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Long guarantees with short duration: The rolling annuity

Søren F. Jarner, VP
ATP Pension Fund



Outline

- ATP and market value accounting
- The problem
- The rolling annuity
- Reserving and hedging
- Implementation at ATP
- Example
- Summing up



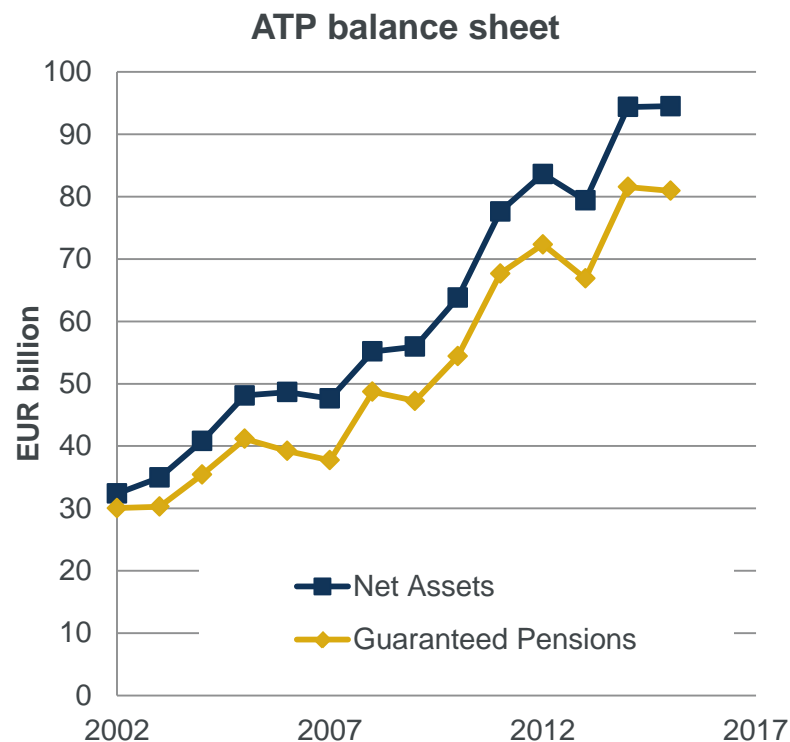
The ATP pension fund

- The Danish Supplementary Labour Market Pension Scheme
 - ... or ATP for short (DK ‘Arbejdsmarkedets Tillægspension’)
 - Founded by law in 1964 as a supplement to State Pension
 - Mandatory scheme for all Danish employees (voluntary for self-employed)
 - Almost 5M members and assets under management of approximately €100B.
- Pension product
 - Life-long nominal annuity receivable from State Pension age (cohort dependent)
 - Discretionary indexation of pensions when the funding ratio of the fund is sufficiently high
 - Individual guarantees purchased for 80 pct. of contributions, remaining 20 pct. are “risk capital”.



Market value accounting and its implications for ATP

- Market value accounting since 2003
 - “Pure” market rate discounting
 - Long-dated liabilities discounted at 30Y rate
 - Allows delta-hedging (in normal markets)
 - Huge interest rate sensitivity
 - Fully hedged in swaps and bonds
 - Decrease in interest rates increased value of liabilities dramatically.
- Discounting curve under Solvency 2
 - Long-dated liabilities valued at UFR
 - *Long-dated liabilities cannot be hedged*
 - Discounted value \neq value of (delta) hedge



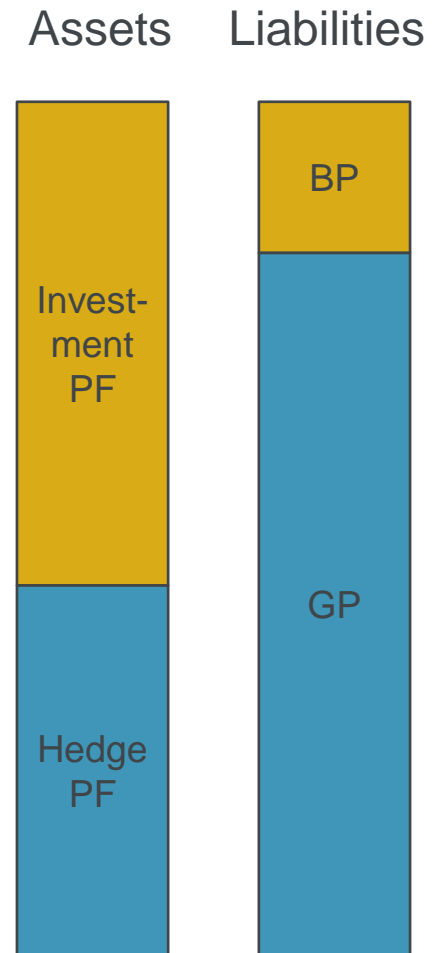
The problem

- The old annuity product at ATP
 - 80 pct. of contribution converted to nominal life-long annuity at the time of payment
 - Annuity level (tariff) updated annually to reflect current market rates and life expectancy.
- Large hedge demand at long-dated maturities
 - Increasingly difficult – and costly – to maintain the necessary hedge
 - Long-dated liabilities non-hedgeable (due to “semi” market rate discounting).
- The Board of ATP wants guarantees!
 - Not an option to move to unit-link type products
 - “Could you please design a hedgeable life-long guarantee”.



... and one more thing

- “Please make sure to preserve the business model”
 - The liability side of the balance sheet is very simple
 - ... allowing a very sophisticated asset side
 - Accommodation of all guarantees in one (simple) business model.
- Implication 1: Type of guarantee
 - All pension rights in the form of “guaranteed annual pension“
 - No individual unit-link accounts.
- Implication 2: Same status of new and old guarantees
 - Collective risk sharing of financial and biometric risks
 - New and old guarantees should entail same, or at least very similar, risks and have the same “claim” on free reserves (BP).



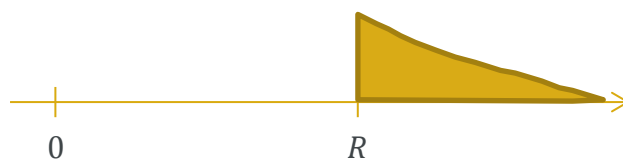
Traditional annuity vs rolling annuity

- Consider a person paying a contribution of 100 at time 0 and retiring at R
 - Denote by $p_t(T)$ the price at time t of a zero-coupon bond (ZCB) maturing at time T

- Traditional (deferred) life-long annuity

- Ignore tax, safety loadings, technical basis etc.
- Guaranteed annuity level = $100 / \int_R^\infty p_0(w) S(w|0) dw$.

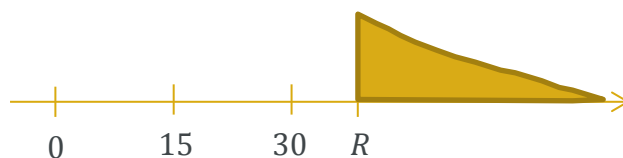
Expected, unit cash-flow = prob. of survival



- The rolling annuity replaces the long interest rate guarantee with shorter ones

- Assume interest rate guarantee of 15 years
- Initial guarantee: $z(0) = \frac{100}{\int_R^\infty S(w|0) dw} \frac{1}{p_0(15)}$
- Guarantee after 15 years: $z(15) = z(0) \frac{1}{p_{15}(30)}$
- Final guarantee: $z(30) = \frac{z(15) \int_R^\infty S(w|30) dw}{\int_R^\infty p_{30}(w) S(w|30) dw}$.

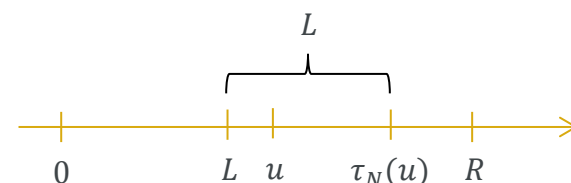
Expected no. of years in retirement = $\int_R^\infty S(w|0) dw$



Market value reserve

- Consider the reserve associated with a contribution paid at time 0

- Let $z(u)$ denote the guarantee at time $u \geq 0$
- Prior to the final guaranteed increase, the reserve is
 - $V(u) = z(u)e(R|u)p_u(\tau_N(u))$,
 - where $e(R|u)$ is the expected no. of years in retirement given survival to time u , and $\tau_N(u)$ is the time of the next increase.



- At or past the final guaranteed increase, the reserve is
 - $V(u) = z(u) \int_{\max\{u,R\}}^{\infty} p_u(w)S(w|u)dw$, i.e. the reserve for an ordinary, life-long annuity.

- Before the final increase, the reserve for a cohort equals the price of a ZCB maturing at $\tau_N(u)$ with principal $\bar{z}(u) \times$ total no. of years in retirement*

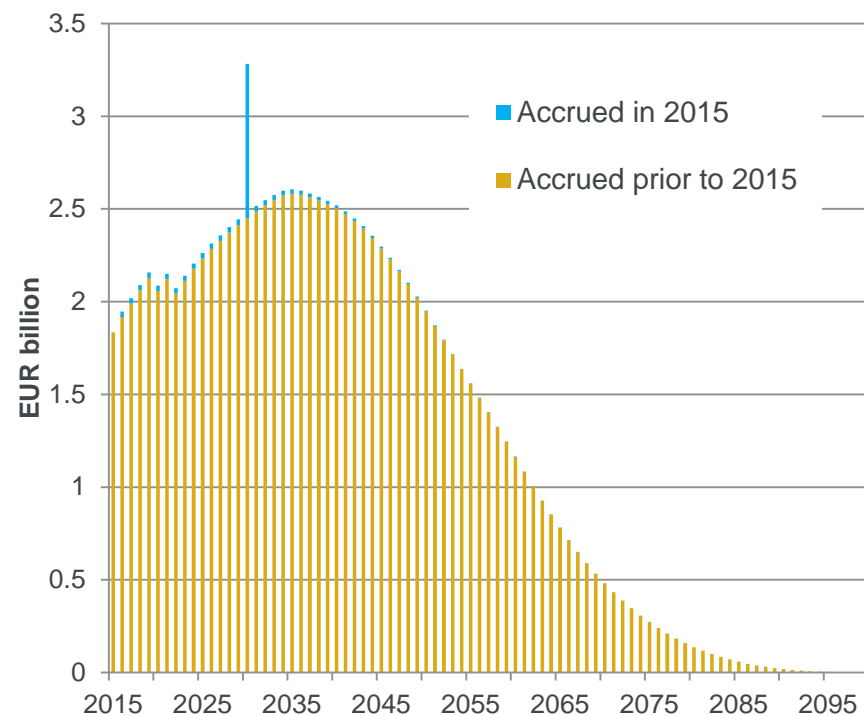
- The liability can be semi-statically hedged, i.e. hedge needs to be adjusted only every L years
- For L up to 20 years, say, the hedge can be implemented in liquid markets
- In practice, the reserve is based on updated mortality assumptions
- Longevity risk is borne collectively, i.e. guarantees are unaffected.



Implementation at ATP

- Rolling annuities were implemented at ATP with effect from 1 January 2015
 - Guarantee period of $L = 15$ years
 - The effect from contributions received in 2015 can be seen as an increased “payment” in 2030
 - The remaining cash flow stems from ordinary annuities; both old guarantees and guarantees written in 2015 for members within 15 years of retirement.
- Hedgeable at large scale
 - The bulk of the (rolling annuity) cash flow is at maturities where market liquidity is high
 - Ordinary life-long annuities are issued only close to retirement.

ATP "hedge cash flow" for annuities



Example: Longevity risk

- Table shows the relative reserve increase when applying a 20% mortality stress
 - GM mortality law* : $\mu(x) = 1.5 \cdot 10^{-5} \exp(0.1 \cdot x) + 2 \cdot 10^{-4}$
 - Stressed mortality law: $\tilde{\mu}(x) = 0.8 \mu(x)$
 - Flat yield curve : $p_t(T) = \exp(-(T - t)r)$, for some fixed r
 - Single premium at age x , age of retirement $R = 65$ yrs, and guarantee period of $L = 15$ yrs.

$\Delta V/V$	Age (x)						
Rate (r)	25	45	55	65	75	85	100
0%	11.4%	11.0%	10.5%	9.0%	11.6%	14.9%	19.9%
2%	11.4%	11.0%	8.7%	7.3%	10.0%	13.5%	19.1%
4%	11.4%	11.0%	7.3%	5.9%	8.7%	12.3%	18.3%

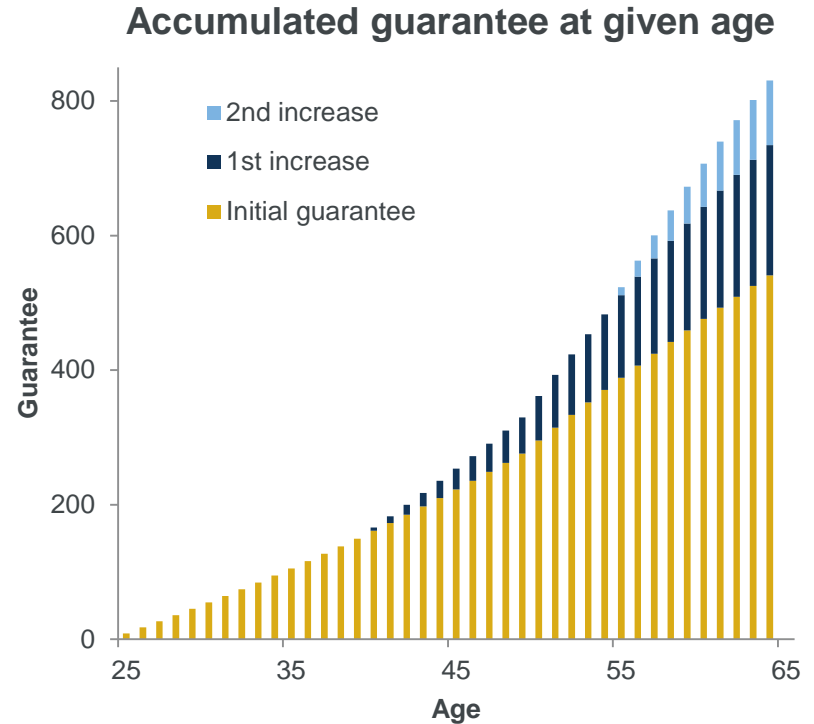
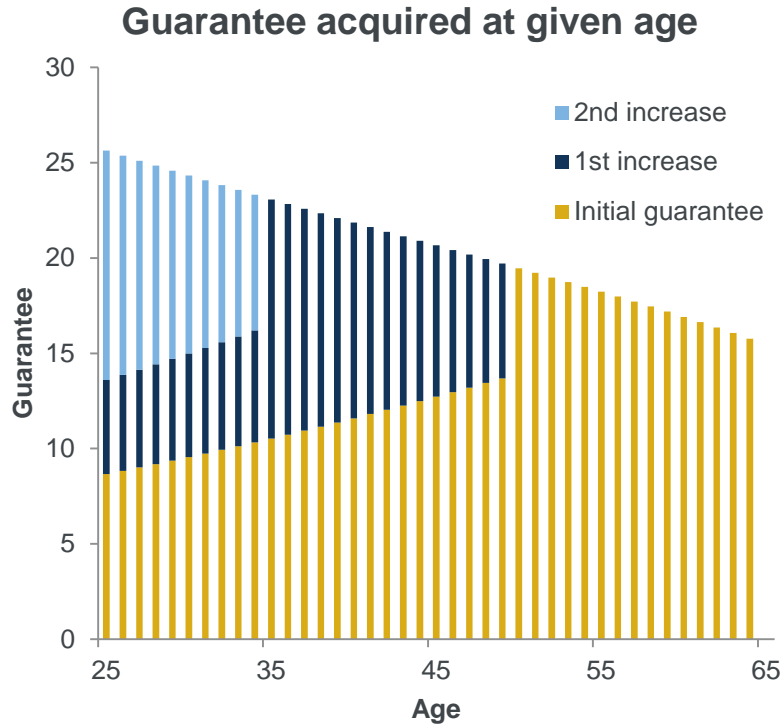
Independent of interest rate



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* Gompertz-Makeham law fitted to Danish unisex population mortality for 2011

Example: Building up of guarantee

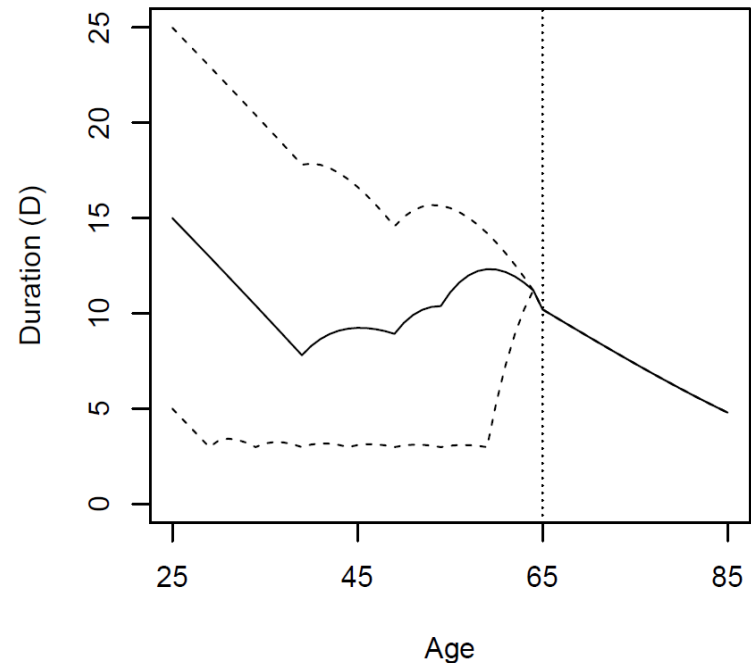
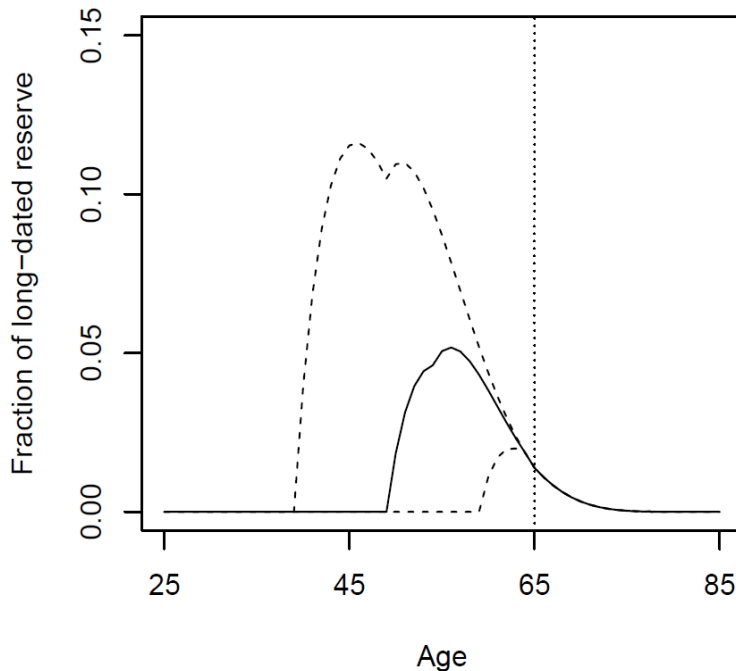


*Annual contribution of 100 indexed by inflation of 2% from age 25 to age 64.
Interest rate of 3%, and guarantee period of $L = 15$ years.*



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Example: Duration



Left plot : Reserve for maturities over 30 years as fraction of the total reserve.

Right plot : Duration of total reserve measured in years.

In both plots the solid line represents a guarantee period of $L = 15$ years, while the dashed lines represent guarantee periods of 5 and 25 years, respectively.

The vertical dotted line at age $R = 65$ years marks the age of retirement.



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Summing up

- Initial minimum guarantee and subsequent guaranteed increases prior to retirement
 - Prior to the final increase, the reserve equals a zero-coupon bond maturing at the next increase
 - Rolling annuities can be hedged at large scale for guarantee periods of up to, say, 20 years
 - Keeping the duration below 20 years imply very similar financial and regulatory (S2) value
 - This simplifies risk management considerably
 - Rolling annuities have been implemented at ATP with a guarantee period of 15 years.
- Longevity risk can be reduced by weakening the “life expectancy guarantee”
 - However, the rolling annuities at ATP have full longevity risk (similar to existing annuities).
- Rolling annuity guarantees are intended as part of a with-profits contract
 - A complementing return-seeking portfolio is essential to obtain broad market exposure
 - The guarantees entail both longevity risk and hedging risk and thus can apply to only part of contributions
 - At ATP, rolling annuities are acquired for 80 pct. of contributions.



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