The Actuarial Profession
making financial sense of the future

Highlights of the 2010 life conference seminar
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Aggregation techniques for Solvency II:
a practical example

2011

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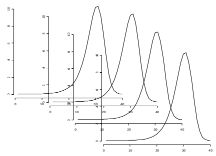
Building blocks of capital models

Desired properties

Modelling of individual risks



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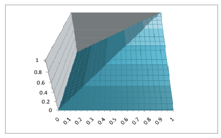
Modelling of dependence structure



Fat tails
Asymmetry
Non-normality

Tail dependence
Interactions
Diversification

Modelling capital required to survive extreme events

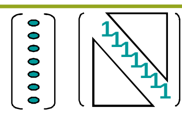
Key concepts explained

Concept	Explanation
Correlation matrix	Matrix used to describe the dependence between pairs of random variables, eg lapse and interest rates
Non linearity/interaction	Non-linearity describes the effect whereby the impact of stresses occurring together differs from the sum of the impacts of the individual stresses
Copula	An approach by which the marginal distributions of a set of variables are combined together into a single multivariate distribution
Marginal distribution	Distribution of one variable obtained when ignoring all other variables of the joint distribution
Tail dependence	Measure of extreme co-movements in the tail of a joint distribution

Overview of main approaches

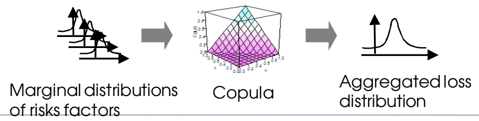
Method	Individual risk calibration	Dependency structure	Aggregation subject	Transformation	Output
Medium Bang	• Assumed normal distribution. • Mean and standard deviation calibrated.	Correlation Matrix	Vector of net asset stresses by risk factor	MAX … Medium Bang Consolidation at Y% Confidence Interval with no diversification	• Capital requirement at given confidence level. • Non-linearity allowed for through medium bang.
Risk Geographies	• Elliptically contoured distributions (e.g. Normal, multivariate T)	Correlation Matrix	Vector of net asset stresses by risk factor	Likely locus; Best estimate assumptions; Least Solvent Likely Event "LSLE"; Ruin locus	• As above. • Non-linearity picked up directly.
Input Copula (Copula applied to marginal risk distributions)	• Marginal distribution.	Copula	Marginal distributions of individual risk drivers	Marginal distributions of individual risks → Copula → Joint distribution of risk drivers → Aggregated loss distribution	• Full distribution of capital requirements. • Non-linearity picked up directly
Output Copula (Copula applied to loss distributions)	• As above.	Copula	Loss functions (P&L Impact) of individual risks	Loss functions of individual risks → Copula → Aggregated loss distribution	• Full distribution of capital requirements. • Non-linearity not picked up.

Aggregation with Variance/Covariance



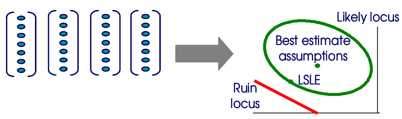
Method	Variance / Covariance
Brief description	• Similar to Standard Formula under Solvency II, based on **a correlation matrix** • Most common method currently used by insurance companies to perform capital calculations (see CRO Forum, Jan. 2009), but leading companies are revisiting their choice of this method
Risk factor modeling & calibration	• Risks are **elliptically distributed** and are combined by using a variance / co-variance approach • Assumptions required regarding the mean and standard deviations of the individual risks
Process	• **Closed-form formula** combining the capital requirement calculated for each risk factor at a given level of confidence using a correlation matrix • Approximate adjustment can be made to allow for the non-linearity of different risks (so-called **Medium Bang** approach used in the UK, involving a combined scenario), but it remains ultimately arbitrary
Outputs	• The method produces a **capital result at the required level of confidence**. Results at different levels of confidence can be produced by repeating the calculation process • Results fairly stable over time due to the simplicity of the method
Key technical features	• Non-linearity, fungibility restrictions and asymmetry are not picked up by the method • Assumes that stress tests for risk factors correspond to the tail of the capital distribution

Aggregation with Input Copulas



Method	Copulas
Brief description	• **Monte Carlo approach** • This method combines the marginal distributions of risk factors using copula functions. Fed through an ALM model, these simulations are then used to produce a full distribution of capital requirements
Risk factor modelling & calibration	• Flexibility in assumed distributions of risk factors (including non-normal) and dependency structure • A key challenge will be obtaining **reliable data to calibrate each copula**
Process	• A large number (e.g. 100,000) of scenarios are produced from the risk factor distributions • Marginal distributions of risk factors are combined using copula functions • The ALM model used for generating capital requirements is usually a **simplified model** (for example based on Replicating Portfolios or Formula Fitting), as many scenarios need to be run to achieve statistical convergence
Outputs	• **Full distribution** of capital requirements (does not assume that stress tests for risk factors correspond to the tail of the capital distribution) • Results tend to be **less stable** over time as the approach is simulation based. They will need to be smoothed, which gives rise to a potential issue of managing the impact of smoothing
Key technical features	• Allowance can be made for **non-linearity** between risk factors through the use of an appropriate simplified ALM model • **Tail dependence** and **fat-tailed distributions** can be allowed for • **Fungibility restrictions** can be allowed for as the method is simulation based

Aggregation with Risk Geographies



Method	Risk Geographies
Brief description	• Basically an **enhancement of the Variance / Covariance + Medium Bang approach** which enables the insurer to determine the **exact scenario** which would have the most severe financial impact on its balance sheet at a given level of confidence
Risk factor modeling & calibration	• Not restricted by particular choices of distributions or copulas • However, most of the implementations to date have been in the variance-covariance framework
Process	• The most onerous scenario is determined by means of an **iterative process of stress tests** • The result does not require Monte Carlo simulations of capital requirements and runs with **full ALM model**
Outputs	• The method produces a **capital result at the required level of confidence**. Results at different levels of confidence can be produced by repeating the calculation process • The other main outputs are the **risk scenarios** (LSLE, MLRE) • Results tend to be **fairly stable** over time as the approach is not simulation based
Key technical features	• Risks are combined in such a way that **non-linearity** is allowed for • Allowance for fat tails, tail dependencies and fungibility restrictions is possible but sometimes requires adaptations

Illustration of Risk Geographies & Input Copulas

Input Copula produce relatively smooth capital requirements .....

Business Unit	Risk Geographies	Input Copulas		
		Highest of 5 runs	Lowest of 5 runs	Lowest / Highest
BU 1 – Term Assurances	5,746	5,744	5,729	99.7%
BU 2 – Annuities	15,909	16,102	15,921	98.9%
BU 3 – Pensions	8,659	8,896	8,708	97.9%
Diversification benefit	-11,761	-12,053	-11,754	n/a
Total – post diversification (Group)	**18,553**	**18,689**	**18,604**	99.5%

However, identification of biting scenario is difficult .....

Scenarios driving capital requirements (100,000 simulations)

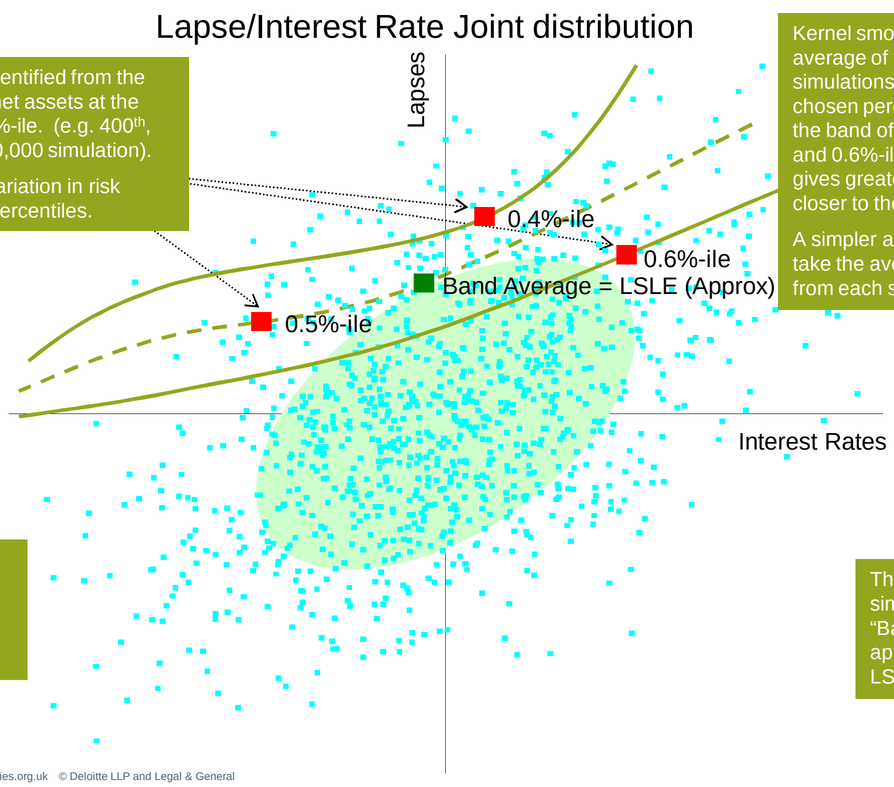
Risk	Risk Geographies	Input Copulas					
	LSLE	Run 1	Run 2	Run 3	Run 4	Run 5	Range
Interest	0.15%	0.77%	0.68%	-0.60%	1.80%	1.93%	2.53%
Credit Spreads	2.24%	2.30%	2.06%	1.70%	2.12%	3.25%	1.55%
Equity Levels	-3.14%	0.86%	-7.11%	-19.35%	-4.45%	8.95%	28.31%
Asset Share Volatility	5.43%	6.27%	7.59%	5.33%	13.70%	2.20%	11.49%
Mortality Level	2.77%	7.48%	5.83%	0.60%	5.06%	3.04%	6.88%

Requires a smoothing mechanism .... Kernel smoothing

Kernel smoothing explained

These are the scenarios identified from the simulations that drive the net assets at the 0.4%-ile, 0.5%-ile and 0.6%-ile. (e.g. 400th, 500th and 600th runs of 100,000 simulation).

It highlights the potential variation in risk factors between different percentiles.



Kernel smoothing takes a weighted average of the net assets from the simulations that lie between 2 chosen percentiles (e.g. shown by the band of points between 0.4%-ile and 0.6%-ile). The kernel function gives greater weighting to points closer to the mid percentile.

A simpler approach could be just to take the average of the net assets from each simulation.

The curves on the distribution represent contours of equal net assets for different percentiles.

The average of the simulations results is the "Band Average" which is approximately equal to the LSLE.

An example of Kernel smoothing

Attribution using Euler method and driving scenarios produces widely varying capital amounts

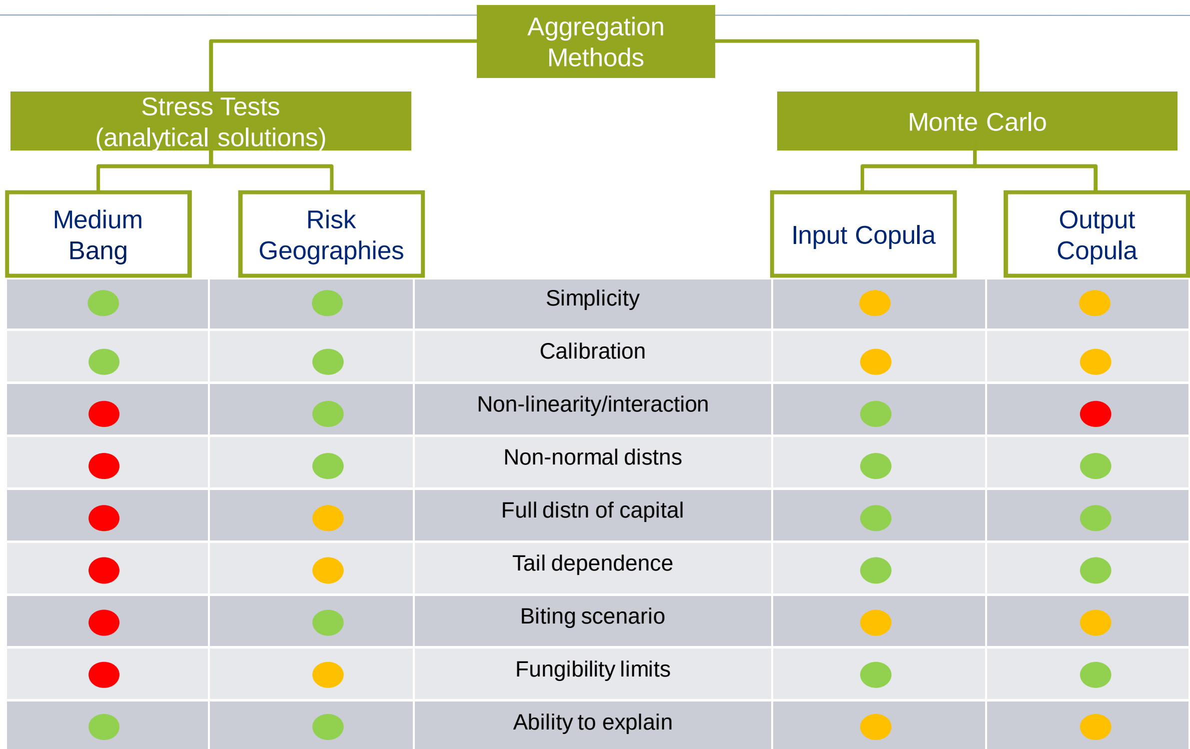
Business Unit	Risk Geographies	Input Copula (100,000 Simulations, Unsmoothed)					
		Run 1	Run 2	Run 3	Run 4	Run 5	Size of range
BU 1 – Term Assurances	2,837	4,636	3,757	2,086	2,303	1,854	2,781
BU 2 – Annuities	13,601	13,219	12,289	10,112	14,143	19,914	9,802
BU 3 – Pensions	2,115	762	2,558	6,452	2,242	-3,081	9,533
Group	**18,553**	**18,617**	**18,604**	**18,650**	**18,689**	**18,687**	**85**

Kernel smoothing reduces variability ....

Business Unit	Risk Geographies	Input Copula (100,000 Simulations, Smoothed)					
		Run 1	Run 2	Run 3	Run 4	Run 5	Size of range
BU 1 – Term Assurances	2,837	2,832	2,864	2,759	2,842	2,854	105
BU 2 – Annuities	13,601	13,531	13,571	13,809	13,567	13,689	278
BU 3 – Pensions	2,115	2,148	2,064	2,013	2,130	2,065	135
Group	**18,553**	**18,512**	**18,500**	**18,581**	**18,539**	**18,608**	**108**

... meaning we are less exposed to random variations

Comparison of methods considered



Risk Geographies & Input Copula selected

L&G experience to date

Selection:

- Monte Carlo with Input Copula and Risk Geographies chosen methods
- Monte Carlo approach was prioritised due to automatic provision of full distribution of capital amounts, and prevalence of the approach
- Also considered to be the most flexible method to implement and to address challenging Solvency II requirements such as extreme events and fungibility constraints

L&G experience to date (continued)

Monte Carlo with Input Copula:

- Randomness of scenarios makes identification of "biting scenario" complex
- Creates challenge for both attribution and communication to Boards
- Algo out-of-the-box smoothing methodology initially used in place of flexible Kernel smoothing

Risk geographies:

- Build on hold pending completion of Monte Carlo
- May be used as a validation tool