Capital Allocation in the Lloyd's Insurance Market

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Market Risk and Reserving Unit

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Part 1

Capital allocation in the Lloyd's market
Risk based capital - Overview

- RBC system applied to corporate members from 1994 and all members from January 1998
- RBC equalises expected loss to the Central Fund per unit of net premium/reserve
- Inputs include:
  - Business mix diversification
  - Profile of reinsurance protection including security
  - Credit for diversification across managing agents
  - Credit for diversification across underwriting years
  - Syndicate specific adjustments

Lloyd's Chain of Security

<table>
<thead>
<tr>
<th></th>
<th>corporate members</th>
<th>individual members</th>
<th>end 2001 £m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premium Trust Funds</td>
<td></td>
<td></td>
<td>£13,462</td>
</tr>
<tr>
<td>Funds at Lloyd's</td>
<td></td>
<td></td>
<td>£7,704</td>
</tr>
<tr>
<td>Other Personal Wealth</td>
<td></td>
<td></td>
<td>£327</td>
</tr>
<tr>
<td>Central Fund</td>
<td></td>
<td></td>
<td>£280*</td>
</tr>
</tbody>
</table>

* an insurance protection as well as an additional callable component is also available

RBC Concept

Note: RBC calculated using illustrative parameters
Syndicate-Specific Parameters

Previously
- RBC has previously used a market average model
- Average means and variances, imputed reserve exposure
- Differences from different portfolios
- Loadings for catastrophe and management risk
- Discounts for syndicate performance

2003
- 2003 YOA model has syndicate-level adjustments for mean and potentially for variance
- Some Cat loadings in model

Operating Risk

- Define OR as "Measurable features of a syndicate that can be shown to be associated with better or worse than average performance"
- Add requirement that these pass the reasonableness test

How to set SSPs : Operating Risk

- Syndicates' actual results not suitable
- Looked instead for Explanatory Variables (EVs)
- 1993 - 2000 years, 50 Risk Groups, all syndicates = 11,000 data points
- 40 potential EVs
- Seven were statistically significant
- Reasonableness checks
**Table of EVs**

<table>
<thead>
<tr>
<th>EV</th>
<th>RBC increases with</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>Smaller syndicates</td>
</tr>
<tr>
<td>UI/W Experience</td>
<td>Less Experience</td>
</tr>
<tr>
<td>UI/W Qualification</td>
<td>No ACII/FCII</td>
</tr>
<tr>
<td>Syndicate growth</td>
<td>Faster growth</td>
</tr>
<tr>
<td>Writing 100% lines</td>
<td>More 100% lines</td>
</tr>
<tr>
<td>Relying on one broker</td>
<td>More from largest broker</td>
</tr>
<tr>
<td>Reinsurance gearing</td>
<td>More reinsurance spend</td>
</tr>
</tbody>
</table>

**Catastrophes**

- Previously potential for loading if certain criteria tripped - based on RDS returns
- Now proposed to use RDS directly in the RBC calculation
- Add 3 specific RDS amounts directly: US Wind, California Earthquake, New Madrid Earthquake
- Old process for others - extend in future years

**Adding Catastrophe Risk**

![Graph showing catastrophe risk distribution](image)
Adding Catastrophe Risk

\[ f_{adj}(x) = f(x-L)p + f(x)(1-p) \]

Part 2

Allocation of risk capital to pooled liabilities
Distortion Principles

- Definition of the risk measure (Denneberg (1990), Wang (1996)):
  \[ \rho(X) = \int_0^\infty g(P(X > x)) dx \]
  \[ g > 0, \quad g'' < 0, \quad g(0) = 0, \quad g(1) = 1 \]
- Distortion principles satisfy the axioms of coherent risk measures, plus the requirement for comonotonic additivity.

Allocation of pooled capital

- n portfolios of stochastic liabilities are pooled.
- The risk capital that the pool must hold is lower than the aggregate capital requirements would be for the non-pooled liabilities.
- Cooperation produces capital savings: how to allocate those to the participants?
- The core of a cooperative game: no disincentives for cooperation.

Example

- 3 Pareto distributed liabilities, \( \alpha = 4, \beta = 3/4 \).
- Correlation matrix and correlations to the aggregate:
  \[ r(X) = \begin{pmatrix} 1 & 0.1 & 0.5 \\ 0.1 & 1 & 0.8 \\ 0.5 & 0.8 & 1 \end{pmatrix} \]
  \[ r(X_1, \Sigma X_1) = 0.64 \]
  \[ r(X_2, \Sigma X_2) = 0.75 \]
  \[ r(X_3, \Sigma X_3) = 0.90 \]
Example (cont’d)

- Aggregate required capital: $\rho(\Sigma, X_i) = 5.06$
- Allocate proportionally: $\rho(X_i) = 1.69$
- Suppose now that only the first two portfolios co-operate.
- Aggregate required capital: $\rho(X_1 + X_2) = 3.29$
- Allocate proportionally: $\rho(X_1) = \rho(X_2) = 1.64$
- The first two portfolios have an incentive to expel the third one! What went wrong?

The ‘fuzzy core’

- Interested in allocations that add up to the aggregate risk and produce no disincentives for cooperation.
- We need to find a vector $d \in \mathbb{R}^n$, such that:
  - $a \Sigma d_i = \rho(\Sigma, X_i)$
  - $b \rho(\Sigma, u, X_i) \geq \Sigma u_i d_i \quad \forall u \in [0,1]$^
- For the distortion principle there is only one such allocation.

A formula for the core allocation

- It turns out that the core allocation is given by:
  $$d_i = E[X_i,g(S_{\Sigma, \{X_i\}})] \quad S_{\Sigma}(z) = P(\Sigma, X_i > z)$$
- We can re-write that formula as:
  $$d_i = E[X_i] \quad \frac{\partial Q}{\partial P} = g(S_{\Sigma, \{X_i\}})$$
- ...and also as:
  $$d_i = \left[ \int \int F_{X_i}(u)g(1-v) dC_{X_i, \Sigma, X_i}(u,v) dudv \right]$$
**Dynamic extension of risk measure and allocation method**

- Let $Z = \sum X_i$. We can write the risk measure as:
  
  $\rho(Z) = \sup_{P \in \mathcal{P}_1} E_P[Z]$ 

- Assume that the underlying risk processes are Markov on $[0,T]$. Let $B_i$ be the event:
  
  $B_i = \{\omega : X_i^1(\omega) = x_1^i, \ldots, X_i^n(\omega) = x_n^i\}$ 

- Then generalise the risk measure by:
  
  $\rho(Z_i | B_i) = \sup_{P \in \mathcal{P}_1} E_P[Z_i | B_i]$ 

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**Explicit formulae**

- Allocated capital to $i$th portfolio:
  
  $d_i = E[D_{i,t} | X_i | B_i]$ 

- Radon-Nikodym derivative:
  
  $D_{i,t} = G_i(P,(Z_t > z | B_i))$ 

- Updated distortion function:
  
  $G_i(s) = \frac{g(sP_i(B_i))}{g(sP_i(B_i)) + 1 - g(1 - s)P_i(B_i))}$ 

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