GIRO conference and exhibition 2010
Alex Marcuson

Solvency II Risk Margins
Cooking up a storm?

14 October 2010
Solvency II Risk Margins
Cooking up a storm?

- Amuse-bouches/dressed for the occasion?
- What’s in the store cupboard?
- Culinary basics
- A look through the utensil drawer
- Ready, steady, cook!
Solvency II Risk Margins
Cooking up a storm?

- A question of heads, hats and tails
- Sources of uncertainty
- Axioms for risk margins
- Available techniques
- A test of a general approach
Solvency II Risk Margins
A few limitations

- Just working undiscounted today
- Lots of unanswered questions here
- Some heroic simplifications used to provide accessible worked examples
- Do all the logical steps hang together?
Solvency II Risk Margins
A question of heads, hats and tails

- Imagine you are an insurance company who under a policy will toss a fair coin in just under 3 years’ time.
- If it comes up heads you pay a claim of £100, if tails you keep the money.
- What reserves do you hold?
- What capital do you hold (under Solvency II)?
- What Solvency II risk margin do you need to hold?
Solvency II Risk Margins
A question of heads, hats and tails (2)

- **Reserves** – the easy bit

- **Capital** – a bit harder…
- The amount is clearly £50 (first order) but is it A or B?

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Solvency II Risk Margins
A question of heads, hats and tails (3)

- It matters because of the impact on the **risk margin**
- Taking 6% of the capital requirement and summing…

![Graph A](image1)

- Option A is akin to using an ultimate view of uncertainty, not a 1-year view…
- …and it increases the risk margin here threefold
Solvency II Risk Margins
A question of heads, hats and tails (4)

- The circularity issue for the risk margin and SCR is nice and simple to resolve in this example.
- Also, consider what happens if we have a loaded and very loaded coins, where the probability of a claim becomes 10% and 1%
- Let RM, SCR be the initial approximate estimates, and RM*, SCR* the true values required

<table>
<thead>
<tr>
<th></th>
<th>Fair coin (p=50%)</th>
<th>Loaded coin (p=10%)</th>
<th>Very loaded coin (p=1%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCR (initial estimate)</td>
<td>50</td>
<td>90</td>
<td>99</td>
</tr>
<tr>
<td>RM (initial estimate)</td>
<td>3</td>
<td>5.4</td>
<td>5.94</td>
</tr>
<tr>
<td>SCR* (circularity resolved)</td>
<td>47.17</td>
<td>84.91</td>
<td>93.4</td>
</tr>
<tr>
<td>RM* (circularity resolved)</td>
<td>2.83</td>
<td>5.09</td>
<td>5.6</td>
</tr>
<tr>
<td>RM*/BEL</td>
<td>5.7%</td>
<td>50.9%</td>
<td>560.4%</td>
</tr>
<tr>
<td>(RM-RM*)/BEL</td>
<td>0.3%</td>
<td>3.1%</td>
<td>33.6%</td>
</tr>
</tbody>
</table>

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Solvency II Risk Margins
A question of heads, hats and tails (4)

Observations

Time horizon
• 1-year horizon is very important in our calculation of risk margin – it requires us to think carefully about what we will know and when we believe we are going to know it
• Getting a good 1-year view of reserve uncertainty can make a big impact on our reserves

Circularity
• The circularity in the SCR and risk margin calculation is challenging to resolve in all but the most straightforward calculations, but…
• It looks like it is may only require a significant adjustment to approximate approach where the distribution is quite skewed, and even then this may be spurious accuracy

<table>
<thead>
<tr>
<th></th>
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<th>Loaded coin (p=10%)</th>
<th>Very loaded coin (p=1%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(RM-RM*)/SCR</td>
<td>0.3%</td>
<td>0.3%</td>
<td>0.3%</td>
</tr>
</tbody>
</table>

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There are a number of different sources of uncertainty that we need to consider:

- Random or process error – *Will it be heads or tails?*
- Parameter error – *How loaded is our coin?*
- Model error – *What if modelling risk margins requires something more complicated than a coin-toss model?*
- Systemic risks / binary events – *Pay £1,000 if the coin lands on its edge?*
- Economic
- ...

It is no longer sufficient just to quantify these, we also need to understand how they manifest themselves over time.
Solvency II Risk Margins
Sources of uncertainty – Random error

• Suppose we now have 10,000 coin tosses, with a probability of a claim of 1% that take place over a three year period

<table>
<thead>
<tr>
<th>Time</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of coin tosses in year</td>
<td>6000</td>
<td>3000</td>
<td>1000</td>
</tr>
<tr>
<td>RM / BEL (approx.)</td>
<td>2.7%</td>
<td>3.6%</td>
<td>5.4%</td>
</tr>
</tbody>
</table>

• If we think of each settlement outcome as one of our coins and the number of coin tosses as representing our run-off pattern, then we have an approximate model of the random component of claim settlement behaviour

• Random error increases proportionately as our reserves run off
Solvency II Risk Margins
Sources of uncertainty – parameter and model error

- We expect these to decrease (absolutely and proportionately) with time, as emerging experience gives us better insight into the way in which our losses will behave.
- Will the uncertainty run off faster than process risk? I think it should.

No more nasty surprises?

<table>
<thead>
<tr>
<th>t=0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Now</td>
<td>Exposure over</td>
<td>Incurred @ 100%</td>
<td>Large claims well understood</td>
<td>Paid @ 100%</td>
</tr>
</tbody>
</table>
Solvency II Risk Margins
Sources of uncertainty – systemic / binary events

- We expect these to decrease (absolutely and proportionately) with time, but the speed of decrease may vary by type of event
  - Catastrophe losses / premium liabilities – quickly
  - Latent claims – very slowly
  - Ogden-type claims – in proportion to claim run-off?
Solvency II Risk Margins
Sources of uncertainty – economic

- Potentially complex to model
- Expected uncertainty decreases, as chance of adverse economic scenario affecting claims experience passes
- But important to understand the impact of inflation and credit spreads on our estimates
Solvency II Risk Margins
Some suggested “axioms” for risk margins

Nomenclature

- Let $C(s,t)$ be a random variable reflecting our claim development result for our reserves between time $s$ and time $t$.
- Let $P(s,t,x\%)$ be the $x\%$ile of the distribution $C(s,t)$
- Then $E_{(t=0)}[C(s,t)]$ is our expected claim development result conditioned on what we know at $t=0$
- Define $RR(s,t,x\%) = E_{(t=0)}[P(s,t,x\%)]$, our expectation at $t=0$ of $P(s,t,x\%)$
- Let $SCR(t,res,w)$ be the expected value at time $w$ of the reserve risk component of the SCR at $t$. Let $SCR(t)=SCR(t,res,0)$
- Then $SCR(t) = RR(t,t+1,99.5\%)$
- Define $\rho(t,\sigma) = RR(t,t+\sigma,99.5\%)$. Then $SCR(t) = \rho(t,1)$
- Finally, our (undiscounted) current risk margin = 6% of $\sum_{i=0}^{\infty} \{\rho(t,1)\}$ and our current ultimate 99.5% reserve risk (ICA style) = $\rho(0,inf)$
Solvency II Risk Margins
Some suggested “axioms” for risk margins

1. \( \rho(t, \sigma) \) is an increasing function of sigma
2. \( \rho(t, \infty) \) is a decreasing function in t
3. \( E_{(k)}[C(s, t)] = 0 \) for all \( k, s, t \)
4. For non-overlapping periods \( (s, t) \) and \( (u, v) \) and \( k < s, t, u, v; \) \( E_{(k)}[C(s, t)] \) is independent of \( E_{(k)}[C(u, v)] \)
Solvency II Risk Margins
Some suggested “axioms” for risk margins

Using a more natural description

1. As the time horizon over which we allow the losses to develop increases, the expected uncertainty increases – *widening funnel of doubt*

2. The ultimate uncertainty is expected to decrease over time – *knowledge gained is not forgotten (and useful)*

3. We expect our best estimates of liabilities at each point in time to be unbiased – *firms won’t set out to systematically over or under-reserve*

4. We expect our claims run-off in each future period to be independent – *we don’t expect a good/bad run-off result won’t affect the following years*
Solvency II Risk Margins
Some suggested “axioms” for risk margins

Using a more natural description

1. As the time horizon over which we allow the losses to develop increases, the expected uncertainty increases
2. The ultimate uncertainty is expected to decrease over time
3. We expect our best estimates of liabilities at each point in time to be unbiased
4. We expect our claims run-off in each future period to be independent

Is this true?
Solvency II risk margins
What might this mean?

- No reason for the proportional proxy to be right (or wrong)
- Can we just add up the run-off results for each year to get our ultimate uncertainties?
- If run-off results are positively correlated, then risk margin might be reduced
Solvency II risk margins
What does this mean? A simple example

• Consider a portfolio with a 5-year run-off of its uncertainties
• Let the run-off in each year i is distributed normally with mean 0 and standard deviation s(i).
• We can see that the run-off to ultimate is therefore also distributed normally with mean 0. Let its standard deviation be S = \sqrt{\sum s(i)^2}
• Suppose we do some analysis that tells us that the 1-year 99.5\%ile of our reserves is 40 above our best estimate, and the corresponding ultimate figure is 60.
• What is our risk margin?

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Solvency II risk margins
What does this mean? An artificially simple example

<table>
<thead>
<tr>
<th>99.5% margin</th>
<th>Option 1</th>
<th>Option 2</th>
<th>Option 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Year 2</td>
<td>45</td>
<td>22</td>
<td>31</td>
</tr>
<tr>
<td>Year 3</td>
<td>0</td>
<td>22</td>
<td>26</td>
</tr>
<tr>
<td>Year 4</td>
<td>0</td>
<td>22</td>
<td>18</td>
</tr>
<tr>
<td>Year 5</td>
<td>0</td>
<td>22</td>
<td>7</td>
</tr>
<tr>
<td>Ultimate</td>
<td>60</td>
<td>60</td>
<td>60</td>
</tr>
</tbody>
</table>

Implied risk margin 5.1 7.8 7.3

Comment:
- In practice not every pair of 1-year and ultimate 99.5%ile combinations will work.
- Important that the run-off of uncertainty makes business sense as well as adding up.
Solvency II Risk Margins
Available techniques – What might we use where?

Some suggestions…

- Mack
- Bootstrap /ODP
- M&W
- L&O

- Policy level models
- Benchmarks
- Curve-fitting
- ESGs

- Scenarios
- IACL
- ACPC methods
- RMS

- Chain-ladder
- Stochastic OT
- Freq-Sev

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Solvency II Risk Margins
Available techniques – Process risk

Some suggestions…

Mack
Bootstrap /ODP
M&W
L&O

Policy level models
Benchmarks
Curve-fitting
ESGs

Scenarios
IACL
Stochastic OT
ACPC methods
Freq-Sev

RMS

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Solvency II Risk Margins
Available techniques – Parameter risk

Some suggestions…

Mack
Bootstrap /ODP
M&W
L&O
Policy level models
Benchmarks
Curve-fitting
ESGs
Scenarios
IACL
ACPC methods
Stochastic OT
Freq-Sev
RMS
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Solvency II Risk Margins
Available techniques – Model risk

Some suggestions…

- Mack
- Bootstrap / ODP
- M&W
- L&O
- Policy level models
- Benchmarks
- Curve-fitting
- ESGs
- Scenario
- Chain-ladder
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- Freq-Sev
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Solvency II Risk Margins
Available techniques – Economic effects

Some suggestions…

- Mack
- Bootstrap / ODP
- M&W
- L&O
- Policy level models
- Benchmarks
- Curve-fitting
- ESGs
- Scenarios
- IACL
- ACPC methods
- RMS
- Chain-ladder
- Stochastic OT
- Freq-Sev

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Solvency II Risk Margins
Available techniques – Systemic risk

Some suggestions…

- Mack
- Bootstrap /ODP
- M&W
- L&O
- Policy level models
- Benchmarks
- Curve-fitting
- ESGs
- Scenarios
- Chain-ladder
- IACL
- Stochastic OT
- ACPC methods
- Freq-Sev
- RMS

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Solvency II Risk Margins
Available techniques – important considerations

• What level of data is available?
• Where can stochastic techniques help us?
• Is benchmarking appropriate and what is available?
• Can we develop suitable scenarios or use expert judgement, particularly in the tail of our distribution?
• Have we captured economic effects suitably?

Comment:
• Remember that we need to consider not just the magnitude of the uncertainty, but how we expect it to manifest itself
Solvency II Risk Margins
Developing a general methodology

- We need to populate a grid that looks something like this:

<table>
<thead>
<tr>
<th>Risk</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
<th>All years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parameter</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Binary</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Economic</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This row gives us our risk margins
This column gives us our ultimate uncertainty by risk type

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Solvency II Risk Margins
Developing a general methodology

- These are the cells that I think I could (probably) populate:

<table>
<thead>
<tr>
<th>“Risk”</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
<th>All years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process</td>
<td>✔️</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✔️</td>
</tr>
<tr>
<td>Parameter</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✔️</td>
</tr>
<tr>
<td>Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✔️</td>
</tr>
<tr>
<td>Binary</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✔️</td>
</tr>
<tr>
<td>Economic</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✔️</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✔️</td>
</tr>
</tbody>
</table>

- There are a lot of gaps to fill… let’s have a go…

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Solvency II Risk Margins
Simple (experimental) worked example…

• Continue our previous example:
  – Claims liabilities = 200, unearned premium = 100
  – Assume that option 3 is our selected process risk for premiums and claims liabilities
  – Parameter and model risk (together) have combined SCR contribution of 60 as well but this manifests itself over only 2 years
  – Binary risk of 100, all arising in year 1
  – No (extra) economic risk
  – All risk types are independent

• What is our risk margin?

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### Solvency II Risk Margins

Simple worked example…

<table>
<thead>
<tr>
<th>“Risk”</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
<th>All years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process</td>
<td>40</td>
<td>31</td>
<td>26</td>
<td>18</td>
<td>7</td>
<td>60</td>
</tr>
<tr>
<td>Parameter</td>
<td>52</td>
<td>31</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>60</td>
</tr>
<tr>
<td>Model</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>Economic</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td><strong>Total SCR</strong></td>
<td>177</td>
<td>44</td>
<td>26</td>
<td>18</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

**Risk mgn.** 16
Solvency II Risk Margins
Still lots of issues to solve…

- Bringing together pricing and reserving risk
- Discounting
- Reinsurance
- Correlations
- Proper distributions – life isn’t Normal
Summary

Do
• Make sure that your future SCRs are consistent
• Consider all risk types
• Make sure it has the right feel

Don’t
• Over-complicate
• Lose sight of what you are trying to achieve

Just like cooking really!

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Questions or comments?

Expressions of individual views by members of The Actuarial Profession and its staff are encouraged. The views expressed in this presentation are those of the presenter.

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14 October 2010