GIRO conference and exhibition 2010
Ben Zehnwirth

Solvency II & Long Tail Liabilities

12-15 October 2010
Introduction and Summary

- We provide our solution to the Solvency II one-year risk horizon, SCR, Technical Provisions (TP) (Fair Value Liabilities), Market Value Margins (Risk Margins) for the aggregate of long tail LOBs
- The solution is non-recursive, non-circular, tractable and satisfies all the directives (requirements)
- IFRS 4 requirements in respect of fungibility and ring-fencing is discussed
- Three types of correlations between LOBs
- How do we know if two LOBs have the same economic drivers?
- Is the economic inflation a principal driver of long tail liability calendar year trends?
Introduction and Summary

- Two LOBs written by the same company rarely have the same trend structure (including in the calendar year direction) and often process (volatility) correlation is either zero or very low. Reserve distribution correlation is often zero and if significant quite low.
- No two companies are the same in respect of trend structure, and process (volatility) correlation is often zero (for the ‘same’ LOB).
- No company is the same as the industry, unless it is a very large proportion of the industry.

- All the above are demonstrated with real life data.
Introduction and Summary

- SII metrics for the aggregate of real life six LOBs compared with SII metrics for the most volatile LOB to illustrate amongst other things risk diversification of SCR and (MVM component) of TP

- **Undiscounted reserves for the aggregate of six LOBs**
  
  \[ \text{Undiscounted reserves} = \text{Technical Provisions} + \text{Solvency Capital Requirement (SCR)} \]
  
  = total liability in Economic Balance Sheet, using a risk free rate of 4% and a spread of 6%.

- No need for additional capital in this example due to risk diversification!

- QIS5 allows for risk diversification credit.

- Conditions for consistent estimates of prior accident year ultimates and SII risk measures on updating. This will explain how to avoid model error “distress”.
Introduction and Summary

- Which probability distributions are required to compute the various risk measures for the aggregate of multiple LOBs?
- VaRs and T-VaRs
- Process Variance versus Parameter Uncertainty
- Reserve risk, underwriting risk and the combined risk
- The ultimate year risk horizon - conceptually much simpler
- Calendar year Payment stream probability distributions – what are the drivers?
Solvency II – Economic Balance Sheet

Ann Hagen in “Solvency II: Brave new world:

"Doing the job

Under Solvency II, the way that work is carried out will change. For example, Solvency II is likely to require different actuarial techniques from the ones currently used. Technical provisions will be estimated as a probability-weighted average of expected future cash flows, taking into account the time-value of money and including a risk margin. Many of us are estimating claims reserves using traditional deterministic actuarial techniques, primarily relying on incurred claims data. Under Solvency II, not only will we need to discount these reserve estimates, requiring projected payment patterns, we will also need to demonstrate a deep understanding of the uncertainty of those reserves. We will additionally be required to apply the same approach to evaluating unexpired risk liabilities currently allowed for in the unearned premium reserves."
Actual payments are made by calendar year

Summing future losses along the calendar year axis produces projections of the cash-flow, and the actual calls on the reserves. This is the dimension in which solvency issues arise.

Using cell distributions and correlations we can compute the distributions for each future year’s cash flow.
Solvency II one-year risk horizon: satisfies three conditions - Summary of decomposing the directives- What are the basic elements?

- Risk Capital is raised at the beginning of each year;
- The analyses are conditional on the first (next) calendar year being in distress (99.5%);
- At the end of the first year in distress, the balance sheet can be “restored” in such away that the company has sufficient technical provisions (fair value of liabilities) to continue business or to transfer the liabilities to another risk bearing entity.

Here follow some relevant articles that lead to above mentioned three conditions
The Concept of Risk - The Fair Value of Liabilities

The Solvency II Framework Directive

- **Article 76**: The value of technical provisions shall correspond to the current amount insurance and reinsurance undertakings have to pay if they were to transfer their insurance and reinsurance obligations immediately to another insurance or reinsurance undertaking.

- **Article 77**: The value of technical provisions shall be equal to the sum of a Best Estimate and a Risk margin ….

- The Best Estimate shall correspond to the probability-weighted average of future cash-flows, taking into account the value of money (expected present value of future cash-flows), using the relevant risk-free interest rate term structure.
The Concept of Risk - The SCR

The Solvency II Framework Directive

**Article 101**: The Solvency Capital Requirement (SCR) shall correspond to the Value-at-Risk (VaR) on the basis own funds¹) ... subject to a confidence level of 99.5% over a one-year period.

- Initial Capital covers at least the potential change in the Fair Value under severe adverse conditions, represented by the 99.5% percentile of the range of possible Fair Values at the end of the selected solvency one-year time horizon; adverse conditions represent a distress scenario for the company.

¹) Essentially, basis own funds defined as the excess of assets over liabilities, both assessed at market value (or capital market consistent value, where a market does not exist)
The Concept of Risk - The Fair Value of Liabilities

- The Best Estimate shall be gross, without deduction of the amounts recoverable from reinsurance contracts and special purpose vehicles. Those amounts shall be calculated separately … (Cf. article 81)
- The Risk Margin shall be such as to ensure that the value of the technical provisions is equivalent to the amount that insurance and reinsurance undertakings would be expected to require in order to take over and meet the insurance and reinsurance obligations. (Cost of providing amount of eligible own funds.)

1. The cost of holding the SCR is assumed to attract a premium over the risk-free interest rate which is called the Cost of Capital
Definition of the One-Year Risk Horizon

For the one-year risk horizon, risk capital is raised at the beginning of each year.

- The cost of raising the risk capital, the Market Value Margin (MVM) or premium on the risk capital, also known as the Risk Margin is paid to the capital providers at the end of each year along with any unused risk capital.

- The sum of the MVMs and the Best Estimate of Liabilities (BELs) for each calendar year (>=2) is the Technical Provision (also referred to as Fair Value of Liabilities).
Definition of the One-Year Risk Horizon

- For an individual year \((k; k>2)\), we can define:
  \[
  TP(k) = BEL(k) + MVM(k);
  \]
  where \(BEL(k)\) and \(MVM(k)\) are the Best Estimate of Liabilities and Market Value Margin for year \(k\).
- Important note: for a future calendar year, \(k\), \(BEL(k)\) and \(MVM(k)\) are additive; VaR\((k)\) is not.

We present a tractable solution to the one-year risk horizon that is not recursive or circular.
The Concept of Risk - The SCR

The Solvency II Framework Directive

Article 101: The Solvency Capital Requirement (SCR) shall... correspond to the Value-at-Risk (VaR) on the basis own funds¹) ... subject to a confidence level of 99.5% over a one-year period.

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¹) Essentially, basis own funds defined as the excess of assets over liabilities, both assessed at market value (or capital market consistent value, where a market does not exist)
As detailed in the Insurance ERM analysis of Solvency II:

“The fair value balance sheet is one of the cornerstones of Solvency II, and its impact is not restricted only to the calculation of fair value assets and liabilities. The concept of market value margin (MVM), and the related one-year risk approach in the calculation of the solvency capital requirement (SCR), find their origin in this fair value driven approach: re/insurance companies should have enough capital on their balance sheet to cover the risks that can emerge over a 12-month timeframe, and allow for a (theoretical) transfer of all (contractual) liabilities at the end of this balance-sheet period. This means that companies have to be able to calculate the impact of such shocks on their end-of-year balance sheets, and value these in such a way that they can be transferred to a third party.”
3.55. The SCR is the difference between the basic own funds over the one year time horizon in the distressed scenario. This implicitly suggests that undertakings should analyse the difference between all component parts of the technical provisions under the stressed scenario, including the risk margin.
The above extracts lead to the following definition: the SCR for the one-year risk horizon is the Value-at-Risk for the first year plus the change in technical provisions (TP) in the subsequent years (suitably discounted), conditional on the first year being in distress.

\[
\text{SCR} = \text{VaR}_{99.5\%}(1) + \Delta \text{TP}(2) + \Delta \text{TP}(3) + \ldots + \Delta \text{TP}(n),
\]

where \( n \) is the limit of run-off.
The Concept of Risk Horizon Perspective

Quantification Requirements- What do we need to compute SII metrics?

• For the calculation of the Technical Provisions, Market Value Margins and SCR for both the One-year Risk Horizon (and Ultimate Year Risk Horizon) for the aggregate of all long-tail LOB’s and each LOB separately the following critical information is required:
  – Probability distributions of paid losses (liability stream) by calendar year \( (k = 1, . . , n) \) and their correlations, for each LOB and the aggregate of all LOB’s
  – Probability distributions of total reserves for each LOB and the aggregate of all LOB’s.
  – Probability distributions of the aggregate paid losses from calendar year \( k \) to calendar year \( n \) for each LOB and the aggregate of all LOB’s. This is required for each \( k \) ranging from 1 to \( n \), where complete run-off is achieved at the ultimate calendar year \( n \)
  – Conditional Probability distributions, conditional on the first (next) calendar year being in “distress”.

• Armed with these distributions any risk measure can be computed, including VaR\((k)\) for the paid losses (total loss) in calendar year \( k \); and Market Value Margins, Technical Provisions and VaRs conditional on the first year in distress, for each LOB and the aggregate of all LOB’s.
Risk Capital – One Year risk Horizon

Simplest Case: Only One Year Runoff

\[ L_1 = \text{projected losses for the year. This is a random variable.} \]

\[ \text{BEL}(1) = \frac{E(L_1)}{(1 + d)^{0.5}} \]

Where \( d \) = interest rate. Losses are paid uniformly through year, so we discount for half a year.

\[ \text{SCR}(1) = \text{VaR}_{99.5\%}(L_1), \text{ i.e. } \Pr(L_1 \leq E(L_1) + \text{SCR}(1)) = 0.995 \]

\( \text{MVM}(1) \) is the cost incurred in having risk fund of \( \text{SCR}(1) \) available for the year. It is paid to capital provider at end of year and so is discounted by a full year.

\[ \text{MVM}(1) = \frac{\text{SCR}(1) \times s}{(1 + d)}, \text{ if the interest on the risk fund is paid directly to capital provider, or } \]

\[ \text{MVM}(1) = \frac{\text{SCR}(1) \times (s + d)}{(1 + d)}, \text{ otherwise.} \]

\[ \text{TP}(1) = \text{BEL}(1) + \text{MVM}(1). \text{ This is the Technical Provision and must be held in company own funds.} \]

We will also let, \( PV(k; d) \), or \( PV(k) \) be used to abbreviate the Present Value factor \( \frac{1}{(1 + d)^k} \)
Next Simplest Case: Two Year runoff, No correlation

$$BEL(1) = E(L_1) \times PV(0.5)$$

$$BEL(2) = E(L_2) \times PV(1.5)$$

$$MVM(1) = VaR_{99.5\%}(1) \times s \times PV(1)$$

$$MVM(2) = VaR_{99.5\%}(2) \times s \times PV(2)$$

The Technical Provision (TP) at inception is the sum of the individual year TPs:

$$TP = TP(1) + TP(2)$$

This amount needs to be available in company own funds to ensure that losses can be met up to a 99.5% or 1/200 risk level in each year. Aggregate losses up to the value of the mean are met out of BEL funds, excess losses are met from the SCR fund, access to which is financed by MVM.
Two-year picture of accounts: In year 1 we require reserves to meet paid loss liabilities for years 1 and 2 and we also need to able to fund the cost of access to the risk capital funds for years 1 and 2, however we only need access to the year 1 risk fund. When year 2 begins our accounts reset, since any cost over-runs from year 1 were paid out of the risk fund and do not degrade our prepared reserves for year 2. Provided the loss over-run is below $\text{RC}(1) = \text{VaR}_{99.5}(L_1)$. 
Risk Capital – One Year Horizon

• This is fine, except for one thing:
  What if the distribution for the losses in year 2 has changed conditional on the losses in year one?

• Simply put, the previous picture assumes there is no correlation between the distributions for years 1 and 2. In other words, whatever the outcome observed after year 1 we are going to remain fixed on our previous course, full steam ahead

  Typically calendar year distributions are positively correlated. The correlations are driven by parameter uncertainty.
If year 1 is in distress at the 99.5th percentile, then our risk fund carries us over into year 2, but the conditional distributions are now different. Year 2 now must be re-evaluated in the light of conditional distributions and these increase the size of the BEL and the MVM, the cost of holding the risk fund. We need to include these adjustments in the year 1 risk fund.
Two-year runoff with first year in distress

Let $\xi = \text{Year 1 in distress}$

Why is $\Delta \text{MVM}(2)$ discounted by 1 year and $\text{MVM}(2)$ by 2 years?

$\text{VaR}(1)$ is consumed.

$\text{MVM}(1) = \text{spread} \times \text{SCR}$ at year end (and returned along with risk free rate).

$\text{VaR}(2|\xi)$ is raised in year 2.
N-year run-off (Correlated)

\[ TP = \sum \text{BEL}(k) + \sum \text{MVM}(k); \ k = 1 \text{ to } n \]
\[ SCR = \text{VaR}_{99.5\%}(1) + \sum [\Delta \text{BEL}(k) + \Delta \text{MVM}(k)]; \ k = 2 \text{ to } n \]
Conditional Statistics from Simulations

Begin with a large number of simulations of the entire forecast table. This provides an equal number of sample paths through all future calendar years.
Conditional Statistics from Simulations

Determine the sample paths corresponding to the distress scenario. If this is “next year at 99.5\(^{th}\) percentile”, then these paths belong in the [99.5, 99.6) order interval.
Conditional Statistics from Simulations

Restricting attention to only these sample paths we can then calculate any conditional statistic, such as $\text{BEL}(k)|\xi$, $\text{MVM}(k)|\xi$, $\text{VaR}(k)|\xi$ etc.
SII metrics for the aggregate of six LOBs compared with SII metrics for the most volatile LOB - risk diversification of SCR and TP

Trends and volatility are unique to each LOB

- LOB4 is the most volatile of the LOBs
SII metrics for the aggregate of six LOBs compared with SII metrics for the most volatile LOB - risk diversification of SCR and TP

- Model for LOB 4
SII metrics for the aggregate of six LOBs compared with SII metrics for the most volatile LOB- risk diversification of SCR and TP

- Diagnostics for LOB4 illustrating normality satisfied
SII metrics for the aggregate of six LOBs compared with SII metrics for the most volatile LOB - risk diversification of SCR and TP

- Forecast table for LOB4
  - Black: Fitted mean
  - Blue: Observed
  - Red: Standard Deviation (log-normal)
SII metrics for the aggregate of six LOBs compared with SII metrics for the most volatile LOB - risk diversification of SCR and TP

- Forecast table for the aggregate of the six LOBs

- **Black:**
  - Fitted mean

- **Blue:**
  - Observed

- **Red:**
  - Standard Deviation

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SII metrics for the aggregate of six LOBs compared with SII metrics for the most volatile LOB- risk diversification of SCR and TP

- LOB4 is the most volatile of the six LOBs
- Aggregate CV is 7.14%
- Substantially more Solvency II risk capital required if LOB4 was written on its own. It has a CV of 49%
Liability stream by calendar year and calendar year correlations for the
Aggregate of the six LOBs
Liability stream by calendar year and calendar year correlations for LOB4- long tail with high correlations
One-year risk horizon
Aggregate of six LOBs

- Aggregate Solvency II capital required (Technical Provisions + SCR) is the same as undiscounted BEL
- Bulk of SCR is the VaR for next year (2010)
- All calculations assume: risk free = 4% and spread = 6%
One-year risk horizon
LOB4

- Solvency II capital required (Technical Provisions + SCR) for LOB4 is substantially higher than for undiscounted BEL

- Bulk of SCR is Delta TP – capital required to restore the balance sheet should the next year be in distress
One-year risk horizon
Comparing the aggregate of six LOBs with LOB4

Aggregate of six LOBs  LOB4 only
One-year risk horizon
Comparing the aggregate of six LOBs with LOB4

Aggregate of six LOBs

LOB4 only
One-Year risk horizon
Aggregate of six LOBs

Which LOB is in distress if the aggregate is in distress?

- LOB3 and LOB1 are in distress if the aggregate is in distress
- Why? LOB3 and LOB1 have the bulk of the payments in the distress year (inset).
Ultimate-year risk horizon
Aggregate of six LOBs

- MVM is calculated based on the VaR ‘to run-off’ for each calendar year
- MVM is around 10x the one-year risk horizon’s MVM
Fungibility and Ring-fencing by example – drawing on the risk fund

In the case of fungibility the risk fund is smaller since it is expected to be supplemented by surpluses from other LOBs/portfolios.

- Drawing on risk fund if LOB1, LOB2 fungible

- Drawing on risk fund if LOB1, LOB2 not fungible; surplus retained in LOB1
Ultimate Year Risk Horizon

- Allocation of capital in the Ultimate Year Risk Horizon framework
Consistent estimates of prior year ultimates and Solvency II Risk Measures on updating

- Total reserve increases from year to year (with same accident (underwriting) exposure as previous year)
- What does a calendar year trend (inflation) of 5% imply in terms of estimates of prior year ultimates, loss reserves and premiums (per unit risk)?
- AXIOM

Calendar year trends (inflation) project (impact) both the prior and future accident (underwriting) years

Here is a simple example that illustrates the main ideas that reserve increases do not represent under-reserving. Indeed, they are necessary in order to maintain consistent estimates of prior year ultimates as the company writes new underwriting (accident) years.)
Consistent estimates of prior year ultimates and reserve increases

- On a logarithmic scale the data were generated as follows

- \[ Y(w,d) = 10 - 0.3d + 0.05 (w+d-1) \] where \( w \) is the accident year 1,\ldots,7 and \( d \) is the development year 0,\ldots,5.

- The numbers down each column increase by 0.05 on a log scale (approximately 5% annual). The numbers along each row decrease by 0.25 \((-0.3+0.05)\) on a log scale. We have assumed that the paid losses run-off after five years. Even if this is the case for 1999, this may not be the case for subsequent accident years especially if inflation is 'high'.
Consistent estimates of prior year ultimates on and SII metrics updating

Reserves and ultimates as at year end 2004

<table>
<thead>
<tr>
<th>Accident Period vs Development Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cal. Per. Total</td>
</tr>
<tr>
<td>1999</td>
</tr>
<tr>
<td>2000</td>
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<tr>
<td>2001</td>
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<tr>
<td>2002</td>
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<tr>
<td>2003</td>
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<tr>
<td>2004</td>
</tr>
<tr>
<td>Total Fitted/Paid</td>
</tr>
</tbody>
</table>

1 Unit = $1

Reserves and ultimates as at year end 2005

<table>
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<tr>
<th>Accident Period vs Development Period</th>
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<tbody>
<tr>
<td>Cal. Per. Total</td>
</tr>
<tr>
<td>1999</td>
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<tr>
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<td>2001</td>
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<td>2003</td>
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<td>2004</td>
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<tr>
<td>Total Fitted/Paid</td>
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1 Unit = $1
## Consistent estimates of prior year ultimates on and SII metrics updating

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Mean Reserve</th>
<th>Ultimate</th>
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<tbody>
<tr>
<td>1999</td>
<td>0</td>
<td>77,359</td>
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<tr>
<td>2000</td>
<td>6,634</td>
<td>81,325</td>
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<tr>
<td>2001</td>
<td>15,930</td>
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<td>2002</td>
<td>28,835</td>
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<td>2003</td>
<td>46,631</td>
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<tr>
<td>2004</td>
<td>71,048</td>
<td>99,331</td>
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<td><strong>Total</strong></td>
<td><strong>169,078</strong></td>
<td><strong>527,873</strong></td>
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</table>

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Mean Reserve</th>
<th>Ultimate</th>
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<td>1999</td>
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<td>77,359</td>
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<td>2000</td>
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<td>2001</td>
<td>6,974</td>
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<td>2002</td>
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<td>2003</td>
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<td>2004</td>
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<td>2005</td>
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<td><strong>Total</strong></td>
<td><strong>177,746</strong></td>
<td><strong>632,298</strong></td>
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### Ratio of year t ultimate to year t-1

1. 1.051267467
2. 1.051275745
3. 1.051266156
4. 1.051269499
5. 1.051277438
6. 1.051273016

### Ratio of Reserves

1. 1.051266279

**N.B.**

2. The ratio of ultimate for year t to year t-1 is 1.05.
3. Increase in total reserves from 2004 to 2005 is 1.05.
Consistent estimates of prior year ultimates on and SII metrics updating

- Each year the company needs to increase its total reserves by at least 5%.
- The ultimates for prior accident years will remain consistent with each increase in total reserves.
- Each year the company needs to increase its premium (price) by at least 5%.
- Ultimates increase by at least 5% from one accident year to the next.
- These are not reserve upgrades
Consistent estimates of prior year ultimates on and SII metrics updating

- Mack and related methods give inconsistent estimates of prior year ultimates (on updating) and inaccurate liability streams by calendar year.

- Bootstrapping the wrong model does not improve the model.

This was all explained on Wednesday!
Conditional Statistics on next calendar period - volatility in ultimates on updating.

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<td>1987</td>
<td>2,964,299</td>
<td>3,460,499</td>
<td>108,346</td>
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<td>0.04</td>
<td>0.03</td>
<td>113,886</td>
<td>205,224</td>
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<td>Total</td>
<td>7,120,007</td>
<td>17,341,201</td>
<td>234,706</td>
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<td>0.01</td>
<td>113,886</td>
<td>205,224</td>
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</tr>
</tbody>
</table>
Updating and monitoring

VaR(2010) ≈ VaR(2009) provided assumptions are “consistent”

Another year 2009
Consistent estimates of prior year ultimates on and SII metrics updating
Consistent estimates of prior year ultimates on and SII metrics updating
Mean Reserve for Calendar Year 2010 is approximately 10% higher than Mean reserve for 2009.
Solvency II calculations with no discounting: MVM for 2010 is almost the same as for 2009, and so is SCR.
Consistent SII metrics on updating

Reduction in TP offsets increase in VaR to produce slightly lower SCR.
Consistent SII metrics on updating

Break-down by future calendar year, shows same pattern but scaled up. Metrics based on the first year being at the 99.5th percentile.
Two LOBs with common drivers - Example 1 - same calendar year trend structure and high process correlation of 0.85
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Process correlation adjusted for the average calendar year trend for each LOB = sum of trend correlation + process (volatility) correlation.
Three LOBs with common drivers - Example 2
Identical trend structure and high process correlation exceeding 0.9!
Process correlation, trend correlation, same trend structure and reserve distribution correlation

• The above two examples are **not** different LOBs!
• The first is E&O D&O gross and net of reinsurance
• The second example involves three layers of a medical malpractice LOB; Lim 1M, Lim 2M and 1Mx1M. The triangles are additive.
• Two LOBs written by the same company rarely have the same calendar year trend structure and often process correlation is either zero or very low. Reserve distribution correlation is much lower.
• No two companies are the same and process correlation often zero (for the ‘same’ LOB)
• No company is the same as the industry
Small company (exposure) versus industry auto BI
New South Wales Australia

Calendar year trend for company is zero, whereas industry it is huge!
(Company also has much higher process volatility)
No two companies are the same in respect of trend structure!
S&P Syn Thesis 2010- RSA: Commercial Fleet vs Employers Liability vs Professional Indemnity vs Product and Public Liability- no relationships!
There are four types of correlations between LOBs:

1. Process (volatility) Correlation (that is, correlation between two sets of residuals)
2. Parameter Correlations
3. Same trend structure (especially along the calendar years)
4. Reserve distribution correlations

#1 induces #2. However, #3 is the 'worst' kind of relationship you can have between two LOBs as it results in very little, if any, risk diversification. It means that in terms of future calendar year trends the two LOBs move together, that is, a trend change in one LOB means a trend change in the other LOB, and is tantamount to the two LOBs having the same drivers. If two LOBs satisfy #3, then #1 and #2 are close to 1.

Fortunately, #3 we have only observed between layers of the same LOB, between segments of the same LOB, and between net of reinsurance and gross data (of the same LOB). #1, #2, #3 induce #4. #4 is typically much less than #1 in the absence of #3.

It is important to recognize that you cannot measure the relationship between two LOBs unless you first identify the trend structure and process variability in each LOB. It is only in the Probabilistic Trend Family (PTF) modelling framework that you can identify a parsimonious model that separates the trend structure in the three directions from the process variability.