A Stochastic Model for Share Earnings

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"Higgledy-Piggledy Growth"
I. M. D. Little, 1962

"Higgledy-Piggledy Growth Again"
A. C. Rayner & I. M. D. Little, 1966

A drunken stagger about a random walk
comment by A. C. Stalker, ca 1980

Wilkie (1986) modelled Share Prices as the Ratio of
Share Dividends to Dividend Yield.

Dividends depended on lagged Price inflation,
plus random walk bits

Dividend Yield modelled as auto-regressive
AR(1) (mean reverting).

Earnings index existed but only since 1962
Too short

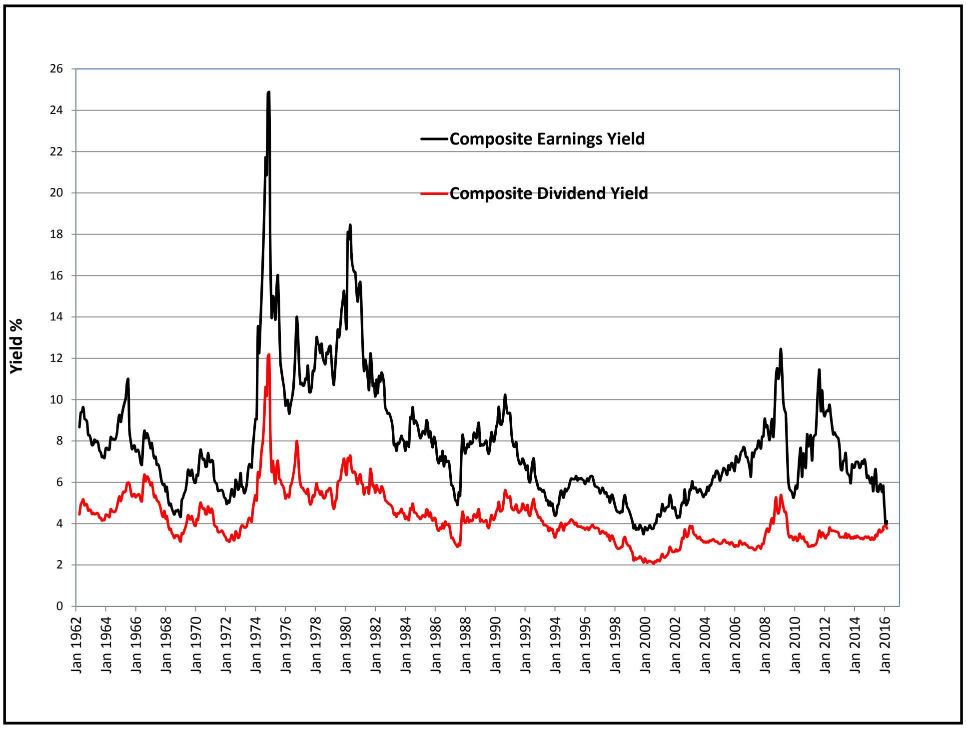
Earnings Index now from 1962, over 50 years

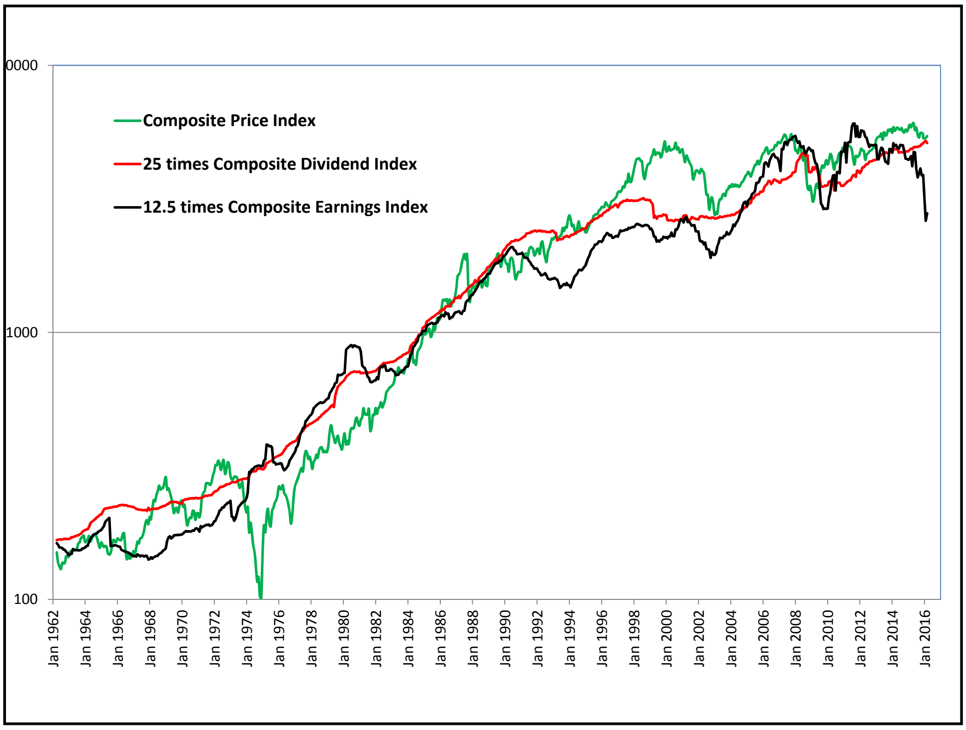
From April 1962 on “Non-financials Index”

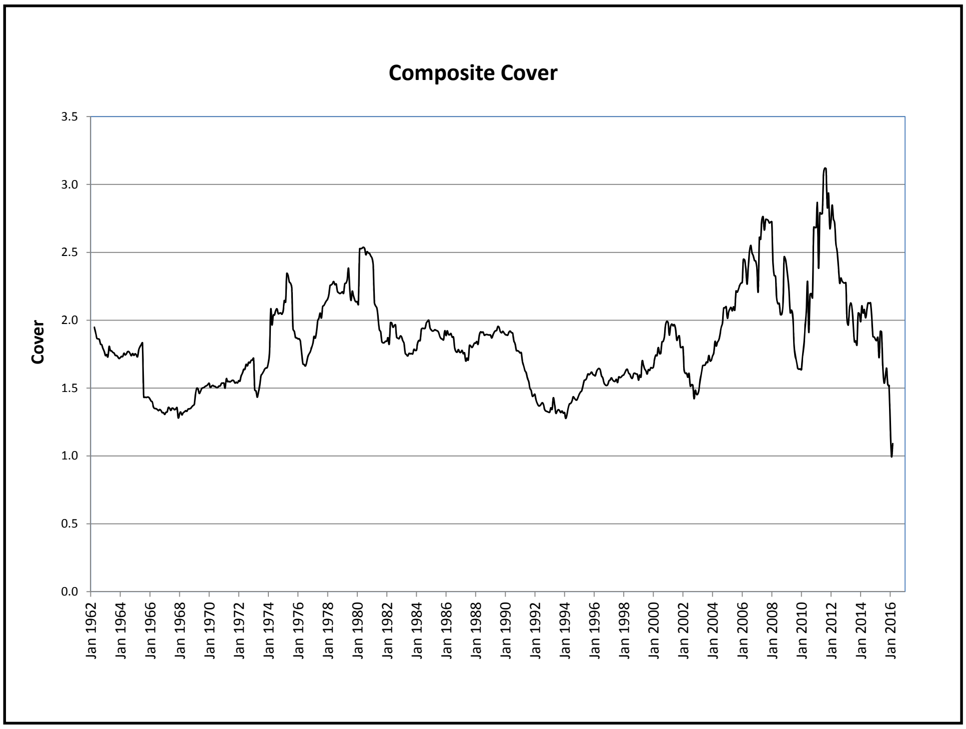
From December 1992 also on All-share Index

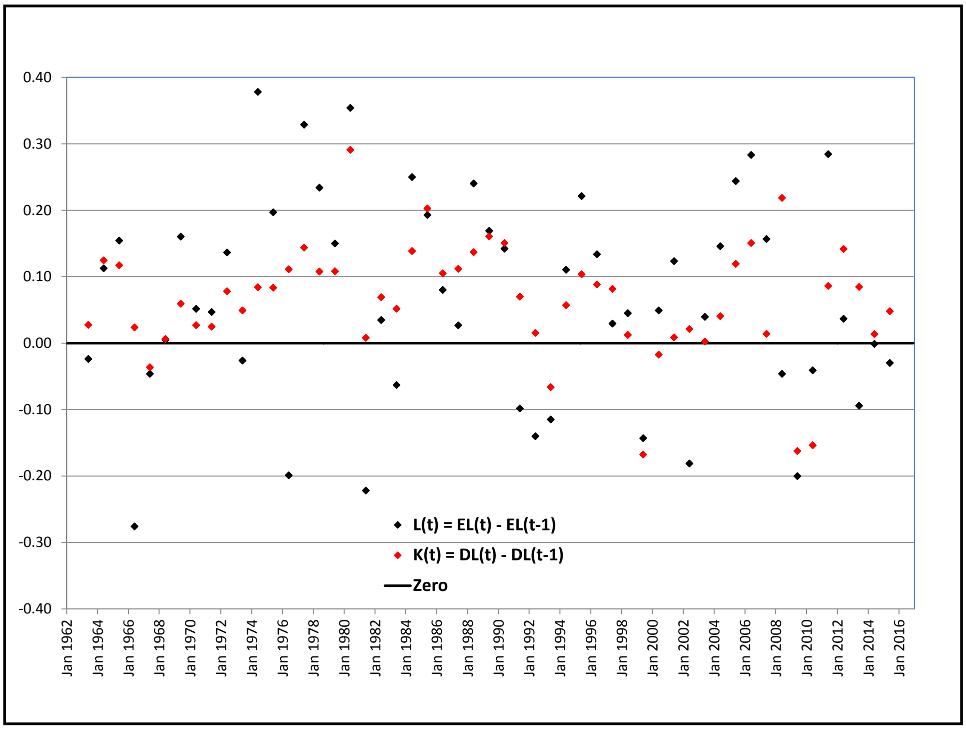
Earnings Yield originally, later P/E ratio

We have spliced NFI and ASI at March 1994
to give Composite Index


Composite Earnings Yield
Composite Dividend Yield
Yield %
26
24
22
20
18
16
14
12
10
8
6
4
2
0
Jan 1962
Jan 1964
Jan 1966
Jan 1968
Jan 1970
Jan 1972
Jan 1974
Jan 1976
Jan 1978
Jan 1980
Jan 1982
Jan 1984
Jan 1986
Jan 1988
Jan 1990
Jan 1992
Jan 1994
Jan 1996
Jan 1998
Jan 2000
Jan 2002
Jan 2004
Jan 2006
Jan 2008
Jan 2010
Jan 2012
Jan 2014
Jan 2016


Composite Price Index
25 times Composite Dividend Index
12.5 times Composite Earnings Index
0000
1000
100
Jan 1962
Jan 1964
Jan 1966
Jan 1968
Jan 1970
Jan 1972
Jan 1974
Jan 1976
Jan 1978
Jan 1980
Jan 1982
Jan 1984
Jan 1986
Jan 1988
Jan 1990
Jan 1992
Jan 1994
Jan 1996
Jan 1998
Jan 2000
Jan 2002
Jan 2004
Jan 2006
Jan 2008
Jan 2010
Jan 2012
Jan 2014
Jan 2016


Composite Cover
Cover
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Jan 1962
Jan 1964
Jan 1966
Jan 1968
Jan 1970
Jan 1972
Jan 1974
Jan 1976
Jan 1978
Jan 1980
Jan 1982
Jan 1984
Jan 1986
Jan 1988
Jan 1990
Jan 1992
Jan 1994
Jan 1996
Jan 1998
Jan 2000
Jan 2002
Jan 2004
Jan 2006
Jan 2008
Jan 2010
Jan 2012
Jan 2014
Jan 2016


0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
L(t) = EL(t) - EL(t-1)
K(t) = DL(t) - DL(t-1)
Zero
Jan 1962
Jan 1964
Jan 1966
Jan 1968
Jan 1970
Jan 1972
Jan 1974
Jan 1976
Jan 1978
Jan 1980
Jan 1982
Jan 1984
Jan 1986
Jan 1988
Jan 1990
Jan 1992
Jan 1994
Jan 1996
Jan 1998
Jan 2000
Jan 2002
Jan 2004
Jan 2006
Jan 2008
Jan 2010
Jan 2012
Jan 2014
Jan 2016

P/E ratio = Price/Earnings = 1/Earnings Yield

Dividend Yield = Dividend/Price = 1/"D/E Ratio"

Cover = Earnings/Dividend = 1/Payout ratio

P/E Ratio × Dividend Yield × Cover = 1 (or 100)

Normally all > 0 on the index, but $E \leq 0$ for some companies, and $D = 0$ for some companies

Use June values, 1962 to 2015

t = year

Present model:

$YL(t) = \text{Ln}(\text{Dividend yield}(t))$

$DL(t) = \text{Ln}(\text{Dividend}(t))$

$K(t) = DL(t) - DL(t-1)$

New model adds:

$EL(t) = \text{Ln}(\text{Earnings}(t))$

$L(t) = EL(t) - EL(t-1)$

$VL(t) = \text{Ln}(\text{Cover}(t))$

L(t) depends on current and last year's Inflation
but not older inflation

$EE(t) = ESD.EZ(\mathrm{t})$
$\mathrm{E}[EZ(t)] = 0$ $\mathrm{Var}[EZ(t)] = 1$, plausibly N(0,1)

$L(t) = EMU + EQ1.I(t) + EQ2.I(t-1) + EE(t)$
$EL(t) = EL(t-1) + L(t)$

We put $EQ1 + EQ2 = 1$, so "unit gain"
$EQ1 = 2.61$, $EQ2 = -1.61$, $EMU = 0.010$, $ESD = 0.130$

$EMU^* = EMU + (EQ1 + EQ2).QMU = 0.01 + QMU$

$VL(t) = EL(t) - DL(t)$
very dependent on Earnings
with influence from price inflation
and AR(1)

$VE(t) = VSD.VZ(\mathrm{t})$
$\mathrm{E}[VZ(t)] = 0$ $\mathrm{Var}[VZ(t)] = 1$, possibly N(0,1)

$VN(t) = VA.VN(t-1) + VE1.EE(t) + VE2.EE(t-1) + VE(\mathrm{t})$

$VL(t) = \mathrm{Ln}(VMU) + VW.I(t) + VN(t)$

VA = 0.90, *VE1* = 0.69, *VE2* = −0.16
VW = 1.63, *VMU* = 1.70, *VSD* = 0.058

Model for *Y*(*t*) unchanged

Parameters for Composite for 1963-2015
YW = 1.50, *YA* = 0.70, *YMU* = 0.037, *YSD* = 0.17

Dividend index *D*(*t*) with *DL*(*t*) = Ln(D(t))

$$DL(t) = EL(t) - VL(t)$$

Share prices *P*(*t*) with *PL*(*t*) = Ln(*P*(*t*))

$$PL(t) = DL(t) - YL(t)$$

P/E ratio *M*(*t*) with *ML*(*t*) = Ln(*M*(*t*))

$$ML(t) = - VL(t) - YL(t)$$

Forecast means and confidence intervals

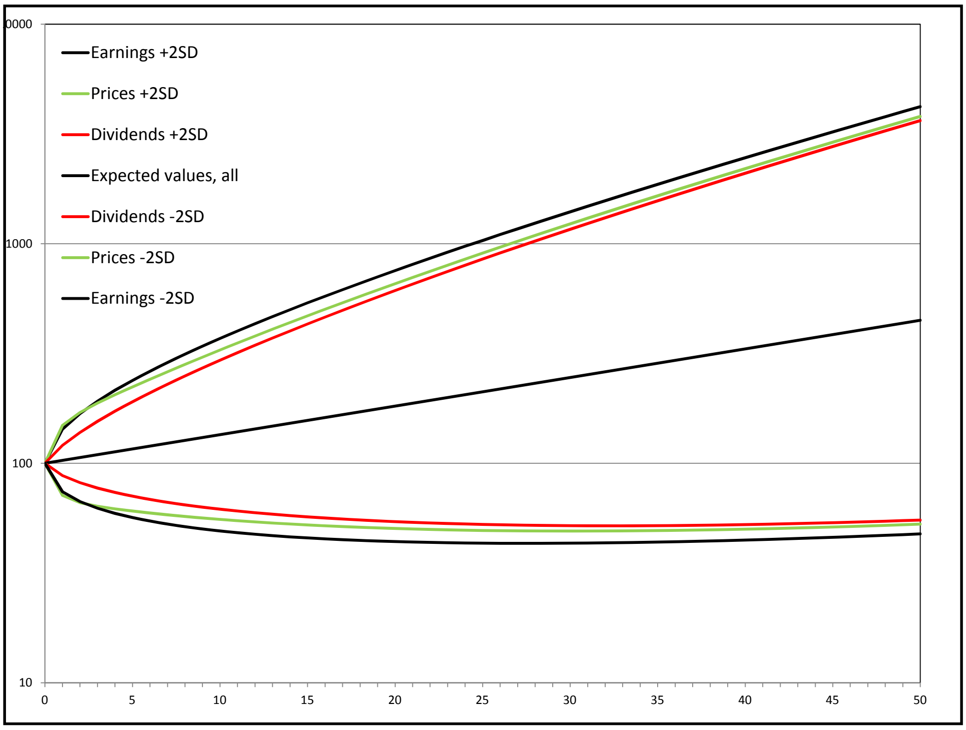
E and E ± 2S.D.

QMU = 0.02 so about 2% inflation

EMU = 0.01 so about 1% real increase in *E*, *D*, *P*

Neutral initial conditions

50 years ahead



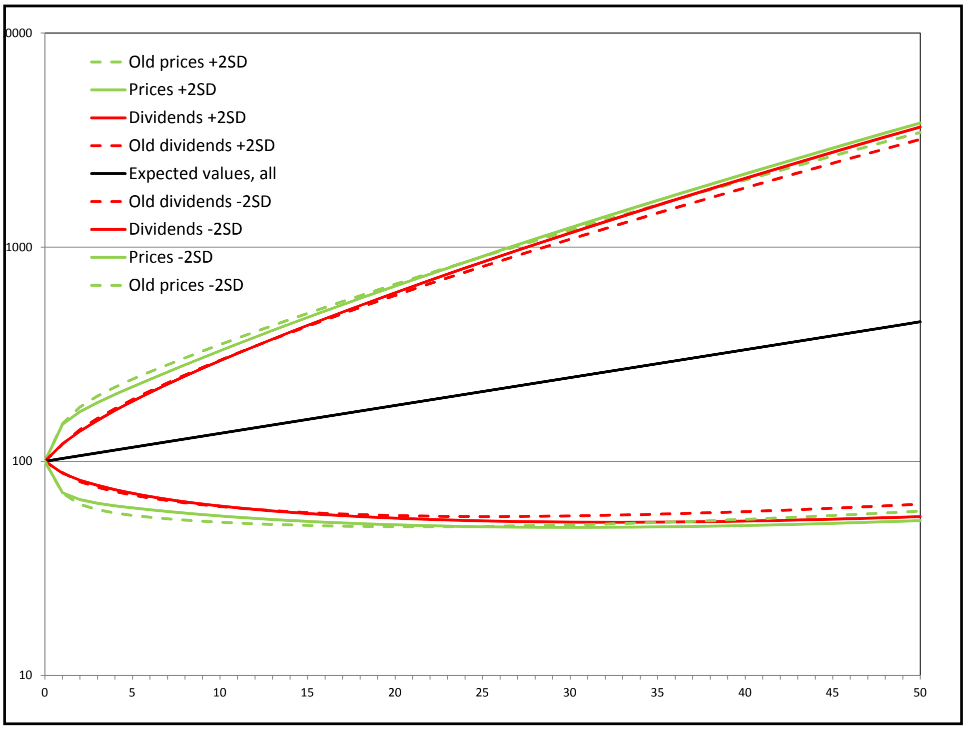
The same for original model

QMU = 0.02 again

DMU = 0.01 so about 1% real increase in *D*, *P*

No Earnings

Expected values all just the same



Two simulations

Same basis

Neutral initial conditions

One 10 times the other

Annual values

Need stochastic interpolation to compare
with actual data

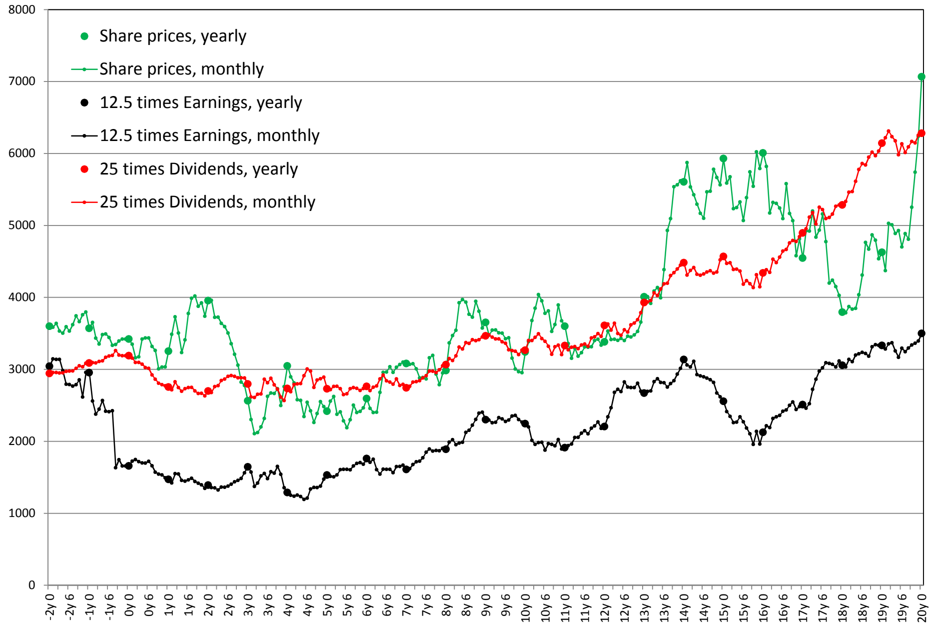
Simulation from "June 2016"

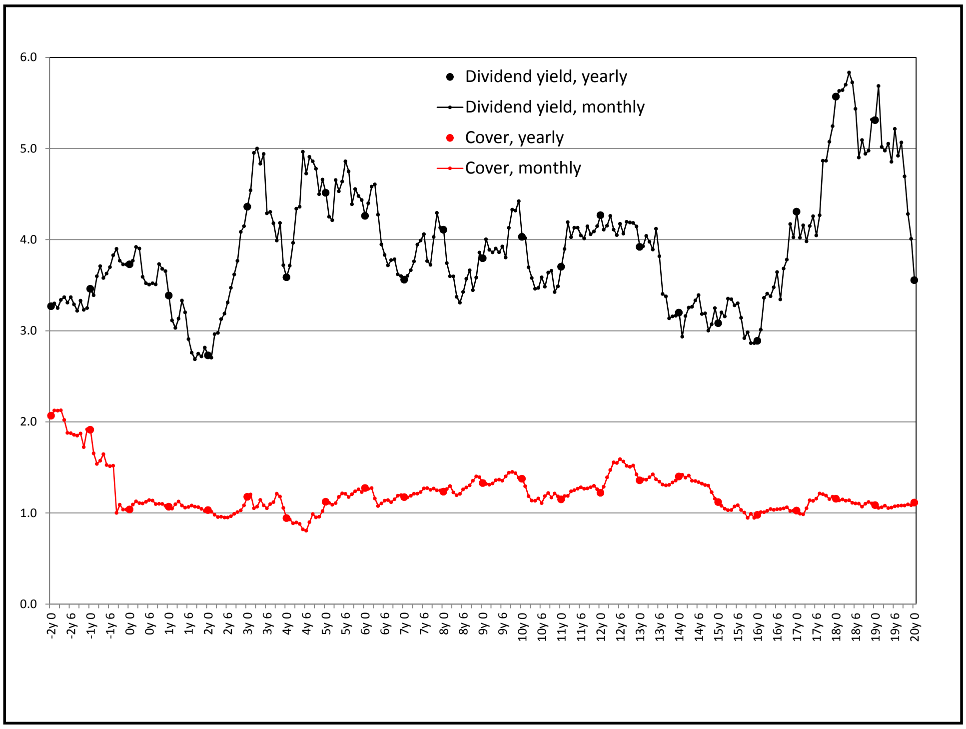
Actual initial conditions for past months

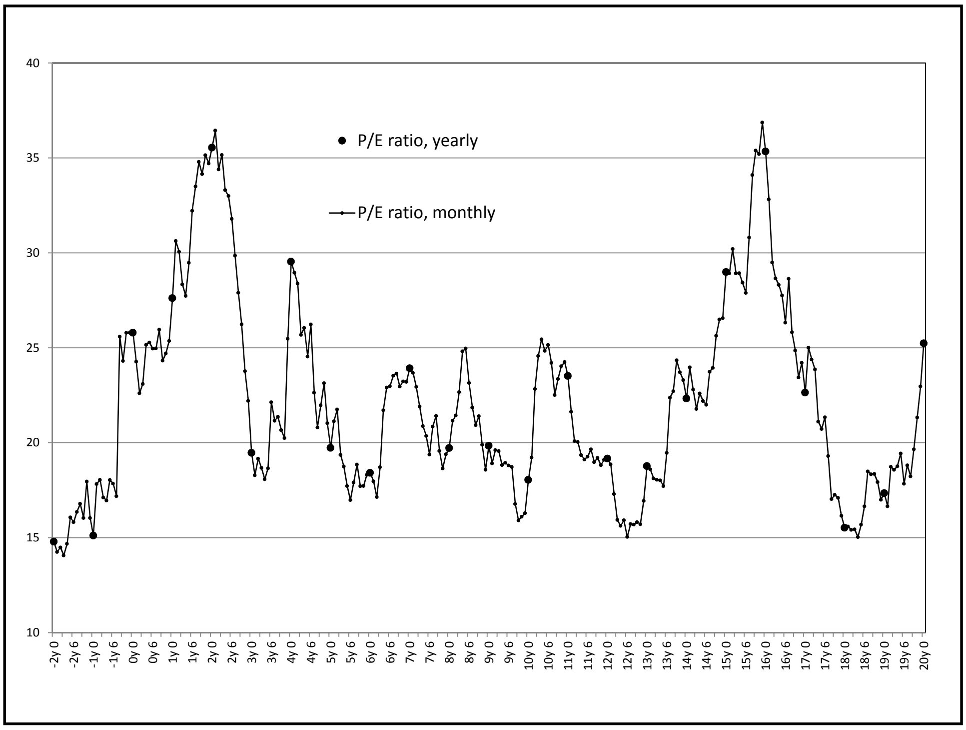
End-April values repeated for end May, end June

Monthly values by stochastic interpolation
with special Brownian bridges

One simulation, 2 past years, 20 future years




Dividend yield, yearly
Dividend yield, monthly
Cover, yearly
Cover, monthly


P/E ratio, yearly
P/E ratio, monthly

Cover has very slow autoregression, VA = 0.90,
so it does not go back quickly to mean.

But can Dividends continue to be very close
to Earnings?

Perhaps we should reconsider the model
when real June 2016 figures available.

Why not VAR model for *EL*, *DL*, *PL* together?

They are co-integrated, levels move together.

Why not co-integrated model?

Differences (ratios) *VL*, *YL*, *ML* stationary.

Three series are independent
we choose *E*, *V*, *Y*, but others are possible