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Putting the science in actuarial science

Simon Margetts (EY) & Jacob Clark (Hiscox)

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Agenda

- History
- Motivation behind our work
- Models that we are considering
- Empirical approach
- Results and implications
- Implications
- Limitations

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History

- Mack & Murphy were the first to produce a statistical model for the case of volume-weighted link ratios
- Barnett and Zehnwirth (2000) consider alternative volatility structures as a part of their modelling framework.
- Bardis, Majidi and Murphy (2009) develop a “flexible factor model” to model reasonable link ratios

Motivation

- Existing approaches are theoretical and data is not considered in developing the model
- What would an empirical approach tell us about this problem?
- Implications on link ratio and volatility estimators?
- Implications on reserve and CoV estimates?

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Mack/Murphy model

Mack/Murphy model:

$$C_{ij+1} = \lambda_j C_{ij} + \sigma_j \sqrt{C_{ij}} \epsilon_{ij+1}$$

$$E[C_{ij+1} | C_{ij}] = \lambda_j C_{ij}$$

$$V[C_{ij+1} | C_{ij}] = \sigma_j{}^2 C_{ij}$$

- Maximum likelihood estimation (with a normal distribution) of the parameters gives the volume-weighted chain ladder estimators

$$\hat{\lambda}_j = \frac{\sum_i C_{ij+1}}{\sum_i C_{ij}}$$

$$\hat{\sigma}_j{}^2 = \frac{1}{n} \sum_i \left(\frac{C_{ij+1}}{C_{ij}} - \hat{\lambda}_j\right)^2 C_{ij}$$

- C_{ij} = Cumulative incurred claims in origin i, dev period j
- λ_j , σ_j = dev period specific parameters to be estimated
- $E(\epsilon_{ij}) = 0$, $Var(\epsilon_{ij}) = 1$

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Flexible Factor Chain Ladder model

Flexible Factor Chain Ladder model ('FFCL'):

$$C_{ij+1} = {}^{c}\lambda_j C_{ij} + {}^{c}\sigma_j {C_{ij}}^{x_j} \epsilon_{ij+1}$$

$$E[C_{ij+1}|C_{ij}] = {}^{c}\lambda_j C_{ij}$$

$$V[C_{ij+1}|C_{ij}] = {}^{c}{\sigma_j}^{2} {C_{ij}}^{2x_j}$$

- Superscript c identifies company specific parameters, x_j is a 'global' parameter, constant across all companies for a specific development period
- Maximum likelihood estimation of the parameters gives the following formula for the ${}^{c}\lambda_j$ and ${}^{c}\sigma_j$ parameters

$${}^{c}\hat{\lambda}_j = \frac{\sum_i [(\frac{C_{ij+1}}{C_{ij}}) {C_{ij}}^{2-2x_j}]}{\sum_i [{C_{ij}}^{2-2x_j}]}$$

$${}^{c}{\hat{\sigma}_j}^{2} = \frac{1}{n} \sum_i (\frac{C_{ij+1}}{C_{ij}} - {}^{c}\hat{\lambda}_j)^2 \, {C_{ij}}^{2-2x_j}$$

- C_{ij} = Cumulative incurred claims in origin i, dev period j
- ${}^{c}\lambda_j$, ${}^{c}\sigma_j$ = dev period specific parameters to be estimated, specific to the company
- $E(\epsilon_{ij}) = 0$, $Var(\epsilon_{ij}) = 1$

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Comments on FFCL model

- Special cases when $x_j = 1$, 0.5 and 0 giving the simple average, volume-weighted and square volume-weighted chain ladder methods

$$^{c}\hat{\lambda}_j = \frac{\sum_i [(\frac{C_{ij+1}}{C_{ij}}) C_{ij}^{\,2-2x_j}]}{\sum_i [C_{ij}^{\,2-2x_j}]}$$

- The formula for $^{c}\lambda_j$ shows that as x_j increases from 0 to 1, less relative weight is given to the link ratios which come from high volume years compared with low volume years
- So the value of x_j can give an indication as to the relative importance of high and low volume years
- Simple average = equal importance

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Maximum likelihood solution (Normal model)

- No closed form solution for x_j exists, but solutions follow the equation for x_j given below:

$$\sum_i \ln(C_{ij})(1 - \frac{(C_{ij+1} - {}^{c}\hat{\lambda}_j C_{ij})^2}{{}^{c}\hat{\sigma}_j{}^2 C_{ij}{}^{2x_j}}) = 0$$

- Along with the solutions for λ_j^c and σ_j^c:

$${}^{c}\hat{\lambda}_j = \frac{\sum_i [(\frac{C_{ij+1}}{C_{ij}}) C_{ij}{}^{2-2x_j}]}{\sum_i [C_{ij}{}^{2-2x_j}]}$$

$${}^{c}\hat{\sigma}_j{}^2 = \frac{1}{n} \sum_i (\frac{C_{ij+1}}{C_{ij}} - {}^{c}\hat{\lambda}_j)^2 \, C_{ij}{}^{2-2x_j}$$

- Solve these to give the estimators ${}^{c}\lambda_j$, ${}^{c}\sigma_j$ and x_j

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Empirical approach

- PRA returns – multi-company analysis
- Estimate the parameters ${}^{c}\lambda_j$, ${}^{c}\sigma_j$ and x_j using maximum likelihood (normal model)
- Three lines of business considered (as per PRA definition):
  - Household – 15 companies
  - Employers Liability – 23 companies
  - Personal Accident – 13 companies
- Only extreme residuals were excluded
- Convergence issues when ${}^{c}\lambda_j$ is close to 1

<u>Results</u>

- Calculated the value of x_j for each of these lines of business and for as many development periods as the data would allow
- Calculated the best estimate reserve and bootstrap CoV ('Coefficient of Variation') for each of the companies using the three different approaches:
  - Simple average
  - Volume weighted
  - FFCL model

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Key results

Table of x_j calculated by development period and line of business:

Line of business	Value of x_j			
	j = 1	j = 2	j = 3	j = 4
Household	0.80	n/a	n/a	n/a
Employers Liability	0.93	0.76	0.80	0.83
Personal Accident	0.83	0.66	n/a	n/a

- Values of x_j between 0.5 and 1
- Suggest potential value could be around 0.8
- No indication as to whether x_j differs by line of business

- Suggests that less emphasis should be given to volume when calculating link ratios than purely performing a volume average

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Reserve estimation

Reserve estimate relative to volume-weighted estimate	Case 1 example			Case 2 example		
	Volume-weighted	Simple-Average	FFCL model	Volume-weighted	Simple-Average	FFCL model
Household	-	2%	2%	-	9%	6%
Employers Liability	-	-	-	-	3%	3%
Personal Accident	-	-1%	-1%	-	17%	10%

- Implication on reserves ranged from two extremes
  - FFCL model had a low impact for Case 1 lines (typically 1-2%)
  - Case 2 lines – significant impact ranging up to 10%

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CoV estimation

CoV estimate relative to volume-weighted estimate	Case 1 example			Case 2 example		
	Volume-weighted	Simple-Average	FFCL model	Volume-weighted	Simple-Average	FFCL model
Household	-	-1%	-3%	-	-5%	-9%
Employers Liability	-	1%	-	-	33%	11%
Personal Accident	-	5%	2%	-	-16%	-14%

- Similar results deduced:
  - FFCL model had a low impact for Case 1 lines (typically 1-3%)
  - Case 2 lines – significant impact ranging up to 15%

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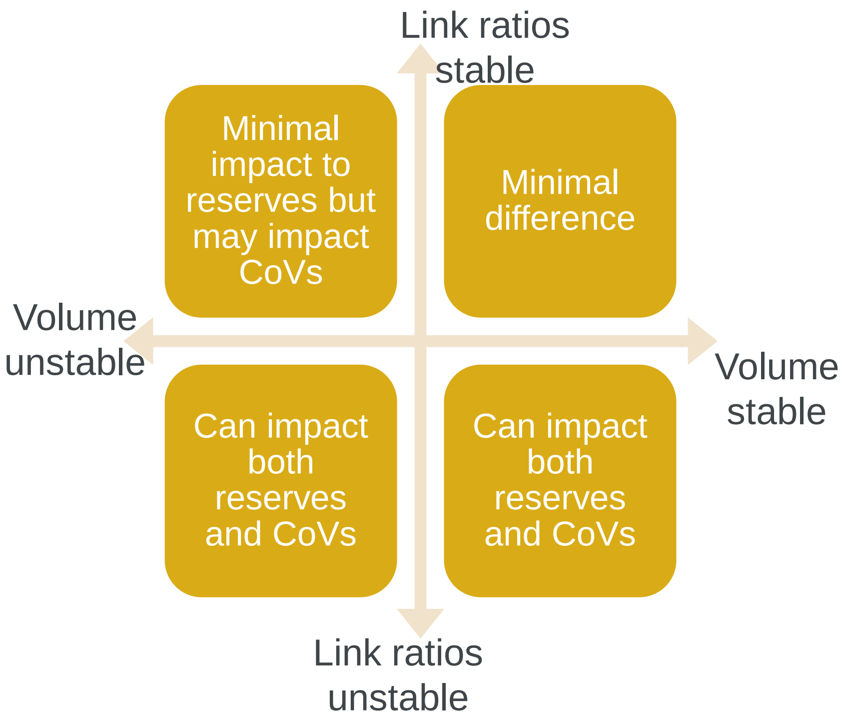
Comments

- There is an impact when using the FFCL method instead of alternatives
- The impact is greater when triangles are not ‘regular’
- But hard to say which way the impact would be



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Limitations

We may have got different results if we:

- Considered other lines of business
- Defined the grouping of data differently
- Obtained data from more companies
- Assumed a different error distribution function
- Considered ‘one-year’ CoVs as well as ‘to-ultimate’
- Ran the bootstrap procedure on a greater number of simulations

In particular, we haven’t:

- Considered the statistical significance of our results
- Compared the results between lines of business
- Investigated the data in the PRA templates

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Final remarks

- Is the value of x_j between 0.5 and 1? – Our results suggest that x_j lies in this range but hard to assert the significance of this
- In practice it will be difficult to calculate
- Possible to use formulas to assess the sensitivity of reserving exercises to the value of x_j
- Or can use the stable / unstable rule-of-thumb
- No assertion about whether this error structure is appropriate, would an alternative structure where error doesn’t tend to zero be possible?

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Questions

Comments

Expressions of individual views by members of the Institute and Faculty of Actuaries and its staff are encouraged.

The views expressed in this presentation are those of the presenter.