Institute and Faculty of Actuaries

CM1 Specimen Questions and Solutions

July 2020

Q1 Calculate ${}_{10|4}q_{[27]+1}$ using AM92 mortality. [3]

(Note: You should show your working, but intermediate steps can be shown using numerical values - no additional notation is required)

Adapted from CM1 April 2019 Q1

Solution:

The question is asking for the probability that a life currently aged [27]+1 will die between the ages of 38 and 42

10|4q([27]+1) = 10p([27]+1) x 4q38

= L38/(L([27]+1)) x (1 - L42/L38)

= (L38 - L42) / L([27]+1)

= (9872.8954 - 9837.0661) / 9936.3549 *[2]*

= 0.003606 *[1]*

Only the last two lines are required for full credit.

Q2 Calculate ${}_{2.75}q_{84.5}$ using the method of uniform distribution of deaths.

Basis:
Mortality ELT15(Females) [4]

(Note: You should show your working, but intermediate steps can be shown using numerical values - no additional notation is required)

CM1 April 2019 Q2 (although an extra mark has been allocated to allow for typing)

Solution:

2.75q84.5 = 1 - 2.75p84.5
= 1 - (0.5p84.5) x (2p85) x (0.25p87) *[1/2]*

0.5p84.5 = 1 - (0.5 x q84) / (1 - (0.5 x q84))
= 1 - (0.5 x 0.08757) / (1 - 0.5 x 0.08757) = 0.95421 *[1]*

2p85 = L87/L85
= 30651 / 38081 = 0.80489 *[1/2]*

0.25p87 = 1 - (0.25 x q87)
= 1 - (0.25 x 0.11859) = 0.97035 *[1]*

2.75q84.5 = 1 - (0.95421 x 0.80489 x 0.97035)
= 1 - 0.74526 = 0.25474 *[1]*

Full marks should be awarded if no notation is shown, provided the method used is clear.

Q3 The force of interest $\delta(t)$ is a function of time, and at any time t, measured in years is given by the formula:

$$\delta(t) = \begin{cases} 0.24 - 0.02t & 0 < t \leq 6 \\ 0.12 & 6 < t \end{cases}$$

(i) Find an expression for $A(t)$, the accumulated amount at time t of a unit investment made at time $t = 0$ for $0 < t \leq 6$. [2]

For $6 < t$, $A(t)$ can be written in the form:

$$A(t) = e^{\boldsymbol{a+bt}}$$

(ii) Derive the values of a and b. [4]

(iii) Calculate the present value of $100 due at time $t = 7$. [2]

[Total 8]

Solution:

(i) A(t) =exp[INT(0,t): (0.24 - 0.02t)dt] *[1]*

= exp[0.24 x t - 0.01 x (t^2)] *[1]*

(ii) For t<6, A(t) = exp[0.24 x t - 0.01 x (t^2)] *[1]*

So A(6) = exp (1.080) *[0.5]*

Then for t>6, A(t) = A(6) x exp[0.12 x (t - 6)]

= exp (1.080) x exp[0.12 x (t - 6)]

= exp(0.36 + 0.12t) *[1.5]*

So a=0.36, b=0.12 *[1]*

(iii) Present value = 100/ A(7)

= 100 exp (- [0.36 + 0.12 x 7]) = 100exp(-1.2) *[1]*

=$30.12 *[1]*

Q4 A life insurance company issues 25-year decreasing term assurance policies to lives aged 40 exact. The death benefit, payable at the end of the year of death, is $500,000 in the first policy year, $480,000 in the second policy year thereafter reducing by $20,000 each year until the benefit is $20,000 in the twenty-fifth and final policy year. Premiums are payable annually in advance for 25 years or until earlier death.

Show that the annual premium per policy is approximately $643 using the basis below.

Basis:

Mortality	AM92 Ultimate
Rate of Interest	4% per annum
Expenses	Ignore

[6]

Adapted from CT5 September 2018 Q13(a) (although extra marks have been allocated for typing)

Solution:

P = (520000 x TA:40:<25> - 20000 x I(TA):40:<25>) / adue:40:<25> [1.5]

Where TA:40:<25> = EA:40:<25> - v^25 x 25p40

= 0.38907 - 0.37512 x (8821.2612/9856.2863)

= 0.38907 - 0.33573 = 0.05334 *[1.5]*

And I(TA):40:<25> = IA:40 - v^25 x 25p40 x [25 x WL:65 + IA:65]

=7.95699 - 0.33573 x [25 x 0.52786 + 7.89442] = 0.87612 *[2]*

P= (520000 x 0.05334 - 20000 x 0.87612) / 15.884 = $643.06 *[1]*