Agenda

• Introduction to compartmental reserving modelling framework
• Modelling the mean claims process
• Modelling reserve uncertainty
• Case study using R/Stan & ‘brms’
• Summary
Compartmental Reserving Models

Introduction to compartmental reserving modelling framework
The compartmental reserving modelling framework

• Key idea: Start by fitting model to data, not data to model

• At the centre of the framework is to think about the data generating process
  – Begin by simulating artificial data that shares the expected real data characteristics

• Use “compartments” to reflect different business processes
  – Exposure being underwritten
  – Claims being reported
  – Payments being made
The compartmental reserving modelling framework

• Expert knowledge required to model and parameterise
  – A Framework *not* a Method!

• Benefits:
  – Transparent model that can be criticised
  – Provides additional insight into business processes
  – Practitioner knowledge can be incorporated into model easily
## Relation to other models / frameworks

<table>
<thead>
<tr>
<th>Chain-ladder methods</th>
<th>Cumulative growth curves</th>
<th>Incremental growth curves</th>
<th>Hierarchical compartmental model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-parametric</td>
<td>Parametric</td>
<td>Additive</td>
<td></td>
</tr>
<tr>
<td>Multiplicative</td>
<td>Multiplicative</td>
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<td></td>
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</tbody>
</table>

- **Cumulative triangles**: Cumulative triangles
- **Paid or incurred**: Paid or incurred
- **Mean + StdError**: Mean + StdError (Max. Likli.)
- **Full distribution (Bayes)**: Full distribution (Bayes)
- **Tail extrapolation**: Tail extrapolation

- **Extensions**:
  - BF Methods
  - Munich-chain-ladder
  - Bayes chain-ladder

- Similarities:
  - Fairly robust, but projections on paid and incurred data can differ

- Differences:
  - Consistent with paid and outstanding data
Hierarchical compartmental reserving models in a nutshell

• Use differential equations to model the mean claims process through time
• Consider which data generating distribution gave rise to the mean process, e.g. Gaussian, Log-normal, Negative-binomial, Tweedie
  – Which variance metric can be considered constant across claims development periods, if any? E.g. coefficient of variation
• Use expert knowledge to set priors on parameters
• Generate data from model: do simulations capture expected features?
• Update model with actual observations to obtain posterior parameter estimates and predictive distributions
Compartmental Reserving Models
Modelling mean claims process
Compartmental models

- Popular tool in multiple disciplines to describe the behaviour and dynamics of interacting processes using differential equations

- Each compartment relates to a different stage or population of the process, usually modelled with its own differential equation

- Examples are found in:
  - Pharma, to model how drugs interact with the body
  - Electric engineering, to describe the flow of electricity
  - Biophysics, to explain the interactions of neurons
  - Epidemiology, to understand the spread of diseases
  - Biology, to describe interaction of different populations
Simple Compartmental claims development model

\[
\frac{dEX}{dt} = -k_{er} \cdot EX
\]
\[
\frac{dOS}{dt} = k_{er} \cdot RLR \cdot EX - k_{p} \cdot OS
\]
\[
\frac{dPD}{dt} = k_{p} \cdot RRF \cdot OS
\]

The parameters describe:

- \( k_{er} \): the rate at which claim events occur and are subsequently reported to the insurer
- \( RLR \): the reported loss ratio
- \( RRF \): the reserve robustness factor, the proportion of outstanding claims that eventually are paid
- \( k_{p} \): the rate of payment, i.e. the rate at which outstanding claims are paid
Analytical solutions can be derived by iterative integration.

Solutions Define Development Patterns

\[
\begin{align*}
\frac{dEX}{dt} &= -k_{er} \cdot EX \\
\frac{dOS}{dt} &= k_{er} \cdot RLR \cdot EX - k_p \cdot OS \\
\frac{dPD}{dt} &= k_p \cdot RRF \cdot OS
\end{align*}
\]

\[
\begin{align*}
EX(t) &= \Pi \cdot \exp(-k_{er}t) \\
OS(t) &= \frac{\Pi \cdot RLR \cdot k_{er}}{k_{er} - k_p} \cdot (\exp(-k_p t) - \exp(-k_{er} t)) \\
PD(t) &= \frac{\Pi \cdot RLR \cdot RRF}{k_{er} - k_p} \cdot \frac{(k_{er} \cdot (1 - \exp(-k_p t)) - k_p(1 - \exp(-k_{er} t)))}{k_{er} - k_p}
\end{align*}
\]
Compartmental model with two claims settlement processes

Single rate of settlement can be too simplistic to capture heterogeneous claims characteristics and hence settlement processes at an aggregated level

\[

dEX/dt = -k_{er} \cdot EX \\
\]
\[
dOS_1/dt = k_{er} \cdot RLR \cdot EX - (k_{p1} + k_{p2}) \cdot OS_1 \\
\]
\[
dOS_2/dt = k_{p2} \cdot (OS_1 - OS_2) \\
\]
\[
dPD/dt = RRF \cdot (k_{p1} \cdot OS_1 + k_{p2} \cdot OS_2) \\
\]
Analytical solutions illustrate different processes

\[ E_X(t) = \Pi \exp(-k_{er} t) \]

\[ OS_1(t) = \frac{\Pi \text{ RLR} \cdot k_{er}}{k_{er} - k_{p1} - k_{p2}} \left[ \frac{\exp(- (k_{p1} + k_{p2}) t) - \exp(-k_{er} t)}{\Pi \text{ RLR} \cdot k_{p2}} \right] \]

\[ OS_2(t) = \frac{k_{p1} (k_{p2} - k_{er}) (k_{er} - k_{p1} - k_{p2})}{\Pi \text{ RLR} \cdot k_{p2}} \left[ \frac{\exp(- (k_{p1} + k_{p2}) t) (k_{er} - k_{p1} - k_{p2}) - \exp(-k_{er} t) (k_{er} - k_{p1} - k_{p2}) - \exp(-k_{er} t) k_{p1}}{\exp(-k_{er} t) k_{p1}} \right] \]

\[ PD(t) = \frac{\Pi \text{ RLR} \cdot RRF}{k_{p1} (k_{p2} - k_{er}) (k_{er} - k_{p1} - k_{p2})} \left[ \frac{(k_{p1} (k_{er} (k_{p1} - k_{er}) - k_{p2} (k_{p1} + k_{p2})) + 2k_{er} k_{p2}) + \exp(- (k_{p1} + k_{p2}) t) (k_{er} (k_{p1} - k_{er} k_{p2} + k_{p2}^2 - k_{p1} k_{p2})) + \exp(-k_{p2} t) (k_{er} k_{p2}) - \exp(-k_{er} t) (k_{p1} (k_{p1} k_{p2} + k_{p2}^2 - k_{er} k_{p2}))}{} \right] \]
Compartment models can be extended easily ...

- To incorporate different claims processes, e.g. a faster and slower settlement process
- Separate earning and reporting processes
- Time dependent parameters
- Calendar year effects

- Analytical solutions may become complex, but can opt for ODE solvers
- Note: Paid claims are scaled integration of outstanding claims
Compartmental Reserving Models
Modelling uncertainty
Be careful with your parameter bookkeeping

In a Bayesian framework we distinguish between:

- **Priors**, before we have actual data:
  - Prior parameter distribution, e.g. Planning Loss Ratio (PLR)
  - Prior predictive distribution, e.g. Capital Model Loss Ratio (CLR)

- **Posteriors**, priors updated with actual data:
  - Posterior parameter distribution, e.g. Expected Loss Ratio (ELR)
  - Posterior predictive distribution, e.g. Ultimate Loss Ratio (ULR)
Which process variance metric can be kept constant?

Simulated Behaviour: Cumulative vs. Incremental Model

- Modelling cumulative paid data directly is problematic
- Modelling incremental paid with constant CoV more realistic
**Parameter Uncertainty + Data Generating Process**

- “Which parameters combinations are consistent with our data and model?”
  - Start with prior assumptions, e.g. $\text{ULR} \sim \logN(\mu, \sigma)$, ...
  - Update prior assumptions via the likelihood, $L(y; \text{ULR}, ...)$
  - Obtain ‘posterior’ parameter distributions, $p(\text{ULR}, ...|y)$

- From posterior ELR to posterior ULR:
  1. Simulate realisations from posterior parameter distributions
  2. Simulate realisations from assumed data generation distribution
  3. Sum future paid increment posterior predictive paths
‘Borrow Strength’ with Hierarchies

• Which parameters vary across different cohorts, e.g. accident years and which are more likely to be fixed?
  – Chain-ladder assumption: shape of curves considered fixed across accident years
  – Ultimate loss ratios vary by accident years

• Hierarchical models allow all parameters to vary across cohort
  – A parameter has greater potential to deviate from the ‘cohort average’ parameter value where data are rich (credibility weighting / shrinkage)
  – Hierarchical priors are used to prevent overfitting (regularization)
Compartmental Reserving Models
Case Study
Example data set: Cumulative paid and incurred

California Cas Group

Full 10 year history for accident years 1988 - 1997
Example data set: Incremental paid and outstanding training triangles for accident years 1988 - 1997
Model process and location parameter

- Let $t$ be the development period
- $y(t, \delta)$ describing paid ($\delta = 1$) and outstanding claims ($\delta = 0$)
- Assume process follows a log-normal distribution, with constant $\text{CoV}_\delta$
- We model the median of the claims process as:

$$y(t_j) \sim \log \mathcal{N}(\mu(t_j), \sigma^2_\delta)$$

$$\mu(t_j) = \log f(t_j; \Theta, \delta)$$

$$f(t_j; \Theta, \delta) = (1 - \delta)OS(t_j; \Theta) + \delta (PD(t_j; \Theta) - PD(t_{j-1}; \Theta))$$

$$\delta = \begin{cases} 
0 \text{ if } y \text{ is outstanding claim} \\
1 \text{ if } y \text{ is paid claim}
\end{cases}$$
Setup analytical solution in Stan/C

```plaintext
myFuns <- "
real paid(real t, real ker, real kp, real RLR, real RRF){
    return(RLR*RRF/(ker - kp) * (ker *(1 - exp(-kp*t)) - kp*(1 - exp(-ker*t))));
}

real os(real t, real ker, real kp, real RLR){
    return((RLR*ker/(ker - kp) * (exp(-kp*t) - exp(-ker*t))));
}
```

Institute and Faculty of Actuaries
real claimsprocess(real t, real devfreq, real oker, real okp, real oRLR, real oRRF, real delta) {
    real out;
    real ker = 1 + exp(oker); real kp = 1 * exp(okp * 0.5);
    real RLR = 0.7 * exp(oRLR * 0.1); real RRF = exp(oRLR * 0.1);

    out = os(t, ker, kp, RLR) * (1 - delta) + paid(t, ker, kp, RLR, RRF) * delta;
    if (delta > 0) && (t > devfreq) { // paid greater dev period 1
        out = out - paid(t - devfreq, ker, kp, RLR, RRF)*delta;
    }
    return(out);
}
Parameter structures

- Parameters assumed ‘fixed’ across accident years
  - $k_{er}$, $k_p$, $\sigma_{pd}$, $\sigma_{os}$

- Parameters assumed to vary ‘randomly’ by accident year
  - $RLR_{[i]}$, $RRF_{[i]}$, allowing for correlation:
    \[
    \begin{pmatrix} RLR_{[i]} \\ RRF_{[i]} \end{pmatrix} \sim \begin{pmatrix} RLR_0 \\ RRF_0 \end{pmatrix} + \mathcal{N} \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right) 
    \]
    
    with $RLR_0$, $RRF_0$ following a log-normal distribution
Create non-linear model formula in R

frml <- bf(loss_train ~ log(claimsprocess(dev_year, 1.0, oker, okp,
                                  oLR, oRRF, delta)),

          oLR ~ 1 + (1 |ID| origin_year),
          oRRF ~ 1 + (1 |ID| origin_year),
          oker ~ 1, okp ~ 1,
          sigma ~ 0 + deltaf,
          nl = TRUE)
Set prior parameter distributions

- Setting sensible priors is crucial for MCMC simulations
- Using standard Gaussians seems to be advisable
  - Standard Gaussian can be transformed to appropriate value ranges
- Example:
  - original oRLR ~ N(0,1)
  - Transformed RLR = 0.7 * exp(oRLR * 0.1), i.e. log-normal distribution
Set prior distribution over parameters

mypriors <- c(prior(normal(0, 1), nlpar = "oRLR"),
              prior(normal(0, 1), nlpar = "oRRF"),
              prior(normal(0, 1), nlpar = "oker"),
              prior(normal(0, 1), nlpar = "okp"),
              prior(normal(-3, 0.2), class = "b",
                     coef="deltafpaid", dpar= "sigma"),
              prior(normal(-3, 0.2), class = "b",
                     coef="deltafos", dpar= "sigma"),
              prior(student_t(10, 0, 0.1), class = "sd", nlpar = "oRLR"),
              prior(student_t(10, 0, 0.05), class = "sd", nlpar = "oRRF"))
Run prior predictive model with ‘brms’ in R/Stan

bla <- brm(frml, data = myDat,
          family = brmsfamily("lognormal", link_sigma = "log"),
          prior = mypriors,
          control = list(adapt_delta = 0.9, max_treedepth = 15),
          file = "models/CaliforniaGasLogNormalIncrPriorCGCana",
          stanvars = stanvar(scode = myFuns, block = "functions"),
          sample_prior = "only", seed = 123, iter = 200, chains = 2)
Review prior predictive output

California Cas Group: 200 prior predictive simulations

- $\delta_{\text{taf}} = \text{paid}$ & $\text{origin\_year} = 1988$
- $\delta_{\text{taf}} = \text{os}$ & $\text{origin\_year} = 1988$

![Graph showing loss ratio over development year](image-url)
Run model with actual data

```r
blafit <- update(bla, newdata=modDT_b[!is.na(loss_train)],
                file="models/CaliforniaGasLogNormalIncrPosterior1CGCana",
                sample_prior="no", seed=123, iter=500)
```
Outstanding data with holdouts

Outstanding Loss Ratio Development by Accident Year
50th, 80th and 95th Posterior Predictive Intervals

1988
1989
1990
1991
1992
1993
1994
1995
1996
1997

Outstanding Loss Ratio

Development Year

0% 25% 50% 75% 100%

0% 25% 50% 75% 100%

0% 25% 50% 75% 100%

0% 25% 50% 75% 100%

0% 25% 50% 75% 100%

0% 25% 50% 75% 100%

0% 25% 50% 75% 100%

0% 25% 50% 75% 100%
Paid data with holdouts

Incremental Paid Loss Ratio Development by Accident Year
50th, 80th and 95th Posterior Predictive Intervals

Paid Loss Ratio

Development Year
Cumulative Paid data with holdouts

Cumulative Paid Loss Ratio Development by Accident Year
50th, 80th and 95th Posterior Predictive Intervals

[Graph showing cumulative paid loss ratio development by accident year from 1988 to 1997 with development years ranging from 2 to 10 and paid loss ratios ranging from 25% to 125%]
Distribution of future payments

Actual Reserve vs. Posterior Reserve Distribution by Accident Year
Actual: Black | Expected: Red

Reserve ($000)
Distribution of future payments

From 500 samples:

- Min. 151,776
- 1st Qu. 174,634
- Median 183,273
- Mean 184,184
- 3rd Qu. 191,748
- Max. 236,694
Compartmental Reserving Models

Summary
Summary

• Compartmental reserving models offer:
  – A flexible and transparent framework to develop parametric non-linear curves to describe development of outstanding and paid claims, simultaneously
  – Insight for a variety of small data sizes, as industry data and expert judgement can naturally be incorporated
  – Intuitive and transferable claims process-linked outputs, e.g. business plan LRs

• Bayesian modelling framework offers flexible approach to model process and parameter distribution
  – Expert judgement required to set prior assumptions and review model output
  – Paid development should be modelled on an incremental basis
References


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