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Debt Portfolio Optimization

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Introduction

- CADES remit consists in the defeasance of the Social Security and Health public debt
- To achieve its task, the institution has been assigned a number of resources, based on taxation
- The first step of the « defeasance » activity is to borrow money from capital markets and pass it on to the Social Security account to clear accumulated deficits
- The second step is to repay debt : providing bondholders with coupon payments and returning their capital in the end
- The purpose of the paper is to show how from the asset and liability management standpoint we can help design both strategies of bond issuance and repayment

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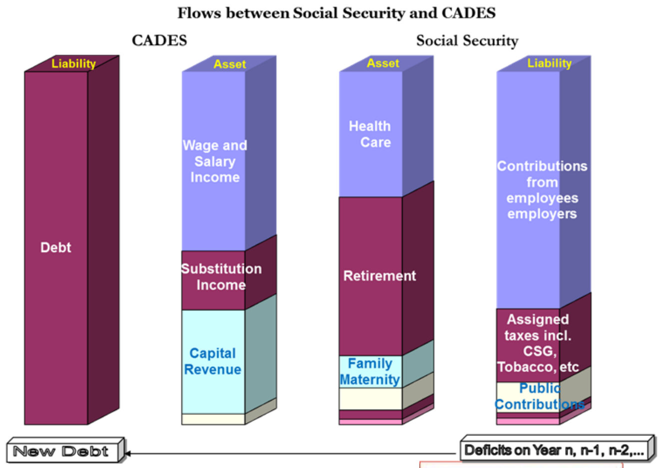
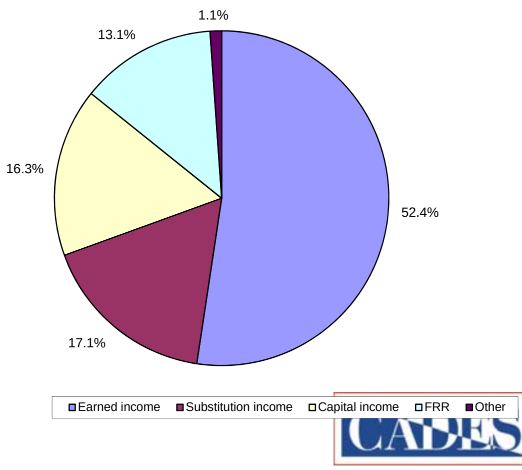
Debt transfer mechanism



Resource breakdown



Modelling the economy

- The initial value of the contribution revenue once assessed, its evolution can be modelled by a real growth rate, and a rate of inflation. Let

  g_t, *real growth rate*
  π_t, *inflation rate*
  and k_t *the value growth rate in continuous-time is the sum* $g_t + \pi_t$

- The economy is also driven by the short-term interest rate. We assume its dynamic follows a Ornstein-Uhlenbeck process, along with the real growth rate and the inflation rate. We specify a Vasicek formulation of the zero-coupon rate curve. For instance the short-term interest rate evolution has the following differential equation :

  $$dr(t) = a(b - r(t))dt + \sigma_r dW_r(t)$$

- Each of the three variables contains a source of risk modelled by a Wiener process ; the three Brownian motions are linked one-another by their cross-correlations. After some transformation one gets a system of three equations with uncorrelated sources of risk. For instance the inflation rate dynamic will be modelled by the following equation :

  $$d\pi(t) = c(d - \pi(t))dt + \rho_{r,\pi}\sigma_\pi dW_r(t) + \sqrt{1-\rho_{r,\pi}^2}\,\sigma_\pi dZ_\pi(t)$$

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Modelling the balance-sheet

- The asset A_t has the following dynamic

  $$dA_t = A_t k(t) dt$$

- Knowing a given liability amount L_t,
  - We compute the disposable as the resources/expenses balance.
  - From there, the net balance is attained by subtracting payables.
  - When positive we can further reduce the existing debt with buy-backs, otherwise a financing requirement is met with an issuance program. The value of the net debt annual variation will fluctuate with the cheap/dearness of the rates environment.

- Our goal is to optimize the yearly net debt reduction, or amortization. We describe the optimization problem in both static and dynamic frameworks

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Optimization in a static framework

- Along with Martellini and Milhau (2009), let us define G_t the asset-liability margin and Q_t the funding ratio, with
  $G_t = A_t - L_t$ and $Q_t = \frac{A_t}{L_t}$
- The institution's goal is to maximize the margin utility expectation at a given horizon T, $E[u(G_T)]$, upon the debt portfolio allocation vector X,
- The optimal solution found by the authors is then (in the absence of a riskless interest rate)

$$X = \frac{\sum_L^{-1} e_b}{C}\left(1 + \frac{Q_0}{\beta} e_b^t \sum_L^{-1} \mu_L - Q_0 e_b^t \sum_L^{-1} \psi\right) - \frac{Q_0}{\beta} e_b^t \sum_L^{-1} \mu_L + Q_0 \sum_L^{-1} \psi$$

with

$$C = e_b^t \sum_L^{-1} e_b$$

- One recognizes the three funds from the separation theorem : the minimum variance portfolio, the "tangent" portfolio, and the asset hedge portfolio

Optimization in a dynamic framework

- Optimizing the asset-liability margin can read

$$\max_X E\left[u\left(G_T - G_0\right)\right]$$

- The optimization program can be written as

$$\max_X E\left[u\left(\int_{t=0}^{T} \left\{A_0 e^{\int_{s=0}^{t} k(s)ds} - L_t X^t \Lambda\right\} + G_0\right)\right]$$

- Under the following constraints
- $dg = gdt + \sigma_g dW_t{}^g$
- $d\varphi = \varphi \pi_t dt$
- $dr = a(b - r)dt + \sigma_r dW_t{}^r$

with

- φ_t the time t price index,
- k_t the nominal revenue rate of growth obtained by compounding the rate of growth in real terms g(.) and the inflation rate π(.),
- r_t the money market rate,

Optimization in a dynamic framework (ctd)

- the temporal variation of the cost of debt dL_t/L_t is expressed in vector form as $X^t\Lambda_.$ For if we name L^1 the debt portion comprising nominal bonds, L^2 the one with inflation-linked notes, L^3 the last one with notes running at the money market rate, then the column vector $\Lambda_.$ reads

$$\Lambda_1 = R(t)dt$$

$$\Lambda_2 = \frac{d\varphi_t}{\varphi_t} + \eta(t)dt$$

$$\Lambda_3 = r(t)dt$$

where R(t) is the cost of the nominal bond

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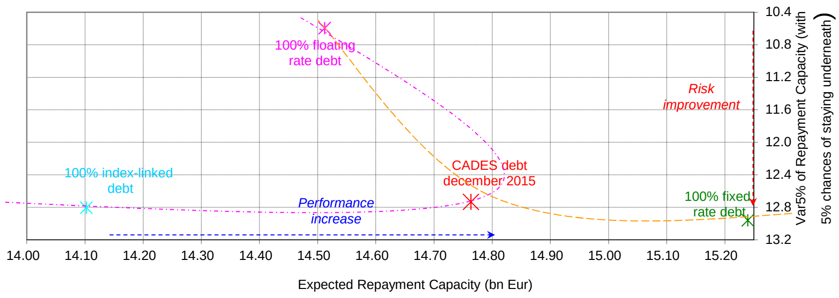
Outputs



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Routes of enhancement

- Bring the debt transfer phenomenon into the model, i.e. change its status from exogenous to endogenous
- Change the temporal step of the model from a yearly frequency to an infra-annual frequency (quarterly e.g.)
- Enhance the debt portfolio representation with a much detailed approach of the short-term notes portfolio

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Debt transfer

- Debt transfer is similar to a “jump” in the process of debt amortization, otherwise a rather smooth process. Usually, two things matter : time (frequency) and magnitude
- Observed facts teach us that deficits accumulation looks like a recurring process. This process bears no obviously perceivable constant frequency, and no constant magnitude either – the magnitude meaning the size of the cumulative deficits at the time public policy decides it must be stopped and transferred to a defeasance agency.

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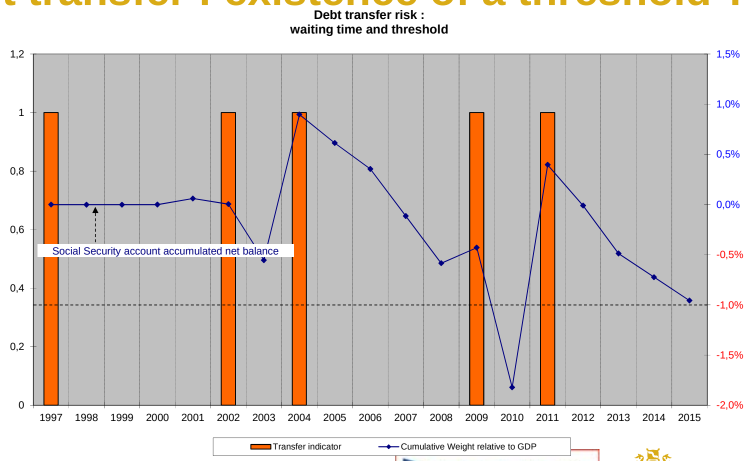
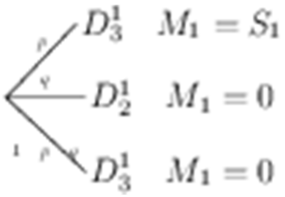
Debt transfer : existence of a threshold ?



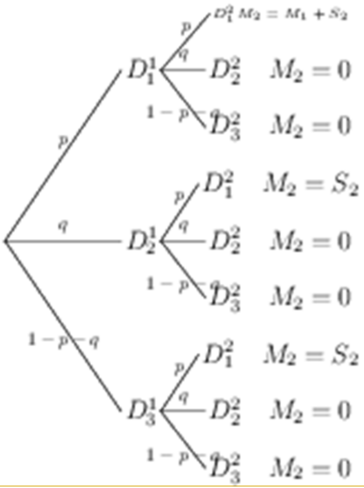
Debt transfer : attempt to modelize the public policy decision rule

- The governing body in charge of social affairs budget facing a new deficit on a given year has a choice between :
  - let the deficits build up, and have the Social Security treasurer finance the deficit by short-term borrowing
  - clear the new deficit by tax levy, or compensate by a cut in spending somewhere else
  - turn the accumulated deficits into a new debt and transfer it to the defeasance agency

Debt transfer : attempt to modelize the public policy decision rule



Debt transfer : attempt to modelize the public policy decision rule



Debt transfer : the "jump option"

- Let at time t_2, S_2 be the new deficit, C^2 the global cost of public choice regarding deficits as of time t_2, p and q the marginal probability values for deciding to postpone debt transfer or clear S_2 deficit by tax levy, respectively,
- We can write the price of the risk of a debt "jump" seen from time t_1, as the cost share associated with solution n°3

$$E_{t_1}(R) = \frac{1}{1+r} \frac{(1-p-q)E\left(\cos t\left(S_2 \middle| D_3^1\right)\right)}{E\left(C^2\right)}$$

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Debt modelization

- A general formulation of the debt process stochastic equation could then read

$$dL_t = L_t\left(\Phi_t + H_t dJ_t\right)$$

- where
- Φ_t represents the amortization process
- H_t is the magnitude of the jump
- J_t the jump process, defined by

$$dJ_t = (1-p-q)dt$$

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Questions

Answers

Thank you

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Questions

Comments

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