TECHNICAL RESERVES

Introduction

Much has already been written about different theoretical methods of calculating technical reserves for outstanding claims. The first part of the Group's work this year, therefore, was to consider the use of these techniques, in practice, and the problems facing the actuary in these situations.

In Paper 2, Jim Ryan considers the choice of reserving basis for use in various circumstances, and discusses the possibility of conflict between the different values derived. The paper also considers the role that case-estimating now has to play.

George Orros then shows, in Paper 3, the type of report on claim reserves which might be made to corporate management by the general insurance actuary. Use is made of a hypothetical set of claims data, which has been adapted from some actual DoT Returns data.

In addition to this type of report, however, the actuary must also measure the consistency of his claims estimation over time. Lawrence Eagles has therefore shown, in Paper 4, how this may conveniently be done, by reference to the same data set as was used in Paper 3. Of particular importance is the need to monitor the outcome of the original estimates made for each claim cohort.

The second main part of the Group's work was prompted by the thought that most claim projection techniques have been developed as a means of dealing principally with inflation, which has been the main problem facing satisfactory claim reserving in recent years. Stripping the inflation effect from these methods, are we left with very rigorous statistical techniques?

Clearly, in the time available to this Group, it was not possible to develop the study of statistical techniques to any great depth. It is clear, however, that actuaries should be considering how best the profession can improve methods of projection in the main problem areas.

The need for statistical methods is particularly acute in the case of risk classes subject to large random variation or where data is limited, and Papers 5 and 6 have therefore considered this problem.

In Paper 5, Chris Mellor outlines the problem of estimating reserves for classes with limited data, with particular reference to extension of the statistical method recently developed by Harry Reid in his Institute paper. The possibility of using information derived from case-estimates is also suggested.

John Ryan (Paper 6) considers practical methods of rate-making where data is limited and of low credibility, including the possibility of grouping data, or of using market data. The paper is illustrated by application to a long-tail liability account.

A final paper has been supplied by Henry Karsten (Paper 7), suggesting an adapted cost/claim projection method, where past experience is given credibility dependent on length of development. The method is an extension of the cost/claim procedure set out by Bennett & Taylor in their Hythe paper, which effectively only dealt with the zero credibility position.

John Taylor
June 1978

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Claims Reserving under different circumstances

This paper discusses the use of different bases and methods of estimation with regard to the type of business and the purpose of the investigation concerning outstanding claims reserves.

Solvency considerations

The solvency valuation should obtain a resulting figure for the liability that will prove to be adequate on a reasonable expectation of future events. The figure required is for the class or classes of business to be adequate as a whole and not necessarily for the individual risk groups within that class of business. If the view is taken that the outstanding claims reserves should be adequate and not become a drain on other resources of the company then a conservative valuation should be made so that a release of reserves would occur in the future with the intention of a controlled release though this would not be a large release. The aim is to minimise the margins and to maximise the security at the same time. This is an area where the Actuary should be useful. It does not need an expert statistician to allocate large margins to reserves to ensure their adequacy, but it is a waste of the company's capital tying it up in claims reserves if it is never likely to be needed. What is required are margins adequate most of the time so that there would be a small chance of the reserves being inadequate.

Though the main object of a solvency margin is for the overall account to be adequate, it is essential to know the position of each claim year so that steps may be taken to correct any adverse trend. If the latest claim years are making losses on its reserves held, while the earlier claim years are making large releases the overall account could show a release to profit indicating a healthy position overall financed by the earlier years' releases and hiding the poor later years. An internal valuation would show the true position of each claim year while the published valuation would show the overall position.

It is worthwhile considering whether to make any fluctuation margins in the valuation implicit or explicit. Implicit margins hide the amount of margins in the reserves and even though it is felt that the reserves are adequate by erring on the cautious side during a valuation no attempt is made to quantify the extent of the margins in the reserves. It could also hide the fact that there is very little fluctuation margin in the reserves.

The explicit margin method would involve making a 'best estimate' of the reserves needed, plus explicit margins for statistical and inflationary fluctuations, the company really needs this method for its internal disciplines. In order to make reasonable projections of cash flow, profitability and the future solvency situation an accurate estimate is required of the way the claims payments will behave in the future and the starting point is the most accurate estimate of the claims reserves possible.

There could be some difficulty with the DoT., if explicit margins are used when it comes to making applications for premium rate increases over the amount of margins needed in the projections of reserves. On the other hand, the DoT want to ensure that each company is adequately valuing their reserves which encourages a cautious basis. Also, there may be difficulty in persuading the Inland Revenue to allow explicit margins and lengthy debates could follow on what size these margins should be as more than adequate margins could be a means of deferring tax.

The showing of explicit margins may not please auditors as the valuation may appear inconsistent due to the possibility of margins changing from year to year, although these changes would have to be backed up by explanations.

When making a claims valuation the basis of the valuation could be stated just as the basis is stated in life business. This would be helpful to the authorities and the public and could demonstrate the strength of the company's valuation basis.

Premium considerations

With regard to premium rating, market considerations influence a company's premium rates to a great extent and also whether the company wants quality or quantity of business. For calculation of the reserves for risk premium considerations, an accurate basis needs to be used so that the profitability of the resulting premium rate can be guaged. If an over-conservative basis is used it may be thought that the profit return is not high enough so that profitable business may be turned away. A balance has to be struck between being optimistic and conservative. Another requirement is accurate estimates of the reserves for the risk groups or sub-classes within a class of business thus ensuring that high and low risk sub-classes are allocated reserves reflecting their experience. It is probably impractical to split a class into many risk groups during a valuation because of the small-ness of the data within those groups, it may simply be split into comprehensive and non-comprehensive, as in Motor Private Car.

Economic considerations

Probably the most unstable element in a valuation basis in recent years has been the estimate of future inflation likely to occur over the future run-off of claim payments. The inflation assumptions would start with a narrow range over the following year from optimistic to pessimistic and the range would diverge with each year into the future reflecting the uncertainty of the estimator. A solvency valuation would probably tend to use inflation rates at the pessimistic end of the range while a premium valuation would use a rate nearer the average of the range. The estimation of future inflation rates is much less of a problem for the property classes than the liability classes due to the shorter claims run-off for property classes and the liability classes being related to the more unstable earnings inflation.

Applications to data

At this time there is no test or method available which when performed can give the required degree of confidence that the reserves are accurate. This being so we should use a few methods which look at the data from different angles so a picture is built up of the account under investigation. Some suggested methods may be case estimates, a grossing up method and a claim size distribution method such as in D.H. Reid's recent paper, which all look at the account differently. As all the grossing up methods tend to point in the same direction, to use these methods together would not really tell us something different about the account.

If we consider some examples of data, case estimates could apply to all classes but for a very large class with say 100,000 claims or over, it is probably

impractical for the claims staff to case estimate all of them, so that this method would be used when the number of claims was of more manageable proportions say after two years from the claim year as most claims are settled within the first two years. Statistical methods would be used during the first or first and second years. When there are less claims outstanding as in the earlier claim years more emphasis would be placed on the case estimates and when there is only a small number of claims left of which most are probably large claims, reliance would be placed completely on the case estimates plus any margin thought to be required. When there is a large amount of data which is reasonably consistent, such as private car, grossing up methods are quite successful but these methods cannot be relied upon on their own as these methods do not react quickly to changes in the account as they rely heavily on past experience.

In a rapidly expanding account an investigation into the mix of the portfolio is needed as there could be large changes over a short period of time. If the class is split into risk groups then the proportion of claims in each risk group compared to its historical level may be compared along with the relative differences in cost between the risk groups. Using this information an estimate may be made of how much the overall class may be affected by the change in mix and adjustments made to the reserves. Alternatively, the individual risk groups could be projected using a cost per claim method but the influence of large claims could affect the results. Comparison of the average payments per claim from year to year, such as in Motor for the own damage section, may indicate whether costs are increasing more or less rapidly than inflation and a possible change in mix indicated.

With a very small amount of data, reliance on case estimates would be heavy as there could be large fluctuations from year to year by the impact of large claims. When considering reserves for premium rating it is necessary to make some estimate of the fluctuations which are likely to occur in this small account as the company may over-react to unfavourable underwriting years thus hindering its steady expansion of business by fluctuations in premium rates.

Case estimates

Case estimates where claims staff consider each claim individually and assess the payment to be made on that claim are traditionally required in a valuation. Because the human element is so important the case estimates are treated with caution. Considering that case estimates are so subjective and that claims estimates can be made satisfactorily by formula methods we may consider whether we still need case estimates if we have adequate data. Some uses which case estimates have are:

(1) to estimate the large claims which may take several years to settle. This would be required even if other methods were used to estimate the smaller claims.

(2) provide a measure of cost control for claims settlement departments.

(3) give the first indication of an account changing which formula methods based on historic data might not detect.

(4) splits the claims of a class into the individual risk groups so that the experience of each risk group may be investigated for rating purposes.

(5) claims may be grouped into different claims amount bands so that the effect of different excess levels may be guaged.

To make an estimate for a claim involves estimating the payment to be made as at the date of the estimate plus inflation up to the date of settlement. A case estimate may be made in two ways:

(a) in money values at the date of estimate by the estimator then inflation added in total taking account of the claims run-off for this particular class or risk group if required.

(b) giving the claims estimator an inflation rate or rates for the future and expecting the estimator to give the value of the claim as at his estimated date of settlement.

Either way the result is a subjective estimate of the claim value. Estimates are more difficult to make if a certain type of claim is increasing more than the rate of inflation e.g. if court awards are increasing in real terms with respect to serious injury claims.

Some guide as to the accuracy of the case estimates and hence how accurate the claims staff are, is needed. This can be done using past experience by comparing how accurate the original case estimates are with what has actually happened to date. The original case estimates can be compared with the payments to date plus outstanding case estimates of successive valuations and for each claim year. Estimating accuracy is a subjective assessment as over the years the case estimates are likely to show varying levels of accuracy and allowances should be made for differences between the actual and the expected levels of inflation in the past. The assumption is made that the case estimators have remained consistent in their estimating and have not decided to increase or decrease their estimates because they know that they have been undervaluing or overvaluing the estimates in the past. It is important that the estimators have not been encouraged to change their method of estimating as then past information would be of little value. Consideration of the trends in the accuracy of the case estimates will give some clue to this.

The original case estimates may also split into bands by claim amounts and the amount of drift may be assessed by following those claims in a particular band over all development years so that accuracy by claim amount is judged.

Having found the range of accuracy from the past along with any other statistical methods used, a margin may be added or subtracted to the case estimates to bring the reserve to the required level. This is useful when a breakdown into risk groups is required when the experience of risk groups is being investigated as for rating purposes as this margin will have been applied to all the individual case estimates.

Where the case estimates score over the statistical methods is when a type of claim arises for which there is very little past experience so that the statistical methods which rely heavily on past experience are unsuitable e.g. subsidence claims in the Householders class in the recent past.

J. Ryan

A technical reserves report from the general insurance actuary

Author: G. C. Orros

1. Introduction

This paper attempts to consider the case study of a hypothetical typical general insurance office. The purpose of the paper is to present the type of information that the general insurance actuary to this hypothetical office might have reported to the corporate management. Consideration has been restricted to the reporting of outstanding claim reserves and their implications on the underlying claims experience of the office. No consideration has been given to matters concerning unearned premiums, unexpired risks, expenses, solvency or liquidity; these being considered to be outside the scope of this paper. A brief attempt has, however, been made to measure historical claims fluctuations.

2. Scenario

This hypothetical office commenced writing business in 1970. The risk groups written are Private Cars, Householders (Property), General Liability and Employers Liability. The position is now early 1977, the 1976 Annual Accounts and 1976 Department of Trade (DOT) Annual Returns having just been completed and audited. The general insurance actuary has been supplied with the office's DOT Annual Returns for 1970 to 1976, no supplementary information being available. The actuary has been asked to report to the corporate management, on the basis of the DOT Annual Returns, as to the underlying claims experience and the position regarding outstanding claim reserves.

The remainder of this paper outlines some of the contents of the general insurance actuary's report. The tables at the end of this report summarise some of the results of the investigation. In practice, of course, supplementary information would be available, enabling more comprehensive actuarial investigations.

3. Conclusions

The main conclusions of the general insurance actuary's report might include the following:-

(i) It would appear that delays (weighted by claim amount) to the settlement of claims (from date of accident) may have been lengthening, although these may have been distorted by either the strength of the valuation basis or by the effects of historical inflation. Further investigations would be required to establish the true underlying position.

(ii) A crude measure of the implicit or explicit margins (historically) in the outstanding claims reserves as at 31st December, 1975 indicates that they were overstated by approximately 10%.

(iii) It would appear that there has been a substantial variation in the estimated loss ratios by year claim incurred and risk group. There was, however, a marked deterioration in 1976, although this may have been exaggerated by the apparently strong valuation basis for outstanding claims.

(iv) On the assumption that the expected ultimate loss ratios for each risk group are Normally distributed, a crude attempt has been made to calculate the extent of 2 standard deviations per unit of premium. It was found that these amounted to approximately 16½% (on published 1970 to 1975 experience) or 25% (on "best estimate" of 1970 to 1976 experience) of the premium income. These figures reduced materially (to 6% and 11% respectively) if, alternatively, it was assumed that the overall experience for all risk groups was Normally distributed. It was considered that these results may have important implications as regards solvency margins.

(v) It would be helpful if the office could provide some supplementary information. This might include:-

a) A subdivision of the four risk groups into risk sub-groups.

b) A full statement of the valuation bases adopted each year.

c) Details of the results of alternative valuation bases considered but not adopted for the DOT Annual Returns. In particular, one would like to see comparisons between case estimates and statistical methods of estimating outstanding claims.

d) Details of the projected emergence of the outstanding claims reserves by year of payment. If this were available one would then also consider discounted outstanding claim reserves.

e) Details of explicit or implicit margins in the valuation bases.

f) Explanations of the underlying causes of various historical underwriting results. For example, the 1976 incurred Householder (Property) risk group, the 1974 incurred General Liability risk group, the 1973 incurred Employers Liability risk group, and so on.

(vi) It is appreciated that some of the answers to the points raised in (v) above will not be available or forthcoming. In particular, offices which rely solely on individual case estimates for outstanding claim estimates may not be aware of either margins in valuation bases or of the expected emergence of the outstanding claim reserves. Nevertheless, the answers to these questions should prove instructive and provoke discussion.

4. Summary of Results

The main results of the actuarial investigations have been presented via Tables 1 to 18.

Table 1 (see page 6) provides details of the emerging claim payments to each cohort and risk group. The outstanding claim estimates include incurred but not yet reported (IBNR) claims. The estimated ultimate claims are the sum of the historical claim payments and the estimated outstanding claims.

Table 2 (see page 7) provides details of the cumulative emerging claim payments. This table is merely a cumulative version of Table 1.

Table 3 (see page 8) summarises the annual contributions to the emerging loss ratios. This table is based on the ratios of the emerging claim payments in Table 1 to the earned premiums from Table 3.

Table 4 (see page 9) summarises the emerging loss ratios and estimated ultimate loss ratios. This table is based on the ratios of the cumulative emerging claim payments in Table 2 to the earned premiums from Table 4.

Table 5 (see page 10) summarises the observed cumulative emerging claim payment distributions. This table is based on the figures in Table 2. It would appear that "latest diagonal" estimates indicate longer delays than the "average" estimates. This apparent lengthening of delays to settlement may, however, have been distorted either by the strength of the valuation bases for outstanding claims or by the effects of historical inflation. If, for example, strong outstanding claim reserves were to be adopted for all cohorts then one would expect the "latest diagonal" estimates to indicate longer delays to settlement than the "average" estimates. Also, historical changes in the rate of inflation could lead to misleading delay distributions. Further investigations are required to establish the underlying delay distributions to claims settlement. These further investigations would include preparing similar tables in respect of:-

a) inflation adjusted emerging claim payments;

b) "best estimates" of emerging claim payments, with or without inflation estimates;

c) numbers of claims settled and outstanding, with proper allowances for reopened claims and zero claims (i.e. claims closed at no cost).

Some of these further investigations require supplementary information, which might not be readily available.

Table 6 (see page 11) provides details of numbers of claims, claim payments and average claim amounts for each cohort and risk group. It would appear that the results vary significantly according to office practices regarding "closed" claims, "claims closed at no cost", "claims closed at some cost", "reopened" claims, "payments on account", and so on. Comparisons can, therefore, sometimes prove misleading.

Table 7 (see page 12) provides details, for private cars only, of exposures (vehicle years), claims frequencies, average claim amounts and pure risk premiums. This information is not available from the DOT Annual Returns for the other risk groups.

Table 8 (see page 13) summarises the latest estimates of the ultimate loss ratios for each cohort and risk group. These estimates were published as at 31.12.1976 and may contain implicit or explicit margins. An attempt has also been made to measure the "best estimates", and these have been summarised via Table 17.

Table 9 (see page 14) indicates the extent of the apparent claims fluctuations, based on the published 1970 to 1975 claims experience. On the assumption that the loss ratio for each risk group is Normally distributed, it would appear that 2 standard deviations about the mean represent 16½% of the premium income. If, on the other hand, one bases the calculations on the overall claims experience for all risk groups, and assumes that the overall loss ratio is Normally distributed, then 2 standard deviations represent only 6% of the premium income. This interesting feature may suggest some evidence of "negative correlation", i.e. a "good" year in one risk group is partly matched by a "bad" year in another risk group. No attempt has yet been made to pursue this matter.

Table 10 (see page 15) summarises the outstanding claim reserves as at 31st December, 1976 for each cohort and risk group. These published reserves may include implicit or explicit margins, the extent of which has not been made available.

Table 11 (see page 16) summarises the published outstanding claims reserves, as reported via Schedule 3 of the Department of Trade Annual Returns for 1970 to 1976. This table can be used to measure (historically) any implicit or explicit margins in the reserves.

Table 12 (see page 17) indicates the release of surplus during 1976 on the outstanding claim reserves as at 31st December, 1975 in respect of each cohort and risk group. It would appear (with hindsight) that the office adopted a strong valuation basis for outstanding claims.

Table 13 (see page 18) indicates the movement between outstanding claim reserves at 31st December, 1975 and 31st December, 1976. This table provides an alternative presentation of the results summarised via Table 12.

Table 14 (see page 19) provides the format for a table on the projected emergence of outstanding claims as at 31st December, 1976. These projections are not available via the DOT Annual Returns, but may be available internally by some general insurance offices. The completion of such a table would enable consideration to be given to close monitoring of "actual against expected" and to discounted outstanding claim reserves, with their implications on the profitability of risk groups.

Table 15 (see page 20) indicates the extent to which implicit or explicit margins were included in the outstanding claims reserves as at 31st December, 1975. The information was taken from Table 12. It would appear that the revised estimates at 31st December, 1976 of the claims outstanding at 31st December, 1975 were approximately 91% of the published outstanding claims reserves (at 31st December, 1975). This indicates an apparent 10% overstatement (historically) in the outstanding claims reserves. Of course, the historical evidence may be totally inappropriate for forward projections.

Table 16 (see page 21) indicates the extent to which implicit or explicit margins have (historically) been included in the outstanding claims reserves. The information was taken from Tables 1 and 11. Table 16 is similar to Table 15 but considers all the historical evidence for all risk groups combined. It would appear that valuation surpluses and deficits vary considerably from one year to the next. It is possible that these results may be affected by changes in the rate of inflation (anticipated or actual). Further investigations (requiring supplementary information) would be required to establish the underlying causes of the historical movements in outstanding claims reserves.

Table 17 (see page 22) summarises the "best estimates", according to Table 15, of the underlying ultimate loss ratios for each risk group. The published estimates (including margins), as at 31.12.1976, have been summarised via Table 8. Table 17 is, of course, based on various assumptions regarding "best estimates" which may prove to be inappropriate. It would be better if supplementary information could be made available concerning the implicit or explicit margins in the valuation basis for outstanding claims.

Table 18 (see page 23) indicates the extent of the apparent claims fluctuations, based on the "best estimates" of the 1970 to 1976 claims experience, according to Table 17. Table 9 provides similar calculations based on the published 1970 to 1975 claims experience. On the assumption that the loss ratio for each risk group is Normally distributed, Table 18 indicates that 2 standard deviations about the mean represent approximately 25% of the premium income. If, on the other hand, one bases the calculations on the overall claims experience for all risk groups, and assumes that the overall loss ratio is Normally distributed, then 2 standard deviations represent only 11% of the premium income. These fluctuations are higher than for Table 9, which may be partly caused by the fact that the Householders (Property) risk group had an abnormally high loss ratio for 1976 incurred claims.

5. Data

The data used for this paper was actually taken from the published DOT Annual Returns of various major general insurance offices. For reasons of confidentiality, their results were aggregated and then divided by constants.

It could be argued that the above approach has led to less statistical random variation than would normally be expected from one particular office. On the other hand, the aggregation of results must have introduced an element of heterogeneity.

It could have been argued that the interesting results from Tables 9 and 18 may possibly have been caused by distortions resulting from the aggregation of the results of various offices. This hypothesis is considered, however, to be unlikely. Separate investigations were carried out to produce Tables 9 and 18 for the constituent offices. It was found that similar results were generally obtainable for each office independently, the absolute levels varying between offices.

GCO
20th June, 1978

TABLE 1

Emerging Claim Payments

Cohort Period	Delay 0 years	Delay 1 years	Delay 2 years	Delay 3 years	Delay 4 years	Delay 5 years	Delay 6 years	Estimated Outstanding Claims at 31.12.1976	Estimated Ultimate Claims
	£000's	£000's	£000's	£000's	£000's	£000's	£000's	£000's	£000's
Private Cars									
1970	1,747	978	344	201	127	65	37	50	3,549
1971	2,016	1,073	400	264	154	155		200	4,262
1972	2,369	1,325	459	446	215			394	5,208
1973	3,018	1,913	662	485				865	6,943
1974	3,265	1,997	596					1,403	7,261
1975	3,990	2,131						2,636	8,757
1976	4,746							6,352	11,098
Householders (Property)									
1970	1,026	358	12	-	-	-	-	7	1,403
1971	973	418	18	5	8	1		2	1,425
1972	1,258	532	48	16	9			25	1,888
1973	1,514	865	146	30				51	2,606
1974	2,310	1,132	100					178	3,720
1975	2,879	1,524						770	5,173
1976	5,966							5,704	11,670
General Liability									
1970	201	308	220	162	139	103	83	319	1,535
1971	178	299	290	220	323	165		506	1,981
1972	202	391	293	350	214			880	2,330
1973	266	664	531	357				1,786	3,604
1974	721	965	693					3,298	5,677
1975	451	802						4,128	5,381
1976	420							6,924	7,344
Employers Liability									
1970	100	337	467	263	210	119	57	193	1,746
1971	81	374	373	265	172	174		172	1,611
1972	63	412	493	365	320			457	2,110
1973	85	563	746	548				1,188	3,130
1974	88	542	696					1,914	3,240
1975	99	596						2,959	3,654
1976	94							4,002	4,096

Notes

1. The "delay" is measured from the year claim incurred (i.e. year of accident) to the year of payment.

2. The outstanding claims estimates include IBNR claims.

TABLE 2

Cumulative emerging claim payments

Cohort Period	Delay 0 years	Delay 1 years	Delay 2 years	Delay 3 years	Delay 4 years	Delay 5 years	Delay 6 years	Estimated Outstanding Claims at 31.12.1976	Estimated Ultimate Claims
	£000's	£000's	£000's	£000's	£000's	£000's	£000's	£000's	£000's
Private Cars									
1970	1,747	2,725	3,069	3,270	3,397	3,462	3,499	50	3,549
1971	2,016	3,089	3,489	3,753	3,907	4,062		200	4,262
1972	2,369	3,694	4,153	4,599	4,814			394	5,208
1973	3,018	4,931	5,593	6,078				865	6,943
1974	3,265	5,262	5,858					1,403	7,261
1975	3,990	6,121						2,636	8,757
1976	4,746							6,352	11,098
Householders (Property)									
1970	1,026	1,384	1,396	1,396	1,396	1,396	1,396	7	1,403
1971	973	1,391	1,409	1,414	1,422	1,423		2	1,425
1972	1,258	1,790	1,838	1,854	1,863			25	1,888
1973	1,514	2,379	2,525	2,555				51	2,606
1974	2,310	3,442	3,542					178	3,720
1975	2,879	4,403						770	5,173
1976	5,966							5,704	11,670
General Liability									
1970	201	509	729	891	1,030	1,133	1,216	319	1,535
1971	178	477	767	987	1,310	1,475		506	1,981
1972	202	593	886	1,236	1,450			880	2,330
1973	266	930	1,461	1,818				1,786	3,604
1974	721	1,686	2,379					3,298	5,677
1975	451	1,253						4,128	5,381
1976	420							6,924	7,344
Employers Liability									
1970	100	437	904	1,167	1,377	1,496	1,553	193	1,746
1971	81	455	828	1,093	1,265	1,439		172	1,611
1972	63	475	968	1,333	1,653			457	2,110
1973	85	648	1,394	1,942				1,188	3,130
1974	88	630	1,326					1,914	3,240
1975	99	695						2,959	3,654
1976	94							4,002	4,096

Notes

1. The "delay" is measured from the year claim incurred (i.e. year of accident) to the year of payment.

2. The outstanding claims estimates include IBNR claims.

TABLE 3

Annual contributions to emerging loss ratios

Cohort Period	Earned Premiums	Delay 0 years	Delay 1 years	Delay 2 years	Delay 3 years	Delay 4 years	Delay 5 years	Delay 6 years
	£000's	%	%	%	%	%	%	%
Private Cars								
1970	4,453	39.2	22.0	7.7	4.5	2.9	1.5	0.8
1971	6,071	33.2	17.7	6.6	4.3	2.5	2.6	
1972	7,937	29.8	16.7	5.8	5.6	2.7		
1973	10,881	27.7	17.6	6.1	4.5			
1974	11,980	27.3	16.7	5.0				
1975	13,655	29.2	15.6					
1976	16,399	28.9						
Householders (Property)								
1970	4,497	22.8	8.0	0.3	-	-	-	-
1971	5,081	19.1	8.2	0.4	0.1	0.2	-	
1972	5,975	21.1	8.9	0.8	0.3	0.2		
1973	7,568	20.0	11.4	1.9	0.4			
1974	9,008	25.6	12.6	1.1				
1975	10,812	26.6	14.1					
1976	13,395	44.5						
General Liability								
1970	2,193	9.2	14.0	10.0	7.4	6.3	4.7	3.8
1971	2,751	6.5	10.9	10.5	8.0	11.7	6.0	
1972	3,657	5.5	10.7	8.0	9.6	5.9		
1973	4,738	5.6	14.0	11.2	7.5			
1974	6,040	11.9	16.0	11.5				
1975	7,883	5.7	10.2					
1976	9,595	4.4						
Employers Liability								
1970	2,092	4.8	16.1	22.3	12.6	10.0	5.7	2.7
1971	2,260	3.6	16.5	16.5	11.7	7.6	7.7	
1972	2,632	2.4	15.7	18.7	13.9	12.2		
1973	3,189	2.7	17.7	23.4	17.2			
1974	3,820	2.3	14.2	18.2				
1975	4,484	2.2	13.3					
1976	4,984	1.9						

Notes

1. The "delay" is measured from the year claim incurred (i.e. year of accident) to the year of payment.

TABLE 4

Emerging loss ratios and ultimate loss ratios

Cohort Period	Earned Premiums	Delay 0 years	Delay 1 years	Delay 2 years	Delay 3 years	Delay 4 years	Delay 5 years	Delay 6 years	Outstanding Claims at 31.12.1976 loss ratio	Ultimate loss ratio
	£000's	%	%	%	%	%	%	%	%	%
Private Cars										
1970	4,453	39.2	61.2	68.9	73.4	76.3	77.7	78.6	1.1	79.7
1971	6,071	33.2	50.9	57.5	61.8	64.4	66.9		3.3	70.2
1972	7,937	29.8	46.5	52.3	57.9	60.7			4.9	65.6
1973	10,881	27.7	45.3	51.4	55.9				7.9	63.8
1974	11,980	27.3	43.9	48.9					11.7	60.6
1975	13,655	29.2	44.8						19.3	64.1
1976	16,399	28.9							38.8	67.7
Householders (Property)										
1970	4,497	22.8	30.8	31.0	31.0	31.0	31.0	31.0	0.2	31.2
1971	5,081	19.1	27.4	27.7	27.8	28.0	28.0		-	28.0
1972	5,975	21.1	30.0	30.8	31.0	31.2			0.4	31.6
1973	7,563	20.0	31.4	33.4	33.8				0.6	34.4
1974	9,008	25.6	38.2	39.3					2.0	41.3
1975	10,812	26.6	40.7						7.1	47.8
1976	13,395	44.5							42.6	87.1
General Liability										
1970	2,193	9.2	23.2	33.2	40.6	47.0	51.7	55.4	14.6	70.0
1971	2,751	6.5	17.3	27.9	35.9	47.6	53.6		18.4	72.0
1972	3,657	5.5	16.2	24.2	33.8	39.6			24.1	63.7
1973	4,738	5.6	19.6	30.8	38.4				37.7	76.1
1974	6,040	11.9	27.9	39.4					54.6	94.0
1975	7,883	5.7	15.9						52.4	68.3
1976	9,595	4.4							72.1	76.5
Employers Liability										
1970	2,092	4.8	20.9	43.2	55.8	65.8	71.5	74.2	9.3	83.5
1971	2,260	3.6	20.1	36.6	48.4	56.0	63.7		7.6	71.3
1972	2,632	2.4	18.0	36.8	50.6	62.8			17.4	80.2
1973	3,189	2.7	20.3	43.7	60.9				37.2	98.1
1974	3,820	2.3	16.5	34.7					50.1	84.8
1975	4,484	2.2	15.5						66.0	81.5
1976	4,984	1.9							80.3	82.2

Notes

1. The "delay" is measured from the year claim incurred (i.e. year of accident) to the year of payment.

TABLE 5

Cumulative Emerging Claim Payment Distributions

Cohort Period	Delay 0 years	Delay 1 years	Delay 2 years	Delay 3 years	Delay 4 years	Delay 5 years	Delay 6 years	Ultimate
Private Cars	%	%	%	%	%	%	%	%
1970	49.2	76.8	86.5	92.1	95.7	97.5	98.6	100
1971	47.3	72.5	81.9	88.1	91.7	95.3		100
1972	45.5	70.9	79.7	88.3	92.4			100
1973	43.5	71.0	80.6	87.5				100
1974	45.0	72.5	80.7					100
1975	45.6	69.9						100
1976	42.8							100
average	45.6	72.3	81.9	89.0	93.3	96.4	98.6	100
latest diagonal	42.8	69.9	80.7	87.5	92.4	95.3	98.6	100
Householders (Property)								
1970	73.1	98.6	99.5	99.5	99.5	99.5	99.5	100
1971	68.3	97.6	98.9	99.2	99.8	99.9		100
1972	66.6	94.8	97.4	98.2	98.7			100
1973	58.1	91.3	96.9	98.0				100
1974	62.1	92.5	95.2					100
1975	55.7	85.1						100
1976	51.1							100
average	62.1	93.3	97.6	98.7	99.3	99.7	99.7*	100
latest diagonal	51.1	85.1	95.2	98.0	98.7	99.7*	99.7*	100
General Liability								
1970	13.1	33.2	47.5	58.0	67.1	73.8	79.2	100
1971	9.0	24.1	38.7	49.8	66.1	74.5		100
1972	8.7	25.5	38.0	53.0	62.2			100
1973	7.4	25.8	40.5	50.4				100
1974	12.7	29.7	41.9					100
1975	8.4	23.3						100
1976	5.7							100
average	9.3	26.9	41.3	52.8	65.1	74.2	79.2	100
latest diagonal	5.7	23.3	41.9	50.4	62.2	74.5	79.2	100
Employers Liability								
1970	5.7	25.0	51.8	66.8	78.9	85.7	88.9	100
1971	5.0	28.2	51.4	67.8	78.5	89.3		100
1972	3.0	22.5	45.9	63.2	78.3			100
1973	2.7	20.7	44.5	62.0				100
1974	2.7	19.4	40.9					100
1975	2.7	19.0						100
1976	2.3							100
average	3.4	22.5	46.9	65.0	78.6	87.5	88.9	100
latest diagonal	2.3	19.0	40.9	62.0	78.3	87.5*	88.9	100

Notes

1. * denotes an adjusted value. These adjustments were necessary for (apparently) freak results.

2. Similar tables could also be produced based on numbers of claims or on inflation adjusted claim payments.

TABLE 6

Average Claim Payments

Cohort Period	Number of claims					Claim Payments			Average claim amounts			
	Closed at no cost	Closed at some cost	Outstanding at 31.12.1976	Overall (excluding zero claims)	Overall (including zero claims)	Closed	Outstanding (inc. payments on account at 31.12.1976)	Overall (Ultimate)	Closed at some cost	Outstanding (inc. payments on account at 31.12.1976)	Overall (excluding zero claims)	Overall (including zero claims)
						£000's	£000's	£000's	£	£	£	£
Private Cars												
1970	8,658	29,610	113	29,723	38,381	3,424	125	3,549	116	1,106	119	92
1971	9,784	30,337	77	30,414	40,198	3,854	408	4,262	127	5,299	140	106
1972	10,984	32,318	115	32,433	43,417	4,556	652	5,208	141	5,670	161	120
1973	13,437	36,466	287	36,753	50,190	5,690	1,253	6,943	156	4,366	189	138
1974	12,648	34,456	682	35,138	47,786	5,447	1,814	7,261	158	2,660	207	152
1975	11,073	32,523	2,166	34,689	45,762	5,517	3,240	8,757	170	1,496	252	191
1976	5,458	19,610	19,584	39,194	44,652	3,299	7,799	11,098	168	398	283	249
Householders (Property)												
1970	3,560	41,757	9	41,766	45,326	1,395	8	1,403	33	889	34	31
1971	3,752	35,158	16	35,174	38,926	1,415	10	1,425	40	625	41	37
1972	4,219	39,461	39	39,500	43,719	1,842	46	1,888	47	1,179	48	43
1973	4,185	39,987	112	40,099	44,284	2,455	151	2,606	61	1,348	65	59
1974	5,854	50,278	516	51,794	56,648	3,432	288	3,720	68	558	72	66
1975	5,759	52,324	1,241	53,565	59,324	4,097	1,076	5,173	78	867	97	87
1976	5,330	72,392	19,789	92,181	97,511	5,560	6,110	11,670	77	309	127	120
General Liability												
1970	4,379	8,027	83	8,110	12,489	1,123	412	1,535	140	4,964	189	123
1971	4,360	7,888	138	8,026	12,386	1,102	879	1,981	140	6,370	247	160
1972	5,446	7,800	249	8,049	13,495	1,316	1,014	2,330	169	4,072	289	173
1973	6,381	8,291	597	8,888	15,269	1,538	2,066	3,604	186	3,461	405	236
1974	6,748	7,861	1,255	9,116	15,864	1,947	3,730	5,677	248	2,972	623	358
1975	5,827	6,534	2,537	9,071	14,898	1,018	4,363	5,381	156	1,720	593	361
1976	2,268	3,524	8,754	12,278	14,546	345	6,999	7,344	98	800	598	505
Employers Liability												
1970	2,831	2,268	43	2,311	5,142	1,529	217	1,746	674	5,047	756	340
1971	2,569	1,998	56	2,054	4,623	1,347	264	1,611	674	4,714	784	348
1972	2,813	1,915	108	2,023	4,836	1,573	537	2,110	821	4,972	1,043	436
1973	3,269	2,162	302	2,464	5,733	1,722	1,408	3,130	796	4,662	1,270	546
1974	2,939	1,821	629	2,450	5,389	1,167	2,073	3,240	641	3,296	1,322	601
1975	2,507	1,124	1,192	2,316	4,823	605	3,049	3,654	538	2,558	1,578	758
1976	837	243	3,044	3,287	4,124	76	4,020	4,096	313	1,321	1,246	993

TABLE 7

Private Cars - Pure Risk Premiums

Cohort Period	Exposure (vehicle years)	Claims Frequency	Average Claim Amount	Pure Risk Premium
			£	£
Including claims closed at no cost				
1970	231,078	.166	92	15.4
1971	264,485	.152	106	16.1
1972	275,299	.158	120	18.9
1973	319,848	.157	138	21.7
1974	322,518	.148	152	22.5
1975	321,367	.142	191	27.2
1976	294,236	.152	249	37.7
Excluding claims closed at no cost				
1970	231,078	.129	119	15.4
1971	264,485	.115	140	16.1
1972	275,299	.118	161	18.9
1973	319,848	.115	189	21.7
1974	322,518	.109	207	22.5
1975	321,367	.108	252	27.2
1976	294,236	.133	283	37.7

Notes

1. The claims frequencies were calculated as the ratio of the number of claims (see Table 6, columns 4 and 5) to the exposure (vehicle years).

2. The 1976 claims frequency (excluding claims closed at no cost) will probably decrease in future years as a result of some outstanding claims being closed at no cost. The corresponding 1976 average claim amount will consequently increase; the 1976 pure risk premium remaining unchanged, other things being equal.

TABLE 8

Estimated ultimate loss ratios

Cohort Period	Private Cars	Householders (Property)	General Liability	Employers Liability	All Risk Groups
	%	%	%	%	%
1970	79.7	31.2	70.0	83.5	62.2
1971	70.2	28.0	72.0	71.3	57.4
1972	65.6	31.6	63.7	80.2	57.1
1973	63.8	34.4	76.1	98.1	61.7
1974	60.6	41.3	94.0	84.8	64.5
1975	64.1	47.8	68.3	81.5	62.3
1976	67.7	87.1	76.5	82.2	77.1

Notes

1. The ultimate loss ratio is the ratio of the claims incurred to the corresponding earned premiums. Both the cohort period and the claims incurred refer to the "year of accident" (i.e. Year of Origin, per DOT Annual Returns).

2. The ultimate loss ratio for "all risk groups" was calculated as the ratio of the claims incurred for "all risk groups" to the corresponding earned premiums for "all risk groups".

3. The information for the above table was taken from Table 4. It represents the published estimates as at 31.12.1976 of the ultimate loss ratios. These ratios may include implicit or explicit margins in the outstanding claims estimates.

TABLE 9

Claims fluctuations - based on 1970 to 1975 published experience

Risk Group	Mean loss ratio	standard deviation of loss ratios	variance of loss ratios	1976 earned premiums	weighted variance of loss ratios
				£000	
Private Cars	.673	.068	.0046	16,399	76
Householders (Property)	.357	.074	.0055	13,395	74
General Liability	.740	.106	.0113	9,595	108
Employers Liability	.832	.087	.0076	4,984	38
All risk groups	.609	.030	.00088	44,373	39
Sum of 4 risk groups	.609	.082	.00667	44,373	296

Notes

1. The above table attempts to provide a crude measure of the underlying claims fluctuations. It is based on the published estimates (as at 31.12.1976) of the ultimate loss ratios for 1970 to 1975 incurred claims. The estimated ultimate loss ratios for 1976 incurred claims were excluded, on the grounds that the outstanding claims estimates appear (on historical evidence, which may be invalid) to contain substantial implicit or explicit margins.

2. The information for the above table was taken from Table 8. The "all risk groups" row was based on the final column of Table 8. The "sum of 4 risk groups" row was based on the sum of the weighted variances of the loss ratios, the weights having been chosen as the 1976 earned premiums. Essentially, the "all risk groups" row assumes that the ultimate loss ratios for "all risk groups" are Normally distributed, whereas the "sum of 4 risk groups" assumes a Normal distribution for each risk group independently.

3. It would appear that 2 standard deviations represent 6% of the premium income for "all risk groups" and 16½% of the premium income for the "sum of 4 risk groups". It would appear that the historical combination of risk groups has tended to reduce the overall claims fluctuations per unit of premium income.

4. The "weighted variances of loss ratios" is the product of the "variance of loss ratios" and the "1976 earned premiums". The weights (i.e. the 1976 earned premiums) are somewhat arbitrary, but do reflect the latest risk group mix.

TABLE 10

Outstanding Claims Reserves as at 31st December, 1976

Cohort Period	Private Cars	Householders (Property)	General Liability	Employers Liability	All Risk Groups
	£000's	£000's	£000's	£000's	£000's
1970	50	7	319	193	569
1971	200	2	506	172	880
1972	394	25	880	457	1,756
1973	865	51	1,786	1,188	3,890
1974	1,403	178	3,298	1,914	6,793
1975	2,636	770	4,128	2,959	10,493
1976	6,352	5,704	6,924	4,002	22,982
All Cohorts	11,900	6,737	17,841	10,885	47,363

Notes

1. The above table is based on the 1976 DOT Annual Returns, Schedule 3.

2. The outstanding claims reserves include IBNR claims.

3. The above outstanding claims estimates probably include implicit or explicit margins. The historical evidence for this hypothesis can be seen from Tables 12, 13, 15 and 16.

TABLE 11

Published Outstanding Claims Reserves

Cohort Period	Published Outstanding Claims Reserves as at 31.12.1970	31.12.1971	31.12.1972	31.12.1973	31.12.1974	31.12.1975	31.12.1976
	£000	£000	£000	£000	£000	£000	£000
Private Cars							
1970	1,598	774	492	307	162	95	50
1971		2,207	1,141	736	510	373	200
1972			3,057	1,458	1,044	638	394
1973				4,267	2,097	1,414	865
1974					4,658	2,278	1,403
1975						5,265	2,636
1976							6,352
Householders (Property)							
1970	425	43	21	11	8	8	7
1971		465	64	26	17	7	2
1972			721	151	73	41	25
1973				1,221	287	96	51
1974					1,344	318	178
1975						2,292	770
1976							5,704
General Liability							
1970	1,067	1,095	906	734	588	483	319
1971		1,670	1,600	1,463	1,309	802	506
1972			2,402	2,148	1,736	1,231	880
1973				3,371	2,886	2,233	1,786
1974					5,043	4,190	3,298
1975						5,494	4,128
1976							6,924
Employers Liability							
1970	1,315	1,162	835	633	352	229	193
1971		1,500	1,186	927	647	379	172
1972			1,970	1,649	1,285	943	457
1973				3,039	2,610	1,887	1,188
1974					3,242	2,858	1,914
1975						4,309	2,959
1976							4,002

Notes

1. The information for the above table was taken from the published DOT Annual Returns for 1970 to 1976, Schedule 3.

TABLE 12

Release of surplus on outstanding claims reserves as at 31.12.1975

Cohort Period	Outstanding claims reserves as at 31.12.1975				
	Private Cars	Householders (Property)	General Liability	Employers Liability	All Risk Groups
	£000's	£000's	£000's	£000's	£000's
Previous estimates (at 31.12.1975)					
1970	95	8	483	229	815
1971	373	7	802	379	1,561
1972	638	41	1,231	943	2,853
1973	1,414	96	2,233	1,887	5,630
1974	2,278	318	4,190	2,858	9,644
1975	5,265	2,292	5,494	4,309	17,360
1970 to 1975	10,063	2,762	14,433	10,605	37,863
Revised estimates (at 31.12.1976)					
1970	87	7	402	250	746
1971	355	3	671	346	1,375
1972	609	34	1,094	777	2,514
1973	1,350	81	2,143	1,736	5,310
1974	1,999	278	3,991	2,610	8,878
1975	4,767	2,294	4,930	3,555	15,546
1970 to 1975	9,167	2,697	13,231	9,274	34,369
Release of surplus					
1970	8	1	81	-21	69
1971	18	4	131	33	186
1972	29	7	137	166	339
1973	64	15	90	151	320
1974	279	40	199	248	766
1975	498	-2	564	754	1,814
1970 to 1975	896	65	1,202	1,331	3,494

Notes

1. The "revised estimates (at 31.12.1976)" allow for the actual claim payments during 1976 and the latest estimates of outstanding claims at 31.12.1976.

2. The "release of surplus" is calculated as "previous estimates" less "revised estimates".

3. The outstanding claims reserves include IBNR claims.

TABLE 13

Movement between outstanding claim reserves at 31.12.1975 and 31.12.1976

Cohort Period	O/S claims reserves at 31.12.1975	Payments in 1976	O/S claims reserves at 31.12.1976	Increase in assumed total cost
	£000's	£000's	£000's	£000's
Private Cars				
1970	95	37	50	-8
1971	373	155	200	-18
1972	638	215	394	-29
1973	1,414	485	865	-64
1974	1,278	596	1,403	-279
1975	5,265	2,131	2,636	-498
1976	-	4,746	6,352	0
all cohorts	10,063	8,365	11,900	-896
Householders (Property)				
1970	8	0	7	-1
1971	7	1	2	-4
1972	41	9	25	-7
1973	96	30	51	-15
1974	318	100	178	-40
1975	2,292	1,524	770	+2
1976	-	5,966	5,704	0
all cohorts	2,762	7,630	6,737	-65
General Liability				
1970	483	83	319	-81
1971	802	165	506	-131
1972	1,231	214	880	-137
1973	2,233	357	1,786	-90
1974	4,190	693	3,298	-199
1975	5,494	802	4,128	-564
1976	-	420	6,924	0
all cohorts	14,433	2,734	17,841	-1,202
Employers Liability				
1970	229	57	193	+21
1971	379	174	172	-33
1972	943	320	457	-166
1973	1,887	548	1,188	-151
1974	2,858	696	1,914	-248
1975	4,309	596	2,959	-754
1976	-	94	4,002	0
all cohorts	10,605	2,485	10,885	-1,331

Notes

1. O/S denotes outstanding.

TABLE 14

Projected emergence of outstanding claims at 31st December, 1976

Year of Payment	Private Cars	Householders (Property)	General Liability	Employers Liability	All Risk Groups
	£000's	£000's	£000's	£000's	£000's
1970 incurred claims					
1977 Quarter 1					
1977 Quarter 2					
1977 Quarter 3					
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
1977 or later	50	7	319	193	569
1971 incurred claims					
1977 Quarter 1					
1977 Quarter 2					
1977 Quarter 3					
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
1977 or later	200	2	506	172	880
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
1976 incurred claims					
1977 Quarter 1					
1977 Quarter 2					
1977 Quarter 3					
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
1977 or later	6,352	5,704	6,924	4,002	22,982
All cohorts					
1977 Quarter 1					
1977 Quarter 2					
1977 Quarter 3					
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
1977 or later	11,900	6,737	17,841	10,885	47,363

Notes

1. The projected emergence of claim payments to each cohort and risk group would be entered in the above table. These projections are not currently available to the author, but might in practice be available or capable of computation (on perhaps more than one valuation basis) to the general insurance actuary.

TABLE 15

Outstanding claims reserves as at 31.12.1975

Comparison of estimates at 31.12.1975 and 31.12.1976

Cohort Period	Private Cars	Householders (Property)	General Liability	Employers Liability	All Risk Groups
$\left(\frac{\text{actual}}{\text{expected}}\right)$ outstanding claims					
1970	.92	(.88)	.83	1.09	.92
1971	.95	(.43)	.84	.91	.88
1972	.95	.83	.89	.82	.88
1973	.95	.84	.96	.92	.94
1974	.88	.87	.95	.91	.92
1975	.91	1.00	.90	.83	.90
1970 to 1975	.91	.98	.92	.87	.91
Average (1970 to 1975)	.93	.89	.90	.91	.91

Notes

1. The information for the above table was taken from Table 12. The above figures are the ratios of the "revised estimates (at 31.12.1976)" to the "previous estimates (at 31.12.1975)", according to Table 12. For the Householders risk group, the 1970 and 1971 ratios are unreliable, and have been excluded from the calculation of the average ratio.

2. The above table provides an indication of the implicit or explicit margins included in the published outstanding claims reserves. It would appear that, during 1976, the "best estimate" of the outstanding claims would have been approximately .91 times the published estimate. In other words, the published outstanding claims reserves at 31st December, 1975 appear to have included implicit or explicit margins of approximately 10%.

TABLE 16

Published outstanding claims reserves - all risk groups combined

Comparison of estimates at end of years T and T + 1

Cohort Period	Published outstanding claim reserves as at 31st December					
	1970	1971	1972	1973	1974	1975
$\frac{\text{actual}}{\text{expected}}$ O/S claims						
1970	1.15	1.07	1.03	.94	.99	.92
1971		1.05	1.06	1.03	.89	.88
1972			.99	1.00	.97	.88
1973				1.00	.98	.94
1974					1.00	.92
1975						.90
1970 to 1975	1.15	1.06	1.01	1.00	.98	.91
average (1970 to 1975)	1.15	1.06	1.03	.99	.97	.91

Notes

1. The information for the above table was taken from Tables 11 and 15. The 1975 column is a copy of the final column of Table 15.

2. The above figures are the ratios of the "revised estimates at the end of Year T + 1" to the "previous estimates at the end of Year T". In other words, they attempt to measure the release of surplus.

3. The above table provides an indication of the implicit or explicit margins included in the published outstanding claims reserves. These margins seem to vary from one year to the next. They may be affected by extraneous factors, such as changes in the anticipated rate of inflation.

TABLE 17

"Best estimate" of ultimate loss ratios

Cohort Period	Private Cars	Householders (Property)	General Liability	Employers Liability	All Risk Groups
	%	%	%	%	%
1970	79.6	31.2	68.7	82.6	61.8
1971	70.0	28.0	70.3	70.6	56.9
1972	65.2	31.6	61.6	78.6	56.3
1973	63.1	34.4	72.7	94.8	60.4
1974	59.6	41.1	89.1	80.3	62.5
1975	62.4	47.2	63.5	75.6	59.8
1976	64.2	83.3	70.0	75.0	72.4

Notes

1. The ultimate loss ratio is the ratio of the claims incurred to the corresponding earned premiums. The claims incurred in the above table are "best estimates", rather than "published estimates" (i.e. cautious estimates, historically).

2. The information for the above table was taken from Tables 4, 8 and 15. The .91 factor from Table 15 was used to convert the published outstanding claims estimates to "best estimates". The ultimate loss ratio for "all risk groups" was calculated as the ratio of the claims incurred for "all risk groups" to the corresponding earned premiums.

3. The above estimates are not published via the DOT Annual Returns. They are based on the assumption, which may be inappropriate, that the published outstanding claims reserves at 31st December, 1976 are 10% higher than the "best estimates". This assumption is somewhat arbitrary, and the corporate view of this assumption would be appreciated. Similar calculations could also be carried out on alternative assumptions regarding the strength of the valuation basis as at 31st December, 1976.

TABLE 18

Claims fluctuations - based on 1970 to 1976 "best estimates" of underlying claims experience

Risk Group	Mean loss ratio	standard deviation of loss ratios	variance of loss ratios	1976 earned premiums	weighted variance of loss ratios
				£000	
Private Cars	.663	.067	.0044	16,399	73
Householders (Property)	.424	.192	.0368	13,395	494
General Liability	.708	.090	.0080	9,595	77
Employers Liability	.796	.077	.0060	4,984	30
All risk groups	.614	.054	.0029	44,373	127
Sum of 4 risk groups	.614	.123	.0152	44,373	674

Notes

1. The information for the above table was taken from Table 17. Similar notes apply as for Table 9.

2. It would appear that 2 standard deviations represent approximately 11% of the premium income for "all risk groups" and approximately 25% of the premium income for the "sum of 4 risk groups". It would appear that the historical combination of risk groups has tended to reduce the overall claims fluctuations per unit of premium income.

3. The "weighted variances of loss ratios" is the product of the "variance of loss ratios" and the "1976 earned premiums". The weights (i.e. the 1976 earned premiums) are somewhat arbitrary, but do reflect the latest risk group mix.

CONSISTENCY OF TECHNICAL RESERVES

(1) A standard tool of the actuary has been to compare actual with expected and thus to judge the model he has developed. Therefore once a model has been developed for establishing technical reserves e.g. for outstanding claims, it is important to monitor the original estimates as they progress into a stream of claims. Schedule 3 Part III of the D.o.T. Returns forms the starting point for a procedure that has been found useful.

(2) However a matrix type layout is used, as this prominently displays features of the development e.g. whether a similar pattern is shown for different years of origin. This could be defined either as year of claim or alternatively as policy year of underwriting in which the claim arose, depending on whether one year accounting or three year accounting had been adopted.

(3) The basic method is to make a continuous comparison of the Technical Reserves in respect of outstanding claims (including claims incurred but not reported).

A comparison between the "reserve" (i.e. estimated

outstanding claims) for each year of origin T at the calculation date (in development year t, say) and the previous estimate less claims paid in the interim is made (assuming an annual update of the estimates). We define:

C_t^T actual claims paid in development year t for business attributable to year of origin T

R_t^T estimated "reserve" at end of development year t for year of origin T

$(TP)_t^T$ total premium for year of origin T as estimated at end of development year t. (On a short tail account e.g. motor, this estimate will be firm.) Where the year of origin has been defined as year of claim this will be the earned premium for that year.

For each year of origin T:

(i) R_t^T is compared with $(R_{t-n}^T - \sum_{r=1}^{n} C_{t-n+r})$ (n = 1, 2, ...)

(Normally of course n would be set = 1, but clearly other values can be interesting.)

(ii) $\dfrac{(R_t^T + \sum_{r=1}^{t} C_r^T)}{(TP)_t^T}$ is compared with $\dfrac{(R_{t-n}^T + \sum_{r=1}^{t-n} C_r^T)}{(TP)_{t-n}^T}$ (n = 1, 2, ..

(i.e. the Estimated Ultimate Loss Ratio at t is compared with Estimated ULR at t-n.)

/contd.

(iii) $(R_t^T + \sum_{r=1}^{t} C_r^T)$ is compared with $(R_{t-n}^T + \sum_{r=1}^{t-n} C_r^T)$ (n = 1, 2,...

(i.e. The Estimated Paid plus Outstanding at time t is compared with the same at t-n.)

These comparisons are also made for all the years of origin common to each update combined, in order to assess how large a deviation there has been between the actual experience and the model estimates for a class of business as a whole.

(4) The application of the method is best shown by sets of examples. The first compares results over 8 underwriting years for some Marine Hull business between 1972 and 1974 (i.e. n = 2). The Technical Reserves for Outstanding Claims have been estimated on two different methods, namely the Chain Ladder shown in Table 1A and the Chain Ladder (inflation adjusted) shown in Table 1B. The inflation index used is the U.K. Retail Price Index lagged by 1 year. Post 1974 inflation has been assumed at 20%. A comparison of columns (5) and (6) will show whether the reserving method has led to releases or strains on the business over the period. Further it is possible to check whether the effect is the same for all years of origin. It will be seen that the Chain Ladder method if it had been adopted would have resulted in an overall strain, but this is the net product of surpluses and strains on different years. The Chain Ladder (inflation adjusted) would have produced an overall release over the period, to which each year of origin except 1968 contributed positively. Surpluses and strains

are perhaps seen even more clearly by comparing columns (3) and (4). It is likely that the Chain Ladder (inflation adjusted) contains a margin of caution, which is now, of course, required by the supervisory authority, as well as being desirable. In addition to the actual amounts and movements of reserves the latest estimates of ultimate loss ratios are indicated. Since in this example a three year accounting method has been adopted the reserve in the first two years includes an allowance for unexpired risks.

(5) The method has then been applied to the Model Office described in the technical reserves report. For each pool of business comparisons are first made of the reserves shown in the D.o.T. Returns. However the purpose here is to show not so much the amount as the trend and consistency of surpluses and strains. The results are set out in Tables 2A to 5A. It will be noted that there are substantial releases on all pools and these seem to arise consistently from year to year.

A further step was to construct statistical estimates for the Model Office reserves as at the end of 1975 and 1976 using the Separation Technique (except for Householders where a Chain Ladder has been constructed). The results are displayed in Tables 2B to 5B. For the Liability classes the total run off period has been assumed to be 10 years and additional points have first been extrapolated beyond the run off triangle. It has been assumed that at both end 1975 and end 1976 it would be appropriate to take future inflation

at 15% p.a.

It will be seen that in each case the statistical method has given very consistent results, columns (3) and (4) being virtually identical. However there is a tendency for there to be slight strains. The analysis would however suggest that the statistical techniques are closer to best estimates than the reserves actually set up, which contain margins.

LAWRENCE M. EAGLES

June 1978

Table 1A: MARINE HULL

Movement in Estimated Outstanding Claims, 1972-74

Chain Ladder (without inflation adjustment)

Year of Origin	(1) Estimated Outstanding Claims ("Reserve") at 31.12.72	(2) Actual Paid Claims 1.1.73-31.12.74	(3) Adjusted "Reserve" at 31.12.74 (1)-(2)	(4) Recalculated "Reserve" at 31.12.74	(5) Estimated Ultimate Claims at 31.12.72	(6) Estimated Ultimate Claims at 31.12.74	(7) Estimated Ultimate Loss Ratio at 31.12.72	(8) Estimated Ultimate Loss Ratio at 31.12.74
	£'000	£'000	£'000	£'000	£'000	£'000	%	%
1965	314	298	16	-	8,955	8,939	107.8	107.6
1966	1,118	803	315	265	16,395	16,346	110.4	110.2
1967	1,442	1,506	(64)	428	15,624	16,116	87.1	89.9
1968	2,830	4,433	(1,603)	1,074	20,204	22,882	91.1	103.2
1969	4,750	2,266	2,484	2,153	21,533	21,202	93.1	91.7
1970	6,975	3,951	3,024	3,193	19,035	19,205	82.4	83.1
	17,429	13,257	4,172	7,113	101,746	104,690	92.9	95.6
1971	13,268	8,536	4,732	5,338	21,296	21,901	84.6	87.0
1972	25,684	13,878	11,806	10,255	27,813	26,262	120.5	113.9
	56,381	35,671	20,710	22,706	150,855	152,853	95.6	96.9

Note: Column (1) includes the Unearned Premium Reserve and Unexpired Risk Reserve as at 31.12.72.

Table 1B: MARINE HULL

Movement in Estimated Outstanding Claims, 1972-74

Chain Ladder (with inflation adjustment)

Year of Origin	(1) Estimated Outstanding Claims ("Reserve") at 31.12.72	(2) Actual Paid Claims 1.1.73-31.12.74	(3) Adjusted "Reserve" at 31.12.74 (1)-(2)	(4) Recalculated "Reserve" at 31.12.74	(5) Estimated Ultimate Claims at 31.12.72	(6) Estimated Ultimate Claims at 31.12.74	(7) Estimated Ultimate Loss Ratio at 31.12.72	(8) Estimated Ultimate Loss Ratio at 31.12.74
	£'000	£'000	£'000	£'000	£'000	£'000	%	%
1965	648	298	350	-	9,289	8,939	111.8	107.6
1966	1,756	803	953	296	17,033	16,377	114.8	110.3
1967	2,311	1,506	805	525	16,494	16,213	91.9	90.4
1968	4,552	4,433	119	1,314	21,927	23,122	98.9	104.3
1969	7,456	2,266	5,190	2,816	24,239	21,864	104.8	94.6
1970	10,420	3,951	6,469	4,273	22,481	20,285	97.3	87.8
	27,143	13,257	13,886	9,224	111,463	106,800	101.8	97.5
1971	19,155	8,536	10,619	7,356	27,183	23,920	108.0	95.1
1972	36,845	13,878	22,967	13,614	38,974	29,621	169.0	128.4
	83,143	35,671	47,472	30,194	177,620	160,341	112.6	101.6

Note: Column (1) includes the Unearned Premium Reserve and Unexpired Risk Reserve as at 31.12.72.

Table 2A: EMPLOYER'S LIABILITY

Movement in Estimated Outstanding Claims, 1975-76

Year of Origin	(1) Estimated Outstanding Claims ("Reserve") at 31.12.75	(2) Actual Paid Claims 1.1.76-31.12.76	(3) Adjusted "Reserve" at 31.12.76 (1)-(2)	(4) Recalculated "Reserve" at 31.12.76	(5) Estimated Ultimate Claims at 31.12.75	(6) Estimated Ultimate Claims at 31.12.76	(7) Estimated Ultimate Loss Ratio at 31.12.75	(8) Estimated Ultimate Loss Ratio at 31.12.76
	£'000	£'000	£'000	£'000	£'000	£'000	%	%
1970	229	57	172	193	1,725	1,746	82.5	83.5
1971	379	174	205	172	1,644	1,611	72.7	71.3
1972	943	320	623	457	2,276	2,110	86.5	80.2
1973	1,887	548	1,339	1,188	3,281	3,130	102.9	98.1
1974	2,858	696	2,162	1,914	3,488	3,240	91.3	84.8
1975	4,309	596	3,713	2,959	4,408	3,654	98.3	81.5
	10,605	2,391	8,214	6,883	16,822	15,491	91.0	83.8
1976		94		4,002		4,096		82.2
		2,485		10,885		19,587		83.5

Table 2B: EMPLOYER'S LIABILITY

Movement in Estimated Outstanding Claims, 1975-76

Year of Origin	(1) Estimated Outstanding Claims ("Reserve") at 31.12.75	(2) Actual Paid Claims 1.1.76-31.12.76	(3) Adjusted "Reserve" at 31.12.76 (1)-(2)	(4) Recalculated "Reserve" at 31.12.76	(5) Estimated Ultimate Claims at 31.12.75	(6) Estimated Ultimate Claims at 31.12.76	(7) Estimated Ultimate Loss Ratio at 31.12.75	(8) Estimated Ultimate Loss Ratio at 31.12.76
	£'000	£'000	£'000	£'000	£'000	£'000	%	%
1970	250	57	193	193	1,746	1,746	83.5	83.5
1971	433	174	259	258	1,697	1,697	75.1	75.1
1972	838	320	518	520	2,170	2,172	82.4	82.5
1973	1,673	548	1,125	1,123	3,067	3,065	96.2	96.1
1974	2,476	696	1,780	1,799	3,106	3,125	81.3	81.8
1975	3,069	596	2,473	2,515	3,168	3,210	70.7	71.6
	8,739	2,391	6,348	6,408	14,954	15,015	80.9	81.3
1976		94		2,902		2,996		60.1
		2,486		9,310		18,011		76.8

Table 3A: GENERAL LIABILITY

Movement in Estimated Outstanding Claims, 1975-76

Year of Origin	(1) Estimated Outstanding Claims ("Reserve") at 31.12.75	(2) Actual Paid Claims 1.1.76-31.12.76	(3) Adjusted "Reserve" at 31.12.76 (1)-(2)	(4) Recalculated "Reserve" at 31.12.76	(5) Estimated Ultimate Claims at 31.12.75	(6) Estimated Ultimate Claims at 31.12.76	(7) Estimated Ultimate Loss Ratio at 31.12.75	(8) Estimated Ultimate Loss Ratio at 31.12.76
	£'000	£'000	£'000	£'000	£'000	£'000	%	%
1970	483	83	400	319	1,616	1,535	73.7	70.0
1971	802	165	637	506	2,112	1,981	76.8	72.0
1972	1,231	214	1,017	880	2,467	2,330	67.5	63.7
1973	2,233	357	1,876	1,786	3,694	3,604	78.0	76.1
1974	4,190	693	3,497	3,298	5,876	5,677	97.3	94.0
1975	5,494	802	4,692	4,128	5,945	5,381	75.4	68.3
	14,433	2,314	12,119	10,917	21,710	20,508	79.6	75.2
1976		420		6,924		7,344		76.5
		2,734		17,841		27,852		75.6

Table 3B: GENERAL LIABILITY

Movement in Estimated Outstanding Claims, 1975-76

Year of Origin	(1) Estimated Outstanding Claims ("Reserve") at 31.12.75	(2) Actual Paid Claims 1.1.76-31.12.76	(3) Adjusted "Reserve" at 31.12.76 (1)-(2)	(4) Recalculated "Reserve" at 31.12.76	(5) Estimated Ultimate Claims at 31.12.75	(6) Estimated Ultimate Claims at 31.12.76	(7) Estimated Ultimate Loss Ratio at 31.12.75	(8) Estimated Ultimate Loss Ratio at 31.12.76
	£'000	£'000	£'000	£'000	£'000	£'000	%	%
1970	402	83	319	319	1,535	1,535	70.0	70.0
1971	623	165	458	459	1,933	1,933	70.3	70.3
1972	992	214	778	779	2,229	2,229	61.0	61.0
1973	1,711	357	1,354	1,350	3,172	3,168	66.9	66.9
1974	2,741	693	2,048	2,068	4,427	4,447	73.3	73.6
1975	3,715	802	2,913	2,882	4,166	4,134	52.8	52.4
	10,184	2,314	7,870	7,857	17,462	17,446	64.1	64.0
1976		420		4,186		4,605		48.0
		2,734		12,043		22,051		59.8

Table 4A: HOUSEHOLDERS

Movement in Estimated Outstanding Claims, 1975-76

Year of Origin	(1) Estimated Outstanding Claims ("Reserve") at 31.12.75	(2) Actual Paid Claims 1.1.76-31.12.76	(3) Adjusted "Reserve" at 31.12.76 (1)-(2)	(4) Recalculated "Reserve" at 31.12.76	(5) Estimated Ultimate Claims at 31.12.75	(6) Estimated Ultimate Claims at 31.12.76	(7) Estimated Ultimate Loss Ratio at 31.12.75	(8) Estimated Ultimate Loss Ratio at 31.12.76
	£'000	£'000	£'000	£'000	£'000	£'000	%	%
1970	8	-	8	7	1,404	1,403	31.2	31.2
1971	7	1	6	2	1,429	1,425	28.1	28.0
1972	41	9	32	25	1,895	1,888	31.7	31.6
1973	96	30	66	51	2,621	2,606	34.6	34.4
1974	318	100	218	178	3,760	3,720	41.7	41.3
1975	2,292	1,524	768	770	5,171	5,173	47.8	47.8
	2,762	1,664	1,098	1,033	16,280	16,215	37.9	37.8
1976		5,966		5,704		11,670		87.1
		7,630		6,737		27,885		49.5

Table 4B: HOUSEHOLDERS

Movement in Estimated Outstanding Claims, 1975-76

Year of Origin	(1) Estimated Outstanding Claims ("Reserve") at 31.12.75	(2) Actual Paid Claims 1.1.76-31.12.76	(3) Adjusted "Reserve" at 31.12.76 (1)-(2)	(4) Recalculated "Reserve" at 31.12.76	(5) Estimated Ultimate Claims at 31.12.75	(6) Estimated Ultimate Claims at 31.12.76	(7) Estimated Ultimate Loss Ratio at 31.12.75	(8) Estimated Ultimate Loss Ratio at 31.12.76
	£'000	£'000	£'000	£'000	£'000	£'000	%	%
1970	-	-	-	-	1,396	1,396	31.0	31.0
1971	-	1	(1)	-	1,422	1,423	28.0	28.0
1972	5	9	(4)	1	1,860	1,865	31.1	31.2
1973	19	30	(11)	11	2,544	2,566	33.6	33.9
1974	137	100	37	40	3,579	3,582	39.7	39.8
1975	1,512	1,524	(12)	189	4,391	4,591	40.6	42.5
	1,673	1,664	9	241	15,192	15,423	35.4	35.9
1976		5,966		3,272		9,238		69.0
		7,630		3,513		24,661		43.8

Table 5A: PRIVATE CARS

Movement in Estimated Outstanding Claims, 1975-76

Year of Origin	(1) Estimated Outstanding Claims ("Reserve") at 31.12.75	(2) Actual Paid Claims 1.1.76-31.12.76	(3) Adjusted "Reserve" at 31.12.76 (1)-(2)	(4) Recalculated "Reserve" at 31.12.76	(5) Estimated Ultimate Claims at 31.12.75	(6) Estimated Ultimate Claims at 31.12.76	(7) Estimated Ultimate Loss Ratio at 31.12.75	(8) Estimated Ultimate Loss Ratio at 31.12.76
	£'000	£'000	£'000	£'000	£'000	£'000	%	%
1970	95	37	58	50	3,557	3,549	79.9	79.7
1971	373	155	218	200	4,280	4,262	70.5	70.2
1972	638	215	423	394	5,237	5,208	66.0	65.6
1973	1,414	485	929	865	7,007	6,943	64.4	63.8
1974	2,278	596	1,682	1,403	7,540	7,261	62.9	60.6
1975	5,265	2,131	3,134	2,636	9,255	8,757	67.8	64.1
	10,063	3,619	6,444	5,548	36,876	35,980	67.1	65.4
1976		4,746		6,352		11,098		67.7
		8,365		11,900		47,078		66.0

Table 5B: PRIVATE CARS

Movement in Estimated Outstanding Claims, 1975-76

Year of Origin	(1) Estimated Outstanding Claims ("Reserve"), at 31.12.75	(2) Actual Paid Claims 1.1.76-31.12.76	(3) Adjusted "Reserve" at 31.12.76 (1)-(2)	(4) Recalculated "Reserve" at 31.12.76	(5) Estimated Ultimate Claims at 31.12.75	(6) Estimated Ultimate Claims at 31.12.76	(7) Estimated Ultimate Loss Ratio at 31.12.75	(8) Estimated Ultimate Loss Ratio at 31.12.76
	£'000	£'000	£'000	£'000	£'000	£'000	%	%
1970	87	37	50	50	3,549	3,549	79.7	79.7
1971	184	155	29	105	4,090	4,167	67.4	68.6
1972	424	215	209	279	5,023	5,093	63.3	64.2
1973	1,073	485	588	643	6,666	6,721	61.3	61.8
1974	1,922	596	1,326	1,256	7,184	7,114	60.0	59.4
1975	4,418	2,131	2,287	2,166	8,408	8,288	61.6	60.7
	8,108	3,619	4,489	4,499	34,920	34,932	63.5	63.5
1976		4,746		4,985		9,732		59.3
		8,365		9,484		44,664		62.6

Statistical Approaches to the Estimation of Ultimate Claims

1 Introduction

Previous papers for the General Insurance working parties have generally looked at methods of analysis of classes that are sufficiently large for purely statistical variation to be small in comparison to the potential effects of variation in inflation. Frequently the calculations incorporate conservative assumptions which it is hoped will render the final estimates high enough to be sufficient in most cases. Frequently also, the calculations are in terms of paid or settled claims, since it is felt that these are amenable to a statistical approach, as opposed to the 'subjective' assessment of outstanding claims by the claims department.

However, it is important to analyse small cohorts of business for underwriting and rating purposes, to obtain greater homogeneity of data, and to provide methods for companies with small portfolios. It is also important to distinguish between the underlying result of a cohort, as represented by its expected value, and the actual result which can be viewed as a sample with appropriate sample variation. The Institute paper presented by Dr. D.H. Reid has covered a great deal of the ground, and has provided suggestions for detailed solution of some aspects.

2. Areas of Uncertainty

In order to look at statistical variations, it is assumed that the data are standardised in respect of external factors - e.g. inflation, effects of legislation. We then wish to estimate the outcome of a defined cohort of business both as to the expected amount and number of claims, and as to the distribution of the expected values. A cohort may be defined as the claims reported in a period, or the claims on losses occurring in a period, or as the claims generated by business underwritten or incepting in a period. Dr. Reid's paper has considered the first case, the second case is that most normally reported, and the third is required for underwriting information. Dr. Reid has also restricted his methods to cases with a moderate number of claims and to settlement or paid patterns only. It may be worth considering how the methods might be extended to give a more powerful analysis.

3. Comments on Dr. Reid's paper

The following notes are suggestions for future work and regrettably are not backed up by demonstration of methods.

1. Detailed analysis is made of a base year, and the patterns of later years are related to the pattern analysed for the base year. The method will therefore not be satisfactory with cohorts below a certain size. On the assumption that changes to delay patterns from year to year can be modelled by general rules, a more powerful analysis will result if the information from all cohorts is taken into account jointly. In practice this would mean that standard delay patterns would be a first assumption, until inspection of residuals had revealed whether there had been a significant speeding up in particular periods, or whether there was a significant shift in delay patterns over time.

   The smaller the sample, the less appropriate it would be to make a very detailed analysis of changes in operational time, changes due to inflation, or changes of the distribution of claims by size. It should be assumed that the main effects are general effects to all cohorts, not specific to each observation.

Cont./..

2. Settled claims are considered in Dr. Reid's discussion. However, the extra information relating to outstanding claims becomes more significant as the size of sample is reduced. Even for a large sample, the number of claims remaining unsettled rapidly diminishes so that the outcome of a class is frequently affected by a small number of relatively large claims.

   Hence the forecast distribution of paid claims should be conditional upon the knowledge of notified outstanding claims. The importance given to the information will depend on its reliability historically and on the confidence of control over current claims assessment procedures. However, even if the outstanding claims information for a class is of low reliability, knowledge of an unusual number of extreme claims must improve the quality of forecasting.

3. The fitting of the joint distribution of delay and amount by polynomials and exponential terms is complex and appears somewhat arbitrary. It would seem reasonable to assume that delay is dependent on size (adjusted for inflation) although in some cases there may be an element of bargaining involved resulting in an interdependence. One would also expect the delay distributions to change in a regular fashion with alterations in size. An estimate of the distribution by claim size can be obtained by considering the marginal distribution. It would seem likely that simpler functions of delay would obtain as good a fit to the density function as the formulae adopted in Dr. Reid's paper, and changes in the functions can then be more simply monitored. Evidence that estimates of the moments of the distribution are biased can be investigated and corrected where the sample is large enough. It should be noted that we probably cannot expect, with the coarseness of time intervals and the general lack of homogeneity, to obtain a statistically acceptable level of fit - as is demonstrated in Dr. Reid's examples with the many termed expressions which are required there.

4. The variance of estimates which is calculated is conditional upon the assumed base cohort distribution, which is obviously subject to more or less error. It is important to measure this source of variance.

In summary, it is suggested that a simpler approach to the model required and the assessment of outstanding claims estimates may make the model more powerful for assessing smaller cohorts.

4. Extensions to unreported claims

The discussion by Dr. Reid did not consider in detail the estimation of IBNR or of unexpired risk reserve. However, delay in reporting claims is frequently a large problem, and where it is important to provide underwriting information, an estimate also of the outcome of business underwritten is required.

Since the delay in settling claims seems to be dependent on the delay to notification of the claim it would seem sensible to estimate the compound distribution of delay to advice and delay to settlement. Dr. Reid's paper then provides a framework if the number of claims unsettled is taken as an estimate, not as a fixed value. Further work needs to

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be done on the accuracy of estimates of unnotified claims based on those motified to date.

Given a framework of estimated delays following the date of loss, it would seem sensible to apply this to the unexpired portion of the risk underwritten. However, this is not satisfactory if the nature of claim is dependent on the age of the policy - e.g. contractors' all risk policies - and in this case it may be necessary to inspect the delay patterns for deviations from the expected pattern.

Restrictions to the data.

Even where information regarding numbers and amounts of claims is available in sufficient detail, the data may be affected by changes to procedures in defining individual claims, or by the fact that more than one claim may relate to one loss, or by changes in policy terms. It may then be very difficult to carry out a satisfactory analysis by numbers and size of claim, so that analysis would have to be made of the total amount of claims movement in each period.

It is then important to develop methods of assessing the uniformity of the development patterns of total amounts of claims, and hence to assess the variance of estimates based on the amounts known to date. To this end we should develop methods of estimating the variance of estimates at each period of delay of the eventual development of amounts given an underlying development distribution, and also methods of estimating the variance of estimates of that delay distribution. Unfortunately these two problems are not independent.

It is clear that a method employing information concerning numbers and amounts of claims separately is better placed both to discount for the exceptional occurrence of larger claims, and to reserve for such occurrences in the future. The analyst is then in a position to say whether the actual profitability of a cohort differs from the underlying profitability indicated by analysis.

C.J.Mellor
June 1978

RATE MAKING/CLAIMS RESERVING WITH LIMITED DATA - J P RYAN

This paper discusses reserving when data available is extremely limited. Reserves of this nature will normally be required for ratemaking purposes and the paper is written in that context. In order to make the paper more complete certain ratemaking aspects are also discussed. There is an unfortunate tendency in U.K. literature to only regard outstanding claims reserving in a year end balance sheet context. However unlike life assurance, claims reserves are required for ratemaking purposes. Indeed the financial consequences to the company of severe inadequacy in outstanding claims reserves used for ratemaking purposes are likely except for some long tail lines much greater than when used for balance sheet purposes.

This paper makes a few general observations and incorporates a practical approach. Throughout, it is assumed that the data immediately available is extremely scanty and of very low credibility. Obviously existing data should be used in so far as it is credible. Other approaches are possible and in such a situation judgement is very important. There is unlikely to be a right answer, although there may be a best answer.

The first question to consider is whether there is likely to be any suitable data; examples of where there are not; are the possibility of finding the Loch Ness monster or some nuclear coverages. Also it may be sensible not to use a statistical approach to the problem but to use a technological approach, e.g. detailed fire schedule rating. While the broader classifications can clearly be checked by statistical methods, the more detailed refinements are dependent largely upon fire surveyors' technological expertise.

One solution to lack of data is to get more data. The problem here is this is likely to increase the heterogeneity of the data under consideration. Grouping, by classes of risk or time periods could well increase the credibility of the data. e.g. expanding geographical rating areas for motor insurance or combining solicitor's professional indemnity with Doctor's professional indemnity.

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Grouping of data over different time periods may well involve the use of trending factors and in any event will require consid - able judgement, if any change in experience is suspected. However, in almost all lines of business in which the Casualty Actuary is involved, he suffers from this conflict between homogeneity of data and its credibility and choice of classifications requires much judgement. Even where the Actuary is not involved and the underwriter or case estimator makes all the decisions based on personal experience, it is important to realise that he, subjectively, is applying similar techniques to those that the Casualty Actuary is applying more objectively. Claims experience may be credible for the smaller claims but not for the larger claims. i.e. in liability classes there are generally a few very large claims as well as many more 'normal' claims.

Thus it is possible that the experience may give perfectly adequate information about the 'normal claims' but adjustment has to be made using other methods or data to allow for the larger claims. In this type of business it is appropriate to allocate claim costs and numbers to different layers in exactly the same way as if the business had been reinsured on an excess of loss basis. It is then appropriate to assign different credibilities to the different layers based on the number of claims in each layer.

The following example shows an approach to the problem based on some long tail liability data and illustrates some of the techniques that can be used in such circumstances. At a number of stages, judge- ment is required and different circumstances will require different solutions and it is difficult to define a 'right' approach. In many cases there may be the possibility of obtaining more data, even of a limited extent and this should be considered. It should also be emphasised that this is a technique that has been applied in practice. It is not the only approach.

Claims experience for four policy years was available. Throughout the example the numbers are based on a practical example though in order to preserve anonymity it has been felt advisable to alter the numbers slightly. Because this particular liability line suffers from a number of very large claims, claims experience has been amended so that all claims either actual or estimated are limited to a basic level. This could be of the order of £5,000 but would obviously depend on the Actuary's judgement. Thus where there is a claim of more than £5,000 it is valued at £5,000, where it is less than £5,000, it is taken at the face value of the estimate.

Premium rates are calculated at current rates and adjusted to the limitation of loss experience, i.e. - cover is assumed to be limited to £5,000. If such a premium is not readily available this can be done by taking the premium rate for full cover and then ratioing down by way of a basic limits factor. This can be calculated in exactly the same way as the basic limits factor to gross up the basic limits claims described below.

If a particular company's experience is very sparse it may be appropriate to calculate the premiums using the rates of another company which is likely to have much more meaningful data.

In this particular case incurred losses as of 31.3.77 are available on the case basis normally used by the company. Because this is long tail business it is essential to develop these to ultimate. If, as is possible, development factors are not available for the data concerned, it is necessary to use development factors derived from broader based data and amend judgementally for the case in point. Furthermore, as the data is inadequate we would not wish to apply the developemnt factors to the claims estimates because small variations in the incurred case estimates will have a disproportionate effect on IBNR reserves. For this example, it is appropriate to consider IBNR claims reserves to include development on existing reported claims. The IBNR factor is obtained from the claim development factor and then applied to the premium at present rates on the basis of an expected loss ratio.

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Thus, the IBNR reserve is equal to (1 - 1/ development factor) x expected loss ratio. See Appendix 1 for the algebra. We now have figures for incurred losses at basic limits as well as estimated IBNR claims. For people interested in more detail of this approach to estimating IBNR claims, see Proceedings of the Casualty Actuarial Society, No IX 1972 - "The Actuary and IBNR" by R L Bornhuetter and R E Ferguson. If the company had IBNR factors based on its own data albeit of low credibility, it would be appropriate to use a credibility weighted factor. The complement of the credibility would be applied to the IBNR factor calculated above.

The claims finally have to be trended to the appropriate level to allow for inflation and increases in frequency etc. The loss ratios for each of the four policy years are then calculated. The results are shown in the table below:

Policy Year	Premium at Basic Limits & at current Rate levels	Incurred Losses at Basic Lmts Trended to Current Levels	IBNR Factor	Estimated IBNR (1) x (3)	Total Basic Limits Incurred	Loss Ratio
72 - 73	3.03	1.81	.07	0.21	2.02	.668
73 - 74	3.68	0.75	.20	0.73	1.49	.406
74 - 75	4.51	1.29	.44	1.98	3.28	.727
75 - 76	5.61	0.61	.69	3.87	4.48	.799

Mean (Actual) Loss Ratio = 650

The mean loss ratio is then calculated although different weights could be applied to each years experience if required. In particular greater weight may be given to the latest 2 years especially if it is thought that 73 - 74 experience is untypical.

It is now necessary to decide upon the credibility of the experience. This will generally be based on the number of claims in the period concerned and will obviously vary considerably by line. It is not the purpose of this paper to elaborate further on the application of credibility theory. Having established the credibility of the data the appropriate rate level can now be determined on the basis of the formula:

$$\text{Credit} = \frac{\text{Expected Loss Ratio} - \text{Mean Loss Ratio}}{\text{Expected Loss Ratio}} \times \text{Credibility}$$

The revised loss ratio is then converted into the appropriate premium rate bearing in mind the company's anticipated expense experience.

We now need to re-convert the basic limits premium into a premium providing full coverage. If sufficient data is available in respect of distributions of claims sizes, this is simply a matter of examining the data in the obvious way. However, it is quite possible that this information will not be available, in which case it is necessary to assume a statistical distribution. Obvious choices for the distribution of claims size would be a log normal or pareto. Again judgement is involved.

Claim frequency and average size is a matter of judgement though obviously considerable attention will be paid to the unlimited claims experience. In order to avoid absurdly high claims it is appropriate to limit some of the very large claims. The claims experience is then simulated in order to give an indication of the distribution of amount of claim.

From the simulation it is possible to obtain the ratio of the expected cost of total claims to that of limited claims:

$$\text{i.e.} - \quad \frac{\text{Simulated claims}}{\text{No of simulations}} \Big/ \frac{\text{Simulated claims limited to £5000}}{\text{No of simulations}}$$

$$= \frac{\text{Simulated claims}}{\text{Simulated claims limited to £5000}}$$

This ratio may be of the order of 2 or 3. The final premium is the basic limits premium multiplied by this factor. It is implicitly assumed that expense loadings as a percentage of the gross premium are constant. If this assumption is inappropriate it is straightforward to incorporate an adjustment factor to the final premium.

A gross premium has been found in this way. Obviously if some form of excess of loss reinsurance protection is taken out, the simulation programe will be adapted to adjust for the reinsurance coverage and the reinsurance premium added to the premium derived from the simulation programe. In any event a contingency loading would be added to the premium.

If claims reserves are required for this part of the business, then the incurred basic limits factor is applied to the incurred losses, and claims already paid are deducted. If a contingency margin is required this can be obtained from the simulation programe at the appropriate probability level in the usual way.

APPENDIX I - Elaboration of IBNR formula

It is assumed that appropriate development factors are available to develop reported reserves to ultimate.

Then if an expected loss ratio must be determined based on premiums at present limits. This is obviously subjective but should produce much more stable results where data is scanty. e.g. - consider the reserve for policy year 73 - 74 if an IBNR factor used on reported losses is considered. However it must also be recognised that there is a certain arbitraryness about any assumption. It is also possible to see how sensitive the analysis is to variations in this assumption:

Total incurred reserves = Premiums at present rates + expected loss ratio

As our definition of IBNR includes all development on reported claims.

Reported x (development factor - 1) = IBNR reserve

$$\text{IBNR} \left\{1 + \frac{1}{\text{develop factor} - 1}\right\} = \text{Premiums x expected loss ratio}$$

$$\text{IBNR} = \text{Premiums x expected loss ratio} \left\{1 - \frac{1}{\text{Devlpment facto}}\right.$$

We thus have an expression for IBNR which is independent of reported losses.

ESTIMATION OF OUTSTANDING CLAIMS FROM RUNOFF DATA

by Henry Hansven

Introduction: A new method is given for handling runoff triangles and generating estimates of outstanding claims. The method is based on credibility ideas and is subjective in nature. Three examples are worked out numerically and the results are compared with estimates from the chain ladder method adjusted for inflation (1), the Bennett-Taylor method (2), the G. Taylor method (1)(3), and the full credibility method (given in this paper).

Data: The following is an example of the kind of data for which it is required to generate outstanding claims. This example and a further two data sets are given in Tables 1,2,3. Let C_{ij}=Claims paid in respect of business for period i ($i \geq 0$) during development period j ($j \geq 0$),i.e. claims paid during period (i+j), adjusted by an index to bring to constant purchasing power terms, seasonally adjusted, divided by a measure of exposed-to-risk. It is part of an aviation runoff conducted in sterling.

Year of origin i	Period of development j							
	0	1	2	3	4	5	6	7
0	31.81	3.82	2.66	2.65	2.04	0.87	-	-
1	29.64	4.21	0.17	0.91	2.63	0.35	-	-
2	18.03	4.01	5.42	2.50	9.07	0.50	-	-
3	44.41	6.13	5.77	18.42	2.80	1.12	-	-
4	30.39	4.99	1.97	3.96	0.40	2.55	-	-
5	51.60	15.63	15.68	0.54	1.84	0.34	-	-
6	36.76	3.78	5.60	9.01	1.93	3.09	-	-
7	49.57	2.65	14.96	3.65	1.06	2.78	-	-

There has been some discussion of the problems involved in using data in the above form for forecasting outstanding claims (See (1),(2)). A major problem is that often there is no available measure of exposed-to-risk other than the premium, and this is inconvenient because the relationship of premiums to exposed-to-risk fluctuates over time. Another major problem is the adjustment for inflation which involves a)choosing a price index suitable for the class of business, b)considering whether there is inflation between claim incident and claim payment, c)predicting future values of the price index. Other problems concern nonutilisation of information such as case estimates, knowledge of portfolio changes and changes in company's rating practise.

Despite these problems it is advisable to form estimates from runoff triangles since they are in practise often fairly good and frequently provide the best estimates, sometimes the only estimates.

Some suggestions for handling runoff data are discussed in (1) although they do not give the following method.

Procedure: Choose a period for which sufficient information is available to have a good idea of the runoff pattern. Suppose this is the first m years of origin $0 \leq i \leq m-1$. If necessary for the later years of origin in the period $[0, m-1]$ tail averaging may be used. Then use averaging to obtain a standard table, i.e. use the formula $r_j = \frac{1}{m} \sum_{i=0}^{m-1} C_{ij}$. The set $\{r_j\}$ then constitute the standard table. Suppose that the length of the runoff is k periods. The runoff triangle which is input data is then $\{C_{ij} : 0 \leq i \leq k, 0 \leq j \leq k-i\}$. For any year of origin i we may define a measure of the amount of information concerning that year of origin i. Such a measure is $\left(\sum_{j=0}^{k-i} r_j \Big/ \sum_{j=0}^{k} r_j \right) = Z_i$, say. Z_i may be called the credibility of the experience so far concerning year of origin i.

For predicting future claims, if the experience so far were to have full credibility then a reasonable estimate of outstanding claims would be $\left(\sum_{j=k-i+1}^{k} r_j \right) \left(\sum_{0}^{k-i} C_{ij} \right) \Big/ \left(\sum_{0}^{k-i} r_j \right)$. This is called here the "full credibility" method. I have not seen it previously published but no merits or advantages are claimed for it. On the other hand if the experience so far has zero credibility then a reasonable estimate of outstanding claims would be $\sum_{j=k-i+1}^{k} r_j$. The credibility formula for estimating outstanding claims would be

$$\left(\sum_{j=k-i+1}^{k} r_j \right) \left(Z_i \frac{\sum_{j=0}^{k-i} C_{ij}}{\sum_{j=0}^{k-i} r_j} + (1 - Z_i) \right) \text{ which}$$

simplifies to $\left(\sum_{j=0}^{k-i} C_{ij} + \sum_{j=k-i+1}^{k} r_j \right) \frac{\sum_{j=k-i+1}^{k} r_j}{\sum_{j=0}^{k} r_j}$. This is the formula put forward in this paper for estimating outstanding claims.

Example: The input data will be the upper left triangle of the above data, i.e.

Period of development j

Year of origin i	0	1	2	3	4	5	6	7
0	31.81	3.82	2.66	2.65	2.04	0.87	-	-
1	29.64	4.21	0.17	0.91	2.63	0.35	-	
2	18.03	4.01	5.42	2.50	9.07	0.50		
3	44.41	6.13	5.77	18.42	2.80			
4	30.39	4.99	1.97	3.96				
5	51.60	15.63	15.88					
6	36.76	3.78						
7	49.57							

This data is characterised as $\{C_{ij}: 0 \leq i \leq k,\ 0 \leq j \leq k-i\}$ where k=7.

On inspection of the data it is reasonable to use the first three years of origin for obtaining a standard table. So m=3. The standard table r_j and $\sum_{s=j}^{k} r_s$ are obtained:

j	0	1	2	3	4	5
r_j	26.49	4.01	2.75	2.02	4.58	0.57
$\sum_{s=j}^{k} r_j$	40.42	13.93	9.92	7.17	5.15	0.57

The r_j above were derived by the formula $r_j = \frac{1}{m} \sum_{i=0}^{m-1} C_{ij}$ for $0 \leq j \leq k+1-m$

$$r_j = \frac{1}{k-j+1} \sum_{i=0}^{k-j} C_{ij} \text{ for } k+1-m \leq j \leq k$$

The estimates for outstanding claims are then derived

(1) Year of origin i	(2) $\sum_{j=0}^{k-i} C_{ij}$ Paid so far	(3) $\sum_{j=k-i+1}^{k} r_j$	(4) HK's estimate of o/s claims	(5) Full credibility method
3	77,53	0.57	1.10	1.11
4	41.31	5.15	5.92	7.09
5	83.11	7.17	16,01	17.92
6	40.54	9.92	12.38	13.19
7	49.57	13.93	21.88	26.07

Note columns (4) and (5) above are derived by the formulae

$$(4) = ((2)\ (3))(3) / \sum_{j=0}^{k} r_j$$

$$(5) = (2)/(3)/(\sum_{j=0}^{k-1} r_j - (3))$$

This is the end of the procedure. However it is of interest to compare the above estimates with the truth and also with estimates of outstanding claims produced by different methods. This comparison is presented in Tables 1,2 and 3 where the results are shown for five different methods of estimation as applied to three real data sets obtained from U.... insurers.

Discussion of procedure: For manual calculations the time taken for the procedure suggested by this paper is less than for the chain ladder method and variations thereon as one does not calculate cumulative claims. A period of m years is chosen to establish the standard table of r_j. It is probably unwise to apply the procedure suggested in the paper to r_j calculated from the whole runoff triangle since it would invalidate the idea of proportioning up claims paid so far. The idea of a credibility factor has a theoretical underpinning in that it is a weighted average whose weights are ideally of the order of the reciprocal of the variances: such weighted averages have the property of producing minimum variance estimators of the underlying means provided the original estimators were independent and unbiased.

Conclusion: The procedure suggested in this paper for estimating outstanding claims from a runoff triangle will be useful for some data pools. Further experimentation of this method on various classes of business is required to determine its value.

References: (1) T. Clarke, M.C. Bennett, S.M. Coutts, B.D. Hudson, H. Karsten, C.T. Mellor, G.C. Orros, W. Rowlandson, A.C. Stalker, Technical Reserves, GISG Conference York 1976 deposited in library of the Institute of Actuaries.

(2) M.C. Bennett and J.M. Taylor Motor outstanding claims, GISG Conference Hythe Kent 1977.

(3) G.C. Taylor Separation of inflation and other effects from the distribution of nonlife insurance claim delays, ASTIN 1977 Vol. IX, Parts 1 and 2.

Table 1(a)

The data set below is taken from the aviation runoff for a large insurance company. It is a matrix of C_{ij} where C_{ij}=Claims paid during development year j in respect of claims incurred during year of origin i, divided by earned premiums times a constant.

Year of development j

i=	j= 0	1	2	3	4	5	6	7
0	31.81	3.82	2.66	2.65	2.04	0.87	-	-
1	29.64	4.21	0.17	0.91	2.63	0.35	-	-
2	18.03	4.01	5.42	2.50	9.07	0.50	-	-
3	44.41	6.13	5.77	18.42	2.80	1.12	-	-
4	30.39	4.99	1.97	3.96	0.40	2.55	-	-
5	51.60	15.63	15.88	0.54	1.84	0.34	-	-
6	36.76	3.78	5.60	9.01	1.93	3.09	-	-
7	49.57	2.65	14.96	3.65	1.06	2.78	-	-

Year of origin i

The input data to the procedure is the upper left hand triangle of the above data, i.e.

Year of development j

i=	j= 0	1	2	3	4	5	6	7
0	31.81	3.82	2.66	2.65	2.04	0.87	-	-
1	29.64	4.21	0.17	0.91	2.63	0.35	-	
2	18.03	4.01	5.42	2.50	9.07	0.50		
3	44.41	6.13	5.77	18.42	2.80			
4	30.39	4.99	1.97	3.96				
5	51.60	15.63	15.88					
6	36.76	3.78						
7	49.57							

Year of origin i

The results and comparisons for this data set are given in Table 1(b). (For the procedure suggested in this paper m=5 was used.)

Table 1(b)

The results below refer to the data set given in Table 1(a).

i) Outstanding claims:

Year of origin	The truth actual o/s	HK's	B-T	G.T	Full Cred	Chain Ladder
3	1.12	1.10	.57	.55	1.11	1.11
4	2.95	5.92	4.71	2.75	6.03	4.43
5	2.72	16.01	10.40	5.51	17.92	22.45
6	19.63	12.38	15.71	11.02	13.19	17.66
7	25.10	21.88	21.79	15.97	26.07	34.07

ii) Comparisons of actual with expected

Year of origin	HK's A-E	B-T A-E	G.T A-E	Full Cred A-E	Chain Ladder A-E
3	.02	.55	.57	.01	.01
4	-2.97	-1.76	.20	-3.08	-1.48
5	-13.29	-7.68	-2.79	-15.20	-19.73
6	7.25	3.92	8.61	6.44	1.97
7	3.22	3.31	9.13	-.97	-8.97
Σ(A-E)	-5.77	-1.66	15.72	-12.80	-28.20
$\Sigma\lvert$(A-E)$\rvert$	26.75	17.22	21.30	25.70	32.16

Table 2(a)

The data set below is a weighted average of householders (property) runoffs for three major general insurance offices. It is a matrix of C_{ij} where C_{ij} = Claims paid during development year j in respect of claims incurred during year of origin i, divided by earned premiums times a constant.

		Year of development j			
	i=	j = 0	1	2	3
	0	22.8	8.0	0.3	-
Year of origin i	1	19.1	8.2	0.4	0.1
	2	21.1	8.9	0.8	0.3
	3	20.0	11.4	1.9	0.4

The input data to the procedure is the upper left hand triangle of the above data, i.e.

		Year of development j			
	i=	j = 0	1	2	3
	0	22.8	8.0	0.3	-
Year of origin i	1	19.1	8.2	0.4	
	2	21.1	8.9		
	3	20.0			

The results and comparisons for this data set are given in Table 2(b). (For the procedure suggested in this paper m = 3 was used.)

Table 2(b)

The results below refer to the data set given in Table 2(a).

i) Outstanding claims:

Year of origin	The truth actual o/s	HK's	B-T	G.T	Full Cred	Chain Ladder
1	.1	0	0	0		0
2	1.1	0.3	0.3	0.3	0.3	0.3
3	13.7	8.4	8.4	8.90	8.3	8.0

ii) Comparisons of actual with expected

Year of origin	HK's A-E	B-T A-E	G.T A-E	Full Cred A-E	Chain Ladder A-E
1	.1	.1	.1	.1	.1
2	0.8	0.8	0.8	0.8	0.8
3	5.3	5.3	4.8	5.4	5.7
$\Sigma(A-E)$	6.2	6.2	5.7	6.3	6.6
$\Sigma\|(A-E)\|$	6.2	6.2	5.7	6.3	6.6

Table 3(a)

The data set below is taken from the motor runoff for a medium sized insurance company. It is a matrix of C_{ij} where C_{ij}=Claims paid during development year j in respect of claims incurred during year of origin i, adjusted to constant purchasing power and divided by the number of claims.

		Year of development j						
	i= \	j= 0	1	2	3	4	5	6
	0	53.97	21.52	7.75	4.30	2.57	1.01	-
	1	53.29	21.05	7.75	5.30	2.95	1.45	-
	2	49.13	19.87	6.32	4.91	2.75	1.16	-
Year	3	48.61	21.59	7.45	5.42	2.21	1.16	-
of	4	48.65	21.34	7.47	3.38	3.19	1.64	-
origin	5	47.90	22.25	6.93	4.05	2.36	2.17	-
	6	49.86	21.58	6.04	5.22	2.45	1.56	-

The input data to the procedure is the upper left hand triangle of the above data, i.e.

		Year of development j						
	i= \	j= 0	1	2	3	4	5	6
	0	53.97	21.52	7.75	4.30	2.57	1.01	-
	1	53.29	21.05	7.75	5.30	2.95	1.45	
	2	49.13	19.87	6.32	4.91	2.75		
Year	3	48.61	21.59	7.45	5.42			
of	4	48.65	21.34	7.47				
origin	5	47.90	22.25					
	6	49.86						

The results and comparisons for this data set are given in Table 3(b). (For the procedure suggested by this paper m=3 was used.)

Table 3(b)

The results below refer to the data set given in Table 3(a)

i) Outstanding claims:

Year of origin	The truth actual o/s	HK's	B-T	G.T	Full Cred	Chain ladder
2	1.16	1.16	1.23	0.88	1.16	1.13
3	3.37	3.90	3.99	3.51	3.99	3.86
4	8.21	8.56	8.97	7.89	8.53	8.68
5	15.51	15.60	16.32	15.78	15.48	15.84
6	36.85	35.97	37.59	36.83	35.30	37.12

ii) Comparisons of actual with expected

Year of origin	HK's A-E	B-T A-E	G.T A-E	Full Cred A-E	Chain ladder A-E
2	0	.03	.28	0	.03
3	-.53	-.62	-.14	-.62	-.49
4	-.35	-.76	.32	-.32	-.47
5	-.09	-.81	-.27	.03	-.33
6	.88	-.74	.02	1.55	-.27
Σ(A-E)	- .09	-2.90	.21	.64	-1.53
$\Sigma\|$(A-E)$\|$	1.85	2.96	1.03	2.52	1.59