Putting the science in actuarial science

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Agenda

• History
• Motivation behind our work
• Models that we are considering
• Empirical approach
• Results and implications
• Implications
• Limitations
History

• Mack & Murphy were the first to produce a statistical model for the case of volume-weighted link ratios

• Barnett and Zehnwirth (2000) consider alternative volatility structures as a part of their modelling framework.

• Bardis, Majidi and Murphy (2009) develop a “flexible factor model” to model reasonable link ratios
Motivation

• Existing approaches are theoretical and data is not considered in developing the model
• What would an empirical approach tell us about this problem?
• Implications on link ratio and volatility estimators?
• Implications on reserve and CoV estimates?
Mack/Murphy model

Mack/Murphy model:

\[ C_{ij+1} = \lambda_j C_{ij} + \sigma_j \sqrt{C_{ij}} \epsilon_{ij+1} \]

\[ E[C_{ij+1}|C_{ij}] = \lambda_j C_{ij} \]

\[ V[C_{ij+1}|C_{ij}] = \sigma_j^2 C_{ij} \]

• Maximum likelihood estimation (with a normal distribution) of the parameters gives the volume-weighted chain ladder estimators

\[ \hat{\lambda}_j = \frac{\sum_i C_{ij+1}}{\sum_i C_{ij}} \]

\[ \hat{\sigma}_j^2 = \frac{1}{n} \sum_i \left( \frac{C_{ij+1}}{C_{ij}} - \hat{\lambda}_j \right)^2 C_{ij} \]

• \( C_{ij} = \text{Cumulative incurred claims in origin } i, \text{ dev period } j \)
• \( \lambda_j, \sigma_j = \text{dev period specific parameters to be estimated} \)
• \( E(\epsilon_{ij}) = 0, \text{Var}(\epsilon_{ij}) = 1 \)
Flexible Factor Chain Ladder model

Flexible Factor Chain Ladder model (‘FFCL’):

\[ C_{ij+1} = c\lambda_j C_{ij} + c\sigma_j C_{ij}^{x_j} \varepsilon_{ij+1} \]

\[ E[C_{ij+1}|C_{ij}] = c\lambda_j C_{ij} \]

\[ V[C_{ij+1}|C_{ij}] = c\sigma_j^2 C_{ij}^{2x_j} \]

- Superscript c identifies company specific parameters, \( x_j \) is a ‘global’ parameter, constant across all companies for a specific development period
- Maximum likelihood estimation of the parameters gives the following formula for the \( c\lambda_j \) and \( c\sigma_j \) parameters

\[ c\hat{\lambda}_j = \frac{\sum_i \left( \frac{C_{ij+1}}{C_{ij}} \right) C_{ij}^{2-2x_j}}{\sum_i \left[ C_{ij}^{2-2x_j} \right]} \]

\[ c\hat{\sigma}_j^2 = \frac{1}{n} \sum_i \left( \frac{C_{ij+1}}{C_{ij}} - c\hat{\lambda}_j \right)^2 C_{ij}^{2-2x_j} \]

- \( C_{ij} \) = Cumulative incurred claims in origin i, dev period j
- \( c\lambda_j \), \( c\sigma_j \) = dev period specific parameters to be estimated, specific to the company

\( E(\varepsilon_{ij}) = 0, \) \( \text{Var}(\varepsilon_{ij}) = 1 \)
Comments on FFCL model

- Special cases when \( x_j = 1 \), 0.5 and 0 giving the simple average, volume-weighted and square volume-weighted chain ladder methods

\[
c\hat{\lambda}_j = \frac{\sum_i \left( \frac{C_{ij+1}}{C_{ij}} \right) C_{ij}^{2-2x_j} }{\sum_i [C_{ij}^{2-2x_j} ]}
\]

- The formula for \( c\hat{\lambda}_j \) shows that as \( x_j \) increases from 0 to 1, less relative weight is given to the link ratios which come from high volume years compared with low volume years

- So the value of \( x_j \) can give an indication as to the relative importance of high and low volume years

- Simple average = equal importance
Maximum likelihood solution (Normal model)

- No closed form solution for \( x_j \) exists, but solutions follow the equation for \( x_j \) given below:

\[
\sum_i \ln(C_{ij})(1 - \frac{(C_{ij+1} - \hat{c}_j C_{ij})^2}{c_\sigma_j^2 C_{ij}^{2x_j}}) = 0
\]

- Along with the solutions for \( \lambda_j^c \) and \( \sigma_j^c \):

\[
c_\lambda_j = \frac{\sum_i [(\frac{C_{ij+1}}{C_{ij}})C_{ij}^{2-2x_j}]}{\sum_i [C_{ij}^{2-2x_j}]}
\]

\[
c_\sigma_j^2 = \frac{1}{n} \sum_i (\frac{C_{ij+1}}{C_{ij}} - \hat{c}_j)^2 C_{ij}^{2-2x_j}
\]

- Solve these to give the estimators \( c_\lambda_j, c_\sigma_j \) and \( x_j \)
Empirical approach

- PRA returns – multi-company analysis
- Estimate the parameters $c\lambda_j$, $c\sigma_j$ and $x_j$ using maximum likelihood (normal model)
- Three lines of business considered (as per PRA definition):
  - Household – 15 companies
  - Employers Liability – 23 companies
  - Personal Accident – 13 companies
- Only extreme residuals were excluded
- Convergence issues when $c\lambda_j$ is close to 1

Results

- Calculated the value of $x_j$ for each of these lines of business and for as many development periods as the data would allow
- Calculated the best estimate reserve and bootstrap CoV (‘Coefficient of Variation’) for each of the companies using the three different approaches:
  - Simple average
  - Volume weighted
  - FFCL model
Key results

Table of $x_j$ calculated by development period and line of business:

<table>
<thead>
<tr>
<th>Line of business</th>
<th>Value of $x_j$</th>
<th>j = 1</th>
<th>j = 2</th>
<th>j = 3</th>
<th>j = 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Household</td>
<td></td>
<td>0.80</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>Employers Liability</td>
<td></td>
<td>0.93</td>
<td>0.76</td>
<td>0.80</td>
<td>0.83</td>
</tr>
<tr>
<td>Personal Accident</td>
<td></td>
<td>0.83</td>
<td>0.66</td>
<td>n/a</td>
<td>n/a</td>
</tr>
</tbody>
</table>

- Values of $x_j$ between 0.5 and 1
- Suggest potential value could be around 0.8
- No indication as to whether $x_j$ differs by line of business
- Suggests that less emphasis should be given to volume when calculating link ratios than purely performing a volume average
Reserve estimation

- Implication on reserves ranged from two extremes
  - FFCL model had a low impact for Case 1 lines (typically 1-2%)
  - Case 2 lines – significant impact ranging up to 10%

<table>
<thead>
<tr>
<th>Reserve estimate relative to volume-weighted estimate</th>
<th>Case 1 example</th>
<th>Case 2 example</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Volume-weighted</td>
<td>Simple-Average</td>
</tr>
<tr>
<td>Household</td>
<td>-</td>
<td>2%</td>
</tr>
<tr>
<td>Employers Liability</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Personal Accident</td>
<td>-</td>
<td>-1%</td>
</tr>
</tbody>
</table>
### CoV estimation

<table>
<thead>
<tr>
<th>CoV estimate relative to volume-weighted estimate</th>
<th>Case 1 example</th>
<th>Case 2 example</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Volume-weighted</td>
<td>Simple-Average</td>
</tr>
<tr>
<td></td>
<td>Volume-weighted</td>
<td>Simple-Average</td>
</tr>
<tr>
<td>Household</td>
<td>-</td>
<td>-1%</td>
</tr>
<tr>
<td>Employers Liability</td>
<td>-</td>
<td>1%</td>
</tr>
<tr>
<td>Personal Accident</td>
<td>-</td>
<td>5%</td>
</tr>
</tbody>
</table>

- Similar results deduced:
  - FFCL model had a low impact for Case 1 lines (typically 1-3%)
  - Case 2 lines – significant impact ranging up to 15%
Comments

- There is an impact when using the FFCL method instead of alternatives
- The impact is greater when triangles are not ‘regular’
- But hard to say which way the impact would be
Limitations

We may have got different results if we:

• Considered other lines of business
• Defined the grouping of data differently
• Obtained data from more companies
• Assumed a different error distribution function
• Considered ‘one-year’ CoVs as well as ‘to-ultimate’
• Ran the bootstrap procedure on a greater number of simulations

In particular, we haven’t:

• Considered the statistical significance of our results
• Compared the results between lines of business
• Investigated the data in the PRA templates
Final remarks

• Is the value of $x_j$ between 0.5 and 1? – Our results suggest that $x_j$ lies in this range but hard to assert the significance of this

• In practice it will be difficult to calculate

• Possible to use formulas to assess the sensitivity of reserving exercises to the value of $x_j$

• Or can use the stable / unstable rule-of-thumb

• No assertion about whether this error structure is appropriate, would an alternative structure where error doesn’t tend to zero be possible?
Expressions of individual views by members of the Institute and Faculty of Actuaries and its staff are encouraged.

The views expressed in this presentation are those of the presenter.