GIRO conference and exhibition 2010
Richard Shaw (Horgen Capital & Risk)

The Modelling of Reinsurance Credit Risk

12-15 October 2010
Topics

- Reinsurance Credit Risk
- The Loss Process
- Diversification and Correlation
- Rating Agency Studies
- Modelling Reinsurance Credit Risk Loss
- Numerical Examples
- Modelling Challenges
Reinsurance Credit Risk
What is Reinsurance Credit Risk

Definition
"The risk of loss if another party fails to perform its obligations or fails to perform them in a timely manner."

Examples of Risk Factors
- Failure of individual Reinsurers
- Credit Deterioration (of individual reinsurers)
- Bad Debt provision inadequacy
- Correlation in extreme loss scenarios
- Credit Concentration
- Duration of Recoveries
- Willingness to Pay / Dispute Risk
Reinsurance Credit Risk
Why it is important to Understand

Regulatory Capital Requirements
- ICA Capital – VaR (@99.5%) over 12-months
- Solvency II SCR and ORSA Capital

Risk Management Best Practices
- An understanding of risks and issues might translate into better practices e.g. Regular aged debt analysis → highlight issues with reinsurers (‘Willingness to Pay’)

Capital Markets Solutions
- Securitisation and risk transfer products

Reinsurance Panel Evaluation
- Given a new reinsurance program how should it be placed – 100% with one reinsurer or equal shares with others, what about rating
- Benefits of diversification
The Loss Process

Expected Loss ("EL") and Unexpected Loss ("UL")

Binary Variable
• Let $Y_i$ be a binary variable for obligor $i$ at time 1 year
• $Y_i$ takes values -1 (Default) or 0 (No Default) given non-default state at $t=0$.

Expected Loss
• $EL_i = PD_i \times EAD_i \times LGD_i$

Unexpected Loss
• $EAD_i$ and $LGD_i$ are constant
• $UL_i = [PD_i \times (1 - PD_i)]^{1/2} \times EAD_i \times LGD_i$
  — $EAD = $ Exposure at Default
  — $LGD = $ Loss Given Default (i.e. severity per unit of exposure)
  — $PD = $ Probability of Default
• This further assumes that $PD_i, EAD_i$ and $LGD_i$ are independent
The Loss Process
Expected Loss (“EL”) and Unexpected Loss (“UL”)

Unexpected Loss
• Otherwise
  \[
  UL_i = \left[ PD_i^2 \times EAD_i^2 \times LGD_i^2 + EAD_i^2 \times LGD_i^2 \times PD_i^2 + LGD_i^2 \times PD_i^2 \times EAD_i^2 + \right. \\
  + PD_i^2 \times EAD_i^2 \times LGD_i^2 + EAD_i^2 \times LGD_i^2 \times PD_i^2 + LGD_i^2 \times PD_i^2 \times EAD_i^2 \right]^{0.5}
  \]
The Loss Process

Expected Loss ("EL") and Unexpected Loss ("UL")

<table>
<thead>
<tr>
<th>Obligor</th>
<th>PD</th>
<th>LGD</th>
<th>EAD</th>
<th>EL</th>
<th>UL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obligor 1</td>
<td>2.0%</td>
<td>40%</td>
<td>2,000</td>
<td>16.0</td>
<td>131.7</td>
</tr>
<tr>
<td>Obligor 2</td>
<td>5.0%</td>
<td>60%</td>
<td>2,000</td>
<td>60.0</td>
<td>283.5</td>
</tr>
<tr>
<td>Portfolio</td>
<td>3.80%</td>
<td>50%</td>
<td>4,000</td>
<td>76.0</td>
<td>319.8</td>
</tr>
</tbody>
</table>

| Asset Correlation | 25% | Universality Benefit | 95.5 |
| Joint Default Prob | 0.28% | as % of (UL1 + UL2) | 23.0% |
| Default Correlation | 6.03% | | |

PD = Probability of Default  
LGD = Loss Given Default (%)  
EAD = Exposure at Default  
EL = Expected Loss  
UL = Unexpected Loss

\[ \text{UL}_i = \text{EAD}_i \times \{ \text{LGD}^2_i \times \text{PD}_i \times (1 - \text{PD}_i) + \text{PD}_i \times \text{LGD}_i \times (1 - \text{LGD}_i) / 4 \}^{0.5} \]

\[ \text{ULT} = (\text{UL}_1 + \text{UL}_2 + 2 \times \rho_d \times \text{UL}_1 \times \text{UL}_2)^2 \]

\[ \rho_d = \text{Default correlation between obligor 1 and obligor 2} \]

\[ \sigma^2_{\text{PD}} = \text{PD}_i \times (1 - \text{PD}_i) \]

\[ \sigma^2_{\text{LGD}} = \text{LGD}_i \times (1 - \text{LGD}_i) / 4 \] (and assuming a Beta Distribution)

\[ \text{EAD}_i = \text{constant} \]
The Loss Process
Determining the Probability of Default

Structural Model ("Merton Model")
• Based on the firm’s capital structure and asset return volatility
• Firm defaults when value of assets < value of liabilities at maturity
• Equity is a call option on the assets of the firm – Black-Scholes framework
• The structural approach uses company-specific information and involves the specification of how a company changes values over time

Reduced Form Model
• The reduced form approach bypasses the company’s financial fundamentals and deals directly with market information.
• Price or spread of a defaultable bond is directly related to a risk-free bond through default and recovery rates that are exogenous.
• The approach is considered mathematically more tractable
• If rating is important then can use can be made of rating agency studies
The Loss Process
Loss Severity

Two ways of modelling loss severity

- Recovery % amount is constant
- Recovery % amount is variable
- Beta Distribution is often used to model Loss Severity in this situation

\[ f(x) = x^{(\alpha - 1)} (1 - x)^{(\beta - 1)} \times \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha) \times \Gamma(\beta)} \quad \text{for} \quad 0 < x < 1 \]

\[ \mu = \frac{\alpha}{\alpha + \beta} \]

\[ \sigma^2 = \frac{(\alpha \cdot \beta)}{(\alpha + \beta)^2 \times (\alpha + \beta + 1)} \]
The Loss Process
Reinsurance Credit Exposure

Reinsurance Exposures are Stochastic
• Reinsurance Recoveries – Function of Gross losses and Payment patterns
• Prior year and Current year – different loss dynamics

Reinsurance – Current Year Exposure
• More accurate modelling of Stochastic Gross to Net Losses
• Detailed knowledge of current reinsurance structures
• Gross Loss calibration – Attritional and Large (Frequency / Severity)

Reinsurance – Prior Year Exposure
• Mix of reinsurers different to Current year
• Average credit rating likely to be lower (rating downgrades)
• Gross to Net Process Loss relationship less accuracy unless modelling prior year reinsurance treaties
The Loss Process
Loss Paradigms and Economic Capital

Mark-to-Market Loss Paradigm
• A loss (or gain) also occurs if there is a change in the credit quality
• Values being determined by the discounting of cash flows using credit curve

Mark-to-Model Loss Paradigm
• A slight variation on the Mark-to-Market paradigm
• None or limited secondary market – Value estimated by model

Default Loss Paradigm
• A loss is only recognised on default
• e.g. reinsurance default

Economic Capital
The Loss Process
Credit Risk Modelling Challenges (vs Market Risk)

The lack of a liquid market
- Makes it difficult to price products
- Time horizon tends to be longer than for market risk
- Requirement for more refined simulation techniques (evolution of exposures)

“True” default probabilities cannot be observed - need to be estimated
- Historical experience of credit ratings
- Market Prices
- Subjective assessment criteria

Default Correlations are difficult to measure for Risk Aggregation
- Sparse data

Economic Capital calculations
- Tails of asymmetric fat-tailed distributions
Diversification and Correlation
The Aggregation of Risks

Overview
• Default loss is sparse making it difficult to estimate default correlations
• Instead use a model utilising the concept of asset return correlation
• A multivariate distribution is needed.

Copulas
• A way of dealing with this difficulty is to split the problem into two parts:
  • Stand alone marginal distribution
  • Dependency structure between the risk variables i.e. the copula of the distribution

Single and Multi-Factor Model
• Copula approach involves Monte Carlo simulation and is computationally intensive
• Simplifications can be achieved by imposing more structure on the model by consideration of single or multi-factor models
• A useful starting point is the multivariate normal distribution
Diversification and Correlation
One-Factor Model

\[ AR_i = \left[R^2_i\right]^{0.5} x X + \left[1 - R^2_i\right]^{0.5} x \varepsilon_i \]

Where:

- \( \varepsilon_i \) = Obligor Specific (Non-Systematic) component
- \( X \) = State of the Economy
- \( R^2_i \) = Obligor asset return correlation with the Economy
- \( \rho_A = \text{Corr} (AR_1, AR_2) = \left[R^2_1\right]^{0.5} x \left[R^2_2\right]^{0.5} \)

**Example:**

- \( R^2_1 = 50\% \) and \( R^2_2 = 25\% \) then \( \rho_A = 35.4\% \)

The common economic factor \( X \) and the obligor specific component are assumed to be standard normal.

Obligors tend to downgrade and default when the economy is in a downturn.
Diversification and Correlation
Asset Return and Default Correlation relationship

\[ Y_i = 1 \Leftrightarrow X_i \leq D_i \Leftrightarrow AR_i \leq K_i \]

Where:

- \( X_i \) = Value of the Assets for obligor i at the end of time t.
- \( D_i \) = Value of the Asset Threshold (or cut-off level) for obligor i at the end of time t.
- \( AR_i \) = Asset Return for obligor i over time t.
- \( K_i \) = Asset Return threshold for obligor i over time t

Number of defaults within a portfolio of M obligors = \( \sum_{i=1}^{M} Y_i \)
Diversification and Correlation
Multi-year modelling – Correlated Credit Migration

Correlated Credit Migration
- The same logic can be used to determine future rating states
- Consider the case of a counterparty currently rated ‘BBB’. The rating thresholds in one year’s are such that the areas of the standard normal distribution between ratings are equivalent to the credit rating transition probabilities for a bond ‘BBB’
Diversification and Correlation
Multi-Factor Model

Multi Factor Model Example – Moody’s KMV
- The systematic risk component is replaced by a linear function of risks factors $x_i$ with coefficients equal to $\beta_i$
- These risk factors consisting of primarily (i) country and (ii) industry specific features
- Also relates to states of the economy
Rating Agency Studies
Cumulative Probability of Default

<table>
<thead>
<tr>
<th>Rating</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.00%</td>
<td>0.03%</td>
<td>0.14%</td>
<td>0.26%</td>
<td>0.39%</td>
<td>0.51%</td>
<td>0.58%</td>
<td>0.68%</td>
<td>0.74%</td>
<td>0.82%</td>
</tr>
<tr>
<td>AA</td>
<td>0.02%</td>
<td>0.07%</td>
<td>0.14%</td>
<td>0.24%</td>
<td>0.33%</td>
<td>0.43%</td>
<td>0.52%</td>
<td>0.60%</td>
<td>0.67%</td>
<td>0.74%</td>
</tr>
<tr>
<td>A</td>
<td>0.08%</td>
<td>0.21%</td>
<td>0.35%</td>
<td>0.53%</td>
<td>0.72%</td>
<td>0.95%</td>
<td>1.22%</td>
<td>1.46%</td>
<td>1.70%</td>
<td>1.97%</td>
</tr>
<tr>
<td>BBB</td>
<td>0.26%</td>
<td>0.72%</td>
<td>1.23%</td>
<td>1.86%</td>
<td>2.53%</td>
<td>3.20%</td>
<td>3.80%</td>
<td>4.40%</td>
<td>5.00%</td>
<td>5.60%</td>
</tr>
<tr>
<td>BB</td>
<td>0.97%</td>
<td>2.94%</td>
<td>5.27%</td>
<td>7.49%</td>
<td>9.51%</td>
<td>11.48%</td>
<td>13.19%</td>
<td>14.75%</td>
<td>16.21%</td>
<td>17.45%</td>
</tr>
<tr>
<td>B</td>
<td>4.93%</td>
<td>10.76%</td>
<td>15.65%</td>
<td>19.46%</td>
<td>22.30%</td>
<td>24.57%</td>
<td>26.47%</td>
<td>28.06%</td>
<td>29.44%</td>
<td>30.82%</td>
</tr>
<tr>
<td>CCC/C</td>
<td>27.98%</td>
<td>36.95%</td>
<td>42.40%</td>
<td>45.57%</td>
<td>48.05%</td>
<td>49.19%</td>
<td>50.26%</td>
<td>51.09%</td>
<td>52.44%</td>
<td>53.41%</td>
</tr>
<tr>
<td>Investment</td>
<td>0.13%</td>
<td>0.35%</td>
<td>0.60%</td>
<td>0.91%</td>
<td>1.24%</td>
<td>1.58%</td>
<td>1.90%</td>
<td>2.20%</td>
<td>2.50%</td>
<td>2.80%</td>
</tr>
<tr>
<td>Speculative</td>
<td>4.44%</td>
<td>8.68%</td>
<td>12.42%</td>
<td>15.46%</td>
<td>17.90%</td>
<td>19.96%</td>
<td>21.72%</td>
<td>23.25%</td>
<td>24.67%</td>
<td>25.96%</td>
</tr>
<tr>
<td>All rated</td>
<td>1.63%</td>
<td>3.23%</td>
<td>4.67%</td>
<td>5.89%</td>
<td>6.90%</td>
<td>7.79%</td>
<td>8.55%</td>
<td>9.23%</td>
<td>9.86%</td>
<td>10.45%</td>
</tr>
</tbody>
</table>

Observation

- There are some inconsistencies by rating within term
  - Top-left: Higher rating, shorter time horizon (Rates need to be smoothed)
  - Function of the methodology - Static Pool Methodology
- Corporate Debt statistics – Adaptability for reinsurance default process?
- Consider use of ‘stressed’ default rates – Impairment, Willingness to Pay
## Rating Agency Studies

### Volatility of Cumulative Probability of Default

The Cumulative Default Probabilities are volatile, more so for lower ratings.

<table>
<thead>
<tr>
<th>PD</th>
<th>Expected</th>
<th>Expected - 1 SD</th>
<th>Expected + 1 SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00%</td>
<td>1.00%</td>
<td>0.00%</td>
<td>3.00%</td>
</tr>
<tr>
<td>0.50%</td>
<td>1.10%</td>
<td>0.50%</td>
<td>3.10%</td>
</tr>
<tr>
<td>1.00%</td>
<td>1.20%</td>
<td>1.00%</td>
<td>3.20%</td>
</tr>
<tr>
<td>1.50%</td>
<td>1.30%</td>
<td>1.50%</td>
<td>3.30%</td>
</tr>
<tr>
<td>2.00%</td>
<td>1.40%</td>
<td>2.00%</td>
<td>3.40%</td>
</tr>
<tr>
<td>2.50%</td>
<td>1.50%</td>
<td>2.50%</td>
<td>3.50%</td>
</tr>
<tr>
<td>3.00%</td>
<td>1.60%</td>
<td>3.00%</td>
<td>3.60%</td>
</tr>
</tbody>
</table>

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Rating Agency Studies
Annual Corporate Default Rates

- Default Rates vary markedly by:
  - Industry and
  - Calendar Year
- Insurance default rates are low (perhaps higher debt initial ratings)
Rating Agency Studies
One Year Default Rates

- Default Rates are very cyclical
## Transition Matrices

**One Year**

<table>
<thead>
<tr>
<th>From / To</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC/C</th>
<th>D</th>
<th>NR</th>
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<tbody>
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<td>0.1%</td>
<td>0.1%</td>
<td>0.0%</td>
<td>0.1%</td>
<td>0.0%</td>
<td>3.3%</td>
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<tr>
<td>AA</td>
<td>0.6%</td>
<td>86.6%</td>
<td>8.1%</td>
<td>0.6%</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>4.0%</td>
</tr>
<tr>
<td>A</td>
<td>0.0%</td>
<td>2.0%</td>
<td>87.1%</td>
<td>5.5%</td>
<td>0.4%</td>
<td>0.2%</td>
<td>0.0%</td>
<td>0.1%</td>
<td>4.8%</td>
</tr>
<tr>
<td>BBB</td>
<td>0.0%</td>
<td>0.1%</td>
<td>3.8%</td>
<td>84.2%</td>
<td>4.1%</td>
<td>0.7%</td>
<td>0.2%</td>
<td>0.3%</td>
<td>6.7%</td>
</tr>
<tr>
<td>BB</td>
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<td>0.1%</td>
<td>0.2%</td>
<td>5.2%</td>
<td>75.5%</td>
<td>7.5%</td>
<td>0.8%</td>
<td>1.0%</td>
<td>9.8%</td>
</tr>
<tr>
<td>B</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.2%</td>
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<td>5.4%</td>
<td>72.7%</td>
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<td>4.9%</td>
<td>11.8%</td>
</tr>
<tr>
<td>CCC/C</td>
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<td>0.3%</td>
<td>0.9%</td>
<td>11.3%</td>
<td>45.0%</td>
<td>28.0%</td>
<td>14.4%</td>
</tr>
</tbody>
</table>

**Rating Agency Studies**

**Transition Matrices**

- Probability of moving from rating now to one at a future time horizon e.g. one year
- Largest values are along the diagonal
  - Values fall off very quickly moving off the diagonal
- Investment Grade companies tend to exhibit lower ratings volatility
- Transition matrices are based on historical rating changes
  - There is volatility in transition rates from year to year – macroeconomic etc.
- Often used for multi-year modelling of future states - \( M_T = (M_1)^T \)
  - where \( M_T \) = T-year transition matrix
  - assumes Markov Process for transition rates – a convenient modelling approach

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# Rating Agency Studies
## Transition Matrices – Conditional vs Unconditional

### Comparison Of Conditional Versus Unconditional Transition Matrices—One Year (1981-2006) (%)

<table>
<thead>
<tr>
<th>Form / To</th>
<th>AAA</th>
<th>AA+</th>
<th>AA</th>
<th>A+</th>
<th>A</th>
<th>BBB+</th>
<th>BB+</th>
<th>B</th>
<th>FICO</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.18</td>
<td>0.33</td>
<td>1.58</td>
<td>6.15</td>
<td>22.83</td>
<td>0.04</td>
<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
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<td>0.00</td>
<td>1.20</td>
<td>3.93</td>
<td>1.00</td>
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<td>3.75</td>
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</tr>
<tr>
<td>A+</td>
<td>0.00</td>
<td>0.00</td>
<td>0.29</td>
<td>0.07</td>
<td>0.06</td>
<td>1.50</td>
<td>1.74</td>
<td>0.34</td>
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<td>4.45</td>
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<td>BBB+</td>
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<td>1.00</td>
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<tr>
<td>BB+</td>
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<td>0.00</td>
<td>2.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.00</td>
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<tr>
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<tr>
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<td>1.00</td>
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<tr>
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<td>0.00</td>
<td>2.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Values > 1.0 → Probability of Default greater if downgrade in a prior period.**

**Markov process for transition matrix assumes only current rating is important.**

### Conditional – Experienced a ratings downgrade in prior period
- If Value = 1.0: Transitions conditioned on prior downgrade are no different
- If Value > 1.0: Future ratings depends on Current AND Prior ratings

Rating Agency Studies
Recovery Rates

Discounted Recovery Rates By Instrument Type (1987-2009)

<table>
<thead>
<tr>
<th>Instrument Type</th>
<th>Mean</th>
<th>Median</th>
<th>Std Dev</th>
<th>CV</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term loans</td>
<td>69.4%</td>
<td>80.4%</td>
<td>32.9%</td>
<td>47.4%</td>
<td>616</td>
</tr>
<tr>
<td>Revolving credit</td>
<td>78.0%</td>
<td>95.4%</td>
<td>29.5%</td>
<td>37.9%</td>
<td>617</td>
</tr>
<tr>
<td>All loans/facilities</td>
<td>73.8%</td>
<td>87.5%</td>
<td>31.3%</td>
<td>42.4%</td>
<td>1,233</td>
</tr>
<tr>
<td>Senior secured bonds</td>
<td>57.2%</td>
<td>58.2%</td>
<td>30.9%</td>
<td>54.1%</td>
<td>299</td>
</tr>
<tr>
<td>Senior unsecured bonds</td>
<td>43.0%</td>
<td>39.2%</td>
<td>32.8%</td>
<td>76.4%</td>
<td>1,084</td>
</tr>
<tr>
<td>Senior subordinated bonds</td>
<td>28.3%</td>
<td>16.6%</td>
<td>32.5%</td>
<td>114.7%</td>
<td>495</td>
</tr>
<tr>
<td>All other subordinated bonds</td>
<td>19.4%</td>
<td>8.3%</td>
<td>29.9%</td>
<td>154.0%</td>
<td>425</td>
</tr>
<tr>
<td>All bonds</td>
<td>37.4%</td>
<td>29.3%</td>
<td>32.6%</td>
<td>87.3%</td>
<td>2,303</td>
</tr>
<tr>
<td>Total defaulted instruments</td>
<td>50.1%</td>
<td>47.9%</td>
<td>36.5%</td>
<td>73.0%</td>
<td>3,536</td>
</tr>
</tbody>
</table>

- Recovery rates are conditional on the level of debt seniority
- Higher security → greater expected recovery
- Standard deviation High
- Measurement does not ‘neutralise’ impact of economic cycle
Rating Agency Studies
Default Rate vs Recovery Rate

- Inverse relationship between Probability of Default and Recovery Rate
Rating Agency Studies
Impairment Rates – A.M. Best Studies

• A.M. Best rated U.S. domiciled insurance companies
• General Corporate Bond Default Rates are inappropriate for insurance:
  – Unique regulatory and accounting environments
• Impairment is a wider category of financial duress than default
  – Impairment often occurs when insurer able to meet policyholder obligations
  – Impairment rates > Default rates for a given rating
• Definition of Impairment
  – Financially Impaired Company (“FIC”) - First official regulatory action taken
    – Ability to conduct normal insurance operations is adversely affected
    – Capital and Surplus inadequate to meet legal requirements
    – General financial condition has triggered regulatory concern
• State Actions – Regulatory Supervision, Rehabilitation, Liquidation, Receivership
Modelling Reinsurance Credit Risk Loss

Assumptions

Loss Process
• Loss only due to default

Time Horizon
• 12-months (as per Solvency II)
• Duration mean-term liabilities (proxy for 12-monthly intervals with rating migration)

Monte Carlo Simulation
• 20,000 Gaussian and t copula simulations using MATLAB
• 20 reinsurers with variable exposure amounts (these assumed to remain constant)
• Variations in
  – Rating
  – Dependency (copulas)
  – LGD – Constant / Variable / Correlated variable
Modelling Reinsurance Credit Risk Loss
Assumptions

<table>
<thead>
<tr>
<th>Reinsurer</th>
<th>Excess</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10,000</td>
</tr>
<tr>
<td>2</td>
<td>15,000</td>
</tr>
<tr>
<td>3</td>
<td>20,000</td>
</tr>
<tr>
<td>4</td>
<td>25,000</td>
</tr>
<tr>
<td>5</td>
<td>30,000</td>
</tr>
<tr>
<td>6</td>
<td>35,000</td>
</tr>
<tr>
<td>7</td>
<td>40,000</td>
</tr>
<tr>
<td>8</td>
<td>45,000</td>
</tr>
<tr>
<td>9</td>
<td>50,000</td>
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<tr>
<td>10</td>
<td>55,000</td>
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<td>11</td>
<td>60,000</td>
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<td>12</td>
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<tr>
<td>13</td>
<td>70,000</td>
</tr>
<tr>
<td>14</td>
<td>75,000</td>
</tr>
<tr>
<td>15</td>
<td>80,000</td>
</tr>
<tr>
<td>16</td>
<td>85,000</td>
</tr>
<tr>
<td>17</td>
<td>90,000</td>
</tr>
<tr>
<td>18</td>
<td>95,000</td>
</tr>
<tr>
<td>19</td>
<td>100,000</td>
</tr>
<tr>
<td>20</td>
<td>105,000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rating</th>
<th>PD</th>
<th>E (LGD)</th>
<th>SD (LGD)</th>
<th>Alpha</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.005%</td>
<td>0.030%</td>
<td>0.070%</td>
<td>0.120%</td>
<td>0.160%</td>
</tr>
<tr>
<td>AA</td>
<td>0.020%</td>
<td>0.070%</td>
<td>0.140%</td>
<td>0.240%</td>
<td>0.330%</td>
</tr>
<tr>
<td>A</td>
<td>0.080%</td>
<td>0.210%</td>
<td>0.350%</td>
<td>0.530%</td>
<td>0.720%</td>
</tr>
<tr>
<td>BBB</td>
<td>0.260%</td>
<td>0.720%</td>
<td>1.230%</td>
<td>1.860%</td>
<td>2.530%</td>
</tr>
<tr>
<td>BB</td>
<td>0.970%</td>
<td>2.940%</td>
<td>5.270%</td>
<td>7.480%</td>
<td>9.510%</td>
</tr>
<tr>
<td>B</td>
<td>4.930%</td>
<td>10.760%</td>
<td>15.650%</td>
<td>19.460%</td>
<td>22.300%</td>
</tr>
<tr>
<td>CCC/C</td>
<td>27.980%</td>
<td>36.950%</td>
<td>42.400%</td>
<td>45.570%</td>
<td>48.050%</td>
</tr>
</tbody>
</table>

Loss Severity is assumed to follow a Beta Distribution

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Numerical Examples
A Rating, 1 Yr PD, Gaussian Copula (25% and 50%), Constant LGD

Gaussian 25% zero losses for 98.5% of dist.
VaR (99.5%) = 87x E(Loss)
Numerical Examples
A and BBB Rating, 1 Yr PD, Gaussian Copula 25%, Constant LGD

BBB Gaussian
25% zero losses
for 95.7% of dist.

VaR (99.5%) =
35x E(Loss)
Numerical Examples
A Rating, 1 Yr and 4 Yr PD, Gaussian Copula 25%, Constant LGD
Numerical Examples
BBB Rating, 1 Yr PD, Gaussian 25% and t 5 df 25% Copula, Constant LGD
Modelling Challenges
Assumptions

Setting assumptions for
- Probability of Default (setting “Stressed levels”)
- Loss Given Default
- Asset (or Default Correlation)
- Dependencies
  - Amongst the above e.g. PD and LGD; or Value of Asset Return and LGD
  - Other variables – insurance loss and default rate

Risk Aggregation
- Copulas or Factor Models
- Single vs Multi-Factor Models
- Model Calibration