Parameter Uncertainty in Capital Modelling

Neil Gedalla
Motivation

• Parameter uncertainty – one of the key remaining challenges in capital modelling?

• Three modelling “camps”:
  – Parameter uncertainty well implemented and communicated
  – Little attempt at parameter uncertainty
  – Parameter uncertainty mostly dealt with outside the models

• Objectives:
  – Parameter uncertainty matters
  – As an industry, we are doing more than we think to take account of it…
  – … but further work (and research) is necessary
Agenda

Overview of parameter uncertainty
• Terminology and definitions
• An example

Parameter uncertainty within capital modelling
• Current state
• Future state?

Underwriting risk – some ideas
Overview of parameter uncertainty
A brief note on terminology

These terms (and possibly others) are used interchangeably.
Definitions

- **Process variance:**
  - Caused by the real world nature of the risk, i.e. natural variability in the claims process

- **Parameter uncertainty:**
  - Caused by uncertainty of the values of the parameters chosen

- **Model error:**
  - Caused by having chosen the wrong model

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An example

- A simple game:
  - Entry fee £6
  - Player tosses 11 coins
  - Receives £1 for each coin that lands heads
  - Operating costs £20

- Base case assumptions:
  - All coins used are fair, i.e. $P(\text{Head}) = P(\text{Tail}) = 0.5$
  - Independence of tosses by the same player
  - Independence of tosses by different players
  - 100 players
An example

- Introduce parameter uncertainty:
  - Number of participants $\sim \text{LogN}(100, 20^2)$
  - Coins fall heads with probability $\sim \text{N}(0.5, 0.05^2)$
- Introduce model error:
  - Assume coin tosses are not independent
  - Coin 1 = heads $\Rightarrow$ Coin 2 is more likely to be heads
- Analogy to capital model:
  - Number of participants = Premium volume
  - Number of heads = Risk volatility
  - Dependency = Dependency
  - Profit = Profit

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### An example

#### Results

<table>
<thead>
<tr>
<th>Model</th>
<th>Mean</th>
<th>SD</th>
<th>99.5th xVaR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process</td>
<td>£30</td>
<td>£17</td>
<td>(£39)</td>
</tr>
<tr>
<td>Process, Parameter</td>
<td>£30</td>
<td>£59</td>
<td>(£135)</td>
</tr>
<tr>
<td>Process, Parameter, Model</td>
<td>£30</td>
<td>£108</td>
<td>(£253)</td>
</tr>
</tbody>
</table>

**Capital models:**

Which line are we closest to?

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An example

Summary

Process variance
In general, we think we understand this and are dealing with it well

Parameter uncertainty
This is interesting!

Model error
Beyond the scope of our standard approaches
Parameter uncertainty within capital modelling
## Capital models

### Current state of parameter uncertainty within capital modelling

<table>
<thead>
<tr>
<th>Risk Type</th>
<th>Usually</th>
<th>Sometimes / other approaches</th>
<th>Rarely</th>
</tr>
</thead>
<tbody>
<tr>
<td>Underwriting risk</td>
<td>Budget gross loss ratio</td>
<td></td>
<td>Claim payment patterns</td>
</tr>
<tr>
<td></td>
<td>Volatility parameters</td>
<td></td>
<td>Budget volume</td>
</tr>
<tr>
<td></td>
<td>Premium rate</td>
<td></td>
<td>Written/earned patterns</td>
</tr>
<tr>
<td>Catastrophe risk</td>
<td>Secondary uncertainty (vulnerability)</td>
<td></td>
<td>Event frequency</td>
</tr>
<tr>
<td>Reserve risk</td>
<td>Volatility parameters via bootstrapping</td>
<td></td>
<td>Mean opening reserve</td>
</tr>
<tr>
<td>Application of RI</td>
<td></td>
<td></td>
<td>Budget ceded loss ratio</td>
</tr>
<tr>
<td>Market risk</td>
<td></td>
<td></td>
<td>ESG?</td>
</tr>
<tr>
<td>Credit risk</td>
<td>Transition &amp; default parameters</td>
<td></td>
<td>LGD</td>
</tr>
<tr>
<td>Operational risk</td>
<td></td>
<td></td>
<td>Volatility parameters</td>
</tr>
<tr>
<td>Dependencies</td>
<td>Trigger based dependencies</td>
<td></td>
<td>Correlation parameters</td>
</tr>
<tr>
<td>Other</td>
<td>One-year recognition parameters</td>
<td></td>
<td>EOY Unincepted (Volumes, LR)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Risk margin parameters</td>
</tr>
</tbody>
</table>

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Capital models

*Future state*

- **Options:**
  - Continue as we are
  - Make everything stochastic
  - Middle ground?

- **Some areas seem better candidates than others:**
  - Insurance risk – business plan
  - Market risk/ESG
  - Dependencies – can of worms?
### Capital models

**Insurance risk – current state**

<table>
<thead>
<tr>
<th>Risk area</th>
<th>Model</th>
<th>Sensitivity tests</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Variance</td>
</tr>
<tr>
<td>Underwriting</td>
<td>Rarely</td>
<td>Sometimes</td>
</tr>
<tr>
<td>Reserving</td>
<td>Rarely</td>
<td>Usually</td>
</tr>
<tr>
<td>Catastrophe</td>
<td>Rarely</td>
<td>Usually</td>
</tr>
</tbody>
</table>
Capital models

Market risk

• Unclear to what extent ESGs have parameter uncertainty built in
• Possible to argue that economic variables have lower levels of uncertainty
• Currently, the evidence indicates that this is not the case
• Some variables are easier than others, e.g., exchange rate volatility
• Volatility may be easier to agree on than the mean
• Possible to estimate levels of uncertainty empirically by comparing range of ESGs…
• … but unclear how best to implement this within capital model
Capital models

Dependencies

• May be difficult to implement in current framework
• Implications for run time, model parsimony, model complexity, …
• Gaussian copula – undercorrelated in the tail?
• Difficult to parameterise, and …
• … again, unclear how best to implement this within capital model
Capital models

Dependencies

- Example:
  - Two lines of business, LogN(100, 50²)
  - Base run has 500k trials @ ~25% correlation
  - Instead, run 10x 50k trials with different levels of correlation
  - Idea is to impose parameter uncertainty distribution ~ Normal(0.25, 0.15²)
Capital models

Dependencies

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Underwriting risk – some ideas
**Underwriting risk**

*Easy options*

**Type II sensitivity testing**

Illustrating parameter uncertainty

- “Best guess pick”

Input parameters

Model outcome based on a range of reasonable picks

**Split existing parameters**

Breaking down expert judgement

- Results of statistical analysis
- Application of expert judgement
- Considering uncertainty
- Selected parameter input for the model
Underwriting risk
Slightly harder options

Stochastic mixing distributions

Stochastic modelling of parameter uncertainty

Distribution for underwriting losses

- Deterministic mean parameter
- Stochastic mean parameter

Bootstrap
Summary

• Be clear what sources of risk are and are not included in models, and why
• Both means and variances can have parameter uncertainty
• Some possible areas for further research:
  – Underwriting risk
  – Market risk and ESGs
  – Correlations
    – Are trigger based dependencies an example of secondary uncertainty?
• Better methods to articulate impact of parameter uncertainty, using underwriting risk volatility as a test case
• Introducing explicit margins may be easier than mixing distributions, but communication is key!
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