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Dollar / Ladder Investment and Universal Portfolio for Pension Schemes

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* The views and opinions expressed in this presentation are those of the authors and do not necessarily reflect the opinion, official policy or position of the organization to which the author belongs.

Introduction

- Interaction of knowledge among actuarial world and other industries.
- Firstly,

Dollar cost averaging <--> Accumulation / Decumulation

Bond ladder Investment <--> LDI

Mental accounting <--> Bucket Strategy

- Secondly,

Universal portfolio, which has close relationship of electronic information theory, is expanded.

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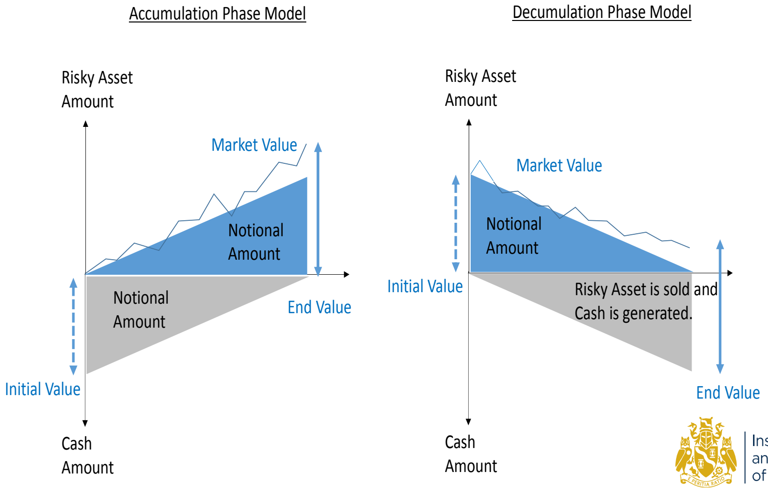
Accumulation/Decumulation phase expressed by Dollar Cost Averaging



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Mathematical Model

An investor has w_0 cash at the beginning and start to invest the cash into a risky asset. The investment amount will be increase as the time goes until end- time t=T. Remaining cash will be hold as it is. We denote total wealth at time t is w_t (its portfolio X_t), a risky asset's characteristics are determined by its price S under geometric Brownian motion (B_t) with constant drift μ and volatility σ (Black-Scholes model, i.e., $dS_t = S_t \mu dt + S_t \sigma dB_t$). The cash flow k is continuous time base and constant. The investment model will be set as follows.

$$dw_t = w_t \mu dt + w_t \sigma dB_t + kdt \, , \; (|k| T = w_0)$$

In case X_0: fully cash, $w_{t-0} = w_0$ and k >0, it is accumulation investment. In case X_0: fully risky asset, $w_{t-0} = w_0$ and k <0, it is decumulation investment.

$$w_t = e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma B_t} \left[w_0 + k \int_0^t e^{-(\mu - \frac{1}{2}\sigma^2)s - \sigma B_s} ds \right]$$

$$E[w_{t=T}] = w_0 e^{\mu T} + \frac{k}{\mu}(e^{\mu T} - 1) = w_0 + (\mu w_0 + k) \frac{e^{\mu T} - 1}{\mu}$$

$$Var[w_{t=T}] = \frac{2}{\mu + \sigma^2} \left[\left\{ (\mu + \frac{1}{2}\sigma^2) w_0 + k \right\}^2 + \frac{1}{2}\sigma^2 (\mu + \frac{1}{2}\sigma^2) w_0 \right] \frac{e^{2(\mu + \frac{1}{2}\sigma^2)T} - 1}{2(\mu + \frac{1}{2}\sigma^2)}$$

$$- \frac{2}{\mu + \sigma^2} (\mu w_0 + k)(\mu w_0 + k + \sigma^2) \frac{e^{\mu T} - 1}{\mu} - (\mu w_0 + k)^2 (\frac{e^{\mu T} - 1}{\mu})^2$$

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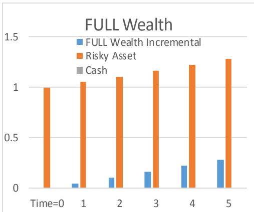
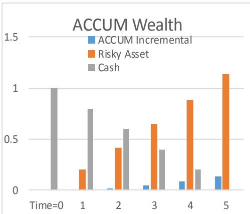
Pros and Cons - Simulation 1

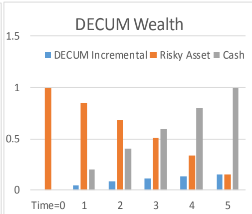
Parameters

Initial Wealth w_0	1	Risky Asset Drift μ	5.0%	In/Out Flow Size k	0.20
End Time T	5	Risky Asset Volatility σ	10.0%		

	Time=0	1	2	3	4	End Value 5	End Value Volatility	Value / Volatility	Increase / Volatility
FULL Wealth	1	1.051	1.105	1.162	1.221	1.284	0.291	23%	0.98
Risky Asset	1	1.05	1.11	1.16	1.22	1.28			
Cash	0	0.00	0.00	0.00	0.00	0.00			
ACCUM (Strategy 1-1)	1	1.005	1.021	1.047	1.086	1.136	0.152	13%	0.89
Risky Asset	0	0.21	0.42	0.65	0.89	1.14			
Cash	1	0.80	0.60	0.40	0.20	0.00			
DECUM (Strategy 1-2)	1	1.046	1.084	1.114	1.136	1.148	0.174	15%	0.85
Risky Asset	1	0.85	0.68	0.51	0.34	0.15			
Cash	0	0.20	0.40	0.60	0.80	1.00			







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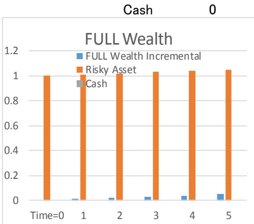
5

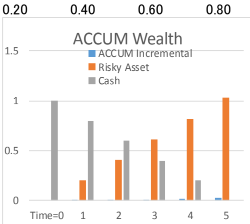
Pros and Cons - Simulation 2

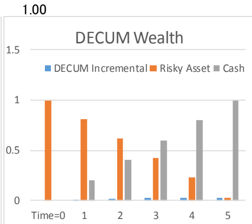
Parameters

Initial Wealth w_0	1	Risky Asset Drift μ	1.0%	In/Out Flow Size k	0.20
End Time T	5	Risky Asset Volatility σ	10.0%		

	Time=0	1	2	3	4	End Value 5	End Value Volatility	Value / Volatility	Increase / Volatility
FULL Wealth	1	1.010	1.020	1.030	1.041	1.051	0.238	23%	0.22
Risky Asset	1	1.01	1.02	1.03	1.04	1.05			
Cash	0	0.00	0.00	0.00	0.00	0.00			
ACCUM (Strategy 1-1)	1	1.001	1.004	1.009	1.016	1.025	0.134	13%	0.19
Risky Asset	0	0.20	0.40	0.61	0.82	1.03			
Cash	1	0.80	0.60	0.40	0.20	0.00			
DECUM (Strategy 1-2)	1	1.009	1.016	1.021	1.025	1.026	0.139	14%	0.19
Risky Asset	1	0.81	0.62	0.42	0.22	0.03			
Cash	0	0.20	0.40	0.60	0.80	1.00			







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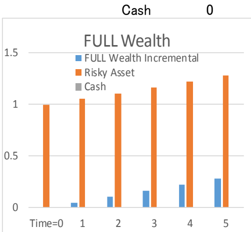
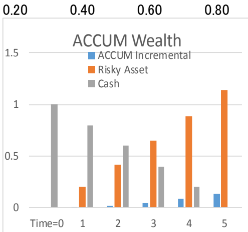
6

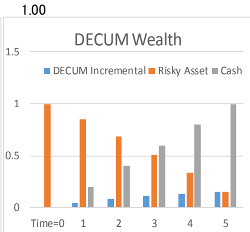
Pros and Cons - Simulation 3

Parameters					
Initial Wealth w_0	1	Risky Asset Drift μ	5.0%	In/Out Flow Size k	0.20
End Time T	5	Risky Asset Volatility σ	5.0%		

	Time=0	1	2	3	4	End Value 5	End Value Volatility	Value / Volatility	Increase / Volatility
FULL Wealth	1	1.051	1.105	1.162	1.221	1.284	0.144	11%	1.97
Risky Asset	1	1.05	1.11	1.16	1.22	1.28			
Cash	0	0.00	0.00	0.00	0.00	0.00			
ACCUM (Strategy 1-1)	1	1.005	1.021	1.047	1.086	1.136	0.076	7%	1.80
Risky Asset	0	0.21	0.42	0.65	0.89	1.14			
Cash	1	0.80	0.60	0.40	0.20	0.00			
DECUM (Strategy 1-2)	1	1.046	1.084	1.114	1.136	1.148	0.086	7%	1.72
Risky Asset	1	0.85	0.68	0.51	0.34	0.15			
Cash	0	0.20	0.40	0.60	0.80	1.00			







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7

Considerations

- Pros: Low Volatility
- Cons: Losing Return Opportunity

- Utility Example

Utility = Profit - λ*σ^2

λ= 2

	Profit	Utility
FULL	0.28	0.11
ACCUM	0.14	0.09
DECUM	0.15	0.15

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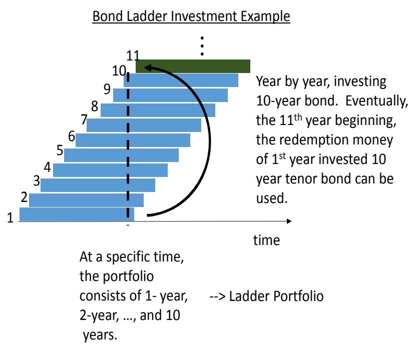
8

Bond Ladder Investment



- Bond ladder investment is interpreted as Bond dollar investment.
- It can be compared to Bond Buy and Hold strategy. For example, to buy 10 year bond and wait for 10 years. Ladder investment is more averagely invest into 10 year bonds in respect to coupon level. (Time diversification)
- Ladder portfolio is useful for Cash Flow matching needs especially for decumulatin phase.

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Considerations

- Negative Interest Rate
  - Why we should invest negative yield bond arises?
- Think about dollar cost averaging and cash flow matching.

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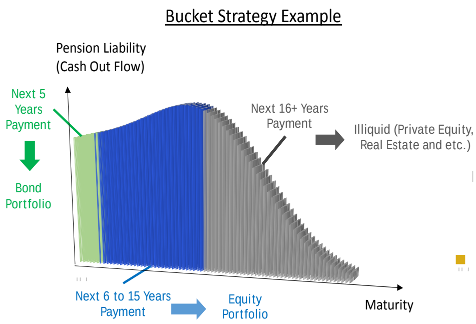
The Bucket Strategy



- "Closed Pension Scheme ALM" + "Mental Accounting"
  - All ways keep the bucket's allocation? or Spending as it doomed?
  - At the end, each bucket should be part of Cash Flow Matching.
  - If so, why it includes equity? Should be Zero-coupon bond.
- Theoretically, it should be

"Best Portfolio" + "Slicing it if necessary"
or ALM/LDI !

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11

Considerations

- Mental Accounting
  - People responsible for the investment reliefed.
  - Pension participants?...

- Fiduciary

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12

Universal Portfolio

Cover (1991) suggested the following online portfolio selection algorithm. Suppose there is m assets (they are m names of stocks). Rebalancing daily and n days are there. (It is not necessarily day periods and we call n steps.) The b denotes portfolio weight vector at k-th step and Sk(b) denotes total portfolio value by weight b at k-th step. The definition indicates gravity of value trends.

$$\hat{b}_1 = \left[\frac{1}{m}, \frac{1}{m}, \frac{1}{m}, \ldots, \frac{1}{m}\right], \quad \hat{b}_{k+1} = \frac{\int b S_k(b) db}{\int S_k(b) db}$$

This algorithm make the ratio between the value of the portfolio $\hat{S}_n$ and the best CRP (constant (weight) rebalanced portfolio in hindsight $S^*{}_n$ the below under the function of n. (Cover, 1998)

$$\frac{\hat{S}_n}{S_n^*} \geq \frac{2}{\sqrt{n+1}},$$

In essence, as usually the best CRP in hindsight is 100% investment into the best beta among the multi asset opportunity, Cover's algorithm, whose result is Universal portfolio, make the portfolio align to the best beta by $1/\sqrt{1+n}$.

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The Models

Min Vol Gravity: Making use of Universal portfolio's gravity averaging,

$$\hat{b}^{MVG}{}_{k+1} = \frac{\int b \sigma_k(b)^{-1} db}{\int \sigma_k(b)^{-1} db}$$

σ_k denotes total portfolio's historical 12 months volatility before the month k+1.

Risk Parity: Risk parity portfolio decided by the followings.

$$\hat{b}^{RP}{}_{k+1} = \left[\frac{\sigma^1{}_k{}^{-1}}{\sigma^1{}_k{}^{-1} + \sigma^2{}_k{}^{-1}}, \frac{\sigma^2{}_k{}^{-1}}{\sigma^1{}_k{}^{-1} + \sigma^2{}_k{}^{-1}}\right]$$

$\sigma^i{}_k$ denotes i-th asset's historical 12 months volatility before the month k+1.

Universal Portfolio: Universal portfolio defined by the followings.

$$\hat{b}^{UP}{}_1 = \left[\frac{1}{2}, \frac{1}{2}\right], \quad \hat{b}^{UP}{}_{k+1} = \frac{\int b S_k(b) db}{\int S_k(b) db}$$

Sk(b) denotes total portfolio value by weight b at k-th month.

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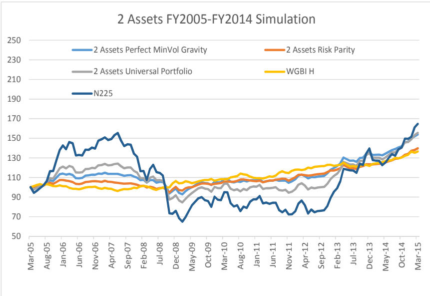
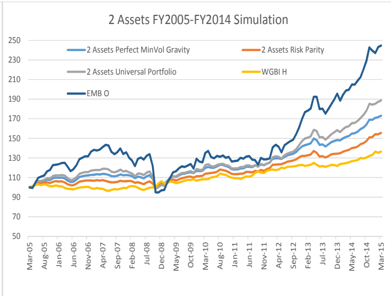
Numerical Simulations 1





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15

Numerical Simulations 2

	2 Assets Risk Parity	2 Assets Universal Portfolio	2 Assets MinVol Gravity
FY2005-2014	0.87	0.47	0.71
Return	3.40%	4.52%	4.40%
Risk	3.89%	9.63%	6.22%
FY2005			
Return	6.34%	21.82%	13.65%
Risk	3.66%	7.89%	5.40%
FY2006			
Return	−0.70%	0.48%	0.00%
Risk	3.13%	6.61%	4.46%
FY2007			
Return	−5.47%	−14.34%	−7.77%
Risk	2.15%	6.91%	3.41%
FY2008			
Return	−0.83%	−15.99%	−8.47%
Risk	7.74%	17.15%	11.74%
FY2009			
Return	6.75%	17.72%	12.85%
Risk	2.36%	8.90%	5.46%
FY2010			
Return	−0.56%	−5.30%	−2.78%
Risk	2.50%	8.56%	4.47%
FY2011			
Return	7.48%	6.02%	7.23%
Risk	3.52%	7.40%	4.86%
FY2012			
Return	5.75%	13.40%	10.25%
Risk	2.30%	8.79%	5.39%
FY2013			
Return	3.77%	10.33%	7.15%
Risk	4.41%	10.13%	6.94%
FY2014			
Return	12.72%	19.41%	15.38%
Risk	2.31%	4.73%	2.94%

	2 Assets Risk Parity	2 Assets Universal Portfolio	2 AssetsMin Vol Gravity
FY2005-2014	1.13	0.97	1.11
Return	4.51%	6.56%	5.63%
Risk	3.98%	6.79%	5.08%
FY2005			
Return	5.02%	11.90%	9.09%
Risk	3.29%	4.26%	3.88%
FY2006			
Return	1.84%	5.20%	3.65%
Risk	3.51%	5.09%	4.25%
FY2007			
Return	−2.15%	−5.45%	−3.23%
Risk	2.32%	4.42%	2.93%
FY2008			
Return	2.61%	−2.25%	0.11%
Risk	7.85%	14.79%	11.14%
FY2009			
Return	5.97%	12.68%	9.49%
Risk	2.79%	5.76%	3.61%
FY2010			
Return	0.07%	−1.60%	−0.68%
Risk	3.14%	3.86%	3.12%
FY2011			
Return	8.21%	9.21%	8.86%
Risk	3.77%	5.47%	3.86%
FY2012			
Return	8.38%	15.45%	11.72%
Risk	1.74%	4.44%	2.69%
1305-FY2014			
Return	2.83%	6.17%	4.57%
Risk	4.96%	7.68%	5.96%
FY2014			
Return	13.20%	16.88%	14.18%
Risk	2.60%	4.05%	2.84%

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16

Numerical Simulations 3

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17

Conclusions

- The characteristics of Dollar cost averaging investment, which is popular among retail mutual fund Investment area, is useful for Accumulation / Decumulation phase pension fund investment consideration.
- Ladder Investment, which is popular among Banks' investment strategy, is a kind of Dollar cost averaging Investment and useful for ALM/LDI of pension funds.
- Bucket Investment, which is getting popular, has to be treated carefully because we should address from behavioural finance's concept "Mental Accounting."
- Cover's Universal portfolio idea is extended into another new return sources of conservative pension fund investment.

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18

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Thank you for your attention!