 | 50 | 7 | 5 | 38 | 10 | 3 | 5 | 50 | 30 | 6 | 50 | 50 | 50 | 50 | 50 | 50 | 50 |
|  | 50 | 70 | － | 64 | 6 | 50 | 50 | 0 | 59 | 5 | 4 | 0 | 50 | 7 | 6 | 5 | 4 | 30 | 34 | 70 | 75 | 50 | 50 |
|  | 50 | 50 | 梁 | 50 | 50 | 5 | 9 | 7 | 6 | S | 坴 | \＄0 | 50 | 50 | 50 | 4 | 50 | 50 | 53 | 90 | 9 | \＄0 | 50 |
|  | 50 | 50 | \＄2 | 5 | \＄0 | 5 | 30 | 7 | 5 | 5 | 70 | 53 | 5 | 50 | 50 | 85 | 50 | 50 | 5 | 50 | 50 | 50 | 50 |
|  | 50 | 50 | 30 | 4 | 70 | 50 | 3 | 3 | 55 | 7 | 65 | 5 | 30 | 70 | 45 | 30 | \＄ | 70 | 30 | 70 | 7 | 50 | 3 |
|  | 50 | 79 | 4 | 4 | 6 | 50 | 30 | 30 | 50 | 0 | 45 | 0 | 50 | 79 | 45 | 5 | 4 | 70 | 50 | 30 | 75 | 50 | 50 |
|  | 50 | 50 | 4 4 | 50 | 50 | 50 | 50 | 70 | 5 | 70 | 20 | 59 | 5 | 50 | 50 | 35 | 50 | 6 | 6 | 35 | 50 | 5 | 50 |
|  | 50 | 50 | 这 | 50 | 60 | 50 | 50 | 7 | 50 | W | \％ | 5 | 30 | 55 | 52 | 30 | 50 | 65 | 53 | 65 | 40 | 50 | 5 |
| ल，${ }^{3}$ | 5 | 56 | 50 | 30 | 50 | 50 | 30 | 7 | 30 | 告 | \％ | 5 | 50 | 5 | 50 | 3 | 30 | 6 | 50 | 65 | \＄ | 50 | 50 |
| 5 | 50 | 68 | 5 | 67 | 65 | 50 | 50 | 59 | 4 | 75 | 65 | 6 | 30 | 70 | 65 | 5 | 50 | 70 | 50 | 70 | 70 | 50 | 30 |
|  | 50 | 50 | 50 |  | 50 | 50 | 50 | 50 | 50 | 70 | 79 | 50 | 3 | 50 | 30 | 50 | 50 | 70 | 50 | 50 | 50 | 5 | 50 |





## APPEADIE 4

## TEE BTMULATTOM PROGRAM

## A生 1 Procran structure and outhine

The simulation program is a simple, single spreadsheet. Each iteration, or simulation, represents a calendar year's losses. A set of parameters is used and these can be changed to re-run the model.

There are three stages to each simulation. These are:
1: Generate the number of events in each period (year); 2: Generate the loss ammount for each event;
3: calculate layer and other costs and accumulate results.

The number of years to be simulated is one of the input parameters. Each stage is described in more detail below.

1. Generate the number of events in each period (year).

A unit uniform random number is generated using the spraadsheet RANDOM function and compared with cumulative poisson distribution values to obtain the number of events during the year. In the worksheet this is achieved by using a Look-Up table with the cumulative frequencies scaled by 1000 and rounded to integer values and the random number generated is multiplied by 1000 and rounded to its integer value and then used to obtain the number of events in the year. The program allows a maximum of geven events in any one year.

The following extract shows a typical calculation with a poisson parameter of 0.9 .

FREQUENCY LOOK-UP TABLE

| CUM FREQ | 0 | 407 | 772 | 937 | 987 | 998 | 1000 |
| :--- | :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| NO EVENTS | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|  |  |  |  |  |  |  |  |
| SIMULATED RANDOM: | .91234 | 2 |  |  |  |  |  |
| NUMBER OF EVENTS: |  | 2 |  |  |  |  |  |

Here the value 912 is used in the look-up table and results in 2 (as 912 exceeds 772 and is below 937 ) being selected.

## 2: Generate the loss ampount for each event.

For each of the losses generated at stage 1 , if any, two more unit randoms are generated. The firgt is used to decide whether the claim is going to be smali, that is less than 10, or large. This is achieved simply by comparing the value with an input parameter. In the example here it is assumed that . 33 of claims are small so that a unit randon at or below 33 results in a small claim and one above in a large ciaim.

The second unit random is then used to "generate" the event loss. The extract below shows some actual values with a year in which two events "occur".

|  | ${ }_{1}^{\text {CLAIM }}$ | $\underset{2}{\text { CLAIM }}$ | $\underset{6}{\text { CLATM }}$ | $\underset{7}{\text { CLAIM }}$ | $\begin{gathered} \text { TOTAL } \\ \text { FGU } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| LARGE/SM | . 35753 | . 13517 |  |  |  |
| SIZE RAN | . 02769 | . 84344 |  |  |  |
| LOSS VAL | 157.50 | 7.7786 |  |  | 165.28 |

For event 1 the claim is large, as the unit random that determines size, .35753 , exceeds the set paramater of .33. The program then useg the second unit random generated, 02769, to calculate the loss amount from the assumed pareto distribution. The calculation here ensuras that only values from 10 to the set upper limit, 500 is used in the example, are produced. This is done by calculating the range in the unit randon variate that this restriction implies and linearly transforming the actual unit random above to this range. The transformed value is .03658 and the simulated parato value obtained using the usual pareto random number generating formula:

$$
\text { Pareto }=\text { Scale * }(1 / \text { RAN })^{*}(1 / \mathrm{p})
$$

where RAN is random unit rectangular variate and $p$ is the Parato parameter. In the case above the scale parameter is 10 , the pareto parameter is 1.2 and the calculation is then

$$
\text { 10* }(1 / .03658) \wedge(1 / 1.2)=157.5
$$

The unit transformation is in two parts. First the value of the unit random corresponding to the set upper limit loss is calculated using the above pareto generation formula. In the example setting this to be $x$, where is the solution of:

$$
500=10 *(1 / x) \wedge(1 / 1.2)
$$

Solving produces $x=0.009146$. Unit random numbers below this value will generate pareto losses in excess of the set limit of 500. The truncated pareto that is required is then obtained by linearly transforming the unit interval ( 0,1 ) to the interval (0.009146, 1). The transformation, using the value above of 0.02769 is :

$$
.009146+0.02679=(1-.009146)=0.03569
$$

as calculated by the program and used to obtain the pareto value.

Different distributions can be generated by using similar techniques. Random numbers of the Weibuli distribution, for example, with scale parameter $b$ and shape parameter $c$ can be computed from random numbers of the rectangular unit variste RAN using the relationship:

$$
W: b, c=b *(-\ln R A N) \wedge(1 / c)
$$

where in denotes the natural logarithm.

The second loss in this particular year is a small (< 10) lose as the generated random, .13517, falls below the large/small loss determining parameter of 0.33 . The actual loss is then calculated by using the second unit random number generated for this loss, .84344 , and the empirical distribution for losses up to 10. The formula used in this particular example was:

Logs $=.45+4.05 * X+5.5 * X * X$, where $X$ is in $(0,1)$
which has a minimum loss of .45 and the maximum of 10.
In the example $X=.84344$ and the calculated value 7.7786 .

3: Calculate layer and other costs and accumulate results.

The final stage is to use the individual loss data to calculate and accumulate the required results for further analysis. This will depend on the purpose of the exercise.

In the actual version used for this paper four sets of "layer" information were collected.

The first is a counter of annual losses that exceed the set priority values, and is used in deriving the aggregate gross loss distributions and return periods. The second calculated and accumulated actual costs, and their squared values, to each of 10 layers of loss arising from the losses in the year. These values measure the insurers gross losses for each of these layers.

The chird set of values is a repeat of the second set with a limit, for any one layer, of two total losses. This measures the reinsurexs (maximum) losses assuming that the cover has one reinstatement. The fourth set of values calculates the amount of loss, for each layer, which would be reinstated under the normal catastrophe contract.

In the example above, as there are only two losses, the gross costs to the insurer are equal to the reinsurers costs. The amount of cover reinstated equals the cost of larger of the two losses. The program calculated values for the two losses above, for the layers chosen for illustration purposes in this paper, are as follows:

| Low | High | Gross | Reduced | Reinstated |
| ---: | ---: | ---: | ---: | ---: |
| 0 | 2.5 | 5.00 | 5.00 | 2.50 |
| 2.5 | 5 | 5.00 | 5.00 | 2.50 |
| 5 | 10 | 7.78 | 7.78 | 5.00 |
| 10 | 25 | 15.00 | 15.00 | 15.00 |
| 25 | 50 | 25.00 | 25.00 | 25.00 |
| 50 | 100 | 50.00 | 50.00 | 50.00 |
| 100 | 150 | 50.00 | 50.00 | 50.00 |
| 150 | 200 | 7.50 | 7.50 | 7.50 |
| 200 | 300 | .00 | .00 | .00 |
| 300 | 500 | .00 | .00 | .00 |
|  |  |  |  |  |

The epreadsheet also accumulates these figures, and their squares, so that averages and standard deviations can be calculated.

## Aㅗ4. 2 Jayers and reinstatements

As seen above it is possible to use the simulation programe to obtain estimates of any layer cost required. The number of reinstatements can be incorporated in the calculations, one is assumed in the above, and both risk rates on line and the impact of reinstatements estimated from output.

## A4.3 Software choice and program design

The spreadsheet package used for these calculations is CA Supercalc 5,1. The main reason for this choice, apart from familiarity, is the very flexible ITERation function available in the package. In this model the ITER function is used in logical statements to accumulate results and to terminate the simulation once a set number of iterations has been reached.

The ITERation function does not appear to be avallable in Lotus 123. Accumulating results would be more cumbersome and could well impact program execution times. No attempt was made to run the complete model in Lotus, or any other spreadsheet package, aithough the simulation part only requires the random number function and could be run in any of the popular spreadsheets.

## A4.4 Run times and limitations

The actual time taken to run any numerically demanding model will depend on hardware configuration as well as the efficiency of the software package used and the efficiency of the actual model design. No attempt was made to speed the calculations of the model described above and the actual spreadsheet was allowed to evolve rather than undergo a complete rewrite once completed.

For indication purposes using a 3865x Notebook running at 20 mhz and fitted with a maths co-processor the model takes approximately one (1) minute to simulate losses for 1000 periods. In order to calculate and accumulate all the information outlined above the time increases to four and a half minutes per 1000 periods.

The actual spreadsheet model (disk) size is 15k, which is very small, and could be fitted on a single screen. It will run casily, but slowly, on an 8086 machine such as the original PC or an Amstrad 1512.

The only (serious) limitation of the model in this environment arises from the use of the spreadsheet RANdom function as there is no facility to "replicate" a series of random numbers by specifying a seed or some starting condition. The actual process used by the spreadsheet to generate its unit random numbers is also not specified in the manuals and its characteristics can only be determined by actually generating and then testing sets of random numbers. No such testing was undertaken in this instance and it was implicitly assumed that any inaccuracy arising from this source would not be material.


[^0]:    Readers are reninded to reviow the many asaunptiona made in deriving all the results before uning these for any purpose.

