

# Equity Investing with Targeted Constant Volatility Exposure

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*AFIR ERM Colloquium of the International Actuarial Association  
Edinburgh, UK*

31 May to 2 June 2016

- Background
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# Background

- Increased emphasis on risk management approach to investment by pension funds following global credit crisis.
- Volatility is more predictable than expected returns.
- Volatility increases during crisis events and then reverts.
- Downside risk and draw-downs have negative impact on long term returns.
- Increased interest in investment strategies that target constant volatility rather than constant mix, with potential to limit downside risk.

# Summary of Main Results

- We show that a targeted constant volatility strategy results in substantial improvement in equity return performance, even after allowing for transaction costs.
- For the US and Australian markets this is around 200 basis points above the stock index return, after transaction costs, with a target volatility approximately equal to the historical market average.
- An investment with a targeted volatility has significantly reduced exposure to downside risk.
- These results hold over a long time period (over 80 years) and also over shorter time periods.

# Approach to Constructing Portfolios

- Funds are fully invested in the market index and portfolio has a value of \$100 million.
- Portfolios are constructed with stocks and stock index futures contracts overlays.
- Winsorized GARCH(1,1) model estimated at day  $t - 1$ , gives one-step ahead volatility forecast and participation ratio  $w_t$  of day  $t$ .
- The weight  $w_t$  invested in the market, also referred to as the Participation Ratio, is given by;

$$w_t = \frac{\text{target volatility}}{\hat{\sigma}_t}$$

where  $\hat{\sigma}_t$  is the volatility forecast for date  $t$ .

- If  $w_t > (<) 1$ , we buy (sell) the nearest maturing futures contracts for a dollar amount of  $|w_t - 1|$  times the current market portfolio value at the close of trade day  $t - 1$ .

# Implementation with Futures

- If the absolute difference between the new target weight of day  $t + 1$  and current target weight of day  $t$  is greater than a  $\delta$  threshold, we adjust the futures position based on the difference between the two weights and the current size of market portfolio.
- Daily interest expenses is charged on any losses on the futures position until the end of each month.
- Transaction cost is \$10 per futures contract (a conservative assumption as actual cost can be substantially less).
- At the end of each month, if the futures position is still open, all cumulative profit/loss and associated fees, i.e. transaction costs and interest charged on losses, are reinvested immediately into the market index.

# Daily Return Calculation

- The daily returns at date  $t$  of the trading strategy are computed as;

$$r_t = w_t(1+r_{market,t}) - 1_{w_t \geq 1}(w_t - 1)(1+r_{f,t}) + 1_{w_t \leq 1}(1 - w_t)(1+r_{f,t}) - 1$$

where  $r_{market,t}$  is the index return at date  $t$ ,  $r_{f,t}$  is the borrowing and lending rate at date  $t$ ,  $1_{w_t \geq 1} = 1$  if  $w_t \geq 1$  and 0 otherwise, and  $1_{w_t \leq 1} = 1$  if  $w_t \leq 1$  and 0 otherwise.

- In the case where we take positions on the futures market, the daily return at date  $t$  of the trading strategy is computed as;

$$r_t = (w_t - 1)r_{futures,t} + r_{market,t}$$

where  $r_{futures,t}$  is the index futures return at date  $t$ .

- The return of the market index (in percentage) at time  $t$  is given by;

$$r_t = \varepsilon_t$$

where  $\varepsilon_t$  is i.i.d  $(0, \sigma_t^2)$ , and the conditional variance  $\sigma_t^2$  is specified as GARCH(1,1).

# Rolling Over Futures

- In regard to rolling a futures position as the contract approaches maturity, we adopt the following rules:
  - If we have to adjust the current futures position within the last 10 trading days of the current contract maturity, we close the current position and simultaneously open the new position on the next most recent expiring futures contract at the close of trade.
  - If a futures position is still open at the close of trade, 5 trading days before the current contract maturity, we close the current position and simultaneously open a new position on the next most recent expiring futures contract.
- When we close futures position, we realize all daily profit/loss and associated fees since the last futures position transaction and reinvest immediately in the market index.



# Annualized returns and volatility

We define the annualized returns,  $\mu$ , and annualized standard deviation,  $\sigma$  as;

$$\mu = 100[(1 + \hat{r})^m - 1] \text{ with } \hat{r} = Y^{\frac{1}{n}} - 1$$

$$\sigma = 100\sqrt{m \frac{\sum_{t=1}^n (r_t - \bar{r})^2}{n - 1}},$$

where  $m$  is the number of trading days in a year,  $n$  is the total number of trading days in the accumulation period, and  $Y$  is the cumulative amount from \$1 invested at the beginning of the accumulation period,  $r_t$  is the daily return, and  $\bar{r}$  is the average daily return in the accumulation period.

# Sharpe ratio and annualized realized volatility

The Sharpe ratio (SR) is expressed as;

$$SR = \frac{\mu}{\sigma}$$

The annualized realized volatility for a given calendar year is computed as;

$$\sigma_{\text{annualized RV}} = 100 \sqrt{\sum_{t=1}^k r_t^2}$$

where  $k$  is a specific number of trading days over a calendar year.

# Data: US and Australia

- US

- CRSP value-weighted market daily returns, 1926 to 2013.
- Settlement prices of futures contracts on the S&P500 from Datastream, 1982 to 2014.
- One month T-Bill and factor returns from the Ken French Data Library.

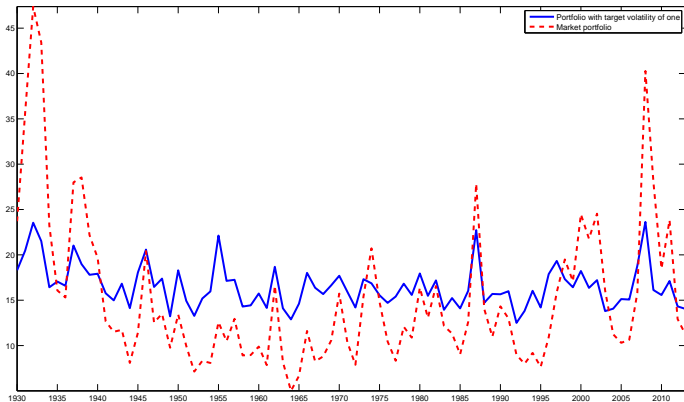
- Australia

- Market index daily returns from Datastream, 1982 to 2013.
- Settlement prices of futures contracts on the ASX200 from Datastream, 2000 to 2013.
- One month Interbank rate from the British Bankers Association.

# US mean and volatility estimates, May 1929 - Dec 2013

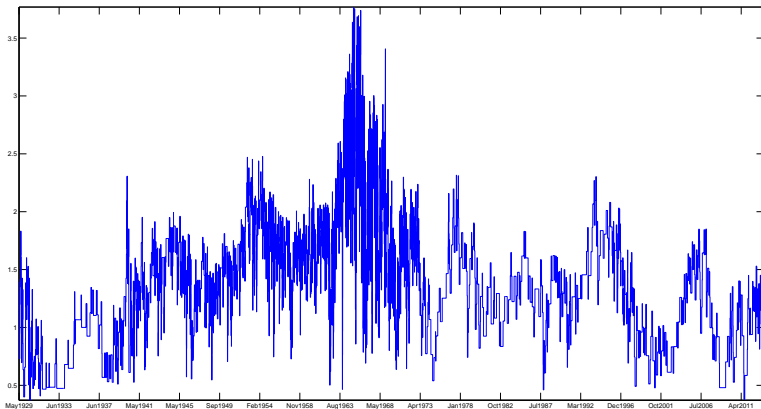
|                             | $\delta = 0$ |          | $\delta = 0.10$ |          | $\delta = 0.20$ |          | $\delta = 0.30$ |          |
|-----------------------------|--------------|----------|-----------------|----------|-----------------|----------|-----------------|----------|
|                             | $\mu$        | $\sigma$ | $\mu$           | $\sigma$ | $\mu$           | $\sigma$ | $\mu$           | $\sigma$ |
| Market portfolio            | 8.8          | 17.1     | 8.8             | 17.1     | 8.8             | 17.1     | 8.8             | 17.1     |
| 0.4 Daily target volatility | 6.6          | 6.6      | 6.7             | 6.6      | 6.6             | 6.7      | 6.7             | 6.8      |
| 0.6 Daily target volatility | 8.1          | 9.9      | 8.2             | 9.8      | 8.4             | 9.9      | 8.1             | 10.1     |
| 0.8 Daily target volatility | 9.5          | 13.2     | 9.5             | 13.0     | 9.6             | 13.1     | 9.8             | 13.6     |
| 1.0 Daily target volatility | 10.8         | 16.5     | 10.8            | 16.3     | 11.1            | 16.4     | 11.0            | 16.6     |
| 1.2 Daily target volatility | 11.9         | 19.8     | 11.8            | 19.6     | 12.2            | 19.6     | 12.1            | 19.7     |
| 1.4 Daily target volatility | 13.0         | 23.1     | 13.2            | 22.9     | 13.1            | 22.8     | 13.2            | 22.9     |
| 1.6 Daily target volatility | 13.9         | 26.4     | 13.9            | 26.2     | 14.1            | 26.1     | 14.5            | 26.2     |
| 1.8 Daily target volatility | 14.8         | 29.7     | 14.8            | 29.5     | 14.8            | 29.3     | 15.3            | 29.4     |
| 2.0 Daily target volatility | 15.5         | 33.0     | 15.6            | 32.8     | 15.5            | 32.6     | 15.5            | 32.5     |
| 2.2 Daily target volatility | 16.0         | 36.3     | 16.1            | 36.1     | 16.1            | 35.9     | 16.3            | 35.9     |
| 2.4 Daily target volatility | 16.5         | 39.6     | 16.6            | 39.4     | 16.4            | 39.2     | 16.9            | 39.1     |
| 2.6 Daily target volatility | 16.8         | 42.9     | 16.9            | 42.7     | 16.8            | 42.5     | 17.1            | 42.4     |
| 2.8 Daily target volatility | 17.0         | 46.2     | 17.0            | 46.0     | 17.5            | 45.7     | 17.4            | 45.6     |
| 3.0 Daily target volatility | 17.0         | 49.5     | 17.1            | 49.3     | 17.3            | 49.0     | 17.2            | 48.9     |

# US Annual Volatility



US annual realized volatility of market and market with daily target volatility of one and  $\delta = 0.2$

# US Daily Participation Ratio



US daily Participation Ratio for a daily target volatility of one ( $\delta = 0.2$ )

# Percentage of days when US Participation Ratio changes

| Daily target | 0.60            | 1.00  | 1.40  | 0.60            | 1.00  | 1.40  | 0.60            | 1.00  | 1.40  |
|--------------|-----------------|-------|-------|-----------------|-------|-------|-----------------|-------|-------|
|              | $\delta = 0.10$ |       |       | $\delta = 0.20$ |       |       | $\delta = 0.30$ |       |       |
| 1930-1933    | 6.67            | 17.05 | 25.65 | 2.70            | 5.74  | 9.54  | 0.84            | 3.21  | 5.15  |
| 1934-1937    | 2.58            | 7.65  | 10.40 | 0.92            | 2.08  | 3.99  | 0.17            | 1.41  | 2.25  |
| 1938-1941    | 11.46           | 21.35 | 30.32 | 3.74            | 8.47  | 14.78 | 1.83            | 3.74  | 7.06  |
| 1942-1945    | 11.80           | 22.18 | 32.97 | 3.63            | 8.85  | 14.92 | 1.18            | 4.64  | 7.42  |
| 1946-1949    | 13.02           | 25.86 | 33.13 | 3.99            | 9.83  | 15.85 | 0.80            | 4.87  | 9.03  |
| 1950-1953    | 14.92           | 28.64 | 41.34 | 4.88            | 11.14 | 18.32 | 2.12            | 6.17  | 10.22 |
| 1954-1957    | 21.15           | 37.44 | 51.14 | 6.45            | 16.68 | 25.32 | 4.07            | 8.54  | 14.50 |
| 1958-1961    | 16.48           | 32.27 | 45.18 | 5.26            | 13.31 | 21.05 | 1.79            | 6.06  | 10.82 |
| 1962-1965    | 41.07           | 63.79 | 75.30 | 20.54           | 33.93 | 49.01 | 9.33            | 22.42 | 31.55 |
| 1966-1969    | 46.27           | 68.03 | 77.94 | 20.84           | 38.30 | 52.50 | 11.75           | 23.80 | 33.91 |
| 1970-1973    | 24.06           | 42.57 | 57.92 | 7.43            | 18.12 | 29.41 | 3.96            | 8.81  | 18.12 |
| 1974-1977    | 6.73            | 18.30 | 24.93 | 1.88            | 4.25  | 9.40  | 1.19            | 2.37  | 4.25  |
| 1978-1981    | 6.33            | 13.25 | 21.56 | 1.58            | 4.06  | 7.52  | 0.99            | 1.98  | 3.26  |
| 1982-1985    | 4.25            | 8.01  | 13.75 | 0.89            | 2.57  | 5.54  | 0.30            | 0.99  | 3.26  |
| 1986-1989    | 6.53            | 13.06 | 19.49 | 1.68            | 4.85  | 7.32  | 0.79            | 2.27  | 3.76  |
| 1990-1993    | 4.64            | 8.69  | 15.50 | 1.38            | 3.95  | 5.73  | 0.79            | 1.48  | 3.06  |
| 1994-1997    | 8.01            | 17.71 | 24.23 | 2.57            | 6.63  | 9.59  | 1.19            | 2.67  | 4.85  |
| 1998-2001    | 6.27            | 14.54 | 22.21 | 1.79            | 4.78  | 7.77  | 1.20            | 2.29  | 4.68  |
| 2002-2005    | 3.57            | 11.01 | 18.55 | 0.99            | 3.67  | 6.25  | 0.30            | 1.29  | 2.38  |
| 2006-2009    | 5.06            | 10.23 | 18.27 | 1.19            | 4.07  | 5.96  | 0.50            | 2.09  | 2.88  |
| 2010-2013    | 8.75            | 18.19 | 25.94 | 2.88            | 6.46  | 10.14 | 0.80            | 3.28  | 5.86  |

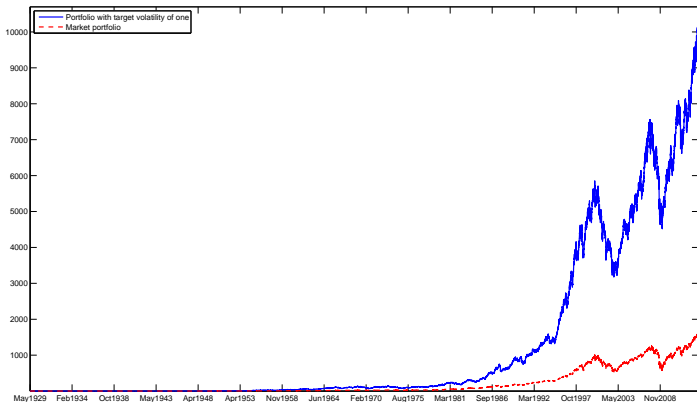
# US return performance summary statistics, 1929-2013

( $\delta = 0.20$ )

|                             | $\mu$ | $\sigma$ | $SR$ | Cumulative   | Max $\sigma_{annRV}$ | Min 1y ret |
|-----------------------------|-------|----------|------|--------------|----------------------|------------|
| Market portfolio            | 8.8   | 17.1     | 0.51 | 1,690.80     | 45.8                 | -64.2      |
| 0.4 Daily target volatility | 6.6   | 6.7      | 0.99 | 290.74       | 14.7                 | -25.3      |
| 0.6 Daily target volatility | 8.4   | 9.9      | 0.84 | 1,219.65     | 17.6                 | -29.5      |
| 0.8 Daily target volatility | 9.6   | 13.1     | 0.73 | 3,328.88     | 20.8                 | -38.6      |
| 1.0 Daily target volatility | 11.1  | 16.4     | 0.68 | 10,698.81    | 23.9                 | -45.8      |
| 1.2 Daily target volatility | 12.2  | 19.6     | 0.62 | 27,120.48    | 28.0                 | -52.5      |
| 1.4 Daily target volatility | 13.1  | 22.8     | 0.57 | 51,500.63    | 32.3                 | -59.0      |
| 1.6 Daily target volatility | 14.1  | 26.1     | 0.54 | 118,605.38   | 36.7                 | -64.8      |
| 1.8 Daily target volatility | 14.8  | 29.3     | 0.51 | 205,787.90   | 41.2                 | -69.0      |
| 2.0 Daily target volatility | 15.5  | 32.6     | 0.48 | 348,833.40   | 45.8                 | -73.5      |
| 2.2 Daily target volatility | 16.1  | 35.9     | 0.45 | 559,582.80   | 51.3                 | -77.4      |
| 2.4 Daily target volatility | 16.4  | 39.2     | 0.42 | 668,595.03   | 55.9                 | -80.3      |
| 2.6 Daily target volatility | 16.8  | 42.5     | 0.40 | 904,557.72   | 60.6                 | -82.9      |
| 2.8 Daily target volatility | 17.5  | 45.7     | 0.38 | 1,609,440.31 | 65.2                 | -85.3      |
| 3.0 Daily target volatility | 17.3  | 49.0     | 0.35 | 1,361,258.62 | 69.9                 | -87.3      |



# Cumulative Value US



Cumulative amount from \$1 invested in the US market from 10 May 1929 to 31 December 2013. Market portfolio with daily target volatility of one and a  $\delta = 0.2$

# US annual returns and transaction costs ( $\delta = 0.20$ )

| Year    | Market  | No cost | With cost | Difference |
|---------|---------|---------|-----------|------------|
| 1993    | 11.891  | 15.028  | 14.903    | 0.12515    |
| 1994    | -0.269  | -5.213  | -5.344    | 0.13068    |
| 1995    | 36.063  | 68.059  | 67.877    | 0.18154    |
| 1996    | 20.453  | 26.407  | 26.191    | 0.21658    |
| 1997    | 31.417  | 30.383  | 30.264    | 0.11846    |
| 1998    | 22.014  | 18.183  | 18.079    | 0.10448    |
| 1999    | 25.788  | 24.274  | 24.248    | 0.02653    |
| 2000    | -10.585 | -13.962 | -14.045   | 0.08264    |
| 2001    | -7.904  | -11.808 | -11.867   | 0.05897    |
| 2002    | -21.185 | -18.426 | -18.466   | 0.03960    |
| 2003    | 29.211  | 27.196  | 27.175    | 0.02161    |
| 2004    | 13.106  | 14.034  | 14.008    | 0.02660    |
| 2005    | 8.406   | 5.002   | 4.939     | 0.06266    |
| 2006    | 14.345  | 18.769  | 18.693    | 0.07611    |
| 2007    | 7.489   | 4.416   | 4.370     | 0.04671    |
| 2008    | -37.401 | -30.479 | -30.501   | 0.02228    |
| 2009    | 27.240  | 18.917  | 18.894    | 0.02271    |
| 2010    | 15.698  | 17.506  | 17.484    | 0.02203    |
| 2011    | -2.165  | -5.851  | -5.867    | 0.01580    |
| 2012    | 13.912  | 12.339  | 12.325    | 0.01465    |
| 2013    | 27.291  | 27.626  | 27.601    | 0.02573    |
| average | 12.306  | 13.667  | 13.569    | 0.09863    |

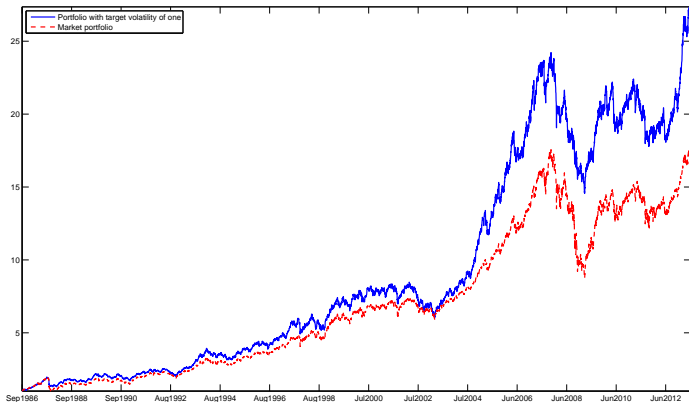
# Return performance summary statistics for Australia, 1986-2013 ( $\delta = 0.20$ )

|                             | $\mu$ | $\sigma$ | SR   | Cumulative | Max $\sigma_{annRV}$ | Min 1y ret |
|-----------------------------|-------|----------|------|------------|----------------------|------------|
| Market portfolio            | 11.1  | 16.7     | 0.67 | 16.67      | 38.2                 | -44.4      |
| 0.4 Daily target volatility | 9.4   | 6.7      | 1.40 | 11.05      | 14.3                 | -13.2      |
| 0.6 Daily target volatility | 10.4  | 10.0     | 1.04 | 14.12      | 19.0                 | -21.6      |
| 0.8 Daily target volatility | 11.7  | 13.0     | 0.90 | 18.94      | 20.8                 | -26.6      |
| 1.0 Daily target volatility | 12.9  | 16.1     | 0.80 | 25.45      | 25.7                 | -33.9      |
| 1.2 Daily target volatility | 13.7  | 19.3     | 0.71 | 30.93      | 30.7                 | -34.7      |
| 1.4 Daily target volatility | 14.3  | 22.6     | 0.63 | 35.77      | 36.3                 | -41.8      |
| 1.6 Daily target volatility | 14.8  | 25.7     | 0.57 | 39.46      | 40.6                 | -46.3      |
| 1.8 Daily target volatility | 15.2  | 29.0     | 0.52 | 43.48      | 46.1                 | -54.3      |
| 2.0 Daily target volatility | 15.3  | 32.3     | 0.47 | 44.70      | 50.8                 | -57.0      |
| 2.2 Daily target volatility | 15.9  | 35.5     | 0.45 | 51.58      | 56.2                 | -62.1      |
| 2.4 Daily target volatility | 15.9  | 38.8     | 0.41 | 51.48      | 61.3                 | -63.8      |
| 2.6 Daily target volatility | 15.7  | 42.0     | 0.37 | 49.09      | 66.3                 | -67.3      |
| 2.8 Daily target volatility | 15.4  | 45.2     | 0.34 | 45.50      | 71.3                 | -69.7      |
| 3.0 Daily target volatility | 15.2  | 48.5     | 0.31 | 44.04      | 76.4                 | -72.8      |

# Return performance summary statistics for Australia, 2000-2013 ( $\delta = 0.20$ )

|                             | $\mu$ | $\sigma$ | SR   | Cumulative | Max $\sigma_{\text{ann}RV}$ | Min 1y ret |
|-----------------------------|-------|----------|------|------------|-----------------------------|------------|
| Market portfolio            | 7.5   | 16.3     | 0.46 | 2.59       | 34.1                        | -44.4      |
| 0.4 Daily target volatility | 6.5   | 7.6      | 0.85 | 2.27       | 14.6                        | -15.5      |
| 0.6 Daily target volatility | 7.7   | 10.1     | 0.76 | 2.64       | 16.6                        | -22.3      |
| 0.8 Daily target volatility | 9.1   | 12.9     | 0.71 | 3.14       | 19.1                        | -27.4      |
| 1.0 Daily target volatility | 10.5  | 15.9     | 0.66 | 3.68       | 22.0                        | -34.3      |
| 1.2 Daily target volatility | 12.6  | 19.0     | 0.66 | 4.70       | 24.4                        | -35.0      |
| 1.4 Daily target volatility | 13.7  | 22.2     | 0.61 | 5.34       | 27.6                        | -41.4      |
| 1.6 Daily target volatility | 14.6  | 25.4     | 0.57 | 5.96       | 31.1                        | -46.0      |
| 1.8 Daily target volatility | 15.2  | 28.8     | 0.53 | 6.39       | 34.7                        | -54.0      |
| 2.0 Daily target volatility | 15.9  | 32.0     | 0.50 | 6.91       | 37.8                        | -56.5      |
| 2.2 Daily target volatility | 16.4  | 35.3     | 0.47 | 7.30       | 41.6                        | -61.8      |
| 2.4 Daily target volatility | 17.4  | 38.6     | 0.45 | 8.12       | 45.4                        | -63.3      |
| 2.6 Daily target volatility | 17.9  | 41.8     | 0.43 | 8.61       | 49.0                        | -66.6      |
| 2.8 Daily target volatility | 18.2  | 45.1     | 0.40 | 8.87       | 52.6                        | -68.9      |
| 3.0 Daily target volatility | 18.4  | 48.4     | 0.38 | 9.06       | 56.3                        | -72.0      |

# Cumulative Value Australia



Cumulative amount from \$1 invested in the Australian market from 23 September 1986 to 31 May 2013. Market portfolio with a daily target volatility of one and a  $\delta = 0.2$

# Australian annual returns and transaction costs

( $\delta = 0.20$ )

| Year    | Market  | No cost | With cost | Difference |
|---------|---------|---------|-----------|------------|
| 2001    | 7.216   | 5.701   | 5.606     | 0.09508    |
| 2002    | -9.832  | -16.201 | -16.296   | 0.09469    |
| 2003    | 12.878  | 16.382  | 16.236    | 0.14524    |
| 2004    | 25.870  | 50.623  | 50.329    | 0.29396    |
| 2005    | 21.420  | 30.294  | 30.076    | 0.21827    |
| 2006    | 22.521  | 30.018  | 29.893    | 0.12528    |
| 2007    | 17.751  | 12.109  | 12.023    | 0.08662    |
| 2008    | -38.443 | -29.636 | -29.795   | 0.15925    |
| 2009    | 39.792  | 33.607  | 33.494    | 0.11352    |
| 2010    | 1.959   | -2.294  | -2.355    | 0.06142    |
| 2011    | -11.231 | -14.255 | -14.342   | 0.08722    |
| 2012    | 18.255  | 19.342  | 19.239    | 0.10253    |
| average | 9.013   | 11.308  | 11.176    | 0.13192    |

## Concluding remarks

- We have shown how a broad equity portfolio with a targeted volatility approximately equal to the historical market average, generates significant returns above the stock index return, after transaction costs.
- This is demonstrated using both US and Australian data over different investment horizons.
- The results reflect a changing market risk return trade-off and a negative link between volatility and expected return.
- Increased emphasis on risk management investment strategies and avoiding significant draw-downs in crisis events means that these strategies are particularly relevant to mutual funds and pension funds that hold large equity portfolios following the broad market.

Thank you for your attention

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[http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=2614828](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2614828)