The Stochastic Full Balance Sheet Model
Bill Curry

Surplus projections

![Graph showing surplus projections over years]

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Surplus projections

Key questions in risk management

- What is the cost to our solvency position of a 20% equity fall?
- What fall in solvency is a 1 in 10 loss?
- What is the probability our capital coverage ratio goes below 120%?
- What is the probability we breach SCR?
- What event is most likely to cause a breach of SCR?
Risk appetite

- Risk appetite buffers take into account the stability of the capital position

Types of group model

- Regulatory Capital
- Economic Capital
- Full Balance Sheet Model
Group model components

Model Structure

Risk Model

Loss Function

Group Model

• Majority of firms use copula simulation models
• Individual distributions specified for each risk

Group model components

Regulatory capital – SII Internal Model

Model Structure

• Model structure part prescribed under SII

Risk Model

Loss Function

• Losses to assets and liabilities normally estimated through proxy functions
Group model components
Regulatory capital – SII Standard Formula

Model Structure
• Model structure mostly prescribed under SII

Risk Model

Loss Function
• Losses represented by linear loss functions fitted to the 1in200 points
• No cross terms to represent interaction between risks

Group model components
Economic capital

Model Structure
• Model structure not prescribed, however commonly similar to the regulatory capital model

Risk Model

Loss Function
• Commonly similar to the regulatory model
Differences could be in:
• MA or VA
• Pension valuation
• Contract boundaries
• Etc.

• May be similar to a firm’s regulatory model
• Other risks could be included
• Different calibrations could be used.
Group model components
Stochastic full balance sheet model

- Model Structure
  - Structure may be close to economic capital model

Risk Model
- Loss Function
  - Represents a firm's best view of risks
  - Likely to be aligned to economic capital model
  - Losses represent changes in the full SII balance sheet rather than just assets and liabilities
  - Need to allow for realistic changes in discount rates (VA, MA, IAS19)

Types of group model
- Regulatory Capital
- Full Balance Sheet Model

Strength of the Capital Position

Stability of the Capital Position
Model Features

Simulation generation

- Simulation generation may use standard copula modelling techniques

We should consider

- Should risk calibrations be Point In Time or Through The Cycle?
Proxy models

- The purpose of a proxy model is to enable fast estimation of balance sheet changes as a function of risk movements.
Proxy models

- The purpose of a proxy model is to enable fast estimation of balance sheet changes as a function of risk movements.

Roll forwards

- Roll forwards techniques are used to estimate how loss functions change
- Interest rate example,
  Change in NAV = X – 20X^2
Roll forwards

- Roll forwards techniques are used to estimate how loss functions change
- Interest rate example,

\[
\text{Change in NAV} = X - 20X^2
\]

Say we have a 1.6% interest rate increase

\[
\text{New Change in NAV} = (X+1.6\%) - 20(X+1.6\%)^2
\]

\[
= 0.36X - 20X^2 + 0.01088
\]
The solvency II balance sheet

- **Transitional Measures**
- **Assets**
  - SCR
  - Risk Margin
  - BEL
- **Surplus**

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Asset and liability modelling

- Asset and liability models typically the same as used in an Internal Model or Economic Capital model

  Difficulties may arise over discount rates used for:
  - Volatility Adjustment (VA) business
  - Matching Adjustment (MA) business
  - Pension liabilities

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Example model

Annuity example - loss model

• 100,000 60 year old annuitants
• Annuity amount £1,000 p.a.
• Expenses of £100, inflating at 1% p.a.
• Mortality as per an example mortality table
• Yield curve flat at 2%
• Risk free fixed interest cash-flows to broadly match the liability run off
Annuity example - risk model

- Normally distributed risks assumed for
  - Longevity
  - Expense
  - Inflation
  - Interest Rate PC1
  - Interest Rate PC2
  - Interest Rate PC3
  - Credit

- Risks aggregated using a Gaussian copula with specified correlations

<table>
<thead>
<tr>
<th></th>
<th>Longevity</th>
<th>Inflation</th>
<th>Expense</th>
<th>PC1</th>
<th>PC2</th>
<th>PC3</th>
<th>Credit</th>
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<tbody>
<tr>
<td>Longevity</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
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<td>Inflation</td>
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<td>100%</td>
<td>0%</td>
<td>50%</td>
<td>0%</td>
<td>0%</td>
<td>-20%</td>
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<tr>
<td>Expense</td>
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<td>0%</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>PC1</td>
<td>0%</td>
<td>50%</td>
<td>0%</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
<td>-25%</td>
</tr>
<tr>
<td>PC2</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>PC3</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
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<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td>Credit</td>
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<td>0%</td>
<td>-25%</td>
<td>0%</td>
<td>0%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Annuity example - principal components analysis

- PCA is a dimension reduction technique commonly used to model yield curve changes

![Diagram showing PCA analysis with PC1, PC2, and PC3 principal components]
Annuity example – fitting approach

- Use Sobol sequence of 1023 fitting points
- Polynomial terms up to order 4 used to fit function to net assets
- Step regression applied to fit the proxy functions
- Out Of Sample (OOS) testing carried out using 100 random points from the risk model.

\[
\text{Proxy change in NAV} = f(L,I,E,PC1,PC2,PC3)
\]
Annuity example – joint exposure PC1 vs longevity

Longevity (change in qx)

0%

15%

-15%

-30%

Threat direction

SCR Proxy Modelling
SCR proxy models – SF

1. Calibration runs
2. Generate proxy functions for each run
3. Use proxy function to get SF capitals
4. Fit SCR proxy function
5. Roll Forward Process
6. OOS testing

SCR proxy models – Standard Formula

1. SCR Risk Model
2. Equity Risk Model
3. Interest Rate Risk Model
SCR proxy models – Standard Formula - longevity

- SF Longevity stress is a 20% fall in $q_x$

\[ SF \text{ Longevity Capital Estimate} = -f(-20\%, 0, 0, 0, 0, 0) \]

SCR proxy models – Standard Formula - expense

- SF Expense stress is a 10% increase in expenses, together with a 1% increase in expense inflation
- Our example model uses a separate expense level risk and inflation risk
- Estimate SF expense capital using a combined expense and inflation event

\[ SF \text{ Expense Capital Estimate} = -f(0, 1\%, 10\%, 0, 0, 0) \]
SCR proxy models – Standard Formula – interest rates

- SF Interest Rate up and down stresses are a function of the current yield curve

**Example, at yields of 2%, SF Yield down Capital Estimate = \(-f(0,0,-1.23,0.25,-1.16)\)**
SCR proxy models – Standard Formula

SCR proxy models – Standard Formula – key exposures
SCR proxy models – Internal Model

Calibration runs → Generate proxy functions for each run → Run SCR model for each run → Fit SCR proxy function → OOS testing

Roll Forward Process

Change in NAV vs Change in Interest Rates

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SCR proxy models – Internal Model

IM SCR Proxy Fit

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SCR proxy models – Internal Model key exposures

Risk Margin Proxy Modelling
Risk Margin proxy models

- Estimate changes in RM for calibration set
- Fit proxy function
- OOS testing for fit performance

Changes in Discount rates
No VA/MA in Risk Margin
Changes in Run-off

Risk Margin proxy models – run off example

- Annuity BEL run off under stress
Risk Margin proxy models – Standard Formula

RM proxy models – Standard Formula – key exposures
Standard Formula example – key exposures

Discount Rates modelling
Liability modelling – Volatility Adjustment

- VA represents a flat addition to the discount curve for applicable long term liabilities
- Designed to protect insurers from the impact of volatility on the insurer’s solvency position
- Calculated as 65% of the spread between the interest rate of the assets in a reference portfolio and the risk free rate, allowing for a fundamental spread
- Published monthly by EIOPA
- Permitted to change under SCR stress by some European supervisors.

How it works in practice

<table>
<thead>
<tr>
<th>Base</th>
<th>Credit stress event</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets</td>
<td>Liabilities</td>
</tr>
</tbody>
</table>

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Liability modelling – Volatility Adjustment

- How it works under (UK) SCR calculations

Liability modelling – dynamic VA model

- VA ~ 65% x (Spread – Fundamental Spread) calculated by rating, maturity
Liability modelling – dynamic VA model

- VA ~ 65% x (Spread – Fundamental Spread) calculated by rating, maturity

### Flat

<table>
<thead>
<tr>
<th>FS</th>
<th>0.0%</th>
<th>0.2%</th>
<th>0.4%</th>
<th>0.6%</th>
<th>0.8%</th>
<th>1.0%</th>
<th>1.2%</th>
<th>1.4%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spread</td>
<td>0.0%</td>
<td>1.0%</td>
<td>2.0%</td>
<td>3.0%</td>
<td>4.0%</td>
<td>5.0%</td>
<td>6.0%</td>
<td></td>
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</tbody>
</table>

### Linear

<table>
<thead>
<tr>
<th>FS</th>
<th>0.0%</th>
<th>0.2%</th>
<th>0.4%</th>
<th>0.6%</th>
<th>0.8%</th>
<th>1.0%</th>
<th>1.2%</th>
<th>1.4%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spread</td>
<td>0.0%</td>
<td>1.0%</td>
<td>2.0%</td>
<td>3.0%</td>
<td>4.0%</td>
<td>5.0%</td>
<td>6.0%</td>
<td></td>
</tr>
</tbody>
</table>
Liability modelling – dynamic VA model

- VA ~ 65% x (Spread – Fundamental Spread) calculated by rating, maturity
Liability modelling – dynamic VA model

- We need realistic models to take into account the movement of these under stress.

Matching Adjustment

IAS19 Discount Rates
Include credit risk as an additional normally distributed risk in the model.

Annuity example - dynamic VA, SF model

Repeat curve fit process for net assets and SCR, RM unchanged
Annuity example - dynamic VA, SF model

Low materiality changes in other risk exposures

Using the Example Model
More assumptions

- Starting surplus = £250m
- Risk Appetite thresholds (based on a one year time frame):
  - Plan to be able to withstand a 1in30 shock
  - We take urgent action if our surplus is unable to withstand a 1in10 shock

Risk appetite

![Risk appetite graph showing density and change in NAV (£m)]
Ruin probabilities

Ruin Prob = 5.9%

Euler allocation by risk – ruin event of £250m loss

• Euler Allocation of 250m loss
Risk A allocation = $E[X|x_{total} = -250m]$
Ruin Events

Euler example

- Risks A and B, multivariate standard normal
- Correlation -99.9%

\[ Change \text{ in NAV} = 2 - (e^A + e^B) \]

1 in 200 capital = 14.8m

Euler allocations:
- Risk A = 7.4m
- Risk B = 7.4m
Ruin events – Most Likely Ruin Event

Ruin region

MLRE

Risk movement

Density

Ruin region

MLRE

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Ruin events – Euler example

- Risk distribution is multivariate normal (-99.9% correlation)
- Density function of A,B is well known \( f(A,B) \)
- Can solve for the maximum of \( f(A,B) \) subject to constraint \[ 2 - (e^A + e^B) = -14.8 \]

Max at \((A,B) = (-2.8,2.8)\) and \((2.8,-2.8)\)
Ruin events – annuity model

- Find MLRE by maximising probability density subject to change in NAV < -£250m
- Risk distribution ~ Multivariate normal so density is well defined
- Change in NAV estimated using proxy functions.

<table>
<thead>
<tr>
<th>Risk</th>
<th>Movement</th>
<th>Percentile</th>
<th>1 in X</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC1</td>
<td>0.437</td>
<td>0.67</td>
<td>3.0</td>
</tr>
<tr>
<td>PC2</td>
<td>-0.209</td>
<td>0.42</td>
<td>2.4</td>
</tr>
<tr>
<td>PC3</td>
<td>-0.128</td>
<td>0.45</td>
<td>2.2</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.6%</td>
<td>0.78</td>
<td>4.6</td>
</tr>
<tr>
<td>Credit (spreads)</td>
<td>-1.8%</td>
<td>0.12</td>
<td>8.1</td>
</tr>
<tr>
<td>Longevity ($Q_x$)</td>
<td>-3.8%</td>
<td>0.31</td>
<td>3.2</td>
</tr>
<tr>
<td>Expense (level)</td>
<td>14.1%</td>
<td>0.77</td>
<td>3.0</td>
</tr>
</tbody>
</table>

![Yield stress graph](image)

Ruin events – annuity model

![Risk as % of 1in10 graph](image)

![Loss by Risk graph](image)
Ruin events – Kernel Density Estimation

- We can use Kernel Density Estimation (KDE) to estimate the density function of the joint risk distribution from the simulations.

Ruin events – annuity model
Ruin cause

<table>
<thead>
<tr>
<th>No.</th>
<th>Event</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Credit</td>
<td>56</td>
</tr>
<tr>
<td>2</td>
<td>Credit / Inflation</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>Inflation</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>Expense</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>Credit / Expense</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Others</td>
<td>11</td>
</tr>
</tbody>
</table>

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Ruin cause

<table>
<thead>
<tr>
<th>No.</th>
<th>Event</th>
<th>Credit (change in spreads)</th>
<th>Inflation (change in RPI)</th>
<th>Expense (change in level)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Credit</td>
<td>-3.5%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Credit / Inflation</td>
<td>-2.6%</td>
<td>0.9%</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Inflation</td>
<td>2.8%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Expense</td>
<td></td>
<td></td>
<td>68.7%</td>
</tr>
<tr>
<td>5</td>
<td>Credit / Expense</td>
<td>-2.5%</td>
<td></td>
<td>19.6%</td>
</tr>
</tbody>
</table>

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Ruin events summary

- Euler is about capital allocation not events
- MLREs give insight into actual events
- KDE can be used to get the density from the simulations
- We can plan what we would do under the ruin events

Uses of the model
The roll forward cycle

Roll Forwards for:
- Run off
- New Business
- Economics
- Basis Changes
- Model changes
- Risk calibration changes

Assess Roll forwards performance
Recalibrate
Roll Forwards

Projections
- Is our balance sheet getting more or less stable over time?
- Is our ruin probability getting better or worse?
Projections

• Proxy functions normally express changes in NAV as a function of risk movements

\[ \text{Proxy change in NAV} = f(L,I,E,PC1,PC2,PC3) \]

• For projections (e.g. of risk appetite), we need to calibrate as a function of risks and time

\[ \text{Proxy change in NAV} = f(L,I,E,PC1,PC2,PC3,\text{Time}) \]

• We can use run off drivers by risk and product to scale the proxy functions over time

Projections - example

• For Risks X and Y

\[ \text{Proxy change in NAV} = aX^2 + bX + cY + d XY \]
What isn't in your model?

- Regulatory risk?
- Changes to business plan?
- Liquidity risk?
- Long term risks?
- Regime changes?

Types of group model

- Regulatory Capital
- Economic Capital
- Full Balance Sheet Model