CMI Technical Note: Comparison of approaches for calculating initial exposure

Introduction

Working Paper 34 describes the methodology and assumptions used for CMI Self-Administered Pension Schemes (SAPS) mortality experience analyses. Whilst it gives an overview of the changes that have recently been made, the paper provides little detail on the reason for the most significant change, namely the move from an initial to a central exposure calculation. This document is intended as a supplementary note and contains a more thorough explanation for the switch.

Rationale for the change from initial to central exposed to risk

Initial exposed to risk was used for all SAPS experience analyses from the start of the investigation in 2003. It was also used as the start-point for the draft graduations, contained in CMI Working Paper 32, for which it was necessary to estimate central exposure from the calculated values of initial exposure.

For the final graduations, central exposure was calculated directly. This direct calculation is clearly a more accurate approach than estimating it from initial exposure. However in its discussions the CMI SAPS Mortality Committee considered more closely the method by which initial exposure had been calculated and felt there were technical issues implicit in its calculation that warranted this document.

In particular, the Committee considered two approaches to calculating initial exposed to risk, both of which have featured in actuarial literature. The difference between the approaches centres on the date at which the exposure for deaths should cease.

Limitations were noted with both approaches and the Committee concluded that neither approach was ideal for the SAPS mortality investigation. These limitations – at high ages for one approach and in dealing with large numbers of new entrants at specific ages in the other – could be avoided if central exposed to risk was calculated directly.

Hence, central exposure was calculated directly for the final graduations (see Working Paper 35) and is intended to be used for all SAPS experience analyses in the future. The Committee will give consideration to when this change will be made and what analysis will be made available to demonstrate its impact.

Overview of approaches

The investigation period for each scheme in the SAPS Mortality data is likely to be a three-year period and span three or four calendar years, depending on the start date of the investigation period. The initial exposed to risk is calculated for each calendar year of the investigation period. The results for each calendar year are then added together to give the results for the overall investigation period. The reason for analysing by calendar year instead of scheme year is to enable analyses of the aggregate experience of schemes that have different scheme years.

The calculation of initial exposed to risk involves determining the first and last days of exposure for each member within each calendar year. The first day of exposure is straightforward to determine and is defined as the latest of:

1. 1st January of the calendar year;
2. The investigation start date for scheme in the calendar year (see paragraphs 3.4 and 3.6 of Working Paper 34 for the historic and amended treatment of investigation start dates); and
3. Date of pension commencement in the calendar year.

The last day of exposure is easy to determine for members who do not exit and those that exit for a reason other than death. For these cases, the last day of exposure is defined as the earlier of:

1. 31st December of the calendar year;
2. The investigation end date for scheme in the calendar year (assuming the valuation date has been provided); and
3. Date of exit (other than due to death) during the calendar year.

It is less clear what approach should be used when a member exits due to death. The initial exposed to risk is calculated for individual ages. For initial exposed to risk where the whole life year can be observed for each individual, it is generally accepted that members that die contribute a full year of exposed to risk for that particular age. However, when calculating exposed to risk for individual calendar years (i.e. where the observation periods may be censored on both sides), the debate arises about when the exposure should be cut-off. The two approaches that were considered are:

Approach 1
For all members that exit due to death, the last day of exposure is the day before the next birthday even if this falls in the next calendar year or after the end of the investigation period.

This approach is based on The Analysis of Mortality and Other Actuarial Statistics by Benjamin and Haycocks (1972 edition).

Approach 2
For members that exit due to death before their birthday in the calendar year, the last day of exposure is the earlier of:

1. Day before birthday in the calendar year; and
2. Investigation end date for scheme in the calendar year

For members that exit due to death on or after their birthday in the calendar year, the last day of exposure is the earlier of:

1. Investigation end date for scheme in the calendar year; and
2. 31st December of the calendar year.

This approach is based on Actuarial Statistics Volume II Construction of Mortality and Other Tables by Anderson and Dow (1948 edition) and was used for all SAPS analyses prior to the re-work of the graduations in 2008. Calculating central exposed to risk does not raise the same issues as initial exposed to risk regarding the last day of exposure for individuals that exit due to death.

Comparison of the two approaches
A comparison of the two approaches is probably best approached using an example.

Example
Consider a number of lives, say 1000. Assuming that births are uniformly spread throughout the calendar year, on average they will reach age x halfway through the year. To simplify the example, we assume that all the lives reach age x exactly halfway through a particular calendar year. During this calendar year these lives will only be observed at age x for the first half of their life year aged x and will be exposed for the remainder of the calendar year, 500 life years in total. The question arises over the treatment of the exposure for those lives that die during the first half of their life year, suppose 50 lives die. The alternatives are:

1. Contribute exposure for a full year, continuing into the next calendar year? For the lives that reach age x halfway through the calendar year the total exposure would be 500 + 25, which is consistent with Approach 1.

2. Contribute exposure for only half a year, to the end of the period during which they are observed? For the lives that reach age x halfway through the calendar year the total exposure would be 500 years, which is consistent with Approach 2. These lives cannot be considered in isolation. The lives who reached age x during the previous calendar year should also be considered. For simplicity, assume that those who reached age x in the previous year did so halfway through that year. These lives would be aged x+0.5 at the start of the calendar year in question.

Information is missing for both groups of lives:

- For the lives who reached age x in the previous calendar year information is missing for the lives that died in the first half of their life year due to censoring that results from starting observations at the start of the calendar year.
- For lives who reached age x during the calendar year information is missing for the lives that survive to age x+0.5 during the second half of the life year, due to censoring that results from ceasing observations at the end of the calendar year.

Approach 1 is appropriate if the experience is stable between cohorts and the cohorts are of similar size. If this is the case then:

- It is assumed that there are equal numbers of lives reaching age x in the previous calendar year (i.e. aged x+0.5 at the start of the calendar year) and the lives reaching age x during the calendar year.
- It is assumed that there is no difference in the mortality experience between lives reaching age x in the previous calendar year and the lives reaching age x during the calendar year.
- The deaths experience for the lives turning age x during the calendar year can be assumed to approximate the missing death information for the lives that reached age x in the previous calendar year. This is achieved by extending the exposure for the lives that reached age x during the calendar year and die before the end of the calendar year to cover a full year, i.e. extending the exposure for these lives beyond the end of the calendar year.

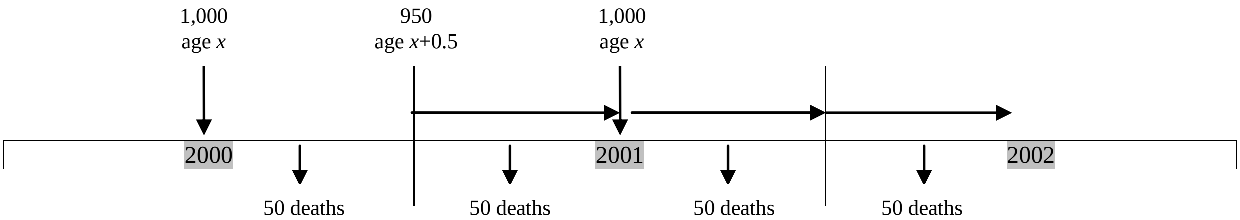
If we assume that 1,000 lives reach age x in the previous calendar year and that the number of deaths from this cohort is 50 during the calendar year in question. If we also assume that the actual value of q_x is 0.1 this implies that there would have been 50 deaths in the previous calendar year and 50 deaths in the next calendar year for the cohorts reaching age x in the previous calendar year and during the calendar year in question, respectively.

Using Approach 1 the value of q_x will be calculated as:

$$\frac{\text{Total deaths aged } x \text{ last birthday}}{\text{Exposure aged } x \text{ last birthday}}$$

Total deaths aged x last birthday = 50 + 50 = 100
Exposure aged x last birthday = 950*0.5 + 1,000 * 0.5 + 50 * 0.5 = 1,000
q_x = 100/1,000 = 0.100

The following diagram illustrates this:

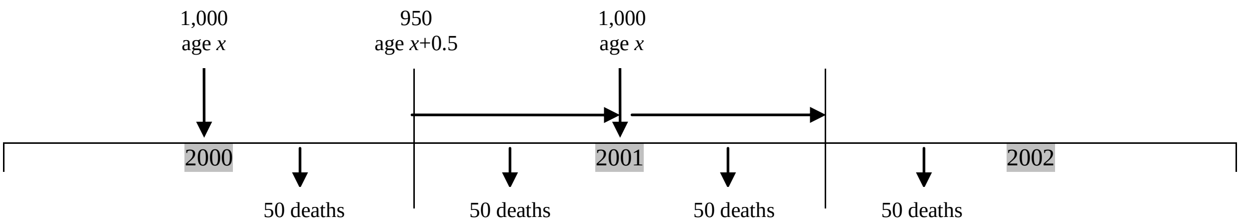


If these conditions hold then Approach 2, which does not attribute a full year of exposure for all deaths, may overstate mortality rates at older ages. This results from higher mortality rates at older ages, whereas the impact at younger ages is not so material due to lower mortality rates.

Using the same example described above, the value of q_x using Approach 2 will be calculated as:

Total deaths aged x last birthday = 50 + 50 = 100
Exposure aged x last birthday = 950*0.5 + 1,000 * 0.5 = 975
q_x = 100/975 = 0.103

The following diagram illustrates this:

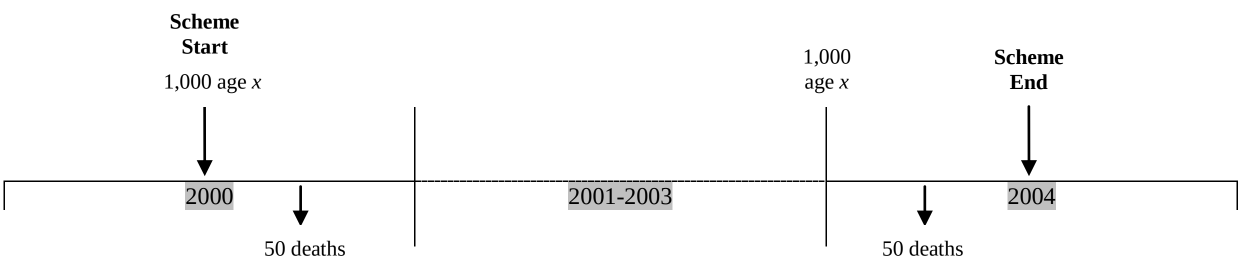


However, Approach 1 is no longer valid if any of the following are true:

1. The two cohorts are not stable in terms of the number of members in each cohort and the number of deaths experienced by each cohort. This can affect all ages and is particularly likely to affect schemes where the age mix is not stable. It is also particularly relevant for data submitted to the SAPS Mortality investigation where the numbers of entrants at normal pension age can vary from year to year.

2. Where the scheme year is not consistent with the calendar year (which is the case for many schemes) then Approach 1 is not valid for the first (and last) calendar year.

   The following diagram illustrates an example that can be used to compare the calculation of q_x for the first and last calendar year (2000 and 2004, respectively, in the diagram below) based on Approach 1 and Approach 2. Calculations of q_x follow the diagram.



   Approach 1
   The figures used for calendar year 2000 and calendar year 2004 are the same:

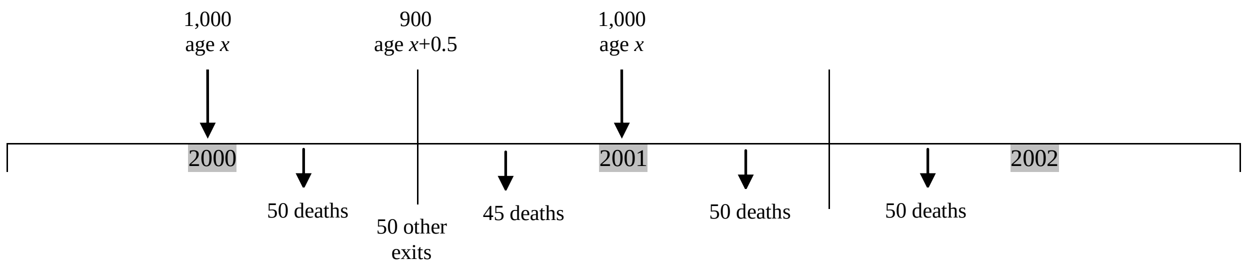
   Total deaths aged x last birthday = 50
   Exposure aged x last birthday = 1,000*0.5 + 50*0.5 = 525
   q_x = 50/525 = 0.095

   Approach 2
   The figures used for calendar year 2000 and calendar year 2004 are the same:

   Total deaths aged x last birthday = 50
   Exposure aged x last birthday = 1,000*0.5 = 500
   q_x = 50/500 = 0.1

3. There are exits for a reason other than death.

   The following diagram illustrates an example that can be used to compare the calculation of q_x where there are exits other than death based on Approach 1 and Approach 2. Calculations of q_x follow the diagram.



   Approach 1
   For calendar year 2001:

   Total deaths aged *x* last birthday = 95
   Exposure aged *x* last birthday = 900*0.5 + 1,000*0.5 + 50*0.5 = 975
   q_x = 95/975 = 0.097

   Approach 2
   For calendar year 2001:

   Total deaths aged *x* last birthday = 95
   Exposure aged *x* last birthday = 900*0.5 + 1,000*0.5 = 950
   q_x = 95/950 = 0.1

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