Institute
and Faculty
of Actuaries

A stochastic Bornhuetter-Ferguson model

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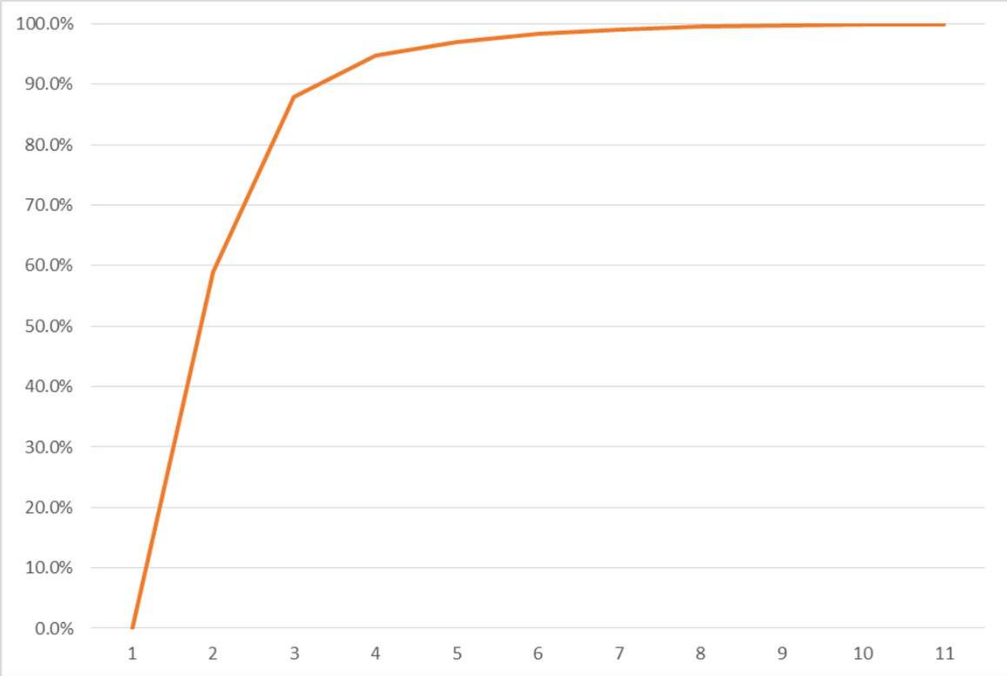
Outline of talk

- The Bornhuetter-Ferguson method
- Introduce numerical example
- Review of ODP model
- Alai, Merz, and Wuethrich's stochastic Bornhuetter-Ferguson (AMW-BF model)
- AMW-BF model applied to numerical example
- Review and critical discussion of AMW-BF model
- Bootstrapping the AMW-BF model
- Actuary-in-the-box and the AMW-BF model

The Bornhuetter-Ferguson method (1/3)



Cumulative Development	Prior Ultimate	Reserve
100.0%	11,653,101	-
99.9%	11,367,306	16,124
99.8%	10,962,965	26,998
99.6%	10,616,762	37,575
99.1%	11,044,881	95,434
98.4%	11,480,700	178,024
97.0%	11,413,572	341,305
94.8%	11,126,527	574,089
88.0%	10,986,548	1,318,646
59.0%	11,618,437	4,768,384

Reserve = (1 – cumulative development %) x prior ultimate

The Bornhuetter-Ferguson method (2/3)

- In practice the cumulative development pattern applied is usually that implied by the basic chain ladder
- In this case the ultimate claims is a weighted average of the basic chain ladder and the prior estimate of the ultimate claims

$$U_{bf} = (1 - cd) \times U_{prior} + cd \times U_{bcl}$$

Reserve: $(1 - cd) \times U_{prior}$

Paid to date: $cd \times U_{bcl}$

- Note: this practice is assuming a different model for the claims development that we have already observed and the future claims development

The Bornhuetter-Ferguson method (3/3)

- C_{ij} – cumulative claims for origin period i, and development period j
- y_j – incremental development proportion for development period j
- U_i – the prior estimate of the ultimate claims for origin period I

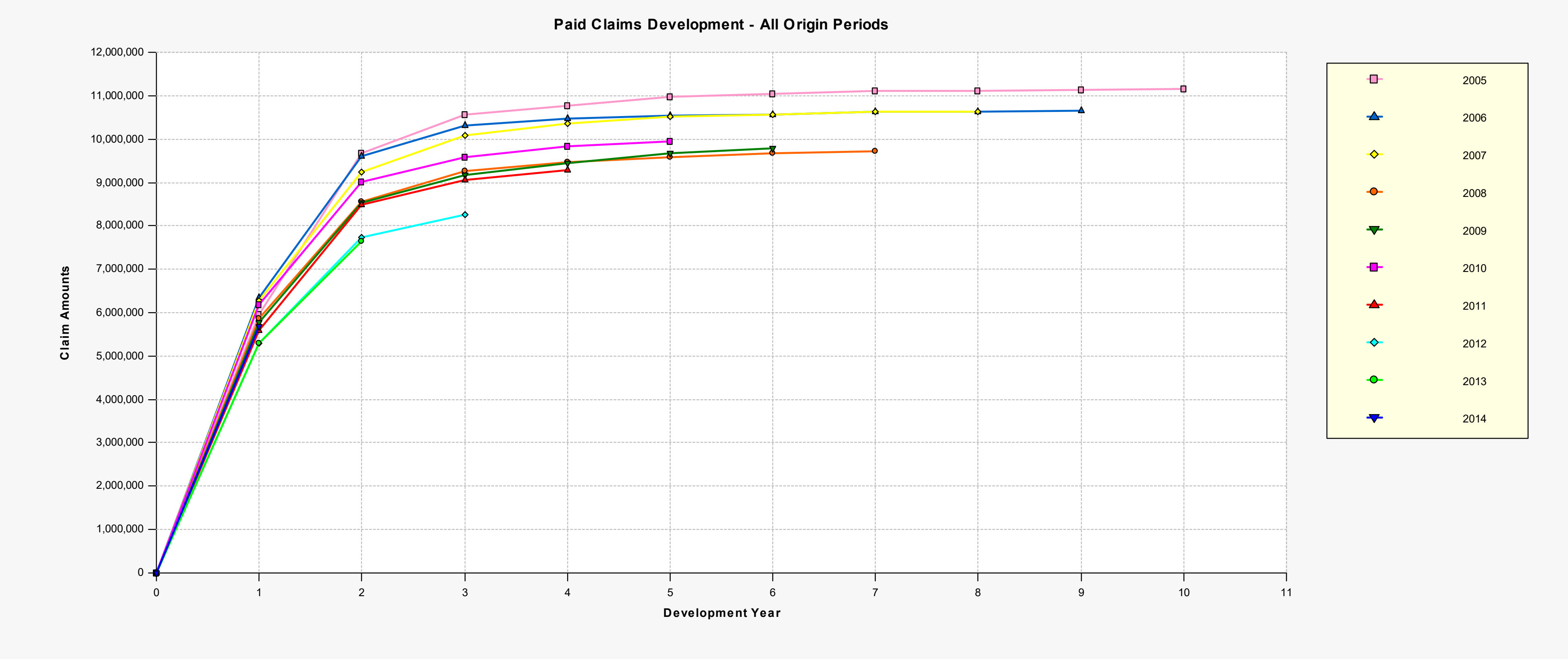
- Then:

$$E[\, C_{ij+1} \mid C_{i1}, \ldots, C_{ij} \,] = C_{ij} + y_j U_i$$

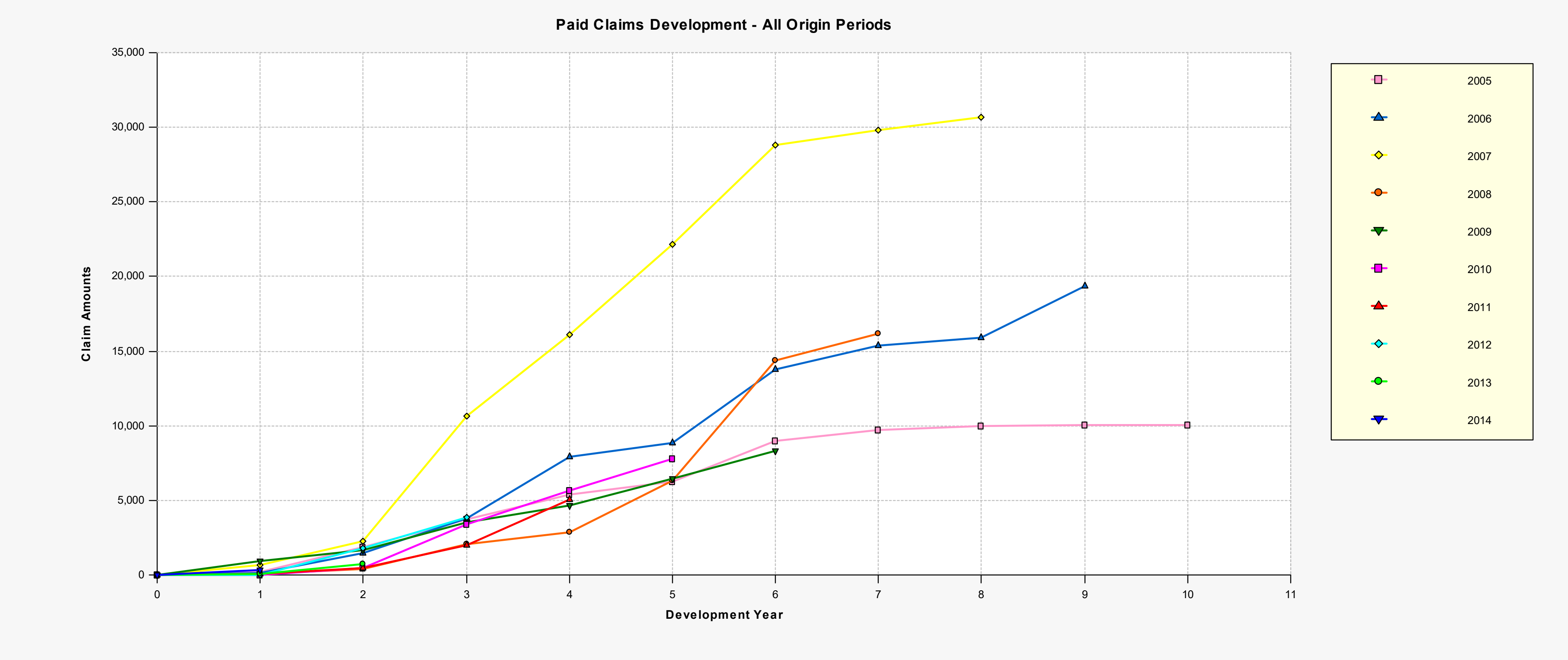
A stochastic Bornhuetter-Ferguson model

- What would a stochastic BF model have to look like?
- Consistent with BF method
  - same reserves
  - applies the same proportions to the prior estimate of the ultimate claims – in practice this means the chain ladder proportions
  - a consistent extension of a stochastic model of the chain ladder
  - allows for uncertainty in the prior estimate of the ultimate claims
- Is such a model even possible?
- If it is not possible should we continue using the Bornhuetter-Ferguson method?

Numerical example A – graph of claims development



Numerical example B – graph of claims development



Example A – ODP bootstrap results

Accident Year	Latest	Reserve	Prediction Error	**Prediction Error%**
2005	11,148	-	-	**0%**
2006	10,648	15	22	**146%**
2007	10,636	26	28	**106%**
2008	9,724	35	30	**87%**
2009	9,787	85	43	**50%**
2010	9,936	156	56	**36%**
2011	9,282	288	75	**26%**
2012	8,256	448	92	**21%**
2013	7,649	1,043	144	**14%**
2014	5,676	3,949	337	**9%**
Total	92,741	6,045	438	**7%**

Example B – ODP bootstrap results

Accident Year	Latest	Reserve	Prediction Error	**Prediction Error%**
2005	10,068	-	-	**0%**
2006	19,333	69	478	**696%**
2007	30,700	4,359	2,690	**62%**
2008	16,201	2,873	1,864	**65%**
2009	8,325	2,249	1,520	**68%**
2010	7,813	7,011	3,214	**46%**
2011	5,062	8,028	3,842	**48%**
2012	3,881	12,643	6,354	**50%**
2013	745	9,831	10,055	**102%**
2014	306	28,566	1,247,687	**4368%**
Total	102,434	75,628	1,247,967	**1650%**

Overview of the Poisson model

	1	2	3	4	5	Ultimate Claims
2011	45	34	7	7	4	x_{2011}
2012	50	25	16	5		x_{2012}
2013	55	49	11			x_{2013}
2014	68	43				x_{2014}
2015	74					x_{2015}
Incremental Development	y_1	y_2	y_3	y_4	y_5	

Incremental claims are independent and Poisson distributed

With mean = $x_i.y_j$

Fitting using maximum likelihood gives exactly the same reserves as the basic chain ladder

Overview of the ODP model

- Model of incremental claims amounts, P_{ij}
- $E[\ P_{ij}\ ] = x_i y_j$
- $(\ y_1, \ldots, y_n\ )$ = incremental development pattern
  x_i = ultimate claim amount for origin period
- Scale parameter allows over-dispersion:
  $Var(\ P_{ij}\ ) = \varphi_j\ E[\ P_{ij}\ ]$
- GLM
  - fit using quasi-likelihood
  - P_{ij} has over-dispersed Poisson distribution
- Expected incremental values must all be positive

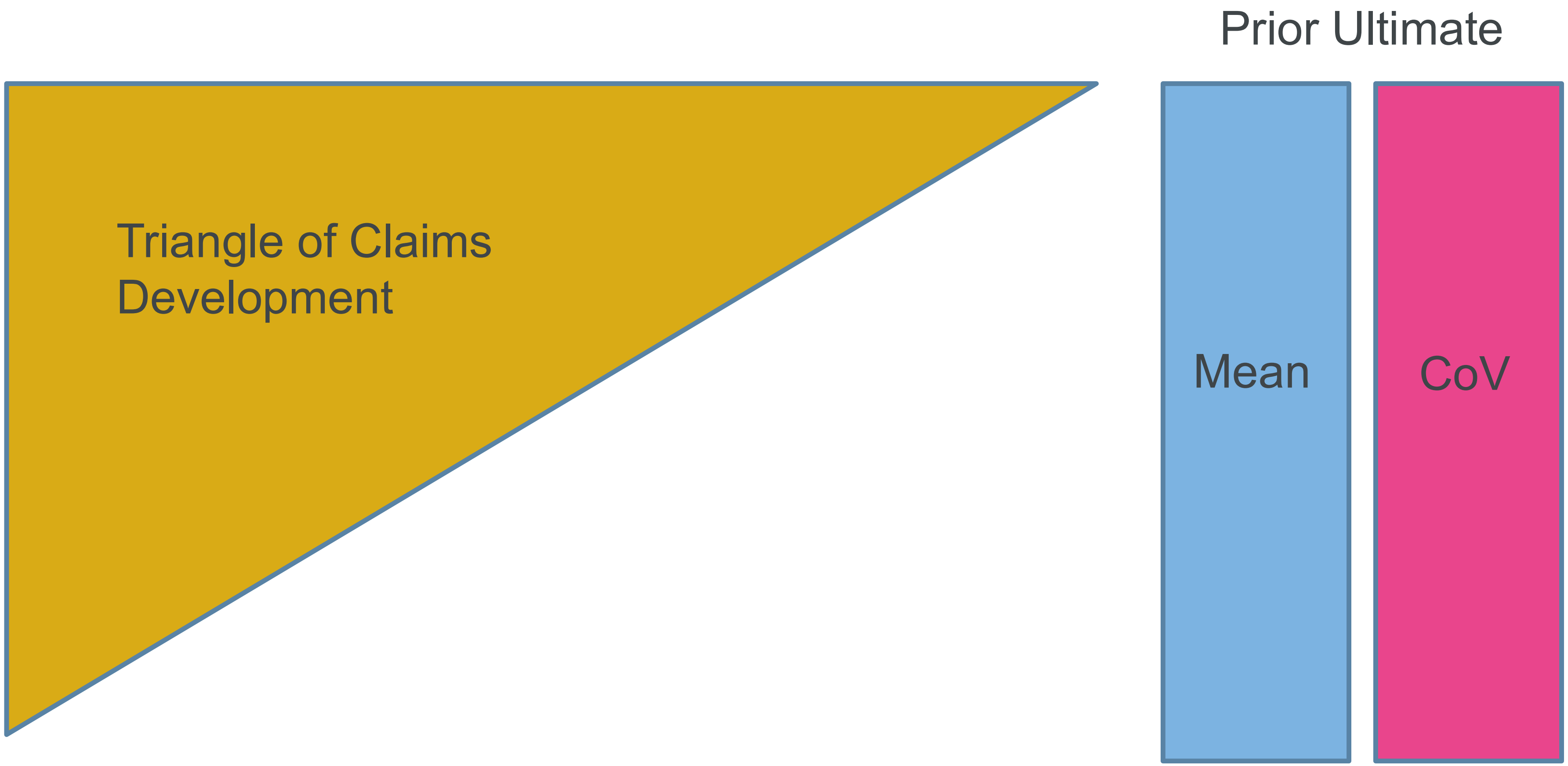
Alai, Merz, and Wuethrich's justification for using ODP

- Cumulative claims
  $C_{ij} = P_{i1} + \ldots + P_{ij}$
- ODP:
  $E[\, P_{ij} \,] = x_i y_j$
- So
  $E[\, C_{ij+1} \mid C_{i1}\, , \ldots, C_{ij} \,] = C_{ij} + E[\, P_{ij} \,] = C_{ij} + x_i y_j$
- Recall that x_i = the estimate of the ultimate claims in the ODP
- This equation has the same form as the equation derived earlier from the BF method assumptions:
  $E[\, C_{ij+1} \mid C_{i1}, \ldots, C_{ij} \,] = C_{ij} + y_j U_i$
  where U_i is the prior estimate of the ultimate claims
- AWM use this to justify using the ODP as a basis for a stochastic BF

AMW-BF model assumptions

- ODP assumptions:
  - There are parameters x_i, and y_j such that $E[\ P_{ij}\ ] = x_i\ y_j$
  - The incremental amounts P_{ij} are independent
  - $Var(\ P_{ij}\ ) = \varphi\ E[\ P_{ij}\ ]$
- The prior estimates v_i of the ultimate claims are independent random variables that are unbiased a priori estimators of the expected value of the ultimate claims C_{in}
- P_{ij} and v_i are all independent

Data inputs for the AMW-BF



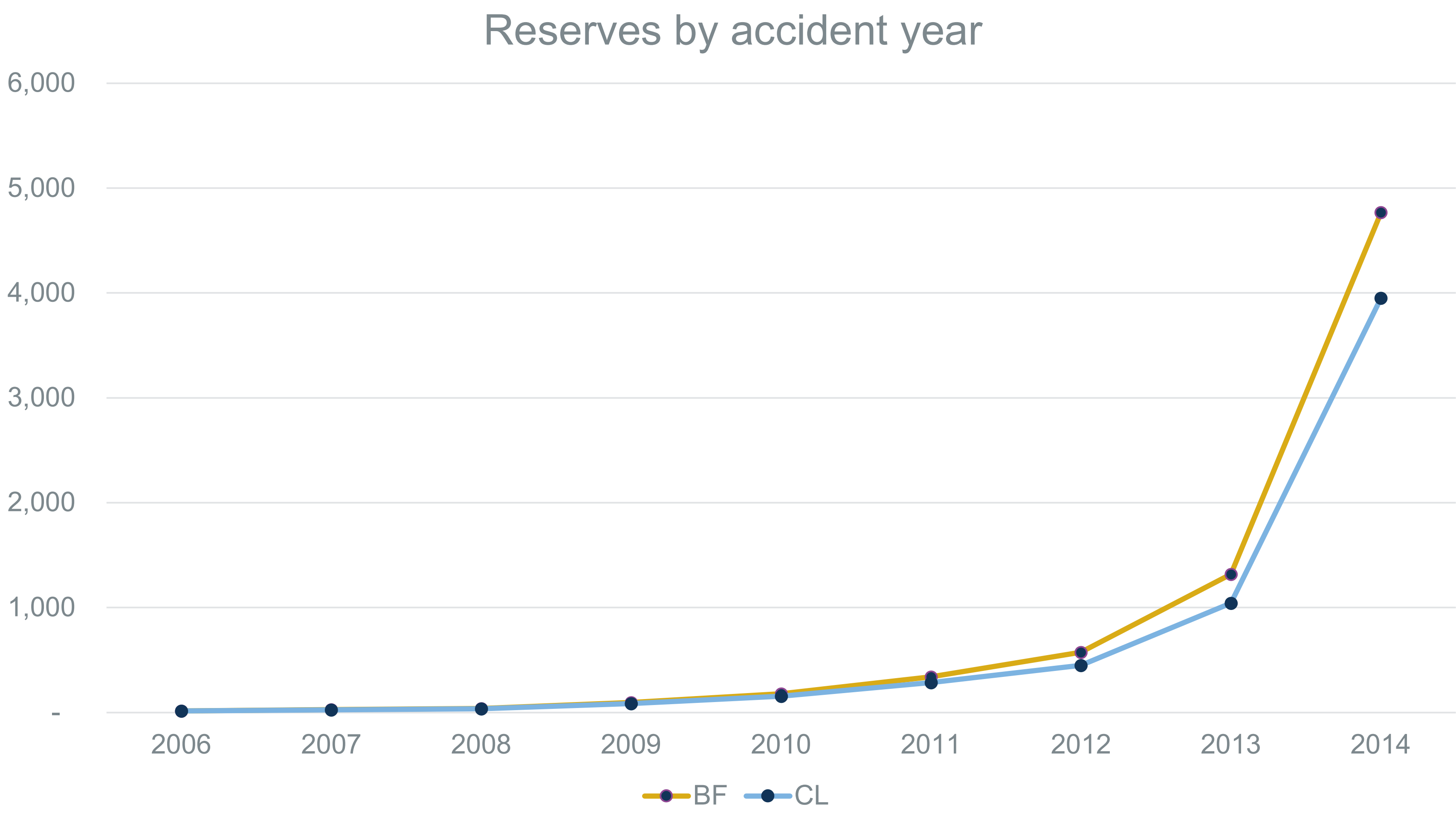
Different sources of error in AMW-BF

Process Error

Parameter Error

Prior Ultimate Error

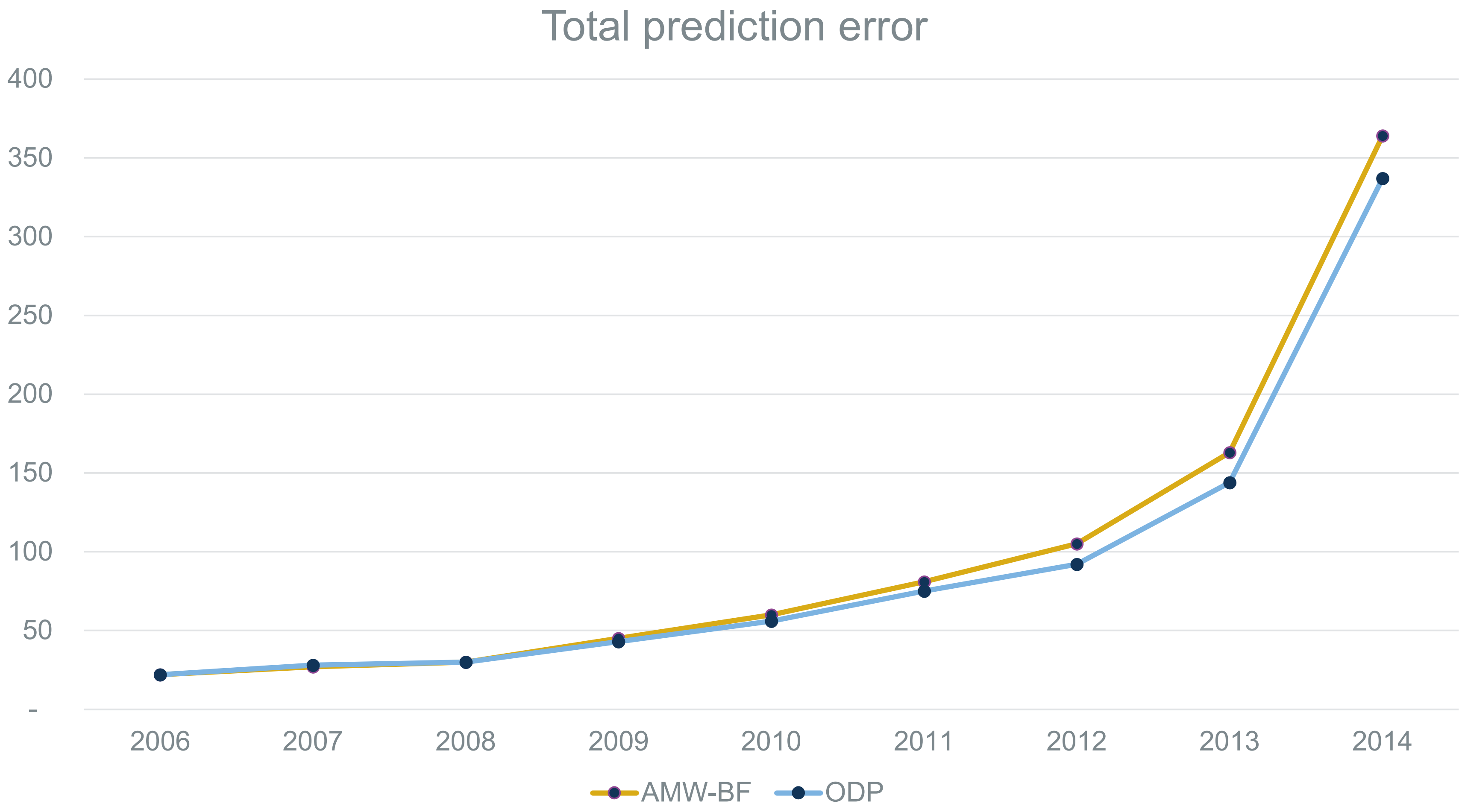
Example A – BF and CL reserves



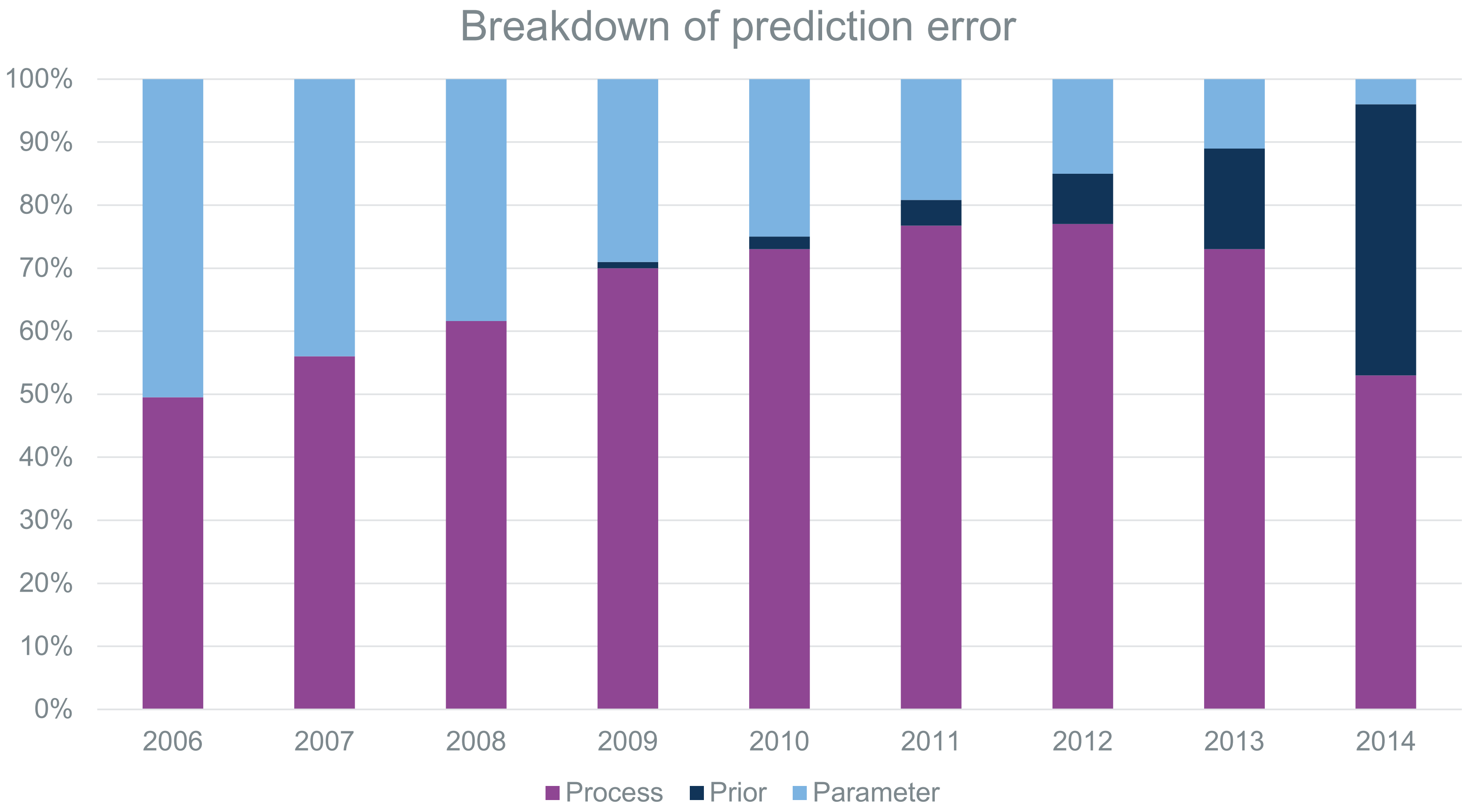
Example A – BF and CL reserves

Accident Year	Latest	Exepected Development	Prior Ultimate Mean	Chain Ladder Reserve	BF Reserve
2005	11,148	0.0%	11,653	-	-
2006	10,648	0.1%	11,367	15	16
2007	10,636	0.2%	10,963	26	27
2008	9,724	0.4%	10,617	35	38
2009	9,787	0.9%	11,045	85	95
2010	9,936	1.6%	11,481	156	178
2011	9,282	3.0%	11,414	286	341
2012	8,256	5.2%	11,127	449	574
2013	7,649	12.0%	10,987	1,043	1,319
2014	5,676	41.0%	11,618	3,952	4,768
Total	92,741	n/a	112,271	6,048	7,357

Example A – BF and CL prediction error results



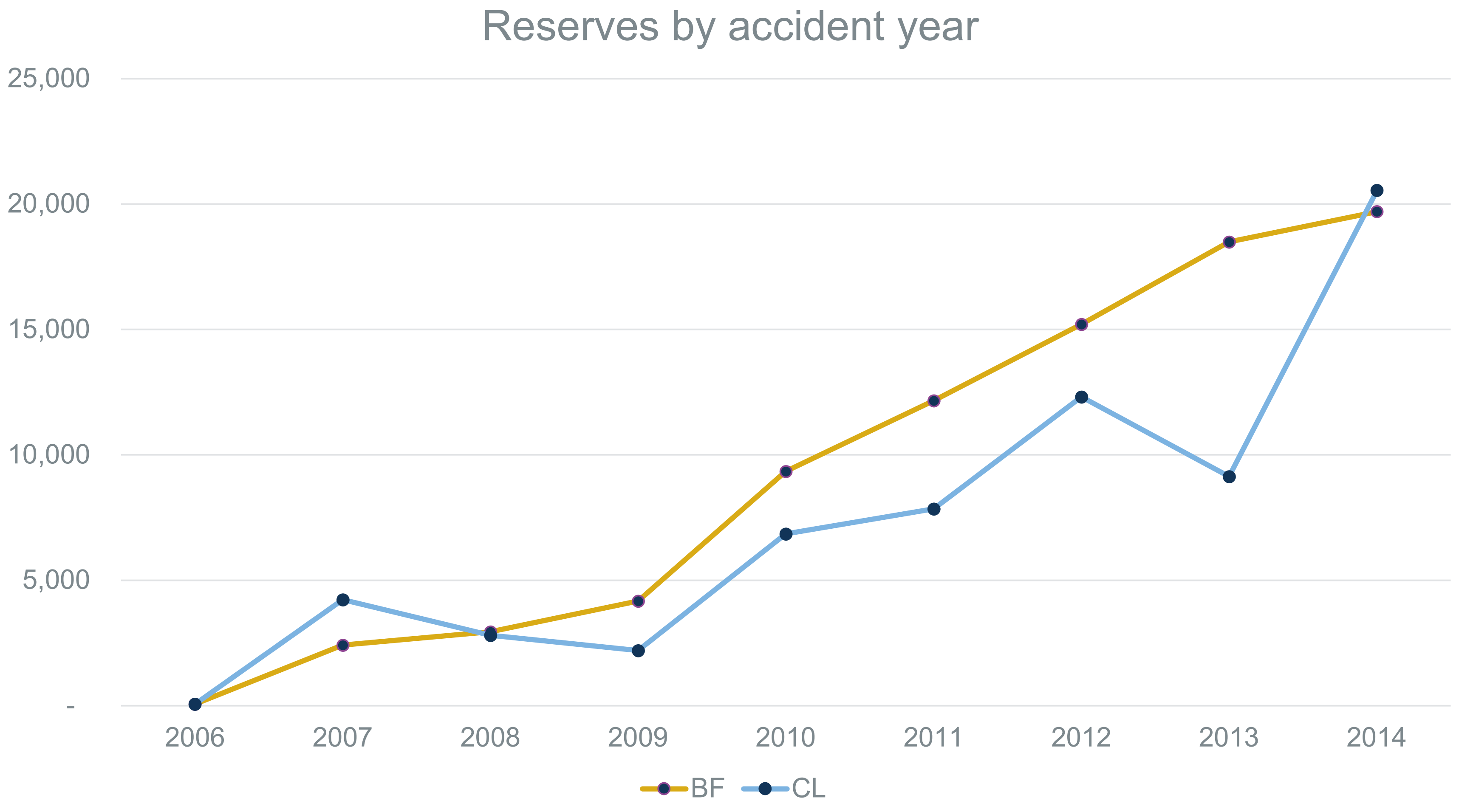
Example A – BF prediction error results breakdown



Example A – BF and CL prediction error results

Accident Year	Prior Ultimate CoV	Process Std Dev	Prior Estimate Std Dev	Estimation Error	Total Prediction Error	Total Prediction Error %	CL Prediction Error	CL Prediction Error%
2005	5%	-	-	-	-	0%	-	0%
2006	5%	15	1	16	22	136%	22	146%
2007	5%	20	1	18	27	99%	28	106%
2008	5%	24	2	19	30	80%	30	87%
2009	5%	37	5	24	45	47%	43	50%
2010	5%	51	9	30	60	34%	56	36%
2011	5%	71	17	36	81	24%	75	26%
2012	5%	92	29	41	105	18%	92	21%
2013	5%	139	66	53	163	12%	144	14%
2014	5%	265	238	76	364	8%	337	9%
Total	n/a	329	250	228	472	6%	438	7%

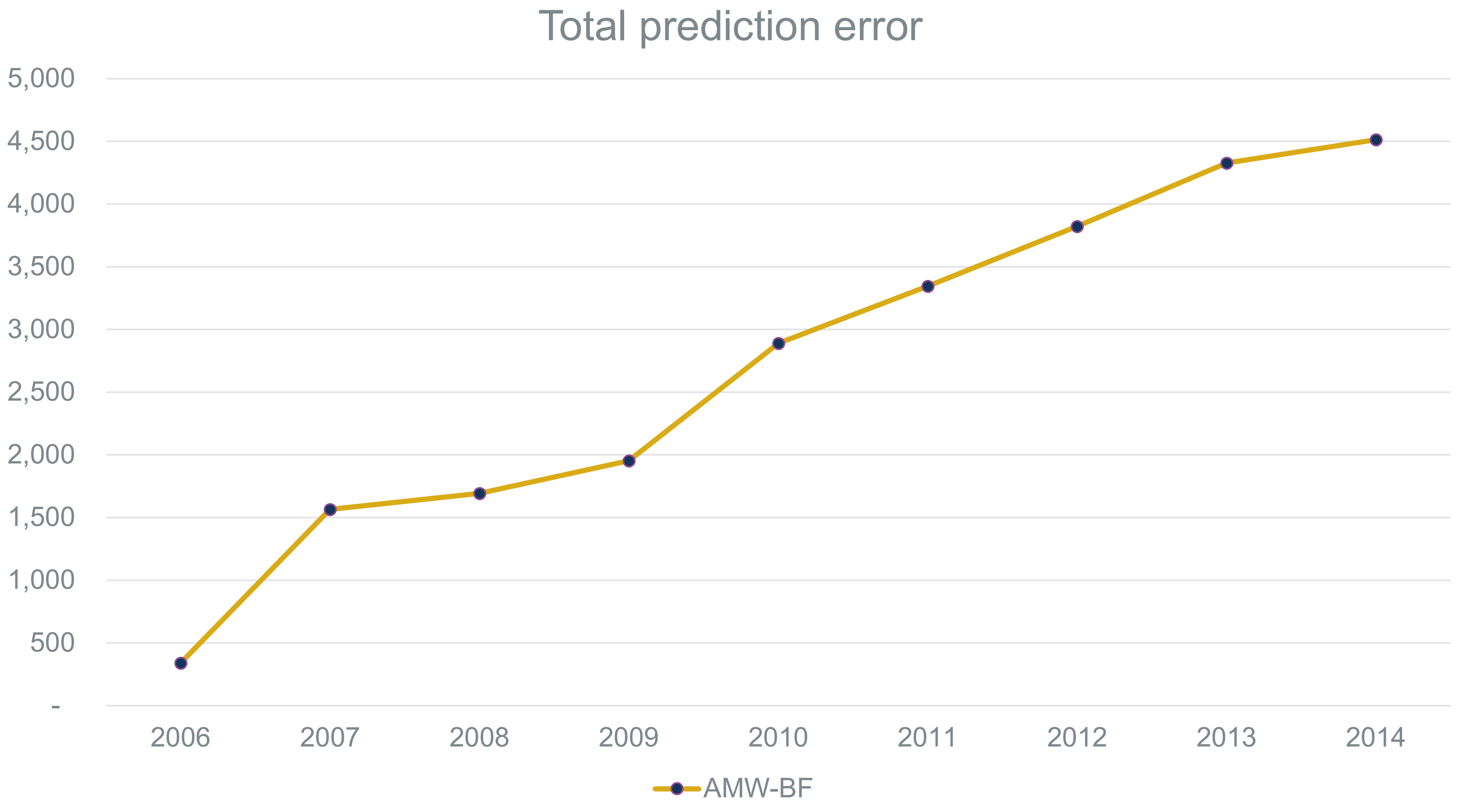
Example B – BF and CL reserves



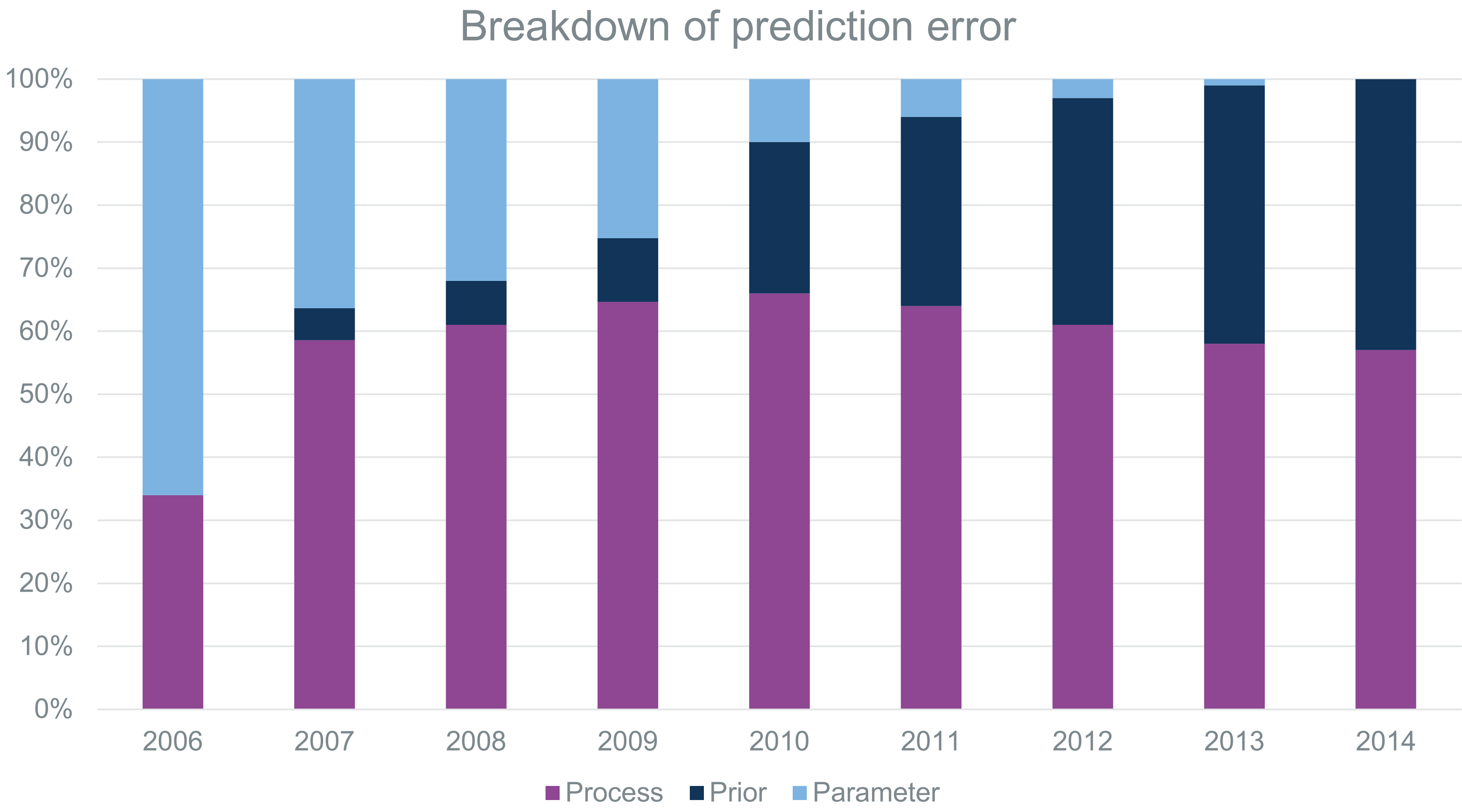
Example B – BF and CL reserves

Accident Year	Latest	Expected Future Development	Prior Ultimate Mean	Chain Ladder Reserve	BF Reserve
2005	10,068	0.0%	20,000	-	-
2006	19,333	0.3%	20,000	64	66
2007	30,700	12.1%	20,000	4,221	2,418
2008	16,201	14.8%	20,000	2,804	2,951
2009	8,325	20.9%	20,000	2,195	4,173
2010	7,813	46.7%	20,000	6,854	9,346
2011	5,062	60.8%	20,000	7,849	12,159
2012	3,881	76.0%	20,000	12,313	15,207
2013	745	92.5%	20,000	9,137	18,492
2014	306	98.5%	20,000	20,551	19,707
Total	102,434	n/a	200,000	65,986	84,517

Example B – BF prediction error results



Example B – BF prediction error results breakdown



Example B – BF and CL prediction error results

Accident Year	Prior Ultimate CoV	Process Std Dev	Prior Estimate Std Dev	Estimation Error	Total Prediction Error	Total Prediction Error %	CL Prediction Error	CL Prediction Error%
2005	15%	-	-	-	-	0%	-	0%
2006	15%	197	10	277	340	518%	478	696%
2007	15%	1,194	363	943	1,564	65%	2,690	62%
2008	15%	1,319	443	965	1,693	57%	1,864	65%
2009	15%	1,569	626	981	1,953	47%	1,520	68%
2010	15%	2,348	1,402	935	2,890	31%	3,214	46%
2011	15%	2,678	1,824	836	3,346	28%	3,842	48%
2012	15%	2,995	2,281	665	3,823	25%	6,354	50%
2013	15%	3,302	2,774	364	4,328	23%	10,055	102%
2014	15%	3,409	2,956	155	4,515	23%	1,247,687	4368%
Total	n/a	7,060	5,258	4,788	10,021	12%	1,247,967	1650%

Review of the AMW-BF model (1/2)

- Stochastic Bornhuetter-Ferguson model
- It is an attempt to create a model consistent with how the Bornhuetter-Ferguson is used in practice
  - Gets the same reserves
  - Applies the chain ladder development pattern
- Based on ODP chain ladder model
- Considers error in the prior estimate of the ultimate claims
- Three sources of error:
  - Process error
  - Parameter error
  - Prior estimate error
- Analytic formulae for the prediction error

Review of the AMW-BF model (2/2)

- Strengths
  - Stochastic model of BF as used in practice
  - Takes account of uncertainty in prior estimate of ultimate
  - Based on well understood model for the chain ladder
  - Analytic formulae for prediction error
- Criticisms
  - Model is ODP with BF assumptions ‘bolted-on’
  - Based on ODP, but BF only applied in practice when chain ladder isn’t a good fit
  - Past and future two different models
  - Independence of prior estimates

General bootstrap process for reserving

1. Define and fit a statistical model to the observed data
2. Calculate the residuals and generate the pseudo data
3. Re-fit the statistical model to the pseudo data
4. Obtain the forecast, including the process error

Bootstrapping AMW-BF model

1. Fit the chain ladder to the triangle of claims
2. Calculate residuals and generate pseudo data:
    1. Calculate residuals and generate pseudo data in exactly the same way as for the ODP
    2. Also generate pseudo prior estimates of the ultimate claims. These are independent of the pseudo triangle data
3. Re-fit the chain ladder to the pseudo data and derive the incremental development proportions
4. Project the future incremental claims as in the ODP model except that the mean incremental claims values are those given by the pseudo prior estimates of the ultimate claims and the pseudo incremental development proportions

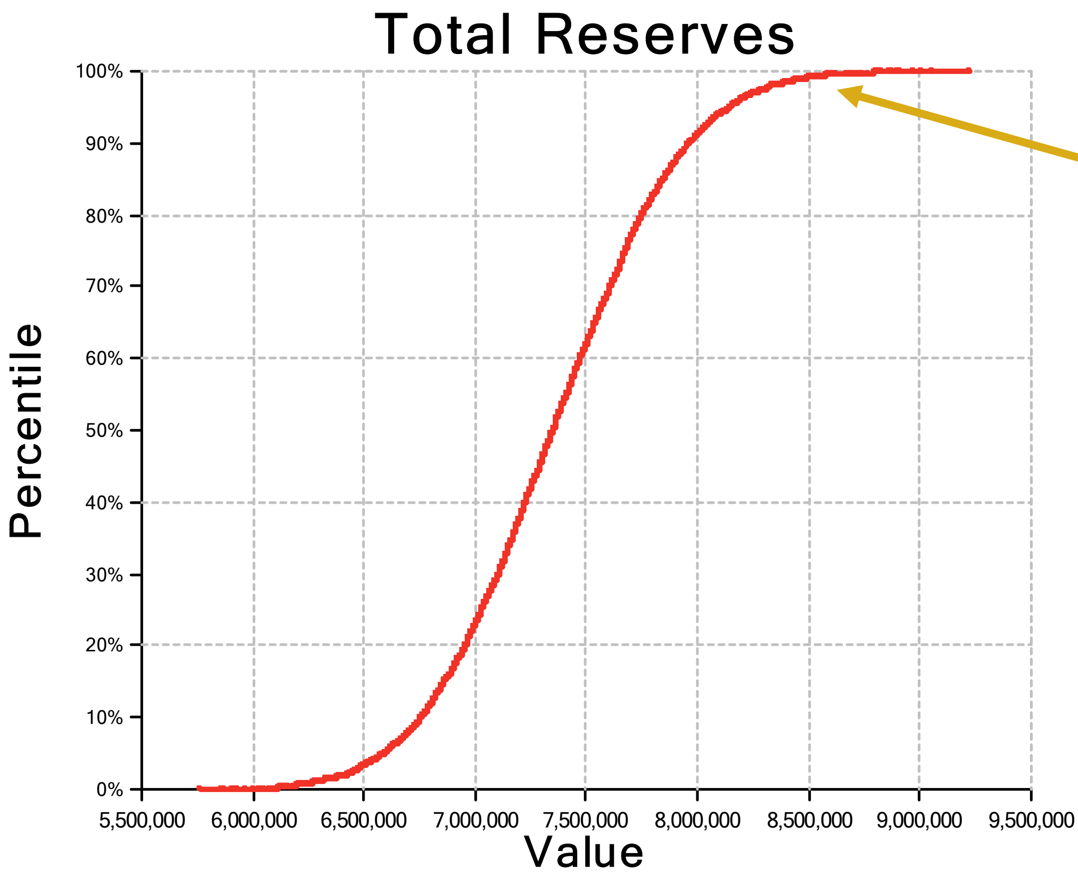
Example A – bootstrap and analytical results

Accident Year	Analytic Results		Bootstrap Results	
	Mean	Error	Mean	Error
2005	0	0	0	0
2006	16	22	16	23
2007	27	27	27	28
2008	38	30	38	31
2009	95	45	95	46
2010	178	60	178	61
2011	341	81	341	81
2012	574	105	576	105
2013	1,319	163	1,321	161
2014	4,768	364	4,765	362
Total	7,357	472	7,355	472

Example B – bootstrap and analytical results

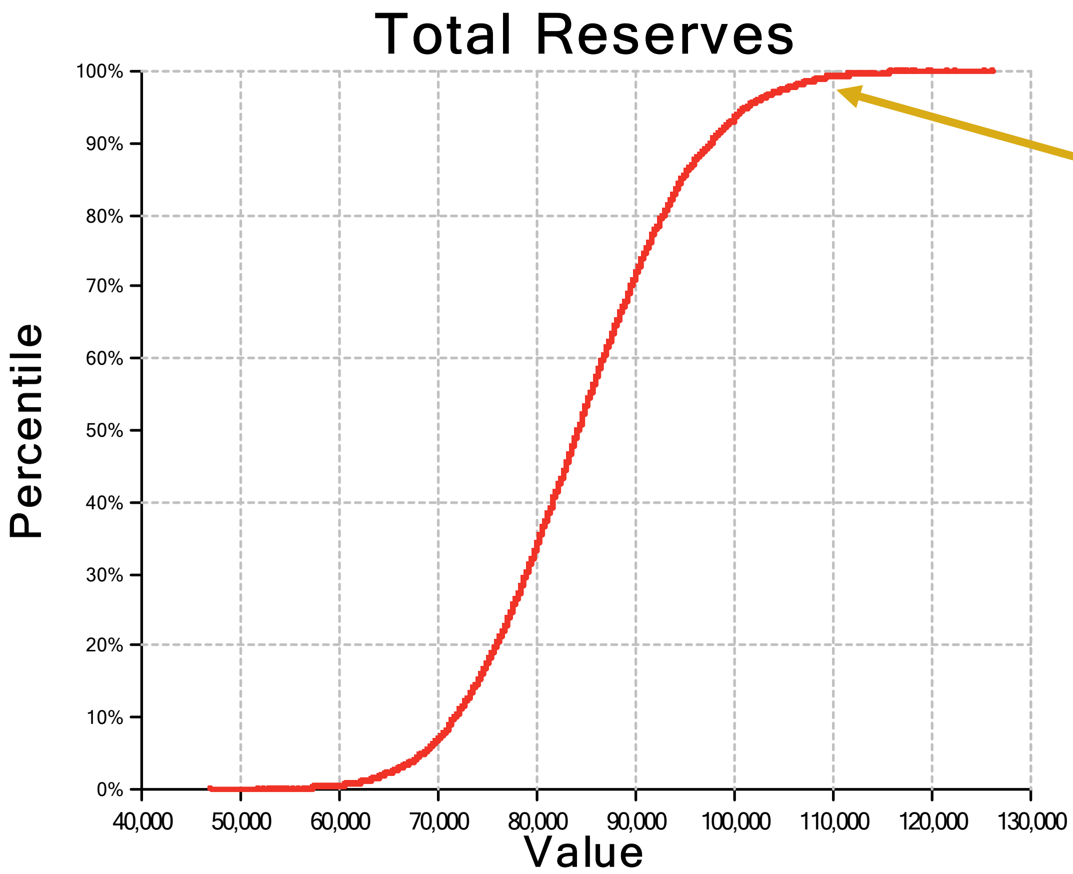
Accident Year	Analytic Results		Bootstrap Results	
	Mean	Error	Mean	Error
2005	0	0	0	0
2006	66	340	68	481
2007	2,418	1,564	2,421	1,620
2008	2,951	1,693	2,937	1,729
2009	4,173	1,953	4,175	1,955
2010	9,346	2,890	9,347	2,870
2011	12,159	3,346	12,152	3,340
2012	15,207	3,823	15,228	3,819
2013	18,492	4,328	18,450	4,291
2014	19,707	4,515	19,688	4,503
Total	84,517	10,021	84,466	10,073

Example A – distribution of total reserves



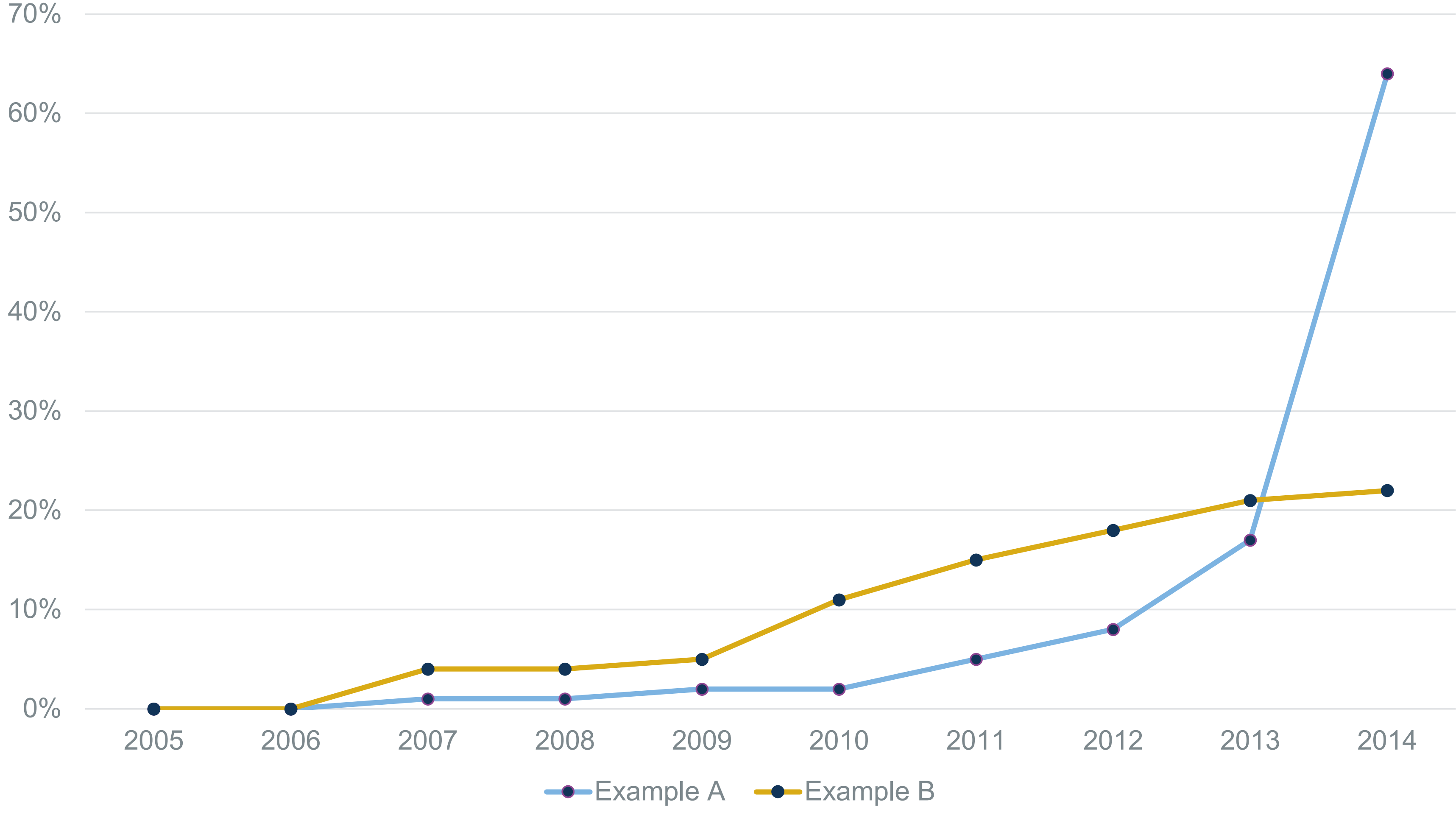
99.5th percentile
= 8,610m

Example B – distribution of total reserves



99.5th percentile
= 112,121

TVaR contribution % of accident years to ultimate risk



General actuary-in-the-box process

1. Carry out a bootstrap procedure
2. Extend the claims data by one year using the bootstrap output
3. Re-fit the statistical model to the extended claims data
4. Calculate the deterministic reserves for the extended claims data

Actuary-in-the-box applied to the AWM-BF model

- Extend the claims triangle as in the chain ladder case
- Re-fit the chain ladder to the extended triangle and calculate the incremental development proportions
- Simulate new prior estimates of the ultimate claims
- Apply the Bornhuetter-Ferguson method to get the reserves
- Calculate the claims development result or other quantities desired

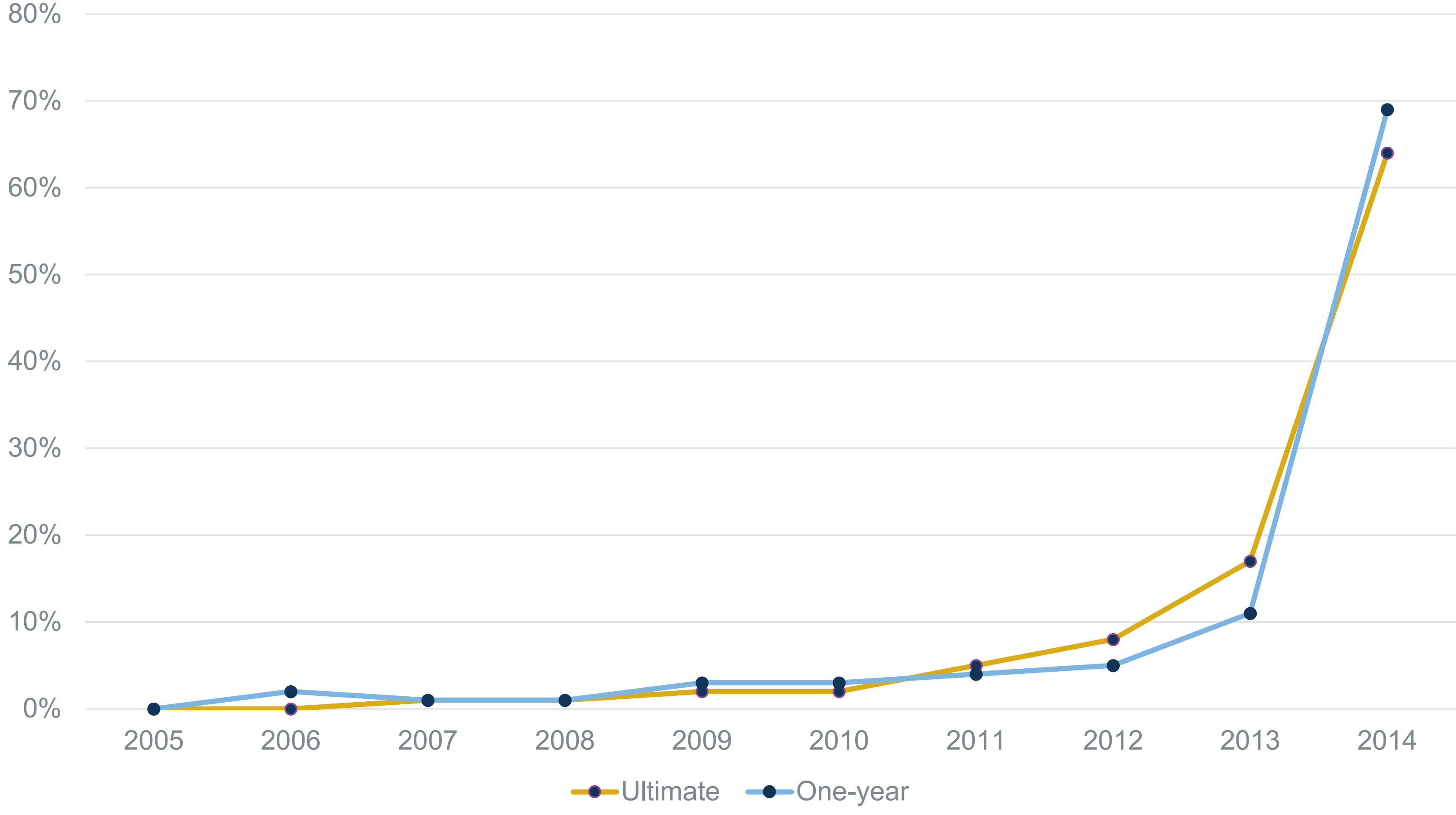
- However…
- It is not at all obvious how to simulate the prior estimate of the ultimate claims

Issues with simulating prior estimate of ultimate claims

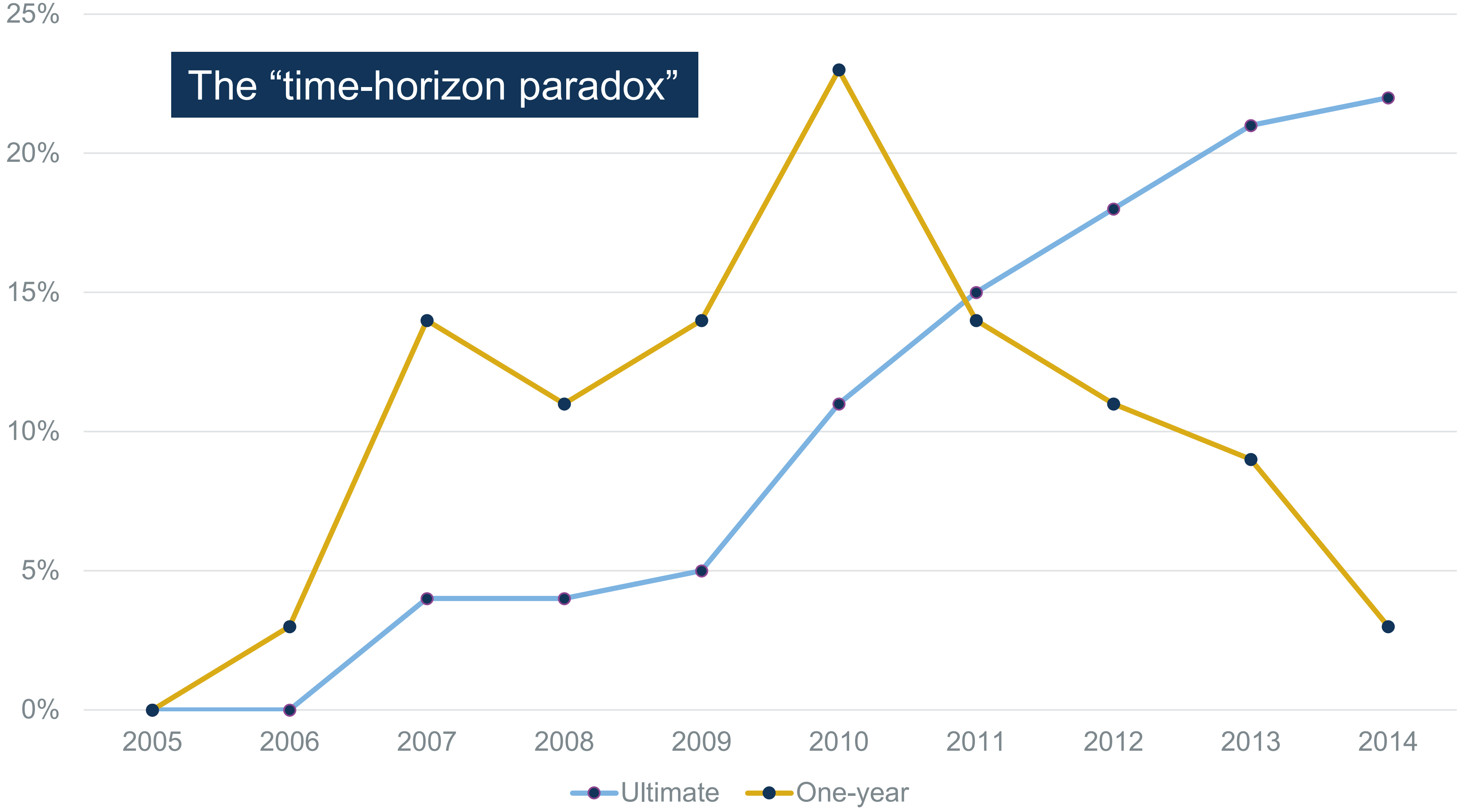
- The mean should be the same as for the ultimo distribution
- However it is not very clear how the CoVs should relate
- The ultimo CoV is parameter error for the prior estimate of the ultimate claims
- The one-year CoV is an estimate of how much the prior estimate could change over the one-year period
- The one-year CoV should probably be smaller than the ultimo CoV
- In a multi-period actuary-in-the-box the sum of the variance of the prior estimates cannot be greater than the ultimo CoV
- But there is no reason why it should be equal to it

- We need an emergence pattern for the prior estimate risk - a prior one-year view for the prior estimate of ultimate claims

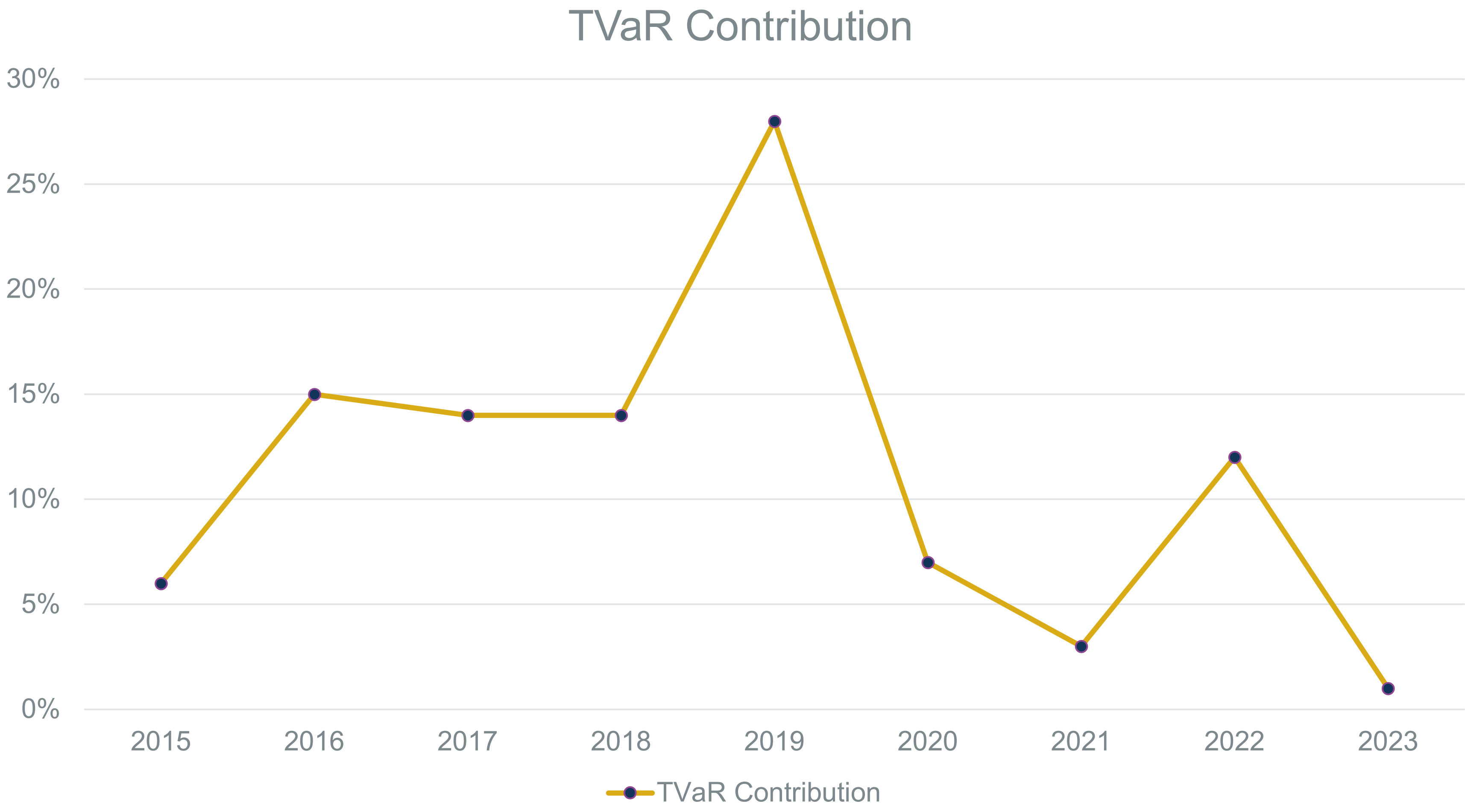
Example A – TVaR contribution ultimate vs one-year



Example B – TVaR contribution ultimate vs one-year



Example B – Future CDR contribution to ultimate risk for 2014 accident year



Bias in the Claims Development Result

	CDR Mean		% of Reserves	
	A	B	A	B
2005	0	0	0%	0%
2006	0	-2	1%	-3%
2007	0	1	-1%	0%
2008	-1	507	-2%	17%
2009	-1	484	-1%	12%
2010	-2	329	-1%	4%
2011	-3	59	-1%	0%
2012	-8	-95	-1%	-1%
2013	-15	-78	-1%	0%
2014	-29	-110	-1%	-1%
Total	-60	1,094	-1%	1%

Discussion of actuary-in-the-box results

- Results are sensitive to assumptions made about prior estimate of ultimate claims for long-tailed classes
- A “time-horizon paradox” can arise:
  - For long-tailed lines of business there can be little development in the first few years of development
  - This can lead to a prediction distribution of the claims development result being tightly spread around zero
  - Which would give a low capital figure for a long-tailed, and in reality quite risky class of business
- There is a bias in the CDR
  - This is because the models for past and future are different
  - Therefore ultimo standard error does not decompose in the nice way that it does for the ODP and Mack models

Conclusions

- The AMW-BF model is a stochastic model of the Bornhuetter-Ferguson as used in practice
- It is a pragmatic compromise between theory and practice
- The model can be bootstrapped and this gives much greater flexibility than the analytic formulae presented by AMW.
- There are number of criticisms that can be made about it
- Many of the criticisms are fundamental to the original Bornhuetter-Ferguson method
- More rigorous Bayesian methods are preferable

Further Reading (1/2)

- A practitioner's introduction to stochastic reserving
  by Alessandro Carrato, Grainne McGuire, and Robert Scarth
- Mean squared error of prediction in the Bornhuetter-Ferguson claims reserving method
  by D. H. Alai, M. Merz, and M. V. Wuethrich
  Annals of Actuarial Science, Vol 4(1), pp. 7-31
- Prediction uncertainty in the Bornhuetter-Ferguson claims reserving method: revisited
  by D. H. Alai, M. Merz, and M. V. Wuethrich
  Annals of Actuarial Science, Vol 5(1), pp. 7-17
- Bayesian over-dispersed Poisson model and the Bornhuetter & Ferguson claims reserving method
  by Peter England, Richard Verrall, and Mario Wuethrich
  Annals of Actuarial Science, Vol 6(2), pp. 258-283

Further Reading (2/2)

- The prediction error of Bornhuetter-Ferguson
  by T. Mack
  ASTIN Bull., Vol 38(1) pp 87-103
- Stochastic claims reserving in general insurance
  by Peter England and Richard Verrall
  B.A.J., 8, III, pp 443-544
- A Bayesian generalized linear model for the Bornhuetter-Ferguson method of claims reserving
  by Richard Verrall
  North American Actuarial Journal, Vol. 8, No. 3, pp 67-89

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