A Systematic Approach to Event Modelling & Clash Pricing

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Lockton Re
Does my tail look big in this?
Agenda

- What is clash
- A brief overview
- Methodology
- Results
- Issues and future improvements
What is Clash?

- Reinsurance
- Protects the insured against multiple losses from the same event
Overview

- Losses are usually modelled on an individual risk basis using a frequency-severity approach.
- Unfortunately this approach doesn’t allow Clash treaties to be modelled, and generates tails that are too thin for Capital modelling.
- We modify the usual simulation methodology to simulate events, which enables us to:
  - Correlate losses within an event.
  - Model risk and clash treaties on a coherent basis.
  - Price Clash treaties.
  - Generate thicker tails to get a more "realistic" view of capital requirements and probability of risk XL horizontal failure.
Methodology
Data

The premium for the excess book would be presented in a typical Limit/Attachment Profile:

<table>
<thead>
<tr>
<th>Limit</th>
<th>Attachment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A₁</td>
</tr>
<tr>
<td>L₁</td>
<td>x₁₁</td>
</tr>
<tr>
<td>L₂</td>
<td>x₂₁</td>
</tr>
<tr>
<td>L₃</td>
<td>x₃₁</td>
</tr>
</tbody>
</table>
Severity Assumptions

Need to assume a distribution for the severity of losses:

• Back solving market ILFs
• Fitting to client’s own experience
Limited Expected Values (LEVs)

We are going to assume that losses are correlated in a given unit, so we first estimate the equivalent premium in each cell by assuming that premium is distributed pro-rata to expected loss.

The Limited Expected Value (LEV) is defined as:

\[ LEV(u) = \int_{0}^{u} x f(x) dx + u(1 - F(u)) \]

where \( F(x) \) & \( f(x) \) are the cumulative and probability density functions of the severity distributions respectively.
Obtain the FGU Premium

<table>
<thead>
<tr>
<th>Colour</th>
<th>RGB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dark blue</td>
<td>R17 G52 B88</td>
</tr>
<tr>
<td>Gold</td>
<td>R217 G171 B22</td>
</tr>
<tr>
<td>Mid blue</td>
<td>R64 G150 B184</td>
</tr>
<tr>
<td>Light grey</td>
<td>R220 G221 B217</td>
</tr>
<tr>
<td>Pea green</td>
<td>R121 G163 B42</td>
</tr>
<tr>
<td>Forest green</td>
<td>R0 G132 B82</td>
</tr>
<tr>
<td>Bottle green</td>
<td>R17 G179 B162</td>
</tr>
<tr>
<td>Cyan</td>
<td>R0 G156 B200</td>
</tr>
<tr>
<td>Light blue</td>
<td>R124 G179 B225</td>
</tr>
<tr>
<td>Violet</td>
<td>R128 G118 B207</td>
</tr>
<tr>
<td>Purple</td>
<td>R143 G70 B147</td>
</tr>
<tr>
<td>Fuscia</td>
<td>R233 G69 B140</td>
</tr>
<tr>
<td>Red</td>
<td>R200 G30 B69</td>
</tr>
<tr>
<td>Orange</td>
<td>R238 G116 B29</td>
</tr>
</tbody>
</table>

The premium for the excess layer is

\[ x_{ij} = \frac{y_{ij} \cdot (LEV(L_i+A_j) - LEV(A_j))}{LEV(L_i+A_j)} \]

The equivalent fgu premium \( y_{ij} \) in each cell is

\[ y_{ij} = \frac{x_{ij} \cdot LEV(L_i+A_j)}{LEV(L_i+A_j) - LEV(A_j)} \]
Expected number of FGU Losses

Given the assumed loss ratio LR, and a cell frequency of $\lambda_{ij}$ we have:

$$\lambda_{ij} \times LEV(L_i + A_j) = y_{ij} \times LR$$

Re-writing this gives:

$$\lambda_{ij} = \frac{LR \times x_{ij}}{LEV(L_i + A_j) - LEV(A_j)}$$

Number of losses per cell is:

<table>
<thead>
<tr>
<th>Limit</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1$</td>
<td>$\lambda_{11}$</td>
<td>$\lambda_{12}$</td>
<td>$\lambda_{13}$</td>
</tr>
<tr>
<td>$L_2$</td>
<td>$\lambda_{21}$</td>
<td>$\lambda_{22}$</td>
<td>$\lambda_{23}$</td>
</tr>
<tr>
<td>$L_3$</td>
<td>$\lambda_{31}$</td>
<td>$\lambda_{32}$</td>
<td>$\lambda_{33}$</td>
</tr>
</tbody>
</table>

Adding up the individual cell frequencies, we get the total frequency:

$$\Lambda = \sum_i \sum_j \lambda_{ij}$$
Obtain the Conditional Distribution

Remember that $\Lambda = \sum_i \sum_j \lambda_{ij}$

<table>
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<tr>
<th>Limit</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1$</td>
<td>$\frac{\lambda_{11}}{\Lambda}$</td>
<td>$\frac{\lambda_{12}}{\Lambda}$</td>
<td>$\frac{\lambda_{13}}{\Lambda}$</td>
</tr>
<tr>
<td>$L_2$</td>
<td>$\frac{\lambda_{21}}{\Lambda}$</td>
<td>$\frac{\lambda_{22}}{\Lambda}$</td>
<td>$\frac{\lambda_{23}}{\Lambda}$</td>
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<tr>
<td>$L_3$</td>
<td>$\frac{\lambda_{31}}{\Lambda}$</td>
<td>$\frac{\lambda_{32}}{\Lambda}$</td>
<td>$\frac{\lambda_{33}}{\Lambda}$</td>
</tr>
</tbody>
</table>
Obtain the Conditional Distribution

\[
c_{11} = \frac{\lambda_{11}}{\Lambda} \\
c_{12} = \frac{\lambda_{11}}{\Lambda} + \frac{\lambda_{12}}{\Lambda} \\
c_{13} = \frac{\lambda_{11} \Lambda + \lambda_{12} \Lambda + \lambda_{13} \Lambda}{\Lambda} \\
c_{21} = \frac{\lambda_{11} \Lambda + \lambda_{12} \Lambda + \lambda_{13} \Lambda + \lambda_{21} \Lambda}{\Lambda} 
\]

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<td>L₃</td>
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</tbody>
</table>

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Specify an Event Distribution

If we assume that the distribution for the number of losses coming from an event is

<table>
<thead>
<tr>
<th>Number of Losses (k)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$p_1$</td>
</tr>
<tr>
<td>2</td>
<td>$p_2$</td>
</tr>
<tr>
<td>3</td>
<td>$p_3$</td>
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<tr>
<td>...</td>
<td>..</td>
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<tr>
<td>...</td>
<td>..</td>
</tr>
<tr>
<td>n</td>
<td>$p_n$</td>
</tr>
</tbody>
</table>

Then the expected number of losses given an event has occurred is:

$$N = \sum_k k \, p_k$$

And the expected number of events is:

$$E = \Lambda / N.$$
Simulation Process

- Sample from the distribution for the total number of events, E, to determine how many events have occurred
- For each event, sample off the Number of losses distribution to determine how many losses have occurred, n
- Sample n independent numbers from a Standard Normal distribution
- Correlate using the Cholesky algorithm and generate losses from the severity distribution
- Use the appropriate conditional $c_{ij}$ distribution to determine the particular excess$(j)$ and limit$(i)$ points for the loss sampled, and therefore the net loss to the insurer

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Simulation Process Continued

- Apply any Risk XL terms to the individual losses to determine these recoveries
- Aggregate claims (capped at max contribution) for each loss, and when all losses from a particular event have been sampled, apply the Clash Excess of loss terms
- Repeat for all Events
- Repeat for each Simulation
Detour — Correlations
What is a Copula?
Correlation

It should be noted that some of the loss distributions modelled are extremely right skew.

A Gaussian copula will generate Normal losses with the required correlation.

Using a Gaussian copula to correlate losses from a right skew distribution will generate lower Pearson correlations than those embedded in the correlation matrix.
Obtain a 40 - 75% Correlation between the simulated losses

Input 75% in Correlation Matrix
Correlation

Obtain a 75% Correlation between the losses

Input 75% - 90% in Correlation Matrix

"Input" Correlation (Gaussian Copula), %

Pearson Correlation Generated, %

CoV 50%  CoV 100%  CoV 200%  CoV 400%  CoV 800%  CoV 1600%
Results
Results

- Coefficient of variance on severity
- Clash Assumptions
- 12 runs

<table>
<thead>
<tr>
<th>CoV</th>
<th>No Clash</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
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<tr>
<td>Low</td>
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Impact of Different CoVs on a Risk Model

Return Periods

Loss Amounts

Millions

Low CoV
Medium CoV
High CoV

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Impact of Clash

![Impact of Clash Graph](image_url)
Impact of Different Clash Assumptions

![Graph showing the impact of different clash assumptions on loss amounts over return periods. The graph includes four lines representing 'No Clash', 'Low', 'Medium', and 'High' scenarios. The x-axis represents return periods ranging from 1 to 1,000, while the y-axis represents loss amounts in millions, ranging from 0 to 400.]
Comparing a Risk Model to an Event Model

Risk

Event

Comparing a Risk Model to an Event Model

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Risk

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Issues
Issues

- Lack of data available to estimate parameters
  - Can cross check your assumptions against those implied by the market price
- Significantly slower than a pure risk XL run because the model simulates losses fgu as opposed to excess of an attachment point
- Assumes same severity distribution for the individual losses that arise as part of a systemic event verses a “non-systemic” losses
- Clash reinsurance varies materially in types of events covered; the “events” considered in determining the distribution in number of losses may not fully match the event definition that will trigger clash reinsurance
Possible Improvements

1. Vary exposure by underwriting year (implicitly assuming exposure flat)

2. Vary loss ratio by Attachment / Limit / Year

3. Explicitly model the different loss processes
Expressions of individual views by members of the Institute and Faculty of Actuaries and its staff are encouraged.

The views expressed in this presentation are those of the presenter.