34TH ANNUAL GIRO CONVENTION
CELTIC MANOR RESORT, NEWPORT, WALES

Claims reserving including count data
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Outline
- Motivation - why use count data
- Data - count data
- A claim - incurred, reported, paid
- IBNR and RBNS
- Stochastic model
- Data study - Auto TPL
- Conclusion
Motivation

An introductory example

This ‘business’ is obviously not very volatile, or?

Maybe it is?!
Example with coin flips

- Monday: 49% tail
- Tuesday: 41% tail
- Wednesday: 53% tail
- Thursday: 59% tail
- Friday: 51% tail

Example with coin flips

- Monday: 49% tail, 2500 flips
- Tuesday: 41% tail, 250 flips
- Wednesday: 53% tail, 500 flips
- Thursday: 59% tail, 300 flips
- Friday: 51% tail, 2100 flips
Data discussion

Which data do we use?
- The number of reported claims
- The aggregate paid amounts
- Both in triangular arrays

Why use this data format?
- Availability
- Relation to existing method
- Both incurred and paid data

Why use this data format?

<table>
<thead>
<tr>
<th>Reported Counts</th>
<th>Paid Counts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incurred Paid</td>
<td>Paid amounts</td>
</tr>
<tr>
<td>(inc. claims estimates)</td>
<td></td>
</tr>
</tbody>
</table>

- Incurred Paid is not ‘real data’ - claim estimates. Cashflow.
- Paid Counts means what? - more payments per claim
- Reported Counts - one claim, one count
- Paid Amounts is ‘real data’ - no claim estimates
A thought up data example

A claim - IBNR and RBNS

The two delays...

- The claim incurred
- The claim is reported
- The claim is (fully) paid
A thought up example; Conditional information on future payments

We know ‘something’ about the future.

### IBNRRBNS

<table>
<thead>
<tr>
<th>Reported Counts</th>
<th>Post Occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>12345</td>
<td>12345678</td>
</tr>
<tr>
<td>1</td>
<td>120</td>
</tr>
<tr>
<td>1</td>
<td>500</td>
</tr>
<tr>
<td>1</td>
<td>145</td>
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<tr>
<td>2</td>
<td>125</td>
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<tr>
<td>2</td>
<td>650</td>
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<tr>
<td>2</td>
<td>110</td>
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<tr>
<td>3</td>
<td>132</td>
</tr>
<tr>
<td>3</td>
<td>700</td>
</tr>
<tr>
<td>3</td>
<td>135</td>
</tr>
<tr>
<td>4</td>
<td>450</td>
</tr>
<tr>
<td>4</td>
<td>350</td>
</tr>
<tr>
<td>4</td>
<td>450</td>
</tr>
<tr>
<td>5</td>
<td>132</td>
</tr>
<tr>
<td>5</td>
<td>600</td>
</tr>
</tbody>
</table>

### IBNR-delay

### RBNS-delay

Stochastic model

The Reported Counts

Given \( \theta \),

\[ N_i \sim \text{Pois}(\theta, E_i) \]

where

\[ \sum_{i=1}^{n} E_i = 1 \]

and \( E_i \) is the expected exposure (e.g. the net premium) the \( i \)th period.

**CHAIN LADDER** for the REPORTED COUNTS (given \( \theta \)).

Given (9.), then \( (N_i) \) are mutually independent.
(9.) are iid.
One can also choose value structure to \( \theta | E_i | A_{i+1} \) given \( \theta \), to correct for claims inflation.
The Reported Counts - a remark

Given $\sum_{i=1}^{n} X_i$ we have

$$(N_{11}, ..., N_{n1}) \sim \text{Multi} \left( \sum_{i=1}^{n} X_i, \sum_{i=1}^{n} \mathbb{P}_i \right)$$

Payment of Reported Claims

$$
\begin{align*}
N_{11} & \quad N_{12} & \quad N_{13} & \quad \cdots & \quad N_{1n} \\
\vdots & \quad \vdots & \quad \vdots & \quad \ddots & \quad \vdots \\
N_{n1} & \quad N_{n2} & \quad N_{n3} & \quad \cdots & \quad N_{nn} \\
\end{align*}
$$

Given $(N_{ij})$ we have

$$(N_{11}, ..., N_{1n}) \sim \text{Multi} (N_{1i}, ..., N_{ni})$$

A thought up example - Payment of Reported Claims

- RED AREA - NOT observed
- RED AREA - depend on GREEN AREA
Payment of Reported Claims

\( (Y_{jk}) \) are paid claims and

\[ X_{ij} = \sum_{k=1}^{n} Y_{jk} \]

Data study TPL

Inflation corrected data
Counts and Paid

Model validation

The delay function
Results

- Reserve: 3,546 * 10^8
  - Conf. Bands: (3,001 * 10^8 ; 4,187 * 10^8)

- IBNR: 2,965 * 10^7
  - Conf. Bands: (2,931 * 10^7 ; 4,225 * 10^7)

- RBNS: 3,25 * 10^8
  - Conf. Bands: (2,729 * 10^8 ; 3,865 * 10^8)

- Mean claim: 16.346,96 DKK
  - Single claim distribution: \( \Gamma(0.01568, 9.592 \times 10^{-5}) \)

Comparison to the Bootstrap method on Paid

- Reserve: 3,311 * 10^8
  - Conf. Bands: (3,116 * 10^8 ; 3,924 * 10^8)

Notice: Chain-ladder reserve contains both IBNR and RBNS...
Conclusion

- Data is available
- Gain understanding of the volatility of IBNR, RBNS and the related delays
- Use both paid and incurred data
- Method closely related to chain ladder
- Cashflows can be modelled effectively

Future work...

- Use claims estimates in counts model (delay)
- Generalise to other distributions for the single claims (Gamma)
- Develop model with more payments per claim
- Make models operational