How long is forever?

Using annual capital re-allocation to calculate a rate of return on capital

Eric Pizarro
Disclaimer

THIS PRESENTATION REFLECTS THE VIEWS OF THE SPEAKER AND NOT NECESSARILY THOSE OF ALLIED WORLD
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   Defining the return on capital

II. How do we get there?
   A practical methodology

III. How will we know when we’ve arrived?
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IV. What are the limitations of the approach?
    Pitfalls & considerations

V. Q & A
I. What are we aiming for?
Defining the ROC
**The hindsight view: Return on Equity (ROE)**

\[
ROE = \frac{\text{Net Profit}}{\text{Shareholder’s Equity}}
\]

- The **ROE** compares the profits generated in the year to the equity held.
- Calculated from the Financials.
- A widely used measure of investment performance.

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**Key figures of non-life insurers (in CHF thousands)**

<table>
<thead>
<tr>
<th></th>
<th>2017</th>
<th>2016</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross premiums written</td>
<td>49,241,703</td>
<td>47,967,106</td>
</tr>
<tr>
<td>Claims paid out</td>
<td>25,961,281</td>
<td>25,811,840</td>
</tr>
<tr>
<td>Cost for the change in technical liabilities</td>
<td>597,371</td>
<td>12,564</td>
</tr>
<tr>
<td>Cost for the change in other actuarial liabilities</td>
<td>798,173</td>
<td>645,157</td>
</tr>
<tr>
<td>Costs for underwriting</td>
<td>10,097,966</td>
<td>9,548,200</td>
</tr>
<tr>
<td>Taxes</td>
<td>739,231</td>
<td>757,187</td>
</tr>
<tr>
<td>Gains/losses from investments</td>
<td>6,060,406</td>
<td>6,517,964</td>
</tr>
<tr>
<td><strong>Annual profits</strong></td>
<td><strong>5,319,225</strong></td>
<td><strong>7,018,905</strong></td>
</tr>
<tr>
<td>Balance sheet total</td>
<td>169,054,218</td>
<td>167,036,747</td>
</tr>
<tr>
<td>Investments</td>
<td>150,572,335</td>
<td>148,589,158</td>
</tr>
<tr>
<td>Technical liabilities</td>
<td>84,669,696</td>
<td>83,445,422</td>
</tr>
<tr>
<td><strong>Equity (before profit allocation)</strong></td>
<td><strong>38,987,401</strong></td>
<td><strong>39,385,332</strong></td>
</tr>
<tr>
<td>Return on investments (in %)</td>
<td>4.06%</td>
<td>4.44%</td>
</tr>
<tr>
<td>Return on equity (in %)</td>
<td>13.69%</td>
<td>17.82%</td>
</tr>
<tr>
<td>Loss ratio (in %)</td>
<td>65.7%</td>
<td>61.6%</td>
</tr>
</tbody>
</table>

The prospective view (I)

• Planning objective: to maximize the *reward* (profit earned) vs. the *risk* to the balance sheet from writing the business

• Return on capital (ROC) is a risk/reward metric
  – *Risk* is measured using economic capital
  – *Reward* is the expected (ultimate) profit

• How should we define a ROC for an insurance LOB? Remember that in the real world:
  – The profit may be earned over many years
  – Capital is released as claims are paid
  – Capital must be held as long claims are open
• Let’s look at two potential analogs from finance
  – Risk Adjusted Return on Capital (RAROC)
  – A mortgage APR
RAROC: From banking…

- Concept developed at Banker’s Trust in the 1970s

\[ RAROC = \frac{\text{Revenue} - \text{Costs} - \text{Expected losses}}{\text{Risk capital}} \]

- Like a prospective ROE, using economic capital
Now widely used in insurance, e.g., *Risk-Adjusted Performance Measurement for P&C Insurers* by Goldfarb (on CAS syllabus)

Table 23: Comparison of RAROC - Using Co-CTE Allocation

<table>
<thead>
<tr>
<th></th>
<th>Economic Profit</th>
<th>Co-CTE (99%) Allocated Capital</th>
<th>RAROC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line A</td>
<td>496,000</td>
<td>2,117,082</td>
<td>23.4%</td>
</tr>
<tr>
<td>Line B</td>
<td>880,000</td>
<td>4,225,340</td>
<td>20.8%</td>
</tr>
</tbody>
</table>

Allocated ultimate capital @T0

Earned over the lifetime of the claims “to ultimate”
RAROC assessment

But does it “work” for insurance?

In general – “No”*

- The reward is the ultimate profit
- The risk is the capital held in the first year only
- A mismatch: There can be no cost of capital without the capital

* Goldfarb (2010) presents alternative approaches that account for this; see p. 41
Mortgage APR

• Now for something more familiar…

• In a typical mortgage, for the bank/lender:
  – The *reward* is the interest payments received
  – The (debt) capital at *risk* is the outstanding loan amount
  – The principal repaid is capital returned

![Diagram showing Outstanding Balance and Interest Payments over time]

10 September 2019
Mortgage APR assessment

• How does it compare to insurance? In general – quite well

<table>
<thead>
<tr>
<th></th>
<th>Mortgage</th>
<th>Insurance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital type</td>
<td>Debt (loan)</td>
<td>Economic, GAAP, etc.</td>
</tr>
<tr>
<td>Capital recipient</td>
<td>Home owner</td>
<td>Underwriter</td>
</tr>
<tr>
<td>Investment</td>
<td>Houses</td>
<td>Insurance contracts</td>
</tr>
<tr>
<td>Profit</td>
<td>Interest on loan</td>
<td>UW profit</td>
</tr>
<tr>
<td>Investment horizon</td>
<td>Term of mortgage</td>
<td>Time claims are open</td>
</tr>
<tr>
<td>Maximum loss</td>
<td>Outstanding balance</td>
<td>Capital held at start of year</td>
</tr>
<tr>
<td>Annual rate of return</td>
<td>APR</td>
<td>Annual rate of ROC</td>
</tr>
</tbody>
</table>
Mortgage APR

• Mortgage analogy for RAROC:
  – The initial loan amount and total interest are known, but not the repayment schedule
  – (A bad idea)

• We can build on these comparisons to define an annual rate of ROC for an LOB…

So many different investment opportunities… how can I compare them?
Four principles for $R$, the rate of ROC

1. Capital must be held as long as claims are open \textit{(investment horizon)}

2. The total capital requirement varies by year according to the downside risk of open claims \textit{(amount of total capital)}

3. The amount of capital allocated to a LOB depends on its contribution to the total capital requirement \textit{(amount of allocated capital)}

4. $R$ is the effective annual rate of return paid to the capital providers over the period that their capital is invested \textit{(normalizes for the amount of capital and the investment horizon)}
The UW account

**Underwriting Account**
Receives *premium* and pays *loss and expense*

\[
P_0 - L_0 \\ P_1 - L_1 \\ P_2 - L_2 \\ \cdots \\ P_n - L_n
\]

Year 0  Year 1  Year 2  \cdots  Year n

Ears the risk free rate $r$

A cheque is written for the balance at $n$ years

\[
\sum_{t=0}^{n} (P_t - L_t) \times (1 + r)^{n-t}
\]
Two accounts, one value

**Underwriting Account**
Receives *premium* and pays *loss and expense*

\[
P_0, \quad -L_0 \quad P_1, \quad -L_1 \quad P_2, \quad -L_2 \quad \ldots \quad P_n, \quad -L_n
\]

Year 0, Year 1, Year 2, \ldots, Year n

**Dividend Account**
Receives *\( R \times (Capital \ held \ @ \ BOY) \) at year-end*

\[
R*C_0, \quad R*C_1 \quad R*C_2, \quad \ldots \quad R*C_{n-1}
\]

Year 0, Year 1, Year 2, \ldots, Year n

Earn the risk free rate \( r \)

Cheques of equal amount are written for the balances at \( n \) years

\[
\sum_{t=0}^{n} (P_t - L_t) \times (1 + r)^{n-t}
\]

\[
\sum_{t=0}^{n-1} R \times C_t \times (1 + r)^{n-t-1}
\]
**Solving for $R$**

- $R$ is the value that makes the two cheques equal
- Dividing by $(1+r)^n$ to get present values:

\[ R \times \sum_{t=0}^{n-1} C_t \times (1 + r)^{-t-1} = NPV(UW\ Profit) \]

- Simple! Looks just like the S2 risk margin formula
  - $R \leftrightarrow$ CoC (6%)
  - NPV (UW Profit) $\leftrightarrow$ Risk Margin
Solving for $C_t$ is the central challenge

• Except now:
  – We are solving for $R$
  – $C_t$ is for a LOB

• How do we solve for $C_t$ for each LOB?

• We can’t just run-off $C_0$ as we might for the SCR
  – The size of the total portfolio capital pie changes, AND
  – The relative share of each LOB changes
Running off $C_0$ doesn’t always work

• A hypothetical two-line portfolio example:
  – Large Property LOB, paid out after 5 years
  – Small Casualty LOB, paid out after 10 years
  – The Casualty LOB could have small/nil/negative allocated capital @ T=0
  – Not so for years 6+

• Allocation applies *at a point in time*
II. How do we get there?

A practical methodology
Path of the projected ultimate
LOB modelled ultimates @ T=0

Terra incognita for Ultimate analysis
Path and length of Ultimate not modelled

co-TVaR ultimate @ T0

Allocated Capital Required C(t)

13.4
0 1 2
Year
The algorithm

**Question**: What capital would we expect to allocate at the start of each year if the true final ultimate is the co-TVaR ultimate @ T=0?

**Answer**: At $T = 1, 2, \ldots$, for each LOB:

1. Recognize a share of the deterioration from best estimate to co-TVaR
2. Parameterize a lognormal using a CoV and the updated best estimate from step 1
3. Simulate an ultimate for each LOB and in total
4. Re-allocate capital to each LOB
Proposal for re-allocating LOB capital

- Claims experience converges to co-TVaR ultimate modelled @ T=0
- At each time step:
  1. A distribution is parameterized
  2. Claims are simulated by LOB and aggregated
  3. Capital is re-allocated
A simplified LOB example

### Model outputs @ T0

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premium</td>
<td>110.0</td>
</tr>
<tr>
<td>Expected Loss &amp; ALAE</td>
<td>100.0</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>0.0%</td>
</tr>
<tr>
<td>Expected UW Profit</td>
<td>10.0</td>
</tr>
<tr>
<td>TVaR 99p</td>
<td>180.0</td>
</tr>
<tr>
<td>Co-TVaR 99p</td>
<td>125.0</td>
</tr>
<tr>
<td>Co-TVaR 99p Stress</td>
<td>25.0</td>
</tr>
</tbody>
</table>

### The Cloud of Aggregation (all LOB)

### The Thunder of Allocation

\[ R \times \sum_{t=0}^{n-1} C_t \times (1 + r)^{-t-1} = NPV (UW Profit) \]

\[ R \times (21.9 + 12.1 + ... + 2.7) = 10 \]

\[ R = 19.2\% \]
III. How will we know when we’ve arrived?

Validating and interpreting results
“Validation” of $R$

- $R$ for individual LOB can’t be validated
  - $R$ depends on the portfolio as a whole and the methodology
- But we can have expectations for the portfolio:
  - Extreme outliers (high or low) not realistic
    - Reason: *Competition/UW discipline place upper/lower bounds*
  - Similar LOB should have a similar ROC
    - Reason: *Comparable risk vs. reward*
  - No trend in ROC vs. premium
    - *But more variability expected for smaller LOB*
- A good methodology will satisfy these criteria
Test 1: Fewer extreme outliers

ROC Distribution

No Re-allocation

ROC Distribution

With Re-allocation
Test 2: Efficient frontier

- A fitted line should start at the origin (no risk, no reward)
- Result depends on portfolio and ROC methodology

\[
NPV \ (UW \ Profit) = \sum (Allocated \ Capital) \times (Discount \ Factor) 
\]

\[
\text{Slope} = R
\]
Test 3: No trend in $R$ vs. premium

- Small (or large) LOB shouldn’t have inherently higher/lower ROC
- But it is reasonable to expect more volatility for smaller LOB
IV. What are the limitations of the approach?
Hazards, pitfalls and considerations
ROC pitfalls & considerations

• ...of which there are many

• General
  – Capital is not legally divisible and the allocation method is a choice
  – Many assumptions are arbitrary and can yield very different ROCs

• Specific to this method (to name a few)
  – How to align reserve risk volatilities (low granularity) with UW LOB (high granularity)
  – The capital release pattern requires many simulations
  – Explaining to key stakeholders
Take away

• The first step is always the most important: choose the right metric
  – The ROC should be a rate of return

• Not all methodologies are equal – develop a validation toolkit for separating the good, the bad, and the ugly

• Communication to stakeholders is key – buy-in to the results depends on buy-in to the method
Now I know my invested capital for each year, from now until ultimate…*until FOREVER!*
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