The Impact of Longevity Risk Hedging on Economic Capital

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Outline

- Introduction and Danish males data
- Correlations between national and sub-population mortality
- Capital requirements and index hedging options
- How to assess the impact of a hedge in 22 easy steps
- How many models do you need?
- Hedging example and numerical results

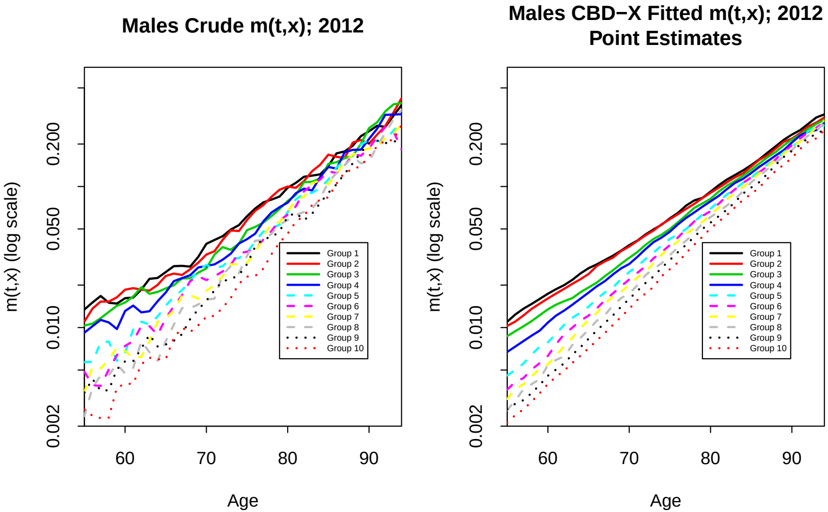
Danish Males Data

- This research: not specifically about Danish insurance problems!
- BUT:
  Denmark has very good quality national data
- Can subdivide population
- Can synthesise many different hypothetical sub-populations
- Allows experimentation with new multi-population models
- We can gain insights into general multi-population hedging problems

Danish Males Data (cont.)

- Males resident in Denmark for the previous 12 months.
- After much experimentation:
- Divide population in year t
  - into 10 equal sized Groups (approx)
  - using *affluence*, A
- Individuals can change groups up to age 67
- Group allocations are locked down at age 67
- Ages 55-94; Years 1985-2012

Model-inferred underlying death rates 2012



Modelling the death rates

$$\log m^{(k)}(t,x) = \beta^{(k)}(x) + \kappa_1^{(k)}(t) + \kappa_2^{(k)}(t)(x-\bar{x})$$

- Model fits the 10 groups well without a cohort effect
- Model structure is essential to preserve group rankings
  - Rankings are evident in crude data
  - *"Bio-demographical reasonableness"*:

    more affluent $\Rightarrow$ healthier

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Forecast correlations

Deciles are quite narrow subgroups

More diversified e.g.

- Blue collar pension plan

  $\Rightarrow$ equal proportions of groups 2, 3, 4
- White collar pension plan

  $\Rightarrow$ equal proportions of groups 8, 9, 10
- Mixed plan

  $\Rightarrow$ proportions $(0, 0, 1, 2, \ldots, 7, 8)/36$ (e.g. amounts)

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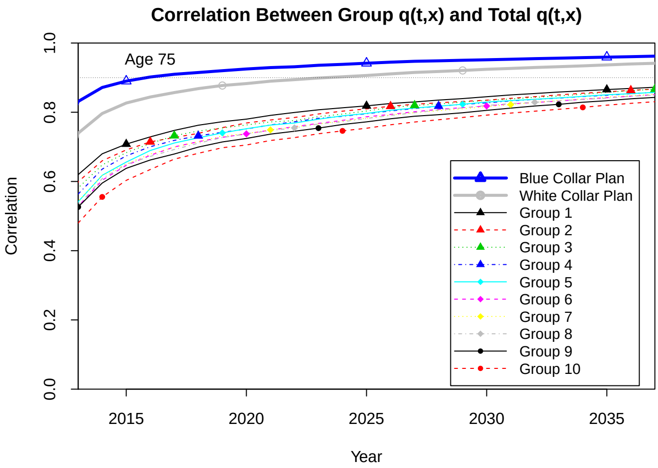
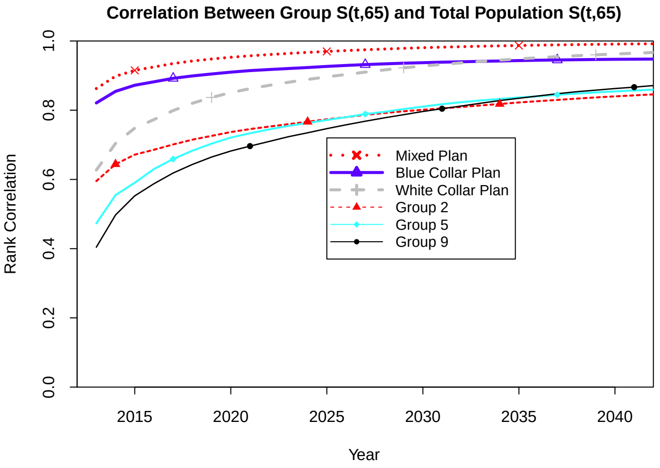
Forecast Correlations: Mortality Rates at Age 75



Forecast Correlations: Cohort Survivorship from Age 65



Capital Requirements

What type of capital calculations?

- e.g. Annuity portfolio
- Solvency II:
  - Solvency Capital Requirement, SCR= difference between
    Best estimate of annuity liabilities (BE) and
    Annuity liabilities following an immediate 20% reduction in mortality
  - or SCR= extra capital required at time 0 to ensure solvency at time 1 with 99.5% probability
  - plus other 'equivalent' variants.

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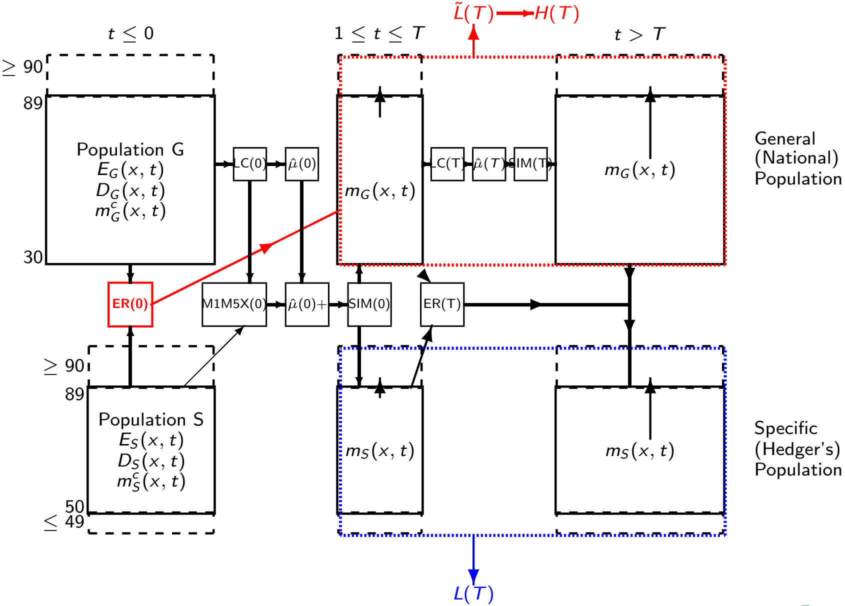
What type of hedge to modify capital requirements and manage risk?

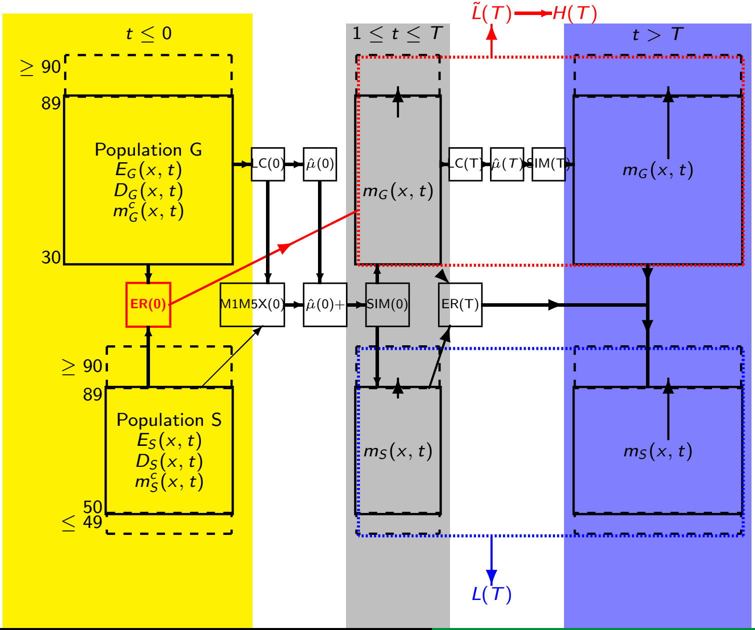
- Customised longevity swap
- Index-based hedge
  - Synthetic $\tilde{L}(T) \approx$ true $L(T)$
  - Swap
  - Bull spread derived from underlying $\tilde{L}(T)$
    Payoff at T:

$$H(T) = \begin{cases} 0 & \text{if } \tilde{L}(T) < AP \text{ (Attachment Point)} \\ \tilde{L}(T) - AP & \text{if } AP \leq \tilde{L}(T) < DP \text{ (Detachment Point)} \\ DP - AP & \text{if } DP \leq \tilde{L}(T) \end{cases}$$

Anatomy of a Hedging Calculation: 1



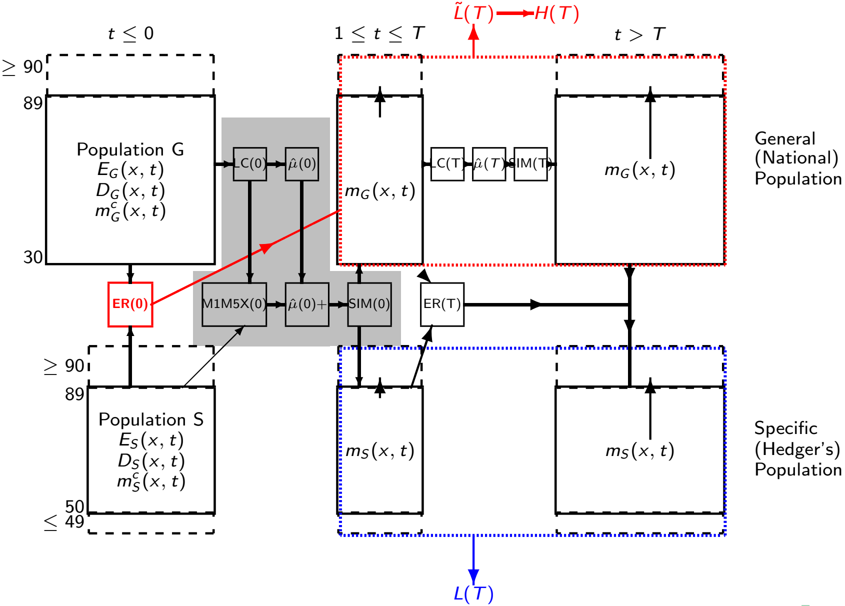
Anatomy of a Hedging Calculation: 2



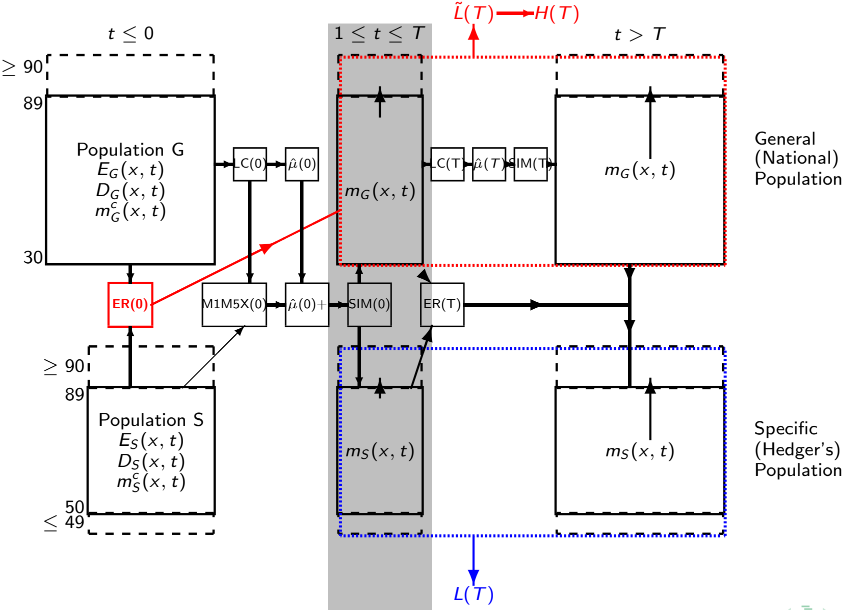
General (National) Population

Specific (Hedger's) Population

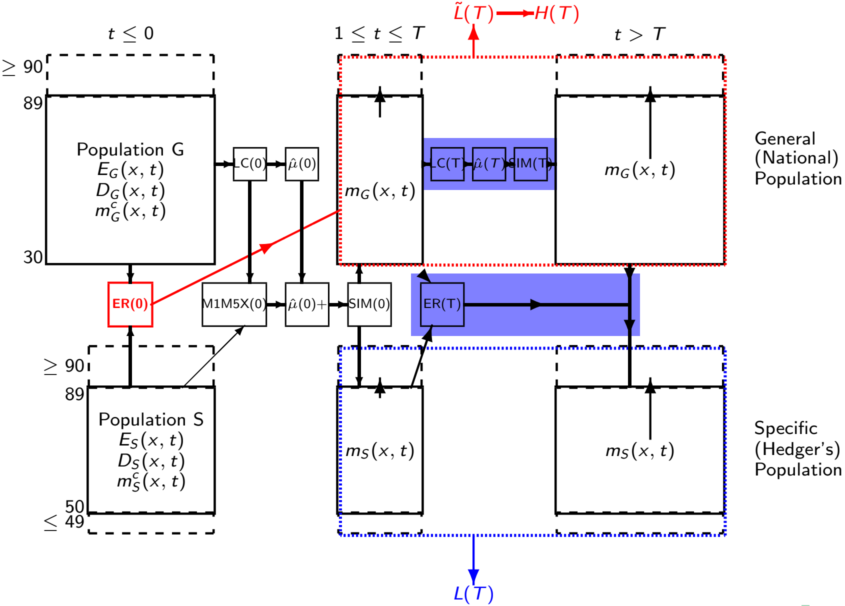
Anatomy of a Hedging Calculation: 3



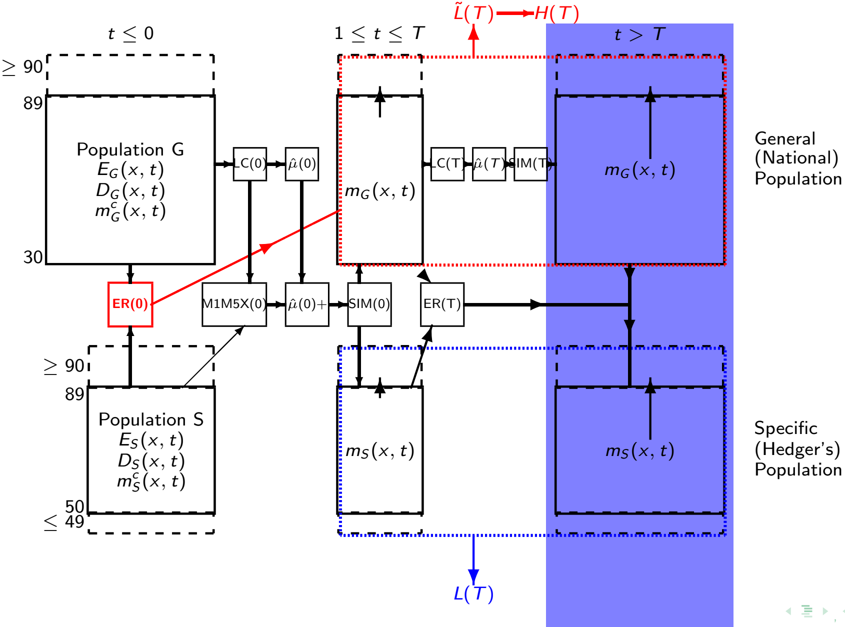
Anatomy of a Hedging Calculation: 4



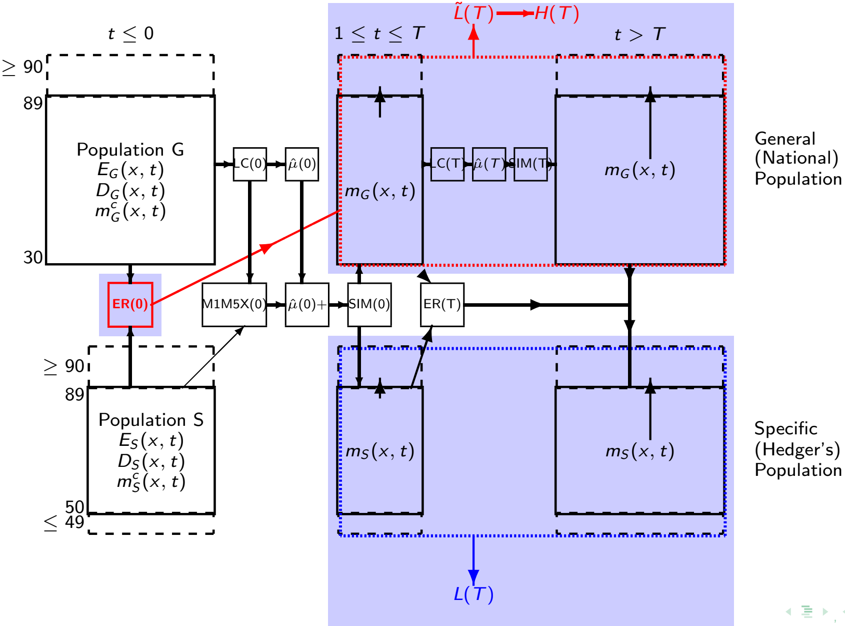
Anatomy of a Hedging Calculation: 5



Anatomy of a Hedging Calculation: 6



Anatomy of a Hedging Calculation: 7



How many models do you need?

In theory: One model

In practice:

- Time 0:
  - Liability valuation model (BE + SCR)
  - Simulation model ($0 \rightarrow T$)
- Time T:
  - Hedge instrument valuation model
  - Liability valuation model
- 'Models' for extrapolating to high (and low) ages

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Time 0 Models

- Unhedged Liabilities:
  Deterministic BE + 20% stress
- Simulation:
  - General: (Lee-Carter/M1)

$$\ln m_{gen}(x,t) = A(x) + B(x)K(t) \quad \text{(Lee-Carter/M1)}$$

  - Specific: (M1-M5X)

$$\ln m_{pop}(x,t) = \ln m_{gen}(x,t) + a(x) + k_1(t) + k_2(t)(x - \bar{x})$$

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Time T models

- Hedge instrument:
  - Lee-Carter (M1) for general population
  - Recalibration on basis specified at time 0

$$q^H_{gen}(x,t) \to q^H_{gen}(x,t) \times ER(x,0) \to \tilde{L}(T) \to H(T)$$

- Liability: specific (hedger's) population
  - Lee-Carter (M1) for general population
  - Possible different calibration from the hedge instrument
  - $q^L_{pop}(x,t) = q^L_{gen}(x,t) \times ER(x,T) \to L(T)$

Hedging Example

- Portfolio of deferred and immediate annuities, ages 50-89
  - Deferred to age 65
  - Amounts/weights: 25 for $x \leq 65$
  - Amounts/weights: $90 - x$ for $65 < x < 90$
- Before and after:

  Compare $L(T)$ with $L(T) - H(T)$
- SCR = 99.5% quantile − mean

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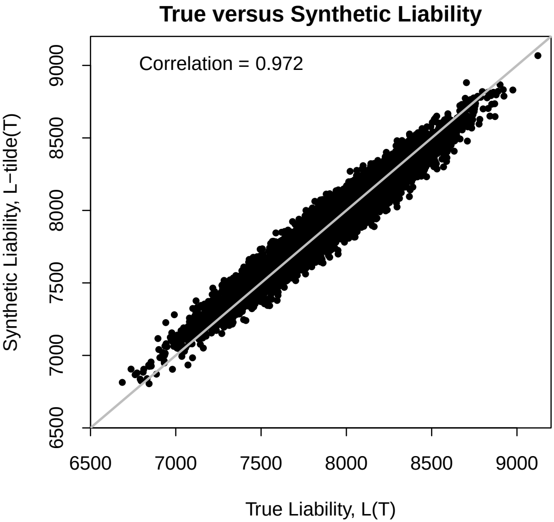
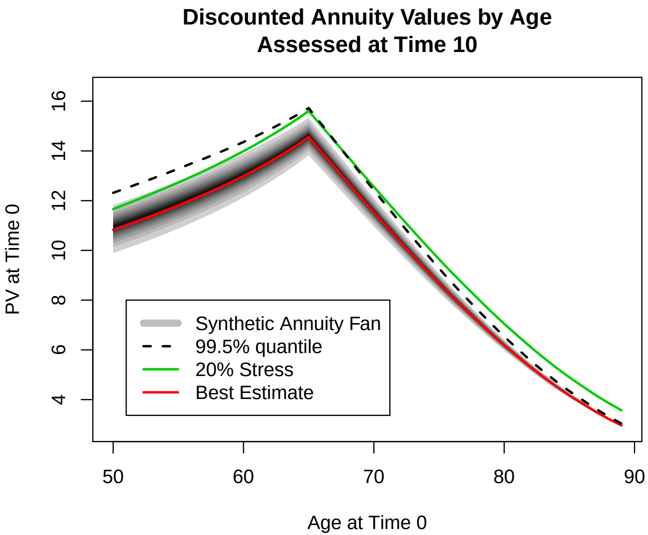
Hedging Example



Hedging Example



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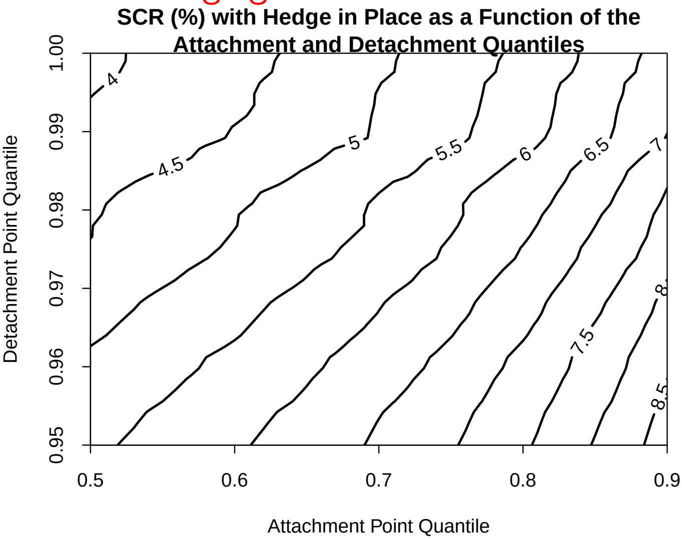
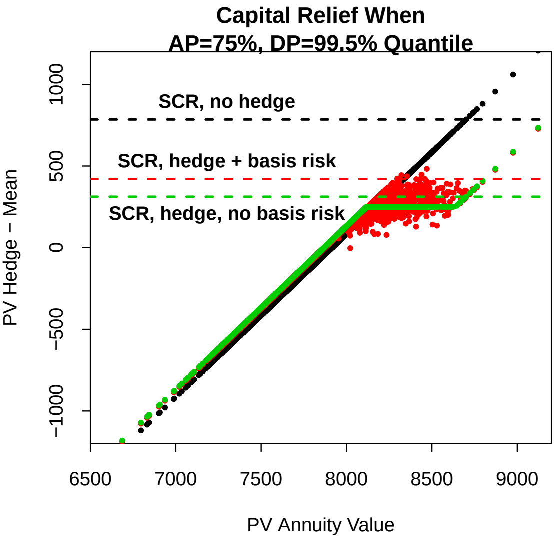
SCR with no hedging = 9.9% of mean



Hedging Example



Bull Spread $\longrightarrow$ Longevity Catastrophe Bond

- Hedger
- Bull spread
- Special Purpose Vehicle
- Cat Bond Payoff $= (DP - AP) - H(T)$
- Cat Bond Holders
  - Bond nominal $= AP - DP$ (for example)
  - Maximum loss $= 100\%$

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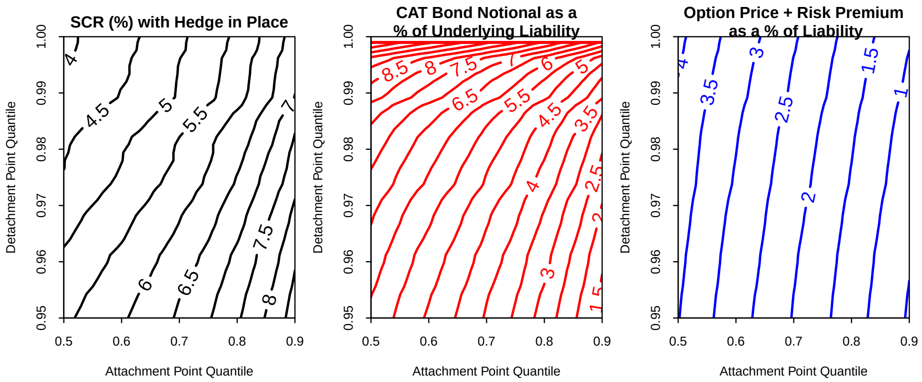
Hedging Example: SCR, Cat Bond Notional, Option Price



Tradeoffs and Other Considerations

How to choose Maturity, AP and DP?

- Reduction in SCR $\nearrow$
- Cat Bond nominal $\searrow$
- Bull spread price $\searrow$
- Shareholder value added $\nearrow$
- Insurer risk appetite, hedging objectives etc.

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Theory vs Practice: Bridging the Gap

OR

Try to avoid this:

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Theory vs Practice: Bridging the Gap

Where we are now?

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Thank You!

Questions?

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