HOW A SINGLE-FACTOR CAPM WORKS IN A MULTI-CURRENCY WORLD: RESULTS FROM THE LATEST RESEARCH

Presenters: Rob Thomson and Taryn Reddy
University of the Witwatersrand
Johannesburg
South Africa

Co-author: Şule Şahin

18 July 2016

This work is a joint effort

Rob Thomson
Şule Şahin

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We have come a long way

• AFIR Colloquium 2013
• “Why the capital asset pricing model fails in a multi-currency world”

Why use the CAPM?

• It’s an equilibrium model.
• It assumes homogenous expectations.
• It’s a simple model.
• Nobody has ever proved it’s wrong.

Global CAPM (GCAPM)

\[ E \{ R_i \} = R_F + \beta_i^W \left[ E \{ R_W \} - R_F \right] \]
International CAPM (ICAPM)

\[ E(R_i) = R_f + \beta_i \left[ E(R_m) - R_f \right] + \gamma_i \left[ E(R_m^c) - R_f^c \right] \]

What do we want?

- A model that:
  - Treats variance as the measure of risk, regardless of source, and
  - That produces the same price.

We had an idea

But we ran into some problems

- Convergence problems
  - Number of constraints > number of unknowns

So back to the drawing board!

Single-factor multi-currency CAPM (SFM-CAPM) assumptions

- (1) Investors who measure their investment returns in currency \( c \) (i.e. ‘currency-c investors’) have indifference curves in mean–variance space, the means and variances being those measured in that currency.

- (2) All investors, regardless of the currency in which they measure returns, have homogeneous expectations of the means, variances and covariances of:
  - the returns in each currency on assets issued in that currency; and
  - rates of strengthening of each currency.
The SFM-CAPM

\[ E \{ R_{dt}^c \} = R_F^c + \beta_{dt}^c \left[ E \{ R_M^c \} - R_F^c \right] \]

SFM-CAPM constraint

If the SFM-CAPM applies in a multi-currency world
then, for any currencies \( c \) and \( e \):

\[ k_{e}^c = k_{e}^c \]

where:

\[ k_{e}^c = \frac{\sigma_{e,M}^c - \sigma_{e,LM}^c}{\sigma_{M,M}^c} \left( \mu_M^c - r_i \right) \]

SFM-CAPM mark 1

Minimise:

\[ D^2 = \frac{1}{Q} \sum_{e} \left[ \sum_{c} \left( \hat{\mu}_c^{(S)} - \hat{\mu}_e^{(G)} \right)^2 + \sum_{c} \left( \hat{\mu}_c^{(S)} - \hat{\mu}_e^{(G)} \right)^2 \right] \]

subject to the constraints:

\[ k_{e}^c = \frac{\sigma_{e,M}^c - \sigma_{e,LM}^c}{\sigma_{M,M}^c} \left( \tilde{\mu}_M^c - r_i \right) = k_{e}^c \]

SFM-CAPM mark 2

Minimise:

\[ D^2 = D_e^2 + hD^2 \]

where:

\[ D_e^2 = \frac{1}{Q} \sum_{c} \left[ \sum_{i} \left( \hat{\mu}_e^{(S)} - \hat{\mu}_i^{(G)} \right)^2 + \sum_{i} \left( \hat{\mu}_e^{(S)} - \hat{\mu}_i^{(G)} \right)^2 \right] \]

\[ D^2 = \frac{1}{Q} \sum_{c} \sum_{i} \left( k_e^c - k_{e}^c \right)^2 \]

Data
The SFM-CAPM

\[ E \left\{ R_{fi}^c \right\} = R_{fi}^c + \beta_{fi}^c \left[ E \left\{ R_{M}^c \right\} - R_{F}^c \right] \]

Why adopt the SFM-CAPM?

- It’s better than the ICAPM.
- It’s better than the GCAPM.
- The difference is material.

A word of advice

- Use real returns rather than nominal returns:
  - It’s closer to the GCAPM, so the adjustments required are smaller.
  - The GCAPM is supposed to be about optimising consumption.
  - Financial mathematicians can’t use real returns; actuaries can.

Teşekkürler

Thank you