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The emergence of risk over one year

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Impact of the risk measure choice

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Same problem – Different context

We expect Risk X to emergence uniformly over the next two years. Therefore in our model we assume that the 1YR is 50% of the ultimate

Do you agree?

Classes A and B are equally risky and independent. The total capital for A and B is 100. Because A and B are equally risky, then the standalone risk capital for A is 50.

Do you agree?

No, capital for A will be around 70

Similarly, 1YR also would be around70 % of ultimate and not 50%.

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One year risk

- 1YR emergence should include changes in expectations of future experience
- In this presentation we assume that any new information arising is immediately recognised in the internal model
- Risks emerging in different years are independent
  - Paid losses in different years may NOT be independent, but risks emerging in different years ARE.
  - Information emerging in the first year which affects future years is captured in the changes in future expectations element

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I am not sure. For example, if reserves deteriorate in a year, we have noticed that it is likely that they will deteriorate in subsequent years. How can risks emerging in different years be independent?

1. In this case, the expected future deteriorations should be recognised in the first year. 1st year risk will be the deterioration of the 1st year plus any expected future deteriorations, given 1st year's deterioration.

2. Then any future deviations from the expected future deteriorations will be independent of what happened in the 1st year.

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One year risk: Some mathematics

- Risk R ult R1 R2 R3

Time =0 Time =1 Time =2 Time =3

- $R_{ult} = R_1 + R_2 + \cdots + R_n$
- $Cor(R_i, R_j) = 0, \quad for\ i \neq j$
- $V(R_{ult}) = V(R_1) + \cdots + V(R_n)$
- $VaR_{99.5}(R_{ult}) \neq VaR_{99.5}(R_1) + \cdots + VaR_{99.5}(R_n)$
- Hence, statements like “if 40% of ultimate emerges in year one, then the other 60% will emerge after year one” are generally incorrect.

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One year risk: Effect of risk measure

- We assume that a loss of amount 1 could occur in each year. Losses are assumed to be independent.

		Var99.5th			Standard Deviation		
probability of loss Yr1	probability of loss Yr2	Var99.5th Year 1	Var99.5th ultimate	1 Yr/ult ratio	StDev Year 1	StDev Ultimate	1Yr/ult Ratio
1.00%	1.00%	1.00	1.00	1.00	0.10	0.14	0.71
0.51%	0.51%	1.00	1.00	1.00	0.07	0.10	0.71
0.49%	0.49%	0.00	1.00	0.00	0.07	0.10	0.71

- Uniform emergence of risk
- 1 Yr/Ultimate Ratio depends on risk measure and p
  - Could be 1 or 0 or .71, or something else

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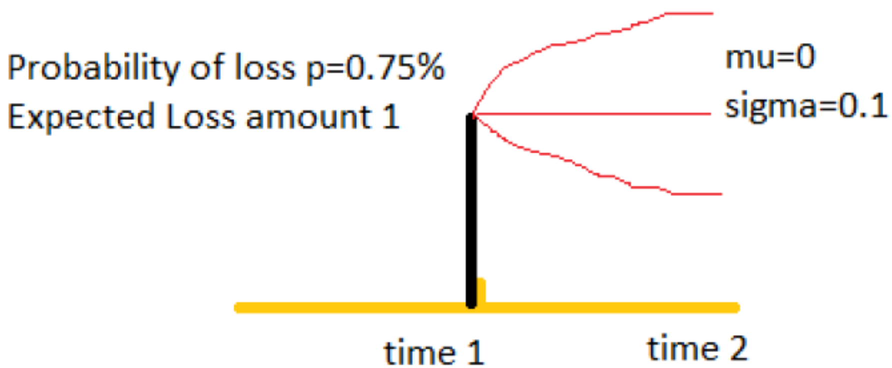
Can one year risk be higher than Ultimate?

- If we adjust for future expectations and assume that information is recognised immediately, then the 1YR risk can not be higher than the ultimate.
  - However, it can be higher if the risk measure is not a coherent risk measure. Var99.5 is not a coherent risk measure



	1 Year	Ultimate
Var 99.5th	1.00	0.95
Variance	0.0074	0.0075

- Sometimes 1YR VaR 99.5th is higher than ultimate when bootstrapping loss triangles

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I am not sure. We have seen people over-reacting to some new piece of bad news and as a result over-reserving. Subsequently the loss turned out not to be so bad.

…or, what if the risk follows a mean reverting process?

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1. If you believe that over-reacting is happening in some systematic way, then the model should allow for this as part of the future expectations as at the end of the 1st year. The ultimate loss will be reduced by these expected future improvements in the reserves

2. Try a mean reverting process and at the end of the first year allow for the expected mean reverting effect. Then you will see that

a. the 1YR risk is always lower than the ultimate and

b. b. the risk emergence in different years is independent.

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One year risk

1YR includes changes in expectations about future losses

Are these changes in future expectations assessed properly?

Emergence of risk is related, but not the same or proportional, to expected development patterns

Emergence of risk often behaves in counter intuitive ways

1YR can not be higher than ultimate if

- future expectations are taken into account,
- information is assumed to be immediately recognised and
- the risk measure is coherent

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Error in the estimation of the one year to ultimate ratio

An example using the Mack

and Merz-Wuthrich Models

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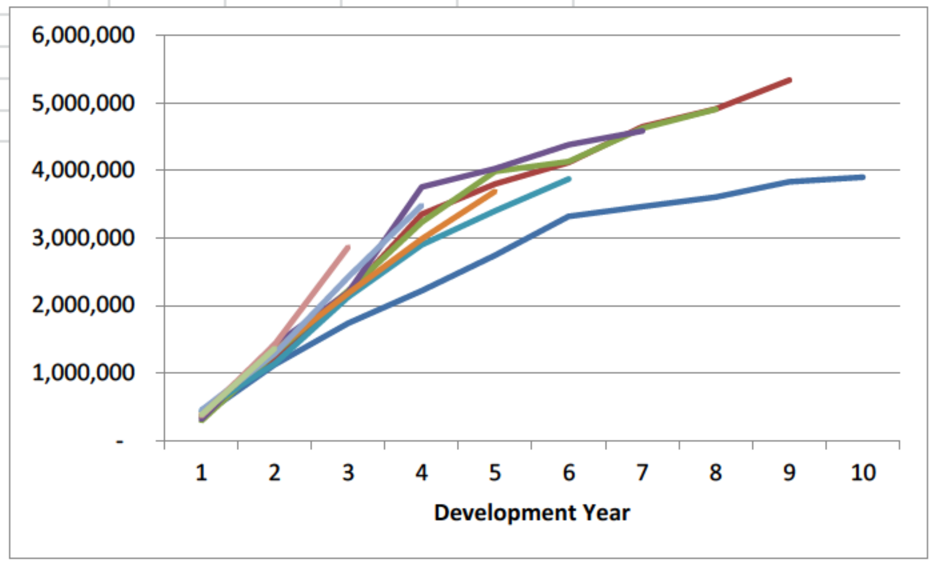
Error in the estimation of one year/ult. ratio

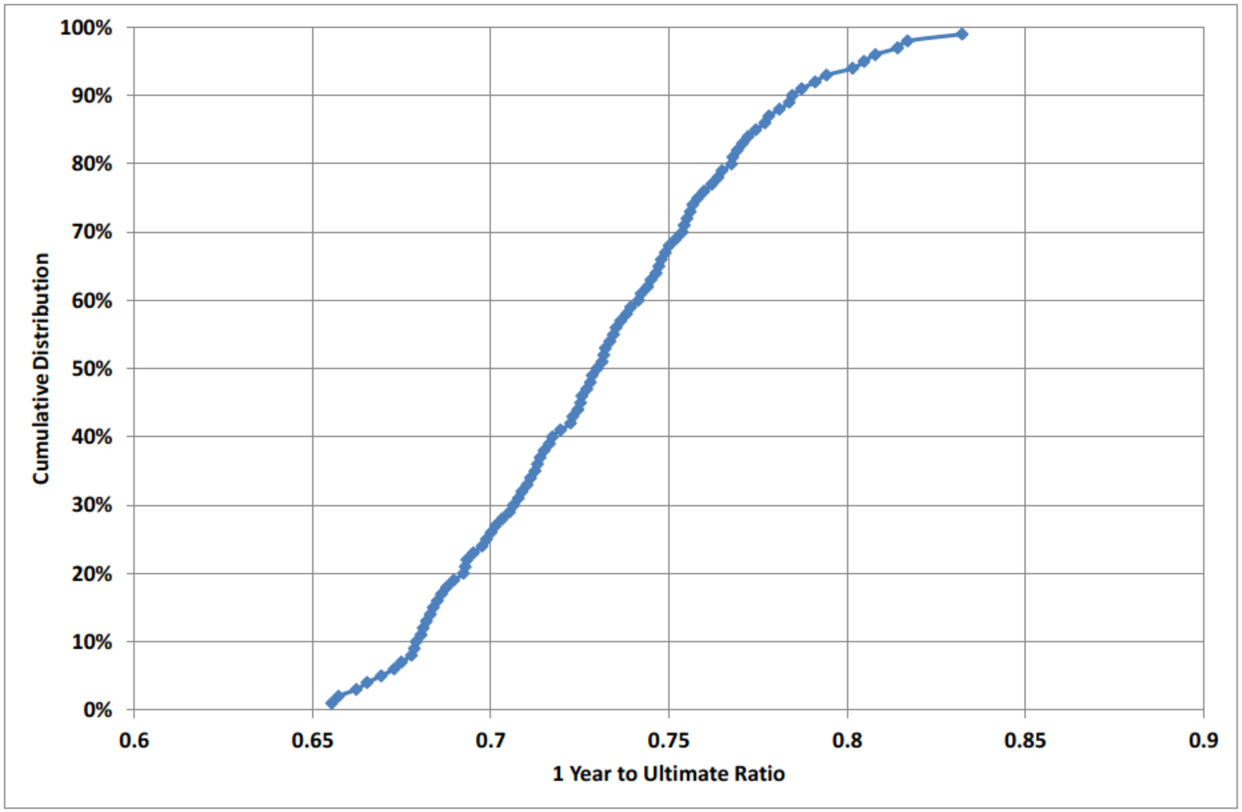
- Do you recognise this loss triangle?

		Development Year									
		1	2	3	4	5	6	7	8	9	10
AY	1	357,848	1,124,788	1,735,330	2,218,270	2,745,596	3,319,994	3,466,336	3,606,286	3,833,515	3,901,463
	2	352,118	1,236,139	2,170,033	3,353,322	3,799,067	4,120,063	4,647,867	4,914,039	5,339,085	
	3	290,507	1,292,306	2,218,525	3,235,179	3,985,995	4,132,918	4,628,910	4,909,315		
	4	310,608	1,418,858	2,195,047	3,757,447	4,029,929	4,381,982	4,588,268			
	5	443,160	1,136,350	2,128,333	2,897,821	3,402,673	3,873,311				
	6	396,132	1,333,217	2,180,715	2,985,752	3,691,712					
	7	440,832	1,288,463	2,419,861	3,483,130						
	8	359,480	1,421,128	2,864,498							
	9	376,686	1,363,294								
	10	344,014									



Error in the estimation of one year/ult.

- I estimated the Mack parameters of the triangle
- I simulated 500 triangles from this model (⟹no model/parameter error)
- Each time I estimated the 1 year to ultimate ratio using M-W



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Error in the estimation of 1YR to Ult. Ratio

- In this example, with no model/parameter error, a 90% Confidence Interval for the 1YR (st. dev.) has a rather wide range of 16% age points

What does this mean for the 1 Year Capital Charges?

Does this type of error diversify?

What does this mean for the use of benchmarks?

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Example of risk emergence

PPOs
Importance of the link between 1YR and risk margin

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PPO example: Assumptions

- 20 annuitants all aged 40 are assumed to follow the mortality of an ELT
- Mortality is assumed to be stochastic.
- Mortality changes are also assumed to be stochastic
  - Mortality changes assumed to follow a random walk
  - A jump process is also assumed to model sudden medical improvements
- Lives are assumed to be independent
  - apart from the dependence introduced by the stochastic changes in mortality which apply to all lives
- For simplicity, stochastic inflation and discounting have been ignored, although their impact may be large

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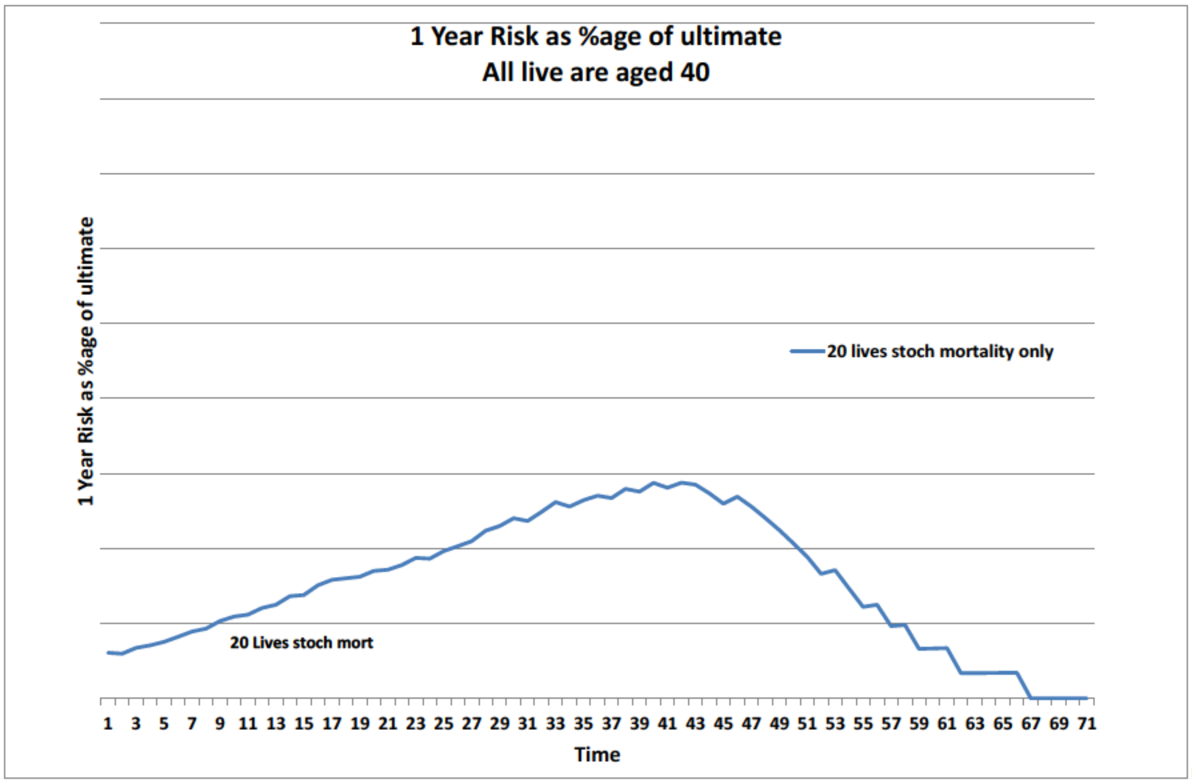
PPO example: Emergence of risk over time

- The emergence of risk in the future is much bigger than in the first year. Risk margin can be significant compared to 1YR



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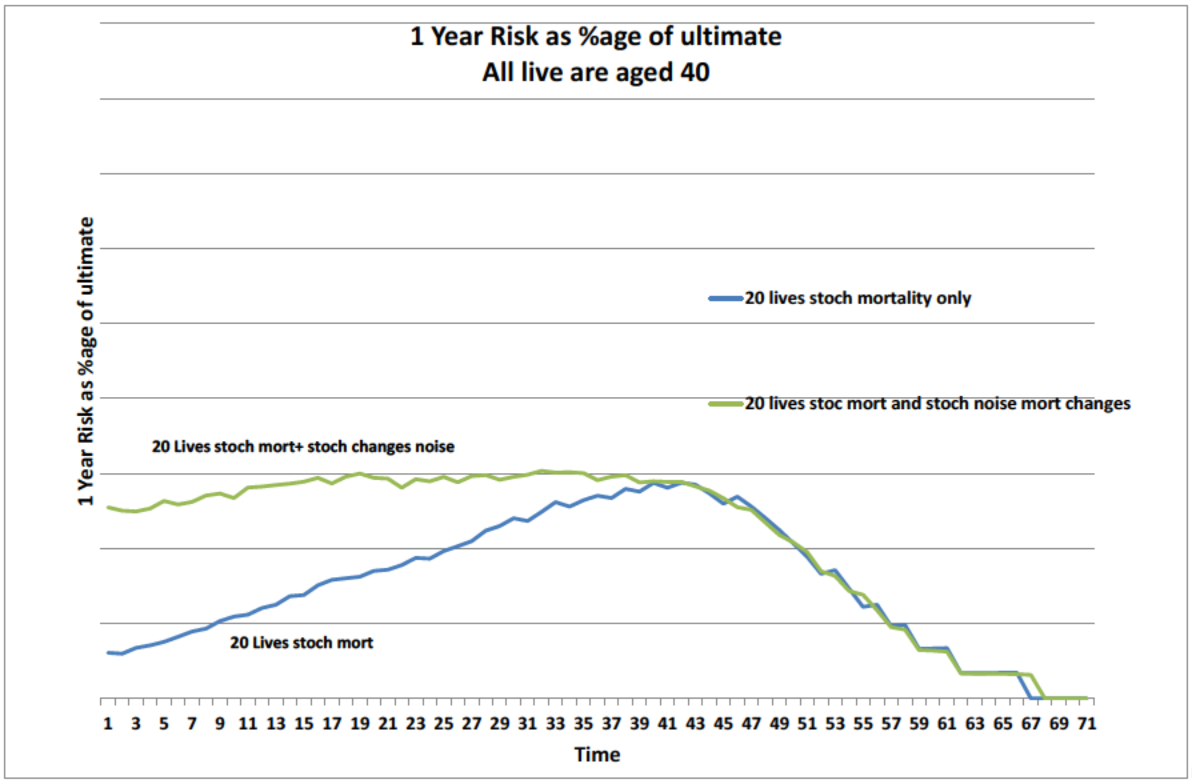
PPO example: Emergence of risk over time

- Both level and pattern of risk emergence varies significantly under different assumptions
- The risk margin can be significant compared to 1YR



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PPO example: Emergence of risk over time

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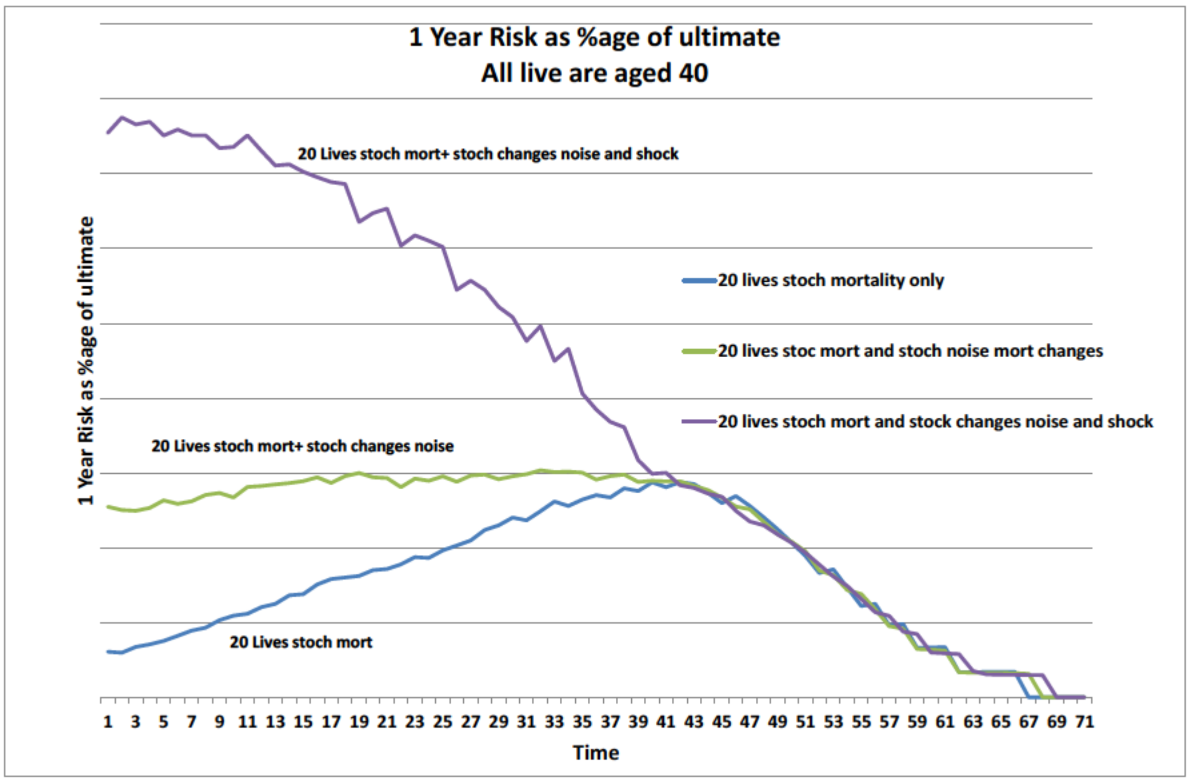

PPO example

The assumptions and the types of volatility in the model significantly affect the level and emergence pattern of risk

The risk margin can be significant compared to the 1YR

The emergence pattern is affected by the risk measure

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Relation between 1 year and ultimate correlations

Mathematical relations and implications

Relation of ultimate and emergence

• Risk X	X1	X2	X3
• Risk Y	Y1	Y2	Y3

Time =0 Time =1 Time =2 Time =3

- Xi and Yj represent the emergence of risk in year i and j
- $X = X_1 + X_2 + X_3$ and $Y = Y_1 + Y_2 + Y_3$
- $Cor(X_i, X_j) = 0, \quad if\ i \neq j$ and similarly for Y
- $Cor(X_i, Y_j) = 0, \quad if\ i \neq j$
- $Cor(X_i, Y_i) = \rho_i$

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Relation between ultimate and 1 year correlations

- The relations in the previous slide imply
- $Cov(X,Y) = Cov(X_1,Y_1) + Cov(X_2,Y_2) + Cov(X_3,Y_3)$ or
- $\rho_{ult} = \frac{\rho_1 \cdot s_{X_1} \cdot s_{Y_1} + \rho_2 \cdot s_{X_2} \cdot s_{Y_2} + \rho_3 \cdot s_{X_3} \cdot s_{Y_3}}{\sqrt{{s_{X_1}}^2 + {s_{X_2}}^2 + {s_{X_3}}^2} \cdot \sqrt{{s_{Y_1}}^2 + {s_{Y_2}}^2 + {s_{Y_3}}^2}}$
- If $\rho_1 = \rho_2 = \rho_3$, then $\rho_{ult} \leq \rho_1, \rho_2, \rho_3$
- If $\rho_1 < \rho_{ult}$, then ρ_2 and/or $\rho_3 > \rho_{ult}$,

  apart from some very unusual cases

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Example: inflation as driver of correlation

- Losses for classes X and Y in years 1 and 2 are assumed to be independent and follow a Lognormal distribution
- Now, we introduce log-normally distributed inflation which introduces dependency

	mean for Yrs 1 and 2	st dev for Yrs 1 and 2
LoB X	91	14
LoB Y	91	14
"inflation"	1.04	0.03

- Results: The 1 Yr. correlation between X and Y is higher than the ultimate

	Correlations
Ultimate	10%
1 Year	15%
Paid in Yr 1&2	5%

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Relation between ultimate and 1 year correlations

If $\rho_1 < \rho_{ult}$, then ρ_2 *and or* $\rho_3, \ldots > \rho_{ult}$.

If 1YR correlation lower than ultimate, then correlations in future years will be higher than ultimate

Is there a justification for this?

Drivers, such as inflation, which act over longer periods

do not necessarily imply higher ultimate than 1 year correlations

If the 1 year/ultimate ratio is large,

then generally 1 year and ultimate correlations will be close

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Summary

1YR includes changes in expectations about the future

1YR and 1yr/ultimate ratio is affected by the risk measure

1YR can not be higher than ultimate if the risk measure is coherent

The estimation error of the 1yr/ultimate ratio can be significant

The emergence of risk affects both the 1 year and risk margin and the two are related

If 1yr correlation is lower than ultimate, then correlations in future years will be higher than ultimate. Are there reasons for this?

Drivers of correlation, such as inflation, which act over longer periods **do not** necessarily imply higher ultimate than 1 year correlations

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Questions

Comments

Expressions of individual views by members of the Institute and Faculty of Actuaries and its staff are encouraged.

The views expressed in this presentation are those of the presenter.

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