GENERAL INSURANCE STUDY GROUP

WORKING PARTY ON SOLVENCY

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THE MANAGEMENT OF UNCERTAINTY IN A GENERAL INSURANCE COMPANY

by

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1. INTRODUCTION

1.1 The Solvency Working Party was established in November 1982 to look at the work that had been carried out by the Finnish Solvency Working Party[1] and to suggest how similar research could be carried out in the UK context.

1.2 A paper was presented to the Bristol Seminar of the GISG in November 1983[2] and subsequently to a Sessional Meeting of the Institute in February 1984[3] reviewing the factors affecting general insurance solvency and focusing on the problems of asset risk, inadequacy of technical reserves, etc. The Working Party, with a substantially changed membership, went on to develop a relatively simple simulation model to look at the impact of asset variability and run-off risk on a company closed to new business and running off its liabilities. Papers were presented to the ASTIN Colloquium in Biarritz[4] and to the Cheltenham Seminar of the GISG in October 1985[5]. The approach was to concentrate on the emerging cash flows, rather than the balance sheet representations of them, and to model the uncertainty in asset values, asset returns, inflation and claim payments using stochastic models and simulation techniques.

1.3 In June 1986 the Solvency Working Party presented a more advanced simulation model of solvency to the first International Conference on Insurance Solvency in Philadelphia[6]. A similar paper was published in the ASTIN Bulletin[7]. This model permitted an analysis of the behaviour of a company under stochastic run-off conditions and also permitted the introduction of further years' business, with subsequent run-off to extinction. The model was designed to enable the impact of a wide variety of different assumptions to be assessed with a single set of simulations.

1.4 The simulation model of solvency was further developed, with further results presented, in an Institute Sessional Meeting paper in February 1987[8]. This provided the opportunity for a broader discussion of the major issues concerning solvency and the assessment of financial strength. In particular, some of the limitations of traditional methods of solvency control were discussed and the advantages of the emerging costs approach, combined with simulation techniques, were set out. It was suggested that solvency control could best be improved by moving towards a requirement for an expert report on the financial strength of the company rather than by attempting to modify the existing solvency requirements to make them reflect to a greater extent the risk profile of individual companies.

The model has been available since April 1988 as a FORTRAN program on a diskette suitable for an IBM-PC or compatible microcomputer, accompanied by an explanatory booklet[9].

2. THE NEXT STEPS

2.1 Although the 1987 Institute paper[8] certainly did not say all that could be said on the subject of solvency, from a statutory or supervisory perspective, the Working Party felt that the focus of attention should move to the broader issue of financial strength, the long term financial well-being of an insurance company. Statutory solvency and Companies Act solvency can be regarded as subsets of this broader concept. For statutory solvency considerations, new business may be assumed to cease, either immediately, or after a short period of continued business. The central concern is with the adequacy of the assets to meet the liabilities in this context. Although we allowed in our 1987 paper[8] for the possibility of two years' new business, little attention was paid to the underlying rationale for the assumption made about the claim ratios achieved on this business, whether the business could be written on such a basis and whether it was consistent with the assumed rate of growth. Business was assumed to cease after two years and then run off; no attempt was made to look at the financial results that would be implied for such a company or what would happen if it were allowed to write business on a continuing basis.

2.2 These are some of the questions to which the Solvency Working Party has turned in the third phase of its work, since 1987. The simulation model of solvency has been transformed into a management model, using similar simulation techniques. Although the basic emerging costs approach is the same, with simulation to model uncertainty, a number of new concepts have been introduced and the model has been greatly elaborated so as to approximate more closely to the operations of a real company and to permit maximum flexibility to someone using the model as a management tool. The basic structure of the model and some preliminary results were presented in a paper to the Second International Conference on Insurance Solvency in May 1988 and at the special meeting on the Capital Needs of Insurance Companies at the start of the Helsinki Congress in July 1988[10] Development work has continued with the model, with a view to producing a software package which will be useful to practitioners; results from the full model will be available shortly. This paper outlines some of the conceptual issues which have been addressed by the Solvency Working Party and discusses the basic structure of the management simulation model, which has been designed to provide practical solutions.

3. EMERGING COSTS PARADIGM

3.1 The accounts of a general insurance company are concerned with measuring what has happened in the past, to give an account of stewardship to shareholders and to provide a modest level of disclosure of the company's financial status to shareholders, potential investors, policyholders, intermediaries, supervisors, etc. The accounts measure the cash flows that have taken place,

but they are not only based on cash, since the fundamental accounting concept of accruals requires that income and expenditure be recognized as they are incurred and matched with one another so far as their relationship can be established or justifiably assumed. Thus premiums received to cover risks in future periods should not be treated as income for the period in question and appropriate provisions need to be set up in respect of claim payments that are still to be made in respect of cover that has already been granted.

3.2 There is a good deal of scope for debate about the size of such provisions, whether they should be conservatively estimated, whether they should be discounted, etc and there is a good deal of variation in what is done in practice. Unfortunately, this is not just an academic question, since the approach adopted and the assumptions made directly affect the company's published financial results and apparent financial strength.

3.3 Problems exist also on the assets side of the balance sheet. Few companies in the UK now show assets at historic cost, an approach which is self-evidently meaningless from the point of view of disclosing actual financial strength and normally produces substantial hidden investment reserves. The market value approach, which has almost entirely replaced book values, does at least have a more relevant and objective meaning, but market values can fluctuate from day to day because of the terms on which the market is willing to buy and sell. This may not reflect any real change in the anticipated stream of income to be generated by the assets; it is on this flow of investment income that the company depends. For an insurance company as a going concern, a forced realization of assets is unlikely to be necessary.

3.4 There is much scope for improving comparability of reporting in insurance company accounts. The Statement of Recommended Practice on Accounting for Insurance Business, issued by the ABI in December 1986[11], is a start along this route. Some standardization may tend to flow from the EC Directive on the Annual Accounts of Insurance Undertakings[12], when it is adopted. However, in our view there are inherent difficulties and shortcomings in the accounting perspective of a company's business which constrain the usefulness of even the most informative accounts from the point of view of an outside observer and mean that management would be very ill-advised to focus too much attention on traditional accounting measures.

3.5 In particular, it is important that a company should be viewed as a dynamic, rather than static, entity since the impact of the business currently being written and to be written in future is fundamental to the financial health of the company. Management should be interested in questions concerning the cash flow impact of writing particular types of business, the impact on the balance sheet as a result of reserving requirements, the capital that is implicitly tied up in writing the business and the rate of return that can be expected on that capital, the risk and reward trade-offs of different investment strategies, the possibility of developing a more or less immunized investment portfolio, the potential variability in projected financial measures such as solvency margin and profit and how to maximize the value of the company as a going concern.

3.6 To answer questions of this sort, a corporate model is required. Such a model needs to focus on future cash flows and to bring together the asset and liability components of the strategy. We call this the emerging costs approach or paradigm. The accounting implications are also important, so that the model needs to operate on two levels: the level of actual cash transactions and the level at which the transactions will be reported using accounting conventions.

3.7 At its simplest, such a model could be a profit-testing model for individual tranches of business, as has been developed in the context of life assurance[13-15]. The profit-testing technique explores the cash flow in and out and the impact of reserving requirements on the rate at which profit can be allowed to emerge. The resulting profit streams can be compared with the notional initial capital that is required if reserves have to be set up which exceed the initial net inflow (premiums received less initial expenses). It could be of interest to see how sensitive the implied rate of return on capital is to the reserving assumptions adopted and to uncertainty in the eventual claim settlement pattern.

3.8 On a rather grander scale, there is a need for corporate planning models which analyse the whole development of the business, again incorporating both assets and liabilities, reporting requirements, etc. The model that has been developed by the second author, and is described in later sections of this paper, is of this genre, but is distinctive in being stochastic rather than deterministic. It is of particular interest because of the way that the company is modelled as part of a market rather than in isolation.

3.9 The emerging costs paradigm lends itself naturally to the modelling process, permitting the most important features of the business to be taken into account in as realistic a way as possible, including taxes, dividend payments, etc as well as premium income, expenses, claim outgo, investment income, asset values and investment strategy. Matters which are largely obscured in the balance sheet, as a result of having to place single values on future streams of cash income or cash outgo, can be modelled in detail, with appropriate allowance for feedback mechanisms and interactions and direct modelling of the uncertainty.

3.10 Uncertainty is involved in the premium rates being charged for the risks being underwritten and the profit which can be expected to be generated by such business, in the claim payments to be made year by year in respect of the outstanding liabilities, and regarding future inflation, investment returns and asset values, if assets need to be realized. Traditional accounting methods have no mechanism for dealing with such uncertainties, except to recognize the outcome after the event. Traditional actuarial methods attempt to manage the problem by the use of conservative margins in reserve estimates, rules of thumb for allowing for potential investment losses when assessing financial strength, cautious estimates of profit margins, etc[16].

3.11 Appropriate stochastic models may not always be available for modelling each type of uncertainty but the emerging costs paradigm provides a structure for analysis which can be developed as better models become available and which in the meantime provides a framework for examining the possible importance of different factors and the potential variability in the outputs which are of direct concern to management.

4. SIMULATION OR SCENARIOS

4.1 The emerging costs approach can be used to explore a single set of assumptions. The analysis can then be re-run after amending one factor or more and a whole range of scenarios can be explored in order to test resilience to extreme assumptions, interactions between assumptions, dangerous combinations, etc. Such a framework does not provide any guidance on how likely particular adverse combinations may be. Indeed, the emphasis of such scenario testing (e.g. as currently proposed by the Canadian Institute of Actuaries for life insurance reporting[17,18]) is often only on the downside potential, as a method of testing the adequacy of implicit and explicit margins. Although this is important from a management point of view, there will also be interest in the potential rewards if the circumstances are not so adverse. A requirement for the company to be capable of surviving all adverse scenarios is likely to produce an ultra-conservative approach.

4.2 Simulation is a form of multiple scenario testing, but in which probability distributions are ascribed to the uncertain elements[19]. In practice these distributions may not be known accurately, or even at all, but simulation permits account to be taken of what knowledge can be gleaned from prior experience and also of the perceptions of the user. The parameters of the distributions can be specified, using a more or less complex distribution, as may be appropriate, and the sensitivity of the results to these factors can be tested.

4.3 In principle the use of probability distributions for particular types of variation might enable the impact on the finances of the company to be explored analytically. This is the subject of risk theory. Considerable progress along the lines of handling complex solvency issues in this way has been made in Pentikäinen and Rantala[1,20] and in Pentikäinen et al[21]. However, the more the models are developed to give a fair representation of reality, the less useful a wholly analytical approach becomes. Greater flexibility can be achieved by using simulation, as has been done extensively by the Finnish Working Party to supplement their analytical development[21].

4.4 With a simulation approach it is not necessary to be able to write down everything in a condensed global set of equations. Each aspect or interaction can be specified separately, with variability where appropriate. The model proceeds for one period at a time and within that time period can be programmed, for example, to solve simultaneous equations, to take specified actions, to take into account specific effects such as taxation or accounting policy, and so on.

4.5 When a large number of simulations are run, the results provide a convolution of a whole series of probability distributions affecting different aspects of the development of the business. This enables probability statements to be made about the outcomes, whether in relation to continued solvency, continued liquidity, free assets, profit, discounted net worth, etc.

5. ASSET/LIABILITY MANAGEMENT

5.1 Central to our considerations in the 1987 Institute paper[8] and to the development of the management model has been the view that assets and liabilities must be considered together. The traditional balance sheet does not encourage this. Assets and liabilities are on two different sides of the balance sheet; in many companies they may be seen as the responsibilities of quite different groups of people. If an actuary is involved, he or she will often be asked to advise on a suitable provision for the liabilities, but someone else will place a value on the assets. If the assets are taken at market value, what meaning can be ascribed to the comparison between the assets and the liabilities? If asset values fall as a result of a change in the market, does the change in surplus give a fair interpretation of a change in underlying financial strength or not? What is the purpose of the valuation?

5.2 Most readers of insurance company accounts would expect the valuation to tell them something about the adequacy of the assets to meet the liabilities, having regard to the incidence of the respective cash flows. In simple terms, if cash income exceeds cash outgo in each future period, the assets are clearly adequate. More commonly, income will exceed outgo only in certain periods, with the reverse applying in other periods. Account then needs to be taken of the investment of surplus income and the realization of assets when there is a shortfall. This is done by the use of a discount factor, which effectively represents a global rate of return at which surpluses can be invested and the terms on which assets can be disinvested when they are needed to meet shortfalls. The basic process of valuation is, therefore, one of comparing cash flows into and out of the entity being valued. For consistency, both income and outgo need to be discounted at the same valuation rate of interest, since it is really the net amounts in or out which are being discounted. However, the result will normally be quite sensitive to the rate of interest assumed, unless there is perfect matching or strict immunization, in the sense proposed by Redington[22] or matching as proposed by Wise[23-25]. To our knowledge the application of these techniques to general insurance has not yet been explored, although Tilley[26] referred to asset/liability management in the context of general insurance in his remarks at the Helsinki Congress.

5.3 It is interesting that this concept of valuation has been spelt out in the draft statement of valuation principles which is being developed by the Casualty Actuarial Society and which was discussed at the May 1988 meeting at Saddlebrook[27]. The nine valuation principles which they identify are set out in Appendix 1.

5.4 Not all of the principles of valuation proposed by the CAS Committee on Valuation Principles are relevant to this paper and discussion of their usefulness is beyond the scope of our present concern. However, principles 1, 2, 3 and 6 are of particular interest, referring to the underlying cash flow nature of both assets and liabilities, the importance of whether events will occur, when they will occur and what the financial consequences will be, and the importance of the stochastic nature of each aspect.

5.5 If market value is used to value the assets, this does not deflect from the basic cash flow principle; market value is the future expected income from the assets discounted at the appropriate market rate of interest. Basic valuation principles would require the liabilities to be discounted at a consistent rate of interest. However, in general insurance, the liabilities may not be discounted at all, or, if they are, the rate at which they are discounted is unlikely to be the market rate. Indeed, the issues are probably rarely considered from this perspective. If the market value of the assets changes, this may be because the implied market rate of discount has changed, rather than anything fundamental about the expected stream of future income from the assets. The value of the liabilities may, however, be quite independent of this and may not change in response, thus creating spurious fluctuations in the apparent margin of assets over liabilities.

5.6 This is not the place for an extensive discussion on the merits of discounting and its application to general insurance, some aspects of which were discussed fully in the Report of the Discounting Working Party to the Torquay seminar of the GISG in October 1987[28]. Suffice it to say that valuation issues point to the need to look at assets and liabilities together in a consistent fashion and to allow for margins consciously, either explicitly or implicitly, for example through the choice of discount rate.

5.7 In life insurance there has been much discussion on this issue, arising out of the requirement in Regulation 55 of the Insurance Companies Regulations 1981 that appropriate provision shall be included against the effects of possible changes in the value of the assets on their adequacy to meet the liabilities. This additional reserve, which has been described as a "mismatching" reserve, has recently been discussed by the Working Party on the Valuation of Unit-linked Business[29] under the broader heading of "resilience testing". From this perspective, the concept would seem equally relevant to general insurance as to life insurance. The methodology for assessing appropriate reserves of this nature forces consideration of the emerging cash flows, testing of sensitivity to changes in asset returns and values and consideration of the effect of discounting at different rates. Unless the cash flows are "unpacked", it is not possible to assess the impact of changes in the value of assets on their ability to meet the liabilities.

5.8 Different asset holdings of equal balance sheet value may have entirely different characteristics from the point of view of matching or immunizing the liabilities and from the point of view of variability. The emerging costs paradigm focuses attention on the actual assets held and the way in which they might behave. It enables specific assumptions to be made about how cash surpluses are invested or what rules should be followed for selling assets to meet a shortfall. Simulation enables the full variability of the assets to be taken into account by means of stochastic asset models.

6. THE COMPANY IN THE MARKET PLACE

6.1 Although the results in our 1987 Institute paper[8] showed the effect of writing business for a further two years, the model requires the user to make assumptions about the rate of growth of premium income and the expected claim ratio in respect of business written in those two years. Whilst it is clearly possible to make assumptions about these factors, it is unrealistic to assume that they are entirely independent or that they can be chosen at will. The volume of business which is written in a competitive market is dependent upon the price charged, and this in large measure determines the loss ratio. There was a degree of arbitrariness about the assumptions made for the two further years' business which makes that approach unsuitable for a model looking at the company as a going concern.

6.2 The report of the Finnish Working Party[1] was concerned with the viability of an insurance company as a going concern, with business continuing to be written. They approached the problem by allowing for cyclical variation in the profitability of contracts written. Their concern was to test for the resilience of the company to the adverse situation of a worsening business cycle by looking at the additional margins necessary on the assumption of different phases of the profitability cycle.

6.3 A difficulty in practice is to know where in the cycle one is at any particular point in time. In any case, it may not be so serious from the point of view of an individual company if the solvency margins of all companies are depressed by an adverse business cycle. It is more important to know if that particular company's solvency margin is depressed whilst other companies are showing resilience to the effect of the cycle. However, it was clear from the results shown in Pentikäinen and Rantala[1] that long term fluctuations in business results could have a substantial impact on the margin of assets over liabilities required to achieve a given security level for a company as a going concern.

6.4 Both the risk theory approach of the Finnish Working Party[1] and the simulation modelling of emerging costs in our 1987 Institute paper[8] focus on the behaviour of a particular company. Allowance is made for endogenous factors, such as investment strategy and premium rating, and for exogenous factors such as inflation, asset movements and claim experience. In practice, however, premium rating and the rate of growth of business cannot be considered as if the company existed in isolation. Premium

rating is based partly on an analysis of company experience in respect of the type of business being written, having regard to company profitability targets, expense loadings etc. It is also strongly influenced by the behaviour of other companies and by capacity in the market. The company may take a positive decision to undercut the market in order to increase market share or may overprice if it wants to discourage a particular type of business.

6.5 In order to produce sensible results from a simulation of a company writing business for a number of future years, consideration needs to be given to modelling the behaviour of the company in the market. Issues of this sort have been the subject of much research by financial economists in recent years, particularly in the United States.

7. THE APPLICATION OF MODERN FINANCIAL ECONOMICS

7.1 The focus of attention, in the early days of the "financial" approach to insurance problems, was the way in which premiums are regulated in particular states, and the need to develop scientific criteria for a "fair" rate of return which a property-liability company might be permitted to earn on the premiums it charges. Rate of return regulation was based initially on the application of fixed, arbitrary rules of thumb to determine a "fair" profit rate for underwriting per dollar of premium. In the 1970s, insurance commissioners in a number of states in the US began to consider a "fair" profit rate based on total earnings from the business, i.e. including investment income.

7.2 Towards the end of the 1970s Biger and Kahane[30,31], Hill[32] and Fairley[33] showed how the Capital Asset Pricing Model (CAPM) could be used to show what rates of return were "fair" on a rational, scientific basis. "Fair" is taken by the proponents of this theory to mean the return which would be earned in a competitive capital market on securities of equivalent risk. The insurance firm is looked at from the perspective of shareholders owning shares in the insurance firm as part of an efficiently diversified portfolio. The idea is that, in an efficient market, insurance premiums can be set no higher than will give to those shareholders a rate of return consistent with the central CAPM hypothesis. This is that prices in the securities market adjust until expected returns conform to:

$$\bar{R}_i = R + \beta_i \, (\bar{R}_m - R),$$

where $\bar{R}_i$ is the expected return on the ith security, R is the riskless rate of interest and $\bar{R}_m$ is the expected return on the "market" portfolio. The "beta" (or systematic risk) of the ith security is

$$\beta_i = \frac{\mathrm{cov}(\bar{R}_m, \bar{R}_i)}{\mathrm{var}(\bar{R}_m)}$$

7.3 The derivation of a fair premium for a proprietary insurance company in a world without taxes where the CAPM holds gives rise to an expression which is simply the expected cost of claims plus a risk premium, both discounted at the riskless rate of interest The risk premium depends on the co-variance of the claim amounts with the return on the market portfolio. If the claim amounts are not correlated with the return on any other security in the market, the fair premium approximates very closely to the expected amount of claims discounted by the riskless rate of interest. The message is that the variability of the return from writing insurance policies is not correlated with the market return, i.e. the return from holding a market portfolio of securities, so that the carrying on of insurance business does not contribute to the risk of an investor who holds an efficiently diversified portfolio of investments. The conclusion drawn therefore, is that a fair premium should not include any risk loading.

7.4 Hill[32] showed that the fair premiums determined by the CAPM approach would have been lower than the rates which had been permitted using traditional rule of thumb methods for the appropriate profit rate. The CAPM profit rates would in fact have been close to the profit rates actually achieved by companies during the 20 year period examined. This is because actual claims were higher than those allowed for in the original premium rating formulae, thus reducing the profit rates actually achieved, but it would seem that insurance companies were nevertheless able to remain in business. Of course, if only CAPM profit rates had been permitted in setting the premiums, the companies would have ended up with a very poor return or would perhaps have become insolvent!

7.5 A major conceptual difficulty with the whole approach is the definition of fair and the assumption that the price of insurance products is in some way determined by stock exchange investors with diversified portfolios of equities, of which insurance company stocks are a small part. One could argue that quite different factors control the rating process. So far as the company is concerned, its only concern with market rates of return is the possibility that it might need to raise new capital. The price of cover is determined by supply and demand in the insurance market. If premium rates had been dependent only on stock market forces, it is difficult to see how the variations in pricing in recent years, from absurd underpricing to significant overpricing, can be explained.

7.6 The CAPM concept of "fair" premiums is an artificial market concept, introduced because there is not a free market insurance in parts of the United States. However, fair could be defined in other ways and it is difficult to balance the conflicting claims of policyholder, insurer, shareholder, supervisor, injured party, etc. CAPM proved to be a convenient way to keep premiums down, a major concern of insurance commissioners in the United States, but it does not address the issue of security or the social need to maintain a strong insurance industry, even if returns happen to be low compared with alternative avenues of investment.

7.7 Much has been written by academic researchers on the subject of the application of the CAPM to insurance over the course of the last 10 years. The original models contained some fairly restrictive assumptions, such as the absence of taxes, but extensions have been developed to cope with some of these limitations. The early models were also highly simplistic in their treatment of settlement delays. Myers and Cohn[34] developed a more sophisticated way of allowing for the time value of money within the CAPM structure. Kraus and Ross[35] made use of the Arbitage Pricing Theory (APT)[36]. However, a basic problem with all of these methods is that the betas are difficult to estimate from the available data. The values obtained vary dramatically with time and have a large standard error[37].

7.8 Because of these fundamental difficulties, interest has tended to wane in CAPM models, and the focus of attention has moved to diffusion-based models, using similar principles to those developed by Black and Scholes[38] for looking at financial options. An application of this concept to insurance liabilities was presented by Doherty and Garven[39] and subsequently by Cummins[40,41]. Whereas CAPM models tend to assume that the risk of default by the insurance company is negligible, diffusion-based models permit the risk of default to be taken into account directly. Cummins (1988)[40] was concerned with just this problem of default risk. Cummins (1988a)[41] looked more broadly at the question of the capital structure of an insurance company and the implications that this has for the determination of fair profits.

7.9 Although much of this work arises naturally out of material that has been presented in the field of financial economics in recent years, the concepts are relatively unfamiliar to most actuaries. Bühlmann[42] has described actuaries who are familiar with these ideas as "actuaries of the third kind" and there is some expectation that the new AFIR section of the International Actuarial Association will begin to address these problems from the actuarial point of view.

7.10 The problems are not just in the realm of theory. The language of financial economics has acquired considerable currency in the US rate-making environment and actuaries, particularly general insurance actuaries, need to be aware of these developments and to have some understanding of them. A useful introduction to the subject is the book by D'Arcy and Doherty[43]. Cummins and Harrington[44] have also published an anthology of papers by some of the key players in the development of these concepts in relation to insurance.

7.11 The Solvency Working Party has devoted some effort to seeking to understand the approaches deriving from financial economics, with a view to exploring their possible application in conjunction with the modelling work which we have been developing. The financial pricing models have three basic principles in common:

- prices are intended to reflect both supply and demand
- prices should reflect the market price of risk
- prices should recognize the time value of money

7.12 The last of these was already inherent in all of our modelling work but it is in the first two areas that we were faced with some uncertainty as to how we should proceed in order to set coherent premium rates and growth factors. However, we have not as yet found a way of making use of the financial economics theory in our model.

8. THE MANAGEMENT MODEL

8.1 The management model which we have now developed is designed to simulate a more realistic market situation, by modelling the possible reaction, in terms of the volume of business written, to increases or decreases in the premium rates charged by a particular company relative to the average market level. The model involves projecting the behaviour of the market as a whole alongside the behaviour of the particular company under investigation and allows one to test the effects of divergences between the view taken by the market and the view taken by the company.

8.2 Profit margins may be set for each type of business, to represent both the average market situation and the experience of the particular company under investigation. Premium volume is specified both for the market and for the company, as is the extent to which the market and the company set premium rates with a view to recovering past losses or rewarding policyholders in respect of past profits. The company rates can then be set at a specified percentage above or below the market rates and the volume of business written by the company is assumed to increase or decrease according to whether premiums are lower than or higher than the market rates, with appropriate gearing factors to reflect the elasticity of demand.

8.3 As in the previous solvency model[8], the claim ratios actually experienced by the company on the business being written include a stochastic element. Stochastic variation is also allowed for in the aggregate claim payments made year by year, in respect of earlier underwriting years, in the rate of inflation and in asset movements and returns.

8.4 The model allows for business to have been written for up to 20 years prior to the date of investigation. This is to enable an initial portfolio to be established based on the outstanding claims from a number of past years' business which are running off. Expenses (including commission) are allowed for explicitly and an attempt is made to allow more realistically for the payment of taxes and for dividends to shareholders. Although the model is based on the emerging costs paradigm, accounting quantities are monitored as well as cash flows in order to be able to present the results as they might appear in the accounts (if these were drawn up on a "best estimate" basis), to demonstrate solvency margins, calculate tax liabilities, etc.

9. THE MANAGEMENT MODEL - PREMIUM INCOME

9.1 The model allows for six types of business to be written. These need not correspond on a one-to-one basis with particular classes of business or lines. They are characterized by a series of parameters, including claim settlement pattern, volume growth rate, claim variability and the fraction of the market held by

the company. A key feature is the specified profit rate for each liability type. This is the percentage margin for profit in the gross premiums, allowing for expenses and for the expected run-off of claims in accordance with the claim settlement pattern, discounted at a realistic real rate of interest.

9.2 The company under investigation is assumed initially to have a specified share of the market by numbers of policies and to have premium rates exactly equal to those of the market. The market volume is specified separately for each liability type, as is the fraction of that particular part of the market held by the company. Growth rates in market volume are specified both for the past and for the future, with flexibility to allow for different growth rates in two separate past periods and in two separate future periods.

9.3 The premiums written by the market are assumed to be the product of the total volume in terms of numbers of policies and the average market premium in £ sterling. Apart from changes in volume determined by the growth rates, the market premiums are assumed to be set using an estimate of the latest year's inflation, based on the increases in the retail price index over the past four years. This formula is specified by the user and may involve factors which are positive or negative and greater than or less than one. Thus it is possible to project the past trend of inflation linearly, quadratically or using a weighted or unweighted average of the past four years' retail price index ratios. When the next year's premiums come to be set, the actual simulated rate of inflation is used, instead of the formula, to arrive at a notional base for premiums in the previous year. The current year's inflation is again estimated using the formula. The market may set premiums higher than otherwise in order to attempt to recoup past losses, or may pass on to policyholders the benefit of past profits.

9.4 The company under observation is assumed to operate within the framework of the market which we have described, starting initially from a neutral position, with a specified market share. It is then assumed to deviate from the market in three principal ways:

(a) as a matter of policy, with a view to increasing profits or in order to grow or contract, it will aim to set its general premium levels at a stated percentage above or below the market level.

(b) it will estimate the future rate of inflation on the basis of past increases in the retail price index, using a formula which may differ from that implicit in the behaviour of the market as a whole.

(c) it may attempt to recoup past losses by charging higher premiums, or pass on the benefits of past profits to policyholders in terms of lower premiums, to a different extent than the market.

9.5 In addition, the model allows for a random variation in the level of company premiums relative to the market, since in practice the company may use a different formula for projecting past inflation for the purpose of setting premium rates.

9.6 Provision is made for the claim ratios to vary on a cyclical basis. The length and shape of the cycle may be specified, and the amplitude and phase may be specified separately for each liability type.

9.7 If the company ends up charging higher premiums than the market, it may expect to lose business volume. Similarly, if it charges lower premiums than the market, it will expect to gain business volume. The relationship between premium differentials and volume response, or the elasticity of demand in economic parlance, is controlled by gearing factors which are specified by the user, separately for premiums above and below the market level. Further details of the operation of the gearing factors are given in Appendix 2.

9.8 The company is assumed to be small enough for its behaviour not to have a feedback effect on the market as a whole. Up to 20 years' future business can be assumed. There would be no problem in principle in extending this, but it is difficult to obtain stable results over long periods without incorporating additional feedback mechanisms, so it is doubtful whether useful results would be obtained on a longer timescale.

9.9 Unearned premiums reserves are set up at the end of the year as specified percentages of gross premiums written during the year. These percentages may differ between liability types and between the company and the market.

10. THE MANAGEMENT MODEL - EXPENSES

10.1 Unlike the solvency model[8] and the previous version of the management model[10], expenses are now taken into account explicitly. Expenses at the market level are assumed to be a specified percentage of actual gross written premiums. It is thus assumed that there are no productivity gains for the market as a whole. There are two main influences on expenses which might be taken into account. The first is improvements in efficiency or productivity. The second is the impact of wage inflation as opposed to price inflation and real growth of premium income. In practice these two may largely balance out for the market as a whole, so that an assumption that expenses are a fixed percentage of gross premium income may not be unreasonable.

10.2 For the company, on the other hand, it is desirable to have greater flexibility. The model provides for the company expenses to be specified in terms of a fixed component, that goes up only in line with price inflation, and a component that is directly proportional to current gross written premiums.

11. THE MANAGEMENT MODEL - CLAIM OUTGO

11.1 In order to estimate the claims which are expected to be incurred in respect of the business written, it is assumed that the premium rate charged by the market, when applied either to the market's or to the company's volume of business, in terms of the number of policies written, and after allowing for the assumed inflation and the assumed level of profitability and

expenses, gives a mean estimate of the claims likely to be incurred. The profit rate as a fraction of gross premiums assumes that a real rate of return will be achieved on the investment of provisions at the same rate as the rate at which they are to be discounted. A random element in the claim ratios is then allowed for. The variability is assumed to be normal, with standard deviations specified by the user separately for each liability type for the market as a whole and for the company. When calculating actual claim payments in future years, as each tranche of business runs off, allowance is made for the inflation in those future years, as generated by the model, rather than the estimate of inflation that was taken into account in setting the premiums.

11.2 The resulting claims are subdivided into claim amounts in successive years in accordance with the specified settlement patterns for each liability type, and are added to claim payments in those years of development from other underwriting years. In this process, allowance is made for the claim payments in each successive year to be inflated in accordance with the retail price index. In practice this means that initially outstanding claims are in terms of the money values of the date of investigation. In each future year of development the claims to be paid and the amounts still outstanding are inflated by a year's increase in the retail price index. Corresponding claim amounts for each future year, arising from the new earned premiums, are then added in. The total claim outgo in any particular future year is assumed to be subject to random variation, in accordance with a normal distribution with standard deviation $aX+b\sqrt{X}$, where a and b are suitably chosen constants, which may be specified by the user.

11.3 The rationale for this formula was set out in Appendix 5 of our Institute paper[8]. The variability factors have been assumed to be the same for both the total market claim outgo and the company claim outgo, since the major influence relates to secular variation, which will normally be the same for both the company and the market. The same random variables are used for the company and the market in specifying the variation in each year.

11.4 For the purposes of establishing notional "best estimate" balance sheet provisions in respect of outstanding claims, future projected claim payments are taken at their nominal amount (before allowance for inflation) and discounted at a specified real rate of return. The intention is that the discount rate should represent the anticipated actual real rate of return, so as not to incorporate implicit margins in the provisions.

11.5 The above procedures enable premiums, expenses and claims to be projected year by year in terms of cash flow and appropriate provisions to be set up in respect of unearned premiums and outstanding claims. In each simulation, a set of random numbers is chosen, which gives rise to a particular realization, in which the program proceeds forward one year at a time, evaluating the various items in the revenue account and the balance sheet. The receipt of premiums and investment income and the payment of claims, taxes and dividends give rise either to a surplus available for investment or to a shortfall requiring assets to be sold.

12. THE MANAGEMENT MODEL - ASSETS

12.1 In the model described in our Institute paper[8], flexibility was given to the user to specify the asset backing for both technical reserves and asset margin, in cash or deposits, equities and gilt-edged securities. The user could also specify the order in which the different types of assets were to be sold when there was a shortfall of income and assets needed to be realized. The results in that paper turned out not to be particularly sensitive to the asset realization strategy specified and that particular facility has been dropped. However, some additional flexibility has been permitted, both in relation to the types of assets available and the way in which they can be matched to the liabilities.

12.2 Seven different types of asset may be used and the components of the asset backing may be separately specified in respect of the provisions for each of the six liability types as well as for the asset margin. The asset types are as follows:

* cash
* three types of dated gilts (terms to be specified)
* index-linked gilts
* equities
* property

12.3 The tax liabilities determined as at a balance sheet date are assumed to be settled a year later. The assets held against these liabilities are invested in the same proportions as the rest of the asset margin, allowing for the part of the asset margin that is represented by agents' balances.

12.4 A proportionate holding of cash may be assumed for any of the liability types or for the asset margin. Cash is assumed to be interest-bearing, with the rate of interest assumed to be one percentage point lower than the yield on irredeemable gilts as defined by the Wilkie model. Non-interest-bearing cash is assumed to be held in respect of the balances due from agents. These are taken as a proportion of gross written premiums, as specified by the user.

12.5 If the asset margin becomes less than the amount of non-interest-bearing cash, but the company is still solvent, the shareholders' funds are assumed to be able to borrow cash to cover the shortfall. In the first instance this is by offsetting negative cash in the shareholders' funds against positive cash elsewhere in the company but if total cash becomes negative it is assumed to be borrowed, on payment of a rate of interest which is set two percentage points above the current yield on irredeemable gilt- edged securities.

12.6 Gilt-edged securities and equities are assumed to behave in accordance with the series of inter-related stochastic processes proposed by Wilkie[45]. However, Wilkie's model provides only for irredeemable gilts and we have assumed a simple yield curve relationship, whereby 20 year gilts are assumed to have the same gross redemption yield as irredeemable gilts and the yield is assumed to fall linearly to one percentage point below the yield on irredeemables as the term to maturity falls to zero.

12.7 Redeemable gilts are assumed to be purchased at par and sold one year later at the ruling price for stocks with one year less to redemption, to be replaced by gilts of the original term, once more purchased at par.

12.8 Wilkie's model also generates future values for the retail price index and these have been used whenever an estimate of future inflation is needed, either in respect of premiums or claims. The inflation model also provides a basis for simulating the behaviour of index-linked gilts. The real yield on index-linked gilts is taken to be a constant below the dividend yield on equities, to be specified by the user. However, the value of index-linked gilts is assumed to rise in line with the retail price index.

12.9 Equities are provided for using the standard Wilkie model, with the facility for the user to specify the parameters.

12.10 The final asset type is property, for which Wilkie[46] has provided a model similar to, but simpler than, the model for equities and parameters to reflect the less volatile nature of the market. Details of the property model are given in Appendix 3.

12.11 At the end of each year of the projection, the provisions are recalculated in respect of each liability type and the assets are respread in order to start the new year with the correct distribution of assets relative to each liability type. Any assets that need to be sold in order to achieve this are assumed to be disposed of at the current market value.

12.12 In the run-off situation described in our Institute paper[8], the normal situation was that of continuous net outgo. We therefore assumed that cash and investment transactions took place on average in the middle of each year. With a continuing company, however, and allowing for inflation, the normal position is that the steady flow of premium income more or less matches the steady outgo in respect of claims. The key transactions, as explained later, are the payments of dividends to shareholders and tax. These payments are assumed to take place at the end of the year in the case of dividends and at the end of the following year in the case of tax. Furthermore, asset holdings are now defined in relation to the amount of the liabilities, so that, on each occasion that investments are to be rearranged, the provisions need to be recalculated. In the light of this, we have assumed that all transactions take place at the end of the year, so that investment values need to be updated only once a year.

13. DIVIDENDS TO SHAREHOLDERS

13.1 Dividends are assumed to be paid annually, at the end of each year. The initial rate of dividend is specified by the user as a percentage of the initial asset margin and the amount of dividends payable each year, in money terms, is increased annually at the rate of increase of the retail price index. A further percentage increase may be specified by the user if it is desired to give shareholders a better return than merely following the retail price index. This clearly ignores the actual profit or loss achieved in the past year, but seems more

realistic than relating dividends directly to actual current profits and it seems to accord fairly closely with recent practice during some rather violent fluctuations in disclosed insurance profits.

14. PROFIT AND LOSS ACCOUNT

14.1 The resulting figures enable a profit and loss account to be drawn up each year, in order to establish taxable profits and show the results in a form familiar to management. The profit and loss account is built up as follows:

Written premiums
- \- Claim payments
- \- Expenses and commission
- \+ Unearned premium reserve brought forward from previous year
- \+ Outstanding claim reserve brought forward
- \- Unearned premium reserve carried forward at the end of the year
- \- Outstanding claim reserve carried forward
- \+ Investment income
- = Balance subject to corporation tax on trading profits

- \- Taxation for year (provision set up)
- \- Dividends to shareholders
- = Balance retained

14.2 Taxation for the year includes tax on chargeable gains or losses as well as tax on trading profits. The amount required to meet the tax liability is set aside as a provision in the balance sheet and settlement of the liability is assumed to take place 12 months later, just before the next balance sheet is drawn up.

14.3 If there are taxable losses, these are carried forward to the following year unless they can be carried back to offset against profits in previous years. The number of years for which carry back is permitted can be specified by the user, as circumstances may differ in different territories (in the UK only 1 year is permitted but in Canada carry back is allowed for up to 3 years). Taxable losses from previous years may be brought forward to offset against any taxable profits.

14.4 The program output displays a distribution of values of the profit (loss) in a selection of future years. The program tracks the progress both of the company under investigation and the market as a whole and shows distributions of profits for each. The profits (losses) in future years are expressed per thousand units of written premium to produce figures more easily comparable with current profits or losses.

14.5 Gains and losses on investments are not taken into account in determining the balance subject to corporation tax on trading profits, nor in the final profit result for the year. Incorporating changes in asset values resulting from the volatility of the market might introduce a confusing level of additional variability into the already quite variable profit results. A facility which it might be useful to allow for in the program might be the possibility of taking into account a moving

average of unrealized capital gains in the final profit figure, so as to allow the growth of equities to be introduced on a smoothed basis.

14.6 As far as the cash flow in the year is concerned, we have the following:

Written premiums
- Portion of written premiums in the year that is still held by agents at the end of the year
+ Portion of written premiums in the previous year that was held by agents at the beginning of the year
+ Investment income
- Claim payments
- Expenses and commission
- Dividend payment
= Cash available for investment

14.7 The net cash flow gives rise to a net amount of surplus cash available for investment or a net cash requirement for which assets need to be sold. In addition the tax provision brought forward at the beginning of the year is used to meet the tax liability which becomes due at the end of the year.

15. BALANCE SHEET

15.1 The balance sheet may be drawn up as follows:

Funds =
+ Unearned premium reserves
+ Outstanding claim reserves
+ Provision for tax liabilities to be paid at the end of the year
+ Asset margin

15.2 These funds are made up by a mixture of assets, including, in the case of the asset margin, balances held by agents. Once the outstanding claim reserves and unearned premium reserves have been calculated, the requirements for each of the types of assets to match the six liability types and the asset margin can be established and assets bought or sold accordingly. Any capital gains tax liability that is created as a result of these transactions will result in a payment of tax a year later but does not require any further rearrangement of the assets. The program output displays a distribution of values of asset margin i.e. excess of total assets over total liabilities, at the end of a selection of future years. This is shown both for the company under investigation and for the market as a whole, expressed per thousand units of gross written premium.

16. USING THE MANAGEMENT MODEL

16.1 Numerical results are not yet available from the full management model since the final touches were only being put to the programming as this paper was being written. Some results may be available for presentation at Harrogate. However, the program has not been designed with a view to producing a single

set of results on the basis of a hypothetical company, but as a tool which actuaries will be able to use in the context of different companies to explore the sensitivity of future prospects for the company to changes in key parameters. It is hoped that the program can be made available in diskette form before too long.

16.2 The description in this paper has concentrated on the main features of the model, in order to focus attention on the most important aspects. For completeness, however, Appendix 4 gives a list of the parameters which are required for the operation of the model and which the user may specify. Two different types of parameter may be distinguished: variable parameters and scalar parameters. Scalar parameters are each attributed a single value for a particular run of the program. Variable parameters, on the other hand, may be given up to five separate values for each run and results will be produced using the same set of simulations (i.e. the same sets of random numbers) with the variable parameters taking each of the specified values in turn, whilst the other variable parameters are held at a standard value. The concept is similar to that embodied in the solvency simulation program, except that the number of parameters to be treated as variable has been kept to a minimum and additional flexibility has been provided for in that any of the five possible data entries for a particular variable parameter may be specified as the standard value.

16.3 Some of the parameters consist of groups of related parameters which are linked together. Some other parameters also need to be chosen carefully in combination in order to produce realistic scenarios. A useful check on whether the parameter combinations are sensible is to run the program without any stochastic variation. This shows whether there is an inbuilt bias towards abnormal growth or disaster. Different parameter combinations can be explored to find a stable scenario and to discover what features are giving rise to any instability.

16.4 When the program is used in stochastic mode, the same sets of random numbers should be used for each different parameter combination, so that comparison between the results using different parameter values is not affected simply by the choice of random numbers. In order to reduce the dependence of the results on the particular sets of random numbers chosen, and hence to produce more stable distributions, a larger number of simulations can be used. For most practical purposes, runs with 1,000 simulations should provide the right balance between economizing on running time and providing stable estimates of the underlying distributions, at least for making comparisons between the results with different parameter values.

16.5 The authors believe that the model provides an invaluable tool for management to explore the consequences of uncertainty in the operations of a general insurance company. Few models designed to assist in the forward planning process give adequate recognition to the uncertainty inherent not only in the claim outgo but also in future inflation, asset movements and yields. This model enables all those aspects to be taken into account in an integrated fashion.

16.6 Use of the model within a company will focus attention on key parameters for determining the future course of the business and managing the inherent uncertainties. The focus on emerging cash flows enables the assets and liabilities to be taken into consideration together in a coherent way. Management (and actuaries) will need to be educated to an appreciation of the results of projections presented in the form of distributions to show the impact of variability and to appreciate how these can be used to understand the trade-off between expected outcome and uncertainty.

17. FUTURE DEVELOPMENTS

17.1 Others will no doubt develop similar modelling techniques to assist in the management of uncertainty in a general insurance company, or translate these concepts into a life insurance environment. An alternative presentation designed to meet many of the same objectives has been put forward by the Finnish Solvency Working Party[21], the so-called "5 doctors". Their latest paper provides some useful insights into the problem of modelling the behaviour of a company operating within a market. Of particular interest is their exploration of the potential for using control theory techniques in this context. The possible application to the field of insurance of these ideas borrowed from an engineering control theory was first proposed by Balzer and Benjamin[47,48] and taken up by Rantala[49-52] and by Taylor[53].

17.2 Similar simulation techniques for looking at life insurance were first explored by the Faculty of Actuaries Solvency Working Party[54]. However, the Finnish Working Party intend to give this problem more attention in the final version of their paper[21]. Some preliminary ideas have been put forward by Pentikäinen and Pesonen[54].

17.3 Measuring the financial strength of general insurance companies is a problem which involves considerable practical difficulties in getting hold of suitable data and understanding what significance to attach to it, and major technical issues in attempting to analyse it. Over the last six years, the Solvency Working Party has sought to come to grips with these issues in as practical a way as possible. This process has brought us back to the fundamentals of actuarial science and an affirmation of the validity of concepts such as emerging cash flow, asset/liability matching, profit testing, principles of valuation, etc. in general insurance, although some will perceive these as being life insurance techniques.

17.4 The journey has also taken us to the far boundaries of actuarial science where new fields of study such as stochastic control theory, CAPM, option pricing and diffusion models, etc. are opening up and offering a challenge to the actuarial profession. Can we understand what the advocates of these other disciplines are saying and find ways of applying their insights in a way that will further the develpment of actuarial science and the practical skills of actuaries? We must beware of letting other disciplines usurp the role of the actuary and take the initiative, as has begun to happen to some extent with financial

economists in the United States. However, the actuarial tradition of the United Kingdom is one with a strong practical emphasis and there is some suspicion of complex theoretical developments which cannot be understood and applied by actuaries in the practical world. Some of the theoretical ideas which have been developed have yet to achieve a major breakthrough into practical applications, but this may be the agenda for the next decade.

17.5 The Second International Conference on Insurance Solvency in Brighton in May 1988 provided an opportunity for an interchange of ideas between researchers working in the field of the solvency and financial strength of insurance companies, but approaching the problem from a wide variety of different perspectives. It has been interesting to observe, over the past 6 years, an increasing convergence between the concepts being explored by our Solvency Working Party and those presented by the Finnish Working Party. We acknowledge a considerable debt of gratitude to Professor Teivo Pentikäinen, Dr Jukka Rantala, and now to the new members of the team, Dr Martti Pesonen, Dr Heikki Bonsdorff and Dr Matti Ruohonen.

17.6 We have also seen an increasing level of cooperation with colleagues in North America, arising out of the International Conferences on Insurance Solvency. In conjunction with the new Financial Management Study Group of the Institute of Actuaries, we sponsored a seminar on the Applications of Modern Financial Economics on 28-29 March 1988 and invited four American colleagues to give lectures: Dave Cummins, Richard Derrig, Steve D'Arcy and Harris Schlesinger.

17.7 At the two International Conferences on Insurance Solvency, work has been presented by North American colleagues on cash flow modelling and simulation solutions to the problems of solvency and financial strength. The papers by Paulson & Deekshit[56] in 1986 and Paulson[57] in 1988 started from a different perspective to ours but appear to be converging as the research proceeds. We have made available to them a copy of our solvency simulation program and foresee further scope for cooperation.

17.8 It is expected that a Third International Conference on Insurance Solvency will be held in 1990, possibly in the Netherlands, and further research will be commissioned for that Conference, particularly in the hope that progress can be made in building on the respective strengths of the actuarial approach to the subject, whether through cash flow simulation techniques or risk theory, and the approach of financial economics. A major aim will be to achieve a greater synthesis of these ideas.

17.9 There seems little likelihood, therefore, that the supply of promising avenues for research in the field of solvency and financial strength will dry up in the foreseeable future. Many challenges remain to be tackled. At the same time, however, we hope that practitioners will begin to make use of our simulation models, or develop similar ones, and that papers will be written on the practical experiences obtained.

18. ACKNOWLEDGEMENTS

18.1 Although this paper appears under the names of two members of the Solvency Working Party, because the development of the latest management simulation model has been almost entirely in the hands of one author, whilst the paper has been written by the other, the authors wish to acknowledge the substantial contribution made to the evolution of these ideas by other members of the Working Party. A list of the current members of the Working Party is given in Appendix 5.

Chris Daykin
Brian Hey 6 September 1988

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Appendix 1

CAS Draft Statement of Valuation Principles

1. Every asset, obligation or consideration is associated with one or more items of cash flow.

2. The value of an asset, obligation or consideration is equal to the combined values of its constituent set of items of cash flow.

3. The value of an item of cash flow depends upon the values of the following valuation variables, each of which is, conceptually, a random variable for which a probability distribution may be assumed to exist:

    (a) the occurrence of the item of cash flow,

    (b) the amount of the item of cash flow

    (c) the interval of time between the valuation date and the time of occurrence of the item of cash flow; and

    (d) a rate of interest related to the interval of time between the valuation date and the time of occurrence of the cash flow.

4. The value of any of the valuation variables with respect to an identified set of items of cash flow may be determined on the basis of any set of rules or assumptions which are appropriate to:

    (a) the nature of the asset, obligation or consideration made up of that set of items of cash flow,

    (b) recognize suitably the various environments within which the valuation is being performed; and

    (c) the purpose of the valuation.

5. All identifiable factors that may have a material effect on the values of a set of items of cash flow involved in a valuation must be taken into account in establishing the set of rules or assumptions to be used in determining those values.

6. In general, the result of a valuation is a random variable.

7. A valuation may involve only the assets and obligations related to specified events underwritten on or before the valuation date or it may involve both those assets and obligations and the assets and obligations related to specified events projected to be underwritten after the valuation date.

8. If a valuation is performed as of a given valuation date and involves only a specified set of an underwriter's obligations, then the set of assets to be associated with that set of obligations for purposes of the valuation must be explicitly identified; and none of the elements of that set of assets may be associated with obligations that are not elements of the specified set of obligations for purposes of a valuation that is performed as of the same valuation date and applies to a different set of obligations.

9. The results of valuations performed as of different valuation dates may not be consolidated.

APPENDIX 2 The relationship between price and volume of business

1. If the company's premiums are above the market level of premiums, the company is assumed to lose business. The loss of volume is determined by a gearing factor which relates the proportionate change in volume to the proportionate deviation from the market premium level. They are of opposite signs, i.e. if the premium is higher the volume is lower and vice versa. Thus, we have the following:

$$\frac{V_m - V}{V_m} = g_a \frac{P - P_m}{P_m} \qquad (1)$$

where g_a is the gearing factor for premiums above the market level

P_m is the market premium level

P is the actual premium charged by the company

V_m is the premium volume previously being written by the company

and V is the new premium volume

2. Now the actual premium charged by the company may incorporate an element in respect of the recovery of past losses by the company. The adjustment to the premium is set so as to produce the specified recovery with the actual volume of business written. Thus we have:

$$P = P' + \frac{R}{V} \qquad (2)$$

where P' is the premium before adjustment for recoveries
R is the amount to be recovered

3. The above equations (1) and (2) have to be solved for the new premium volume (V). This is given by the roots of the following quadratic equation:

$$V^2 - V.V_m\,(1+g_a-g_aP'/P_m) + g_a\,V_m\,.R/P_m = 0 \qquad (3)$$

4. If the amount to be recovered is too large, equation (3) may have no real solutions. Also if the gearing factor is large, and the first estimate of premium (P') is well above the market, then the resulting volume can be unreasonably low. In such cases we assume that the company's volume of business cannot fall to less than half the volume which would apply if the company still had its initial share of the market. By way of illustration of the impact of the gearing, Table 1 shows the resulting company volume

in terms of thousands of policies for different assumptions. In each of these cases the target percentage above market premium is achieved.

Table 1
Company volume (000s of policies) according to target percentage above market premium and gearing factor assumed

Gearing factor	Target percentage above market premium 0	5	10
1	500	475	450
2	500	450	400
3	500	425	350

Notes The market volume is assumed to be 50 million policies and the company in question is assumed to have 1 per cent of the market. Random variation of the company premiums as compared to the market is ignored. No allowance is made for recovery of past losses.

5. We have allowed for different gearing factors to be used for premiums above and below the market premium level. Because of the complicated interaction of the various factors, in particular the effect of trying to recover past losses or reward policyholders for past profits, it is not always possible to tell in advance whether the resulting premium will be above or below the market. This leads to some awkward programming problems.

6. Table 2 shows the effect of introducing recovery of past losses. In this case the company is assumed to be aiming at 5 per cent above the market premium level, but in addition is trying to recover losses as shown. The actual premium level as a percentage of the market premium is shown in brackets.

Table 2
Company volume (000's of policies) and (in brackets) the actual premium level according to losses to be recovered and gearing factor assumed

Gearing factor	Losses to be recovered (£000) £0m	£1m	£2m	£3m
1	475 (£105)	464 (£107.2)	453 (£109.4)	441(£111.8)
2	450 (105)	427 (107.3)	400 (110.0)	369 (113.1)
3	425 (105)	386 (107.6)	336 (111.0)	225 (118.3)

7. Finally, we show in Table 3 the impact of recovery of past losses when the company is otherwise aiming to be 5 per cent below the market premium level. If the premium charged by the company is below the market level, the company is assumed to gain volume. The formula is the same as equation (3) but the gearing factor might be different.

Table 3
Company volume (000's of policies) and (in brackets) the actual premium level according to losses to be recovered and gearing factor assumed

Gearing factor	Losses to be recovered			
	£0m	£1m	£2m	£3m
1	525 (£95)	515 (£96.9)	505 (£99.0)	495 (£101.1)
2	550 (95)	531 (96.9)	511 (98.9)	489 (101.1)
3	575 (95)	548 (96.8)	517 (98.9)	482 (101.2)

APPENDIX 3 A Stochastic Model of Property Values and Yields

Let R(t) be an index of property rents
and PY(t) the yield on this index, i.e. the rental index at the specified date divided by the price index at that date

The model for PY(t) is

$$\ln PY(t) = \ln PYMU + PYA\,[\ln PY(t-1) - \ln PYMU] + PYSD.\,PYZ(t)$$

where PYZ(t) is a sequence of independent identically distributed unit normal variates.

Approximate values for the parameters are:

PYMU=0.05, PYA = 0.6, PYSD = 0.075

The model for R(t) is

$$\ln R(t) = \ln R(t-1) + RW.RM(t) + RX.\ln Q(t) + RMU + RSD.RZ(t)$$

where RZ(t) is a sequence of independent identically distributed unit normal variates and

$$RM(t) = (1-RD).\,RM(t-1) + RD.\ln Q(t)$$

Approximate values for the parameters are:

RW = 1.0, RD = 0.1, RX = 0.0, RMU = -0.1, RSD = 0.05

The corresponding property price index RP(t) is given by

$$RP(t) = \frac{R(t)}{PY(t)}$$

The basic structure of this model is similar to the other stochastic investment models proposed by Wilkie[45].

APPENDIX 4 List of parameters which user may specify in management model

Variable parameters

Up to 5 values may be specified for each. The number of values being specified has to be stated by the user, with an indication of which is to be the standard value.

Variable parameter	Parameter names	Description
1	FACTM1/4 FACTC1/4	Factors to be used by the market and by the company for weighting the past four years' RPI increases to get the projected RPI increase in current year.
2	AMARGM/C	Initial asset margins for the market and for the company as fractions of written premiums in the year before the date of investigation.
3	DIVRTM/C DIVGRM/C	Initial dividend rates for the market and for the company as fractions of written premium in the year before the date of investigation, and subsequent real growth rates for actual dividend payouts.
4		Not currently used.
5		Not currently used.
6	GEARO/U	Gearing factors for company when premiums are set above or below the market level, to determine elasticity of demand.
7	RECVMP/L RECVCP/L	Recovery rate to be assumed in respect of past losses ("bonus" rate in respect of past profits) for the market and company when setting premiums.

Scalar parameters (miscellaneous)

Parameter number	
BKPFMH BKPFCH	Past profit levels for market and for company for carrying back losses for tax purposes (for up to five past years).
BKCGMH BKCGCH	Past capital gains for market and for company for carrying back losses for tax purposes (for up to five past years).
NYRBK	Number of years for which losses may be carried back.
PSTINF	Past rate of inflation.
BUSM/C	Asset margin level required for market and for company for statutory solvency purposes.
TXINCM/C	Tax rates on profits for market and for company.
TXCGM/C	Tax rates on capital gains for market and for company.
DISCRM/C	Rate of discount (real rate relative to price inflation) for provisions for outstanding claims.
NFY	Number of further years for which premiums are to be written.
NOLIAB	Number of liability types.
NSIM	Number of simulations required.
IFSTDO	Whether variable parameters are to vary or take only the standard values.
IFWILK	Whether asset values and returns are to vary stochastically.
IFSTOC	Whether claim ratios and claim outgo are to vary stochastically.
IFANAL	Whether analysis of results is required.
IFTEST	Whether test output is required.
LENCYC	Length of business cycle to be assumed (up to 20 years).
CRSIN	Shape of business cycle (number of years to correspond to LENCYC), starting at zero.
Various	Parameters for generating random numbers.

Various	Parameters for Wilkie models.
COVA	Covariance between random numbers for equity and property models.
LGIL1/3	Length of the three different types of gilt-edged security to be used.

Scalar parameters (asset arrays)

ASFRMK ASFRCO	Proportions of assets backing each of the six liability types and the asset margin which are to be held in each of the seven asset types.
CSTM/C	Original cost of assets of each of the seven types held by the market and the company, as fraction of the market value at date of investigation.

Scalar parameters (liability arrays)

Each of the following to be specified for each liability type (up to 6)

RUNOFF	Settlement patterns (for up to 20 years).
VOLMK	Market premium volume (numbers of policies).
PRMMK	Average level of premium in market.
GRTEB1/2	Growth rates before date of investigation.
GRTEA1/2	Growth rates after date of investigation.
IGRYRB	Years prior to date of investigation for which growth rate GRTEB1 holds (GRTEB2 assumed for earlier years).
IGRYRA	Years after date of investigation for which growth rate GRTEA1 holds (GRTEA2 assumed for later years).
PROFM/C	Profit margins in premium rates for market and for company as fractions of gross premiums.
COFRAC	Proportion of market held by the company.
CRAMP	Amplitude of business cycle for claim ratios for both market and company.
ICRCYM/C	Phase of business cycle for claim ratios for market and company (number of years into cycle specified by LENCYC and CRSIN).
CRSDM/C	Standard deviation of claim ratio for market and for company.
EXFIXM/C	Fixed expenses (i.e. not dependent on volume or premium rates) for market and company.

EXVARM/C	Variable expenses (i.e. directly proportionate to gross premiums) for market and company.
UPRPCM/C	Percentage of gross written premium to be held as unearned premium reserve by market and by company.
AGPERM/C	Percentage of gross written premiums during year which are still held by agents at end of year - for market and for company.
OUTSDA/B	Variable factors "a" and "b" in formula aX + b/X for variability in claim outgo.

APPENDIX 5 Members of Solvency Working Party

The members of the GISG Solvency Working Party as at August 1988 were as follows:

C D Daykin (Chairman)
R B Akhurst
G D Bernstein
S M Coutts
E R F Devitt
G B Hey
D I W Reynolds
P D Smith