Mortality Projections Committee

WORKING PAPER 93

CMI Mortality Projections Model: Consultation responses and plans for CMI_2016

December 2016

This Working Paper was originally issued in November 2016. This version corrects the titles of Tables 5.3 and 5.4 and makes a number of minor changes to formatting.

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Executive Summary

The CMI Mortality Projections Committee (“the Committee”) has consulted on proposed changes to the CMI Mortality Projections Model (“the Model”), with a view to implementing those changes in the next version, CMI_2016. The Committee has now considered responses to the consultation and will make a number of changes to our original proposals.

The changes are:

1. calibrating the Model to data for England & Wales;
2. constraining the cohort components of mortality improvements for the youngest and oldest cohorts;
3. making projections for cohorts not contained in the calibration data;
4. not introducing Intermediate parameters;
5. adding an “Extended” layer of parameters, initially containing the period smoothing parameter, $S_\alpha$;
6. moving the "Constant Additional Rate of Mortality Improvement" to the Advanced layer of parameters, rather than removing it; and
7. changing the naming convention, so that it is not necessary to include the period smoothing parameter, $S_\alpha$, if the Core value is used.

Of these, only the first two affect the projected mortality rates, and only the second differs from the illustrative results shown in Working Paper 90.

The Committee intends to release CMI_2016 in March 2017. In this paper we provide an early estimate of CMI_2016, based on data to 14 October 2016. This suggests a mortality improvement of around 2% in 2016; however unexpected mortality experience over the remainder of the year would affect this estimate.
## Contents

Executive Summary .................................................................................................................. 2  
1. Introduction .......................................................................................................................... 4  
2. Changes to the proposals in Working Paper 90 ................................................................. 6  
3. Constraining cohort components of mortality improvements ............................................. 9  
4. Changes in method between CMI_2015 and CMI_2016 .................................................... 12  
5. Early estimate of CMI_2016 ............................................................................................... 13  
6. Consultation responses ....................................................................................................... 16  
References ............................................................................................................................... 26  

Appendix A – Implementation of cohort constraints in the APCI Model .................................. 27
1. Introduction

The Committee has consulted on proposed changes to the Model, with a view to implementing these changes in the next version, CMI_2016. The Committee:

- published Working Paper 90, on 22 June 2016, containing detail of our proposals, together with a number of consultation questions;
- held public meetings, in Edinburgh on 29 June 2016 and in London on 11 July 2016, to discuss our proposals; and
- published Working Paper 91, containing further technical detail, and illustrative software, on 31 August 2016.

The consultation period closed on 30 September 2016. Having considered the responses, the Committee is now in a position to comment on them, and to set out our proposals for CMI_2016.

1.1. Responses to the consultation

The Committee was pleased to receive twenty responses, from insurers, reinsurers, consultancies, academics and an industry body. We would like to thank all of the individuals and organisations who responded to the consultation.

The overall tone of most responses is positive, with questions and comments primarily concerning points of detail rather than fundamental principles. The responses are described in detail in Section 6.

1.2. Software

Working Paper 91 said of the software file “CMI Model consultation software v0.1.xlsm”, issued to accompany the consultation, that:

“The software is intended to allow interested parties to replicate the results in Working Paper 90 and this paper, and to consider the impact of particular parameter choices. It is not intended to offer the full functionality of the proposed model, and more complete software will be released to accompany CMI_2016 in March 2017.”

We reiterate this, as a number of responses commented on the software as if it were a direct replacement for the existing Model.

1.3. Contents of this paper

The sections of this paper are as follows:

1. This introduction.
2. A description of how the Committee’s intentions for CMI_2016 now differ from the proposals made in Working Paper 90.
3. Detail of the most substantive change to the Working Paper 90 proposals – to constrain cohort components of mortality improvements at younger ages.
4. A summary of the changes in method now proposed between the previous version of the Model, CMI_2015, and CMI_2016.
6. A summary of the responses to each consultation question and the Committee’s views.
1.4. Next steps

The Committee intends to publish the next version of the Model, CMI_2016, in March 2017.

In addition, a number of respondents to the consultation asked for help in developing their understanding of the impact of using different values of the period smoothing parameter, $S_n$. The Committee is investigating this, using a number of methods; we intend to provide this either alongside, or in advance of, the release of CMI_2016.

1.5. Committee members

The members of the Mortality Projections Committee involved in the production of this Working Paper are: Tim Gordon (Chair), Steve Bale, Piero Cocevar, Matthew Fletcher, Steven Rimmer, Neil Robjohns and Brian Sewell.
2. Changes to the proposals in Working Paper 90

This section describes those areas in which the method to be used in the next version of the Model, CMI_2016, differs from the proposals made in Working Paper 90.

The changes are:

1. calibrating the Model to data for England & Wales;
2. constraining the cohort components of mortality improvements for the youngest and oldest cohorts;
3. making projections for cohorts not contained in the calibration data;
4. not introducing Intermediate parameters;
5. adding an “Extended” layer of parameters, containing the period smoothing parameter, $S_{\alpha}$;
6. moving the “Constant Additional Rate of Mortality Improvement” to the Advanced layer of parameters, rather than removing it; and
7. changing the naming convention, so that it is not necessary to include the period smoothing parameter, $S_{\alpha}$, if the Core value is used.

Of these, only the first two affect the projected mortality rates, and only the second differs from the illustrative results shown in Working Paper 90; its impact is shown in Section 3.

We consider each of the changes in more detail in the following sections.

2.1. Calibrating to data for England & Wales

Existing versions of the Model were calibrated to national population data for England & Wales rather than the UK, primarily due to the timings of releases of data. In Working Paper 90, the Committee proposed to calibrate to UK data instead, subject to consultation, and if the timing and availability of data allowed. This would better reflect the population that UK actuaries typically apply the Model to.

Three respondents would prefer to use England & Wales data. Reasons given are: earlier availability of data, consistency with the existing Model, and consistency with other commonly-used data (e.g. Office for National Statistics (ONS) data on cause of death and deaths by socio-economic class).

Most respondents support the use of UK data however many of these expressed concern about the risk of delays to the publication of the Model and would only support the use of UK data if these concerns could be addressed.

The Committee has investigated the timing of availability of data for Scotland and Northern Ireland, and has contacted their national statistical bodies to see if we could obtain early access to the data. Unfortunately it will not be possible to obtain the relevant data until the second half of March, and this would be too late to reliably produce the Model in a timely manner, after the England & Wales data becomes available.

Consequently, and somewhat reluctantly, the Committee has decided that future versions of the Model will continue to be calibrated to data for England & Wales, in order to ensure its timely and reliable publication.

2.2. Constraining the cohort components of mortality improvements

Under the approach proposed in Working Paper 90, cohort components of mortality improvements would be taken directly from the Age-Period-Cohort Improvement (APCI) model, without any constraints being placed on them. This proposal differs from the current Model, which constrains the cohort components of mortality improvements to be nil for the youngest and oldest cohorts.
Some responses to the consultation raised the concern that the large cohort components of mortality improvement at young ages, particularly for males, may be an artefact of the particular identifiability conditions used in conjunction with the APCI model.

The Committee has investigated this. We share the concerns and will constrain cohort components in a similar way to the current Model. The changes made, and the impact on the results of the Model, are considered in Section 3.

2.3. Projecting improvements for young cohorts

In Working Paper 90 the Committee proposed that the Model would only make projections for cohorts contained within the calibration data; i.e. those aged 20 and above at the end of the calibration period.

While a majority of respondents are happy with the Committee’s proposal, six think that it would be helpful for the Model to make projections for younger cohorts as well. As a result, CMI_2016 will include projections for younger cohorts.

The method for projecting younger cohorts is the same as for other cohorts (as described in Section 5.1 of Working Paper 91). The cohort component of initial mortality improvements for younger cohorts will be nil, consistent with the approach described in Section 2.2 above.

2.4. No “Intermediate” parameters

Working Paper 90 proposed the introduction of an “Intermediate” layer of parameters. These parameters would not add to the functionality of the Model, but would provide a way to express certain changes to Advanced parameters in a succinct way.

Consultation responses were mixed, and among those respondents who agree with the introduction of Intermediate parameters there is no consensus on which parameters should be included. As a result the Committee feels that Intermediate parameters may add complexity without achieving their aim and we have decided not to introduce Intermediate parameters.

2.5. “Extended” parameter layer

The Committee had proposed that the period smoothing parameter, $S_x$, should be an Advanced parameter. We will now create an “Extended” parameter layer, to lie between Core and Advanced, to contain this parameter (only, initially at least). This is being done to emphasise the status of $S_x$ as a parameter that the Committee would encourage users of the Model to consider carefully; but one for which a default value is provided.

2.6. Constant Additional Rate of Mortality Improvement

The Committee had proposed to remove the Constant Additional Rate of Mortality Improvement parameter, unless respondents asked for it to be retained.

Several respondents confirmed that this parameter can be useful; consequently it will be retained in CMI_2016, but will be an Advanced parameter rather than a Core parameter.

2.7. Naming convention

Working Paper 90 proposed that the naming convention for specific parameterisations of the Model should require the inclusion of the period smoothing parameter, $S_x$.

In response to the consultation responses, the period smoothing parameter does not need to be included in the name if the Core value of 7.5 is used. This is described further in Section 2.8.
2.8. Parameter layers and naming

Table 2.1 compares the Core, Extended and Advanced layers of parameters. It sets out the contents and definitions of these layers; the requirement for describing a use of the Model as being Core, Extended or Advanced; and the standard naming convention, with examples.

**Table 2.1: Description of Core, Extended and Advanced layers of parameters**

<table>
<thead>
<tr>
<th>Layer</th>
<th>Core</th>
<th>Extended</th>
<th>Advanced</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contents</td>
<td>Long-term rate of mortality improvement (LTR)</td>
<td>Period smoothing parameter ($S_\alpha$)</td>
<td>All other parameters</td>
</tr>
<tr>
<td>Definition</td>
<td>The “Core” parameter must be determined by users of the Model; as no default value is provided</td>
<td>The “Extended” parameter has a default value, but the Committee encourages users to consider it carefully</td>
<td>The “Advanced” parameters are those which are less likely to be amended by the user</td>
</tr>
<tr>
<td>“… Model”</td>
<td>The description “Core Model” should be used only if no Extended or Advanced parameter has been changed from its default value</td>
<td>The description “Extended Model” should be used only if (i) the parameter $S_\alpha$ is given a value other than its default, and (ii) no Advanced parameter has been changed from its default value</td>
<td>The description “Advanced Model” should be used if any Advanced parameter has been given a value other than its default</td>
</tr>
<tr>
<td>Naming convention</td>
<td>CMI_yyyy_G [L%]; where: yyyy is model version, G is gender (“M” or “F”), L is the long-term rate</td>
<td>CMI_yyyy_G [L%;S=$S_\alpha$]; where: $S_\alpha$ is the period smoothing parameter</td>
<td>CMI_yyyy_G [L%;S=$S_\alpha$] Advanced and any Advanced parameters that take values other than the default should be described</td>
</tr>
</tbody>
</table>
3. Constraining cohort components of mortality improvements

In Section 2.2 we noted that we will constrain the cohort components of mortality improvements for the youngest and oldest cohorts. This is described in detail in this section.

3.1. Constraints

The cohort components of initial mortality improvements will be constrained to be nil for those cohorts aged 30 and younger or 110 and older in the final year of the calibration data. The current Model applies similar constraints for ages 28 and younger, and ages 124 and older. The changes to these ages reflect a rounding of the current assumption for the lower age, and for the older age a desire for consistency with total mortality improvements tapering to nil at age 110.

Appendix A describes how these constraints will be implemented within the APCI model. The rest of this section describes the motivation for the change, and the impact on the Model.

3.2. Results – mortality improvements

In this section we show results with and without the constraints. We use data to the end of 2015, so the results without the constraints match those of Working Paper 90.

Charts 3A and 3B show, for males and females, the cohort components of initial mortality improvements, plotted by age in 2015. Note that the values shown for ages above 100 (to the right of the vertical line) are not used directly in the Model – improvements at those ages are based on the value at age 100 and taper to nil at age 110.

Constraining the cohort components at the youngest and highest ages materially affects these cohorts and slightly affects the values at intermediate ages.
Charts 3C and 3D show the age-period components of initial mortality improvements. These are indirectly affected by the constraints, although their broad shapes are unchanged.

Chart 3C: Age-period component of initial mortality improvements (males)

Chart 3D: Age-period component of initial mortality improvements (females)

Charts 3E and 3F show the total initial mortality improvements. For most ages these are similar with or without the cohort constraints; however they are materially different for ages below age 40. At those ages, the total mortality improvements are materially lower than in CMI_2015, which has total improvements of nearly 5% for males aged 20.

Chart 3E: Total initial mortality improvements (males)

Chart 3F: Total initial mortality improvements (females)

3.3. Results – life expectancies

Tables 3.1 and 3.2 compare cohort life expectancies with and without constraints on the youngest and oldest cohorts. The life expectancies are all as at end 31 December 2015 using base tables of S2PMA for males and S2PFA for females, and an illustrative long-term rate of 1.5% p.a. This is consistent with the main results in Working Paper 90.
Table 3.1: Cohort life expectancy at 31 December 2015, using S2PMA and a long-term rate of 1.5% p.a. (males)

<table>
<thead>
<tr>
<th>Age</th>
<th>25</th>
<th>35</th>
<th>45</th>
<th>55</th>
<th>65</th>
<th>75</th>
<th>85</th>
<th>95</th>
<th>105</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconstrained</td>
<td>63.826</td>
<td>52.812</td>
<td>41.993</td>
<td>31.872</td>
<td>22.396</td>
<td>13.737</td>
<td>6.775</td>
<td>2.829</td>
<td>1.586</td>
</tr>
<tr>
<td>Constrained</td>
<td>63.983</td>
<td>53.079</td>
<td>42.239</td>
<td>32.050</td>
<td>22.462</td>
<td>13.724</td>
<td>6.753</td>
<td>2.823</td>
<td>1.584</td>
</tr>
<tr>
<td>Difference</td>
<td>+0.25%</td>
<td>+0.51%</td>
<td>+0.59%</td>
<td>+0.56%</td>
<td>+0.29%</td>
<td>−0.10%</td>
<td>−0.33%</td>
<td>−0.21%</td>
<td>−0.15%</td>
</tr>
</tbody>
</table>

Table 3.2: Cohort life expectancy at 31 December 2015, using S2PFA and a long-term rate of 1.5% p.a. (females)

<table>
<thead>
<tr>
<th>Age</th>
<th>25</th>
<th>35</th>
<th>45</th>
<th>55</th>
<th>65</th>
<th>75</th>
<th>85</th>
<th>95</th>
<th>105</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconstrained</td>
<td>65.682</td>
<td>54.744</td>
<td>44.137</td>
<td>34.028</td>
<td>24.329</td>
<td>15.244</td>
<td>7.636</td>
<td>3.268</td>
<td>1.696</td>
</tr>
<tr>
<td>Constrained</td>
<td>65.749</td>
<td>54.842</td>
<td>44.220</td>
<td>34.068</td>
<td>24.323</td>
<td>15.214</td>
<td>7.608</td>
<td>3.252</td>
<td>1.691</td>
</tr>
<tr>
<td>Difference</td>
<td>+0.10%</td>
<td>+0.18%</td>
<td>+0.19%</td>
<td>+0.12%</td>
<td>−0.02%</td>
<td>−0.20%</td>
<td>−0.37%</td>
<td>−0.50%</td>
<td>−0.27%</td>
</tr>
</tbody>
</table>

The impact on life expectancy of constraining cohort components is relatively small: between −0.50% and +0.59% for the ages shown in the tables.

Section 12.1 of Working Paper 90 lists the elements of the original proposal for CMI_2016 that are materially different from current practice. We reproduce this list below, annotated with updates (in red) for changes following the consultation.

1. Calibrating the Model to data for the United Kingdom rather than England & Wales; subject to verifying the feasibility of this with the relevant national statistical bodies.

2. Simplifying the method used to adjust exposure data. The new method retains the broad principle of the previous method – that underlying mortality rates are smooth, and outliers are indicative of artefacts in the data.

3. Defining mortality improvements in terms of \( \log m_{x,t} \) but with results from the Model still being expressed using the existing definition in terms of \( q_{x,t} \).

4. Using a new Age-Period-Cohort Improvement (APCI) model to determine components of historical improvements. This means that:
   a. we can fit historical mortality rates and determine mortality improvements, both in aggregate and split into age-period and cohort components, in a single step;
   b. the fitting process is much quicker in terms of run-time; and
   c. we can implement the Model entirely within Microsoft Excel using Visual Basic for Applications (VBA), which makes it more accessible to users.

5. Removing the "step-back" from the edges of the data when determining historical improvements and, instead, requiring the Model itself to deal with the issue of stability.

6. Allowing and encouraging users to adjust the responsiveness of the Model to new data by using a single "period smoothing parameter" that will be included in an "Extended" layer of parameters.

7. Enabling users to express the pattern of convergence in terms of the slope of mortality improvements ("direction of travel") as an alternative to the current approach of proportion remaining at mid-point.

8. Tapering the long-term rate of age-period mortality improvements to zero between ages 85 and 110, rather than between ages 90 and 120.

9. Shortening convergence periods for the youngest cohorts.

10. Removing Making the "Constant Addition to Mortality Improvements" parameter, which we think is little-used, an Advanced parameter.

11. Adding an Intermediate layer of parameters to make it easier to set and communicate certain Advanced parameters.

Please note that this is a summary – see Working Paper 90, Working Paper 91, and Section 2 of this paper for full details.
5. Early estimate of CMI_2016

Working Paper 90 illustrated the impact of the Committee’s proposals by showing results based on data for calendar years 1975-2015, consistent with CMI_2015. To give an indication of the likely results of CMI_2016 we have calibrated a version of the Model to an estimate of the 1976-2016 dataset that we intend using for CMI_2016. Deaths data for 2016 is based on actual data to 14 October 2016 and estimated data for the rest of the year whilst exposure for 2016 has been estimated by rolling forward the ONS 2015 mid-year population estimate. The methodology for estimating deaths and exposure for 2016 is consistent with the approach used within CMI_2014 and CMI_2015 and is described in more detail in Appendix D of Working Paper 74.

The estimate is intended to give an indication of the likely change between CMI_2015 and CMI_2016. However it may not be an accurate prediction, as mortality in the final part of 2016 is unknown; for example, temperature and the prevalence of influenza could affect the results.

5.1. Results – data for 2016

Before looking at results from the early estimate CMI_2016 itself, we consider the estimated data before applying the Model.

Charts 5A and 5B show the numbers of deaths and exposures over recent years (for males and females combined). We use an open circle for data points for 2016 as a visual reminder that they are estimates. Based on actual data to 14 October 2016 we expect about 1% fewer deaths in 2016 than in 2015, and we expect exposure to be about 1% higher, so the mortality improvement from 2015 to 2016 is expected to be about 2%.

Chart 5A: Total numbers of deaths for ages 20-100

Chart 5B: Total exposures for ages 20-100

Chart 5C shows the progression of the standardised mortality ratio (SMR) as a percentage of the value in 2011, together with the 2000-2011 trend. Although the estimated SMR in 2016 is lower than in 2015, the gap between the SMR and the trend has widened slightly (as the mortality improvement in 2016 is slightly lower than the trend improvement).

In earlier working papers we have calculated SMRs using a reference population equal to the actual population in a particular year (typically 2011). In this paper, and in our future work, we will calculate SMRs using the 2013 European Standard Population. This is consistent with the approach taken by many organisations, including the ONS, and will allow a clearer comparison between males and females.

Chart 5D shows four-year average mortality improvements derived from the SMRs. The four-year improvement in 2016 is higher than in 2015, but still below the improvements observed between 2000 and 2011.
5.2. Results – life expectancies

In this section we compare life expectancies under our early estimate of CMI_2016 to historical versions of the Model, and earlier proposals. Tables 5.1 to 5.4 compare the following:

- “CMI_2014” – the actual published Model;
- “CMI_2015” – the actual published Model;
- “CMI_2015P” – the proposal in Working Paper 90 calibrated to data for 1975-2015. This is the model described as “Proposed” in results in Section 9.6 of Working Paper 90 and Section 7.1 of Working Paper 91;
- “CMI_2015R” – the revised proposal, constraining cohort components of initial mortality improvements as described in Section 3, calibrated to data for 1975-2015; and

The life expectancies are all as at end 31 December 2016 using base tables of S2PMA for males and S2PFA for females, and an illustrative long-term rate of 1.5% p.a. (Note that these values are as at end-2016, to enable comparison with the actual values – when CMI_2016 is released – and hence differ from the values shown in Section 3.3.)

Life expectancies fell between CMI_2014 and CMI_2015 for all of the ages shown, and are expected to fall further between CMI_2015 and CMI_2016.

The expected falls in life expectancy between CMI_2015 and CMI_2016 can be split into two components:

- The rows “2015 to 2015R” show the impact of changes in method, using data for 1975-2015 in each case. The impact is material, and similar to that shown in Working Papers 90 and 91. The impact of imposing cohort constraints announced in this paper (in the rows “2015P to 2015R”) is relatively small compared to those changes announced in Working Paper 90 (in the rows “2015 to 2015P”).
- The rows “2015R to 2016E” show the estimated impact of using data for 1976-2016 rather than 1975-2015, using the revised method in each case. These falls are broadly comparable to those between CMI_2014 and CMI_2015, although with some variations by age.

We emphasise that results for CMI_2016E are an estimate, and could change; dependent on actual mortality in the final part of 2016.
Table 5.1: Life expectancy – males

<table>
<thead>
<tr>
<th>Age</th>
<th>25</th>
<th>35</th>
<th>45</th>
<th>55</th>
<th>65</th>
<th>75</th>
<th>85</th>
<th>95</th>
<th>105</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMI_2014</td>
<td>65.259</td>
<td>54.048</td>
<td>43.051</td>
<td>32.678</td>
<td>22.803</td>
<td>13.930</td>
<td>7.018</td>
<td>3.029</td>
<td>1.656</td>
</tr>
<tr>
<td>CMI_2015</td>
<td>64.979</td>
<td>53.764</td>
<td>42.763</td>
<td>32.397</td>
<td>22.519</td>
<td>13.672</td>
<td>6.880</td>
<td>2.978</td>
<td>1.637</td>
</tr>
<tr>
<td>CMI_2015P</td>
<td>63.946</td>
<td>52.941</td>
<td>42.109</td>
<td>31.959</td>
<td>22.461</td>
<td>13.841</td>
<td>6.857</td>
<td>2.842</td>
<td>1.587</td>
</tr>
<tr>
<td>CMI_2015R</td>
<td>64.098</td>
<td>53.201</td>
<td>42.363</td>
<td>32.143</td>
<td>22.536</td>
<td>13.831</td>
<td>6.833</td>
<td>2.836</td>
<td>1.584</td>
</tr>
<tr>
<td>CMI_2016E</td>
<td>63.798</td>
<td>52.886</td>
<td>42.020</td>
<td>31.837</td>
<td>22.275</td>
<td>13.641</td>
<td>6.735</td>
<td>2.787</td>
<td>1.576</td>
</tr>
</tbody>
</table>

Table 5.2: Life expectancy – females

<table>
<thead>
<tr>
<th>Age</th>
<th>25</th>
<th>35</th>
<th>45</th>
<th>55</th>
<th>65</th>
<th>75</th>
<th>85</th>
<th>95</th>
<th>105</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMI_2015</td>
<td>67.107</td>
<td>55.927</td>
<td>45.035</td>
<td>34.608</td>
<td>24.586</td>
<td>15.295</td>
<td>7.848</td>
<td>3.533</td>
<td>1.769</td>
</tr>
<tr>
<td>CMI_2015P</td>
<td>65.793</td>
<td>54.859</td>
<td>44.238</td>
<td>34.132</td>
<td>24.413</td>
<td>15.344</td>
<td>7.721</td>
<td>3.279</td>
<td>1.696</td>
</tr>
<tr>
<td>CMI_2015R</td>
<td>65.858</td>
<td>54.955</td>
<td>44.323</td>
<td>34.175</td>
<td>24.410</td>
<td>15.314</td>
<td>7.692</td>
<td>3.262</td>
<td>1.691</td>
</tr>
<tr>
<td>CMI_2016E</td>
<td>65.586</td>
<td>54.674</td>
<td>44.036</td>
<td>33.897</td>
<td>24.162</td>
<td>15.123</td>
<td>7.588</td>
<td>3.209</td>
<td>1.682</td>
</tr>
</tbody>
</table>

Table 5.3: Life expectancy comparisons – relative percentage differences, males

<table>
<thead>
<tr>
<th>Age</th>
<th>25</th>
<th>35</th>
<th>45</th>
<th>55</th>
<th>65</th>
<th>75</th>
<th>85</th>
<th>95</th>
<th>105</th>
</tr>
</thead>
<tbody>
<tr>
<td>2014 to 2015</td>
<td>-0.43%</td>
<td>-0.53%</td>
<td>-0.67%</td>
<td>-0.86%</td>
<td>-1.25%</td>
<td>-1.85%</td>
<td>-1.97%</td>
<td>-1.71%</td>
<td>-1.16%</td>
</tr>
<tr>
<td>2015 to 2016E</td>
<td>-1.82%</td>
<td>-1.63%</td>
<td>-1.74%</td>
<td>-1.73%</td>
<td>-1.08%</td>
<td>-0.23%</td>
<td>-2.10%</td>
<td>-6.40%</td>
<td>-3.73%</td>
</tr>
<tr>
<td>2015 to 2015P</td>
<td>-1.59%</td>
<td>-1.53%</td>
<td>-1.53%</td>
<td>-1.35%</td>
<td>-0.26%</td>
<td>+1.24%</td>
<td>-0.34%</td>
<td>-4.56%</td>
<td>-3.09%</td>
</tr>
<tr>
<td>2015P to 2015R</td>
<td>+0.24%</td>
<td>+0.49%</td>
<td>+0.60%</td>
<td>+0.58%</td>
<td>+0.34%</td>
<td>+0.07%</td>
<td>-0.34%</td>
<td>-0.22%</td>
<td>-0.14%</td>
</tr>
<tr>
<td>2015 to 2015R</td>
<td>-1.36%</td>
<td>-1.05%</td>
<td>-0.94%</td>
<td>-0.78%</td>
<td>+0.08%</td>
<td>+1.16%</td>
<td>-0.68%</td>
<td>-4.77%</td>
<td>-3.22%</td>
</tr>
<tr>
<td>2015R to 2016E</td>
<td>-0.47%</td>
<td>-0.59%</td>
<td>-0.81%</td>
<td>-0.95%</td>
<td>-1.16%</td>
<td>-1.37%</td>
<td>-1.43%</td>
<td>-1.71%</td>
<td>-0.53%</td>
</tr>
</tbody>
</table>

Table 5.4: Life expectancy comparisons – relative percentage differences, females

<table>
<thead>
<tr>
<th>Age</th>
<th>25</th>
<th>35</th>
<th>45</th>
<th>55</th>
<th>65</th>
<th>75</th>
<th>85</th>
<th>95</th>
<th>105</th>
</tr>
</thead>
<tbody>
<tr>
<td>2014 to 2015</td>
<td>-0.48%</td>
<td>-0.59%</td>
<td>-0.72%</td>
<td>-0.93%</td>
<td>-1.37%</td>
<td>-1.94%</td>
<td>-2.05%</td>
<td>-2.00%</td>
<td>-1.72%</td>
</tr>
<tr>
<td>2015 to 2016E</td>
<td>-2.27%</td>
<td>-2.24%</td>
<td>-2.22%</td>
<td>-2.05%</td>
<td>-1.73%</td>
<td>-1.12%</td>
<td>-3.31%</td>
<td>-9.17%</td>
<td>-4.96%</td>
</tr>
<tr>
<td>2015 to 2015P</td>
<td>-1.96%</td>
<td>-1.91%</td>
<td>-1.77%</td>
<td>-1.38%</td>
<td>-0.71%</td>
<td>+0.32%</td>
<td>-1.61%</td>
<td>-7.20%</td>
<td>-4.17%</td>
</tr>
<tr>
<td>2015P to 2015R</td>
<td>+0.10%</td>
<td>+0.18%</td>
<td>+0.19%</td>
<td>+0.13%</td>
<td>-0.01%</td>
<td>-0.20%</td>
<td>-0.38%</td>
<td>-0.52%</td>
<td>-0.27%</td>
</tr>
<tr>
<td>2015 to 2015R</td>
<td>-1.86%</td>
<td>-1.74%</td>
<td>-1.58%</td>
<td>-1.25%</td>
<td>-0.72%</td>
<td>+0.12%</td>
<td>-1.99%</td>
<td>-7.68%</td>
<td>-4.43%</td>
</tr>
<tr>
<td>2015R to 2016E</td>
<td>-0.41%</td>
<td>-0.51%</td>
<td>-0.65%</td>
<td>-0.81%</td>
<td>-1.02%</td>
<td>-1.25%</td>
<td>-1.35%</td>
<td>-1.61%</td>
<td>-0.55%</td>
</tr>
</tbody>
</table>
6. Consultation responses

This section considers each of the consultation questions in Working Paper 90 in turn. It summarises the responses received, and gives the Committee’s thoughts, particularly where there is no clear support for the proposals.

<table>
<thead>
<tr>
<th>Q 3.1</th>
<th>Do you have any concerns about the current Model that are neither mentioned in Section 3.3 nor addressed by our proposals?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Responses</td>
<td>There were a wide range of responses to this open-ended question. Individual respondents:</td>
</tr>
<tr>
<td></td>
<td>• would like clarity over the methods used for estimating the last year of data</td>
</tr>
<tr>
<td></td>
<td>• are concerned that the APCI model does not allow for cohort effects reducing with increasing age.</td>
</tr>
<tr>
<td></td>
<td>• would like to make $S_\kappa$ and direction of travel required inputs without default values</td>
</tr>
<tr>
<td></td>
<td>• make a lengthy comment about residuals</td>
</tr>
<tr>
<td></td>
<td>• note that adding an extra year of data can have a material impact on fitted improvements in earlier years (e.g. adding data for 2015 changes views of 1980 improvements)</td>
</tr>
<tr>
<td></td>
<td>• ask the CMI to fully disclose implicit assumptions</td>
</tr>
<tr>
<td></td>
<td>• are concerned about the CMI using the term “default”</td>
</tr>
<tr>
<td></td>
<td>• describe “weaknesses” of the convergence function for some combinations of initial and long-term rates</td>
</tr>
<tr>
<td></td>
<td>• suggest that users may wish to use un-smoothed improvements when rolling forward a base table</td>
</tr>
<tr>
<td></td>
<td>• suggest that cohort improvements need not run off to zero</td>
</tr>
<tr>
<td></td>
<td>• ask what help the CMI can give on basis risk</td>
</tr>
<tr>
<td></td>
<td>• make several comments on the desirability of assessing a model with respect to its predictiveness and asks if the CMI will share its work in this area</td>
</tr>
<tr>
<td></td>
<td>• note that “both the current and proposed projections models rely, in the medium term, on the extrapolation of past trends which will not necessarily reflect expectations about how the underlying drivers may change”. They “are not proposing that the Committee adopts a different approach, but we do recognise that it is a limitation”.</td>
</tr>
</tbody>
</table>

**Comment**

The Committee has noted these comments and questions, some of which have been addressed through this paper; we will consider which of the other areas should be explored further.
### Q 5.1 Would you prefer the Model to be calibrated to data for England & Wales (as is done currently) or the United Kingdom (England, Wales, Scotland and Northern Ireland)?

<table>
<thead>
<tr>
<th>Responses</th>
<th>The most common answer is “yes, but”. Most respondents support the use of UK data however many of these express concern about the increased likelihood of delays to publication of the Model and would only support the use of UK data if these concerns could be addressed. Three respondents would prefer to use England &amp; Wales data because of: earlier availability of data, consistency with the existing Model, and consistency with other material (e.g. cause of death and deaths by socio-economic class). There are differing views on whether multiple datasets should be provided. One respondent would like only a single dataset to be offered, for consistency between users. Another respondent would like datasets for individual countries to be made available. One response mentions the aspiration of using SAPS data in due course.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comment</td>
<td>The Model will continue to be calibrated to data for England &amp; Wales, as discussed in Section 2.1.</td>
</tr>
</tbody>
</table>

### Q 5.2 Do you agree with the proposed new exposure adjustment method?

<table>
<thead>
<tr>
<th>Responses</th>
<th>There is broad support for the proposed method. However individual respondents: · ask whether there may be cohort-specific patterns and whether allowing for cohort patterns when making adjustments would be preferable · disagree with the proposal without suggesting a specific alternative · express concern about the “cliff-edge” all-or-nothing nature of the adjustment · note that the adjustment makes little difference to results and suggest that it adds “additional complexity for little real gain”.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comment</td>
<td>The Committee agrees that the exposure adjustment is less important under the proposed method than for earlier versions. Making an exposure adjustment was material for CMI_2014 and CMI_2015, due to the degree of smoothing being chosen &quot;automatically&quot; based on the quasi-Bayesian Information Criterion. However the exposure adjustment is much less material for CMI_2016, due to the degree of smoothing being set explicitly using parameters. The Committee also notes that more complex methods could be used. However, the Committee will apply the proposed exposure adjustment method in CMI_2016. The method is simple, and makes it clear that the problem has been addressed, rather than it being dealt with in the APCI model.</td>
</tr>
</tbody>
</table>
Q 6.1 Do you agree with the use of mortality improvements in terms of $\log m_{x,t}$ within the APCI model, and in the definition of the long-term rate?

Responses

Most respondents are happy with the Committee’s proposal to use $\log m_{x,t}$ within the APCI Model but five respondents would prefer the LTR to be expressed in $q$-terms; one suggests an approximate conversion $LTR^* = -\log(1 - LTR)$ to achieve this.

Individual respondents:

- comment that they “struggle to understand why this change is deemed necessary by the CMI” and are concerned that this “change in currency effectively weakens the meaning of a given long term rate parameter (e.g. 1.5% p.a.), which actuaries and clients have become very familiar with over the last 7 years.”
- note that the change in the form of the LTR leads to lower liabilities and ask for this to be made explicit
- ask for clarity over which LTR should be input into the Model (in $m$-terms or $q$-terms).

Comment

The Committee will retain the proposed approach, consistent with the views of the majority of respondents.

Q 6.2 Do you agree with the method, described in Section 6.3, of converting to mortality improvements in terms of $q_{x,t}$?

Responses

There is broad support for the proposed method.

One respondent asks if the conversion to $q_x$ could happen after fitting the APCI model and before projection.

Three respondents note an alternative method with $q_{x,t} = \frac{2m_{x,t}}{2 + m_{x,t}}$ that is consistent with an assumption of a uniform distribution of deaths within each age/year cell. One expresses a preference for this, but accepts that the difference is not material; the other two express a preference for the proposed method.

Comment

The Committee will retain the proposed approach, consistent with the views of the majority of respondents.
Q 7.1 Do you agree with the use of the APCI model to determine components of historical mortality improvements?

Responses
There is broad support for use of the APCI model. Individual respondents:
- suggest use of the cohort extension of the Lee-Carter model
- ask for more research into age-moderation of cohort
- note that the Committee has not offered any evidence for the APCI model based on predictive capabilities
- note that “the consultation document shows some large difference between the existing and proposed initial rates” which they would like to understand better.

Comment
The Committee will retain the proposed approach, consistent with the views of the majority of respondents.

Q 7.2 Do you agree with the identifiability constraints used in the APCI model?

Responses
Most respondents agree with the proposal, and one notes consistency with the Cairns-Blake-Dowd framework.

Two respondents raise concerns over the size of cohort effects at the edges of the data. One suggests removing the identifiability constraints on the cohort term and replacing the cohort penalty with $\lambda \gamma \sum_c \gamma_c^2$. The other would like the “number of cohorts to exclude” parameter to be part of the Model.

One respondent asks if the constraint on $\kappa_t$ could be removed as it only affects the split between age and period improvements (as noted in Working Paper 90).

Comment
The changes described in Section 3 should address the concerns over the size of cohort effects at the edges of the data. The Committee thinks that this is preferable to including “number of cohorts to exclude” as an option.

The Committee considers the constraint on $\kappa_t$ is helpful, even if it does not affect the results of the Model, as it helps with the speed of convergence of the APCI model.

Q 7.3 Do you agree with the approach taken in smoothing the APCI model; i.e. the use of regularisation penalties with a different order used for the period terms?

Responses
Most respondents agree with the proposal. Individual respondents:
- refer to their responses to question 10.3 (smoothing $\kappa_t$ outside the model calibration process) and question 7.2 (an alternative penalty for $\gamma_c$)
- ask why a second–order penalty is not used for $\gamma_c$
- ask for more explanation of why a third-order penalty is suitable.

Comment
The Committee has some sympathy with the view that a second-order penalty for $\gamma_c$ should be used. It is not obvious whether a second- or third-order penalty is preferable; however one advantage of a second-order penalty – controlling effects at the edges of the data – is addressed explicitly by the change described in Section 3.
### Q 8.1  Do you agree that we should give users the option of specifying convergence as they choose: either in terms of direction of travel or proportion remaining at mid-point?

**Responses**  Most respondents support the proposal.

One respondent says that direction of travel should not be an option; “Section 5.3 of Working Paper 91 […] casts doubt on the robustness of using this method” and “including the option to specify direction of travel in the CMI model implicitly gives credence to this method”.

Two respondents note that having multiple ways of achieving the same thing could represent “unnecessary complexity”.

**Comment**  The Committee will retain the proposed approach, consistent with the views of the majority of respondents.

### Q 8.2  If you agree with allowing users to specify convergence in terms of direction of travel, should the Core model allow for this or not?

**Responses**  There is broad support for the proposal that direction of travel should be nil in the Core Model.

**Comment**  The Committee will retain the proposed approach, consistent with the views of the majority of respondents.

### Q 8.3  Do you agree with the proposed Core assumption, to taper the long-term rate to zero between ages 85 and 110?

**Responses**  Eleven respondents agree with the proposal, describing it as “more realistic” and commenting that it reflects current practice better than the current Model.

Four respondents disagree:

- three state that they expect mortality improvements to increase for higher age groups over time
- one suggests a taper from age 90 to 110, but with a small non-zero improvement at old ages.

One respondent suggests that improvements for males should taper earlier than for females, but require a greater starting level of long-term rate.

One respondent notes that while they agreed with the proposal, the proposed change would reduce liabilities, and users should arguably increase their LTRs to compensate.

One respondent agrees with the direction of the change, but would welcome more analysis from the Committee.

**Comment**  The Committee will retain the proposed approach, consistent with the views of the majority of respondents (noting that users can choose to adjust the tapering assumptions in the Advanced parameters).
Q 8.4 | Do you agree with the proposal to amend the cohort convergence periods in the way described in Section 8.7?

**Responses**
All bar one of the respondents are happy with the proposal. One respondent uses this question to raise a related point; they strongly disagree with assigning improvements to cohort components at young ages and suggest that cohort improvements should be nil below age 40.

**Comment**
The Committee will retain the proposed approach, consistent with the views of the majority of respondents. The disagreement is addressed by the changes set out in Section 3.

---

Q 8.5 | Would you be happy for the Model to only make projections for cohorts contained within the calibration data; i.e. those aged 20 and above at the end of the calibration period?

**Responses**
A majority of respondents are happy with the Committee’s proposals; however six respondents think it would be helpful to have Model output for younger cohorts as well.

One respondent notes that different subsets of the data can produce quite different cohort effects at younger ages. They suggest forcing cohort effects to be nil for young ages.

**Comment**
The Committee will project younger cohorts in CMI_2016, as discussed in Section 2.3.
### Q 9.1 Do you agree with the proposed Core assumption for $S_\alpha$ of 7.5?

**Responses**
Fifteen responses express a preference regarding the value for $S_\alpha$; of these, ten agree with the Committee’s proposal, two prefer a lower value and three prefer a higher value.

Individual respondents:
- are concerned about the sensitivity of the parameter, noting that “it will be difficult to justify a parameter at a non-round number”. They would like to better understand how a value of 7.5 compares to the responsiveness of the current Model.
- ask if we can provide a more intuitive description of this parameter – e.g. can it be expressed as equivalent to an N-year running average for some N. They suggest that few pension scheme trustees will understand what it means.
- note that “it is difficult to contextualise what this parameter means and it seems quite subjective as to whether the value chosen is appropriate.”, and would like us “to find an alternative way to express this parameter such that it would not lose all meaning when a different data set, time period or age range is used”. They suggest that most users will use the Core value, so this should “ideally be an industry consensus best-estimate.”
- find our analysis simplistic and think that “the value should be chosen to a higher precision and should be targeted at a level that provides the best long term prediction of mortality rates.”

Another respondent notes elsewhere in their response that “it would be helpful if the Committee could articulate for users what the smoothing parameters actually represent”.

**Comment**
The Committee is reassured that most respondents agree with our proposal, and the other suggestions are split between higher and lower values. Accordingly, we will retain the proposed value of 7.5 for CMI_2016.

The Committee notes the demand for help with understanding the parameter, and how it affects the smoothness of different datasets. As noted in Section 1.4, we will address this either alongside, or in advance of, the release of CMI_2016.

### Q 10.1 Do you agree with the removal of the Constant Additional Rate of Mortality Improvement parameter?

### Q 10.2 If you disagree with the removal of the Constant Additional Rate of Mortality Improvement parameter, do you agree with making it an Advanced rather than a Core parameter?

**Responses**
Three respondents ask for the parameter to be retained; one says that it is useful for sensitivity testing; and one notes that (although they would be happy for it to be removed) some competitors use the parameter as part of their statutory basis.

If the parameter is retained, almost all respondents would prefer it to be an Advanced parameter.

**Comment**
The parameter will be retained, but made an Advanced parameter.
### Q 10.3 Do you agree that it is helpful for users of the Model to be able to control the responsiveness of the Model by varying the parameter $S_x$?

**Responses**

Most respondents agree.

One respondent says that using the parameter $S_x$ to control responsiveness is “neither intuitive, nor convenient”. They suggest not smoothing $k_x$ as part of the calibration, but smoothing it outside the calibration (in an unspecified way). They also note that “a major disadvantage of using $S_x$ as a measure of responsiveness is that it requires actuaries wanting to implement this feature (e.g. in a valuation system) to have access to the whole dataset used to calibrate the model, rather than just a calibration. This is burdensome (having already written code to do this ourselves), and therefore we would strongly prefer an approach to adjusting responsiveness that acted solely on the parameters of a calibrated model.”

Another respondent distinguishes between two parts of a projection: bringing a base table up to date, and then projecting it into the future. These have different requirements and they “believe that it would be preferable if the responsiveness of the future projections could be controlled in isolation (without affecting the initial rates to the current date produced by the model)”.

**Comment**

The Committee will retain the proposed approach, consistent with the views of the majority of respondents.

### Q 10.4 Do you agree that $S_x$ should be an Advanced rather than Core parameter?

**Responses**

Most respondents agree that $S_x$ should be an Advanced parameter. There are four exceptions:

- one would like it to be a Core parameter, with no default value;
- one suggests that it should be an Intermediate parameter;
- one says “probably core”, but does not have a strong view; and
- one would like the Committee to clarify its criteria for making a parameter Core, Intermediate or Advanced; perhaps based on materiality and conceptual simplicity.

**Comment**

As described in Section 2.5, $S_x$ will be an “Extended” parameter.

### Q 10.5 Which of the three options in Section 10.2 for expressing the smoothing parameter do you prefer?

**Responses**

No respondents are in favour of Option 1 (i.e. using $k_x$ directly).

Eleven respondents prefer Option 2 (the proposed option, with $S_x = \log_{10} k_x$) and three prefer Option 3 (expressing $S_x$ relative to the Core assumption). Three are indifferent between Options 2 and 3.

One respondent notes that “the scale produced by the base 10 logarithm gives a tight range of consideration between 6.5 to 8 within which changes may appear spurious” and suggests that “the Committee consider adopting natural logarithms for defining this parameter which would produce a wider numerical range”.

**Comment**

The Committee will retain the proposed approach (Option 2), consistent with the views of the majority of respondents.
### Q 10.6

**Do you agree that $S_x$, $S_\beta$ and $S_\gamma$ should be Advanced parameters?**

**Responses**
All respondents agree except one, who suggests that they should be Intermediate.

**Comment**
The Committee will retain the proposed approach, of making these Advanced parameters, consistent with the views of the majority of respondents.

### Q 10.7

**Do you agree with the introduction of Intermediate parameters?**

**Q 10.8**
**Which of the four Intermediate parameters listed in Section 10.2 should be included?**

**Q 10.9**
**Are there any other Intermediate parameters that you would like to be included?**

**Responses**
There is no consensus here and several of the responses are nuanced. Somewhat subjectively there are nine “Yes”, five “No” and four lukewarm or unclear responses to Q10.7. The last group express concerns over increased complexity, and whether the parameters will actually be used.

Among those respondents that agree with Intermediate parameters there is no consensus on which should be introduced, and some respondents suggest slight modifications to our proposed list of parameters.

Three respondents suggest other Intermediate parameters: one suggests a flat multiple to initial rates, another suggests a flat addition to cohort improvements whilst the third suggests an entirely new parameter, of a floor to mortality improvements in individual age/year cells.

Some respondents suggest that if scaling were applied to convergence periods, as proposed; each should be rounded to an integer.

**Comment**
As noted in Section 2.4, the Committee has decided not to introduce Intermediate parameters.

### Q 11.1

**Do you agree with Model numbers being sequential, so the March 2017 release of the model would be CMI_2016?**

**Responses**
A large majority of respondents agree; however one respondent notes that “people [are] already referring to your proposal as CMI_2016”. They, and three other respondents, would prefer it to be CMI_2017, corresponding to the year of release.

**Comment**
The Committee will retain the proposal, using CMI_2016 for the March 2017 release, consistent with the views of the majority of respondents.

### Q 11.2

**Do you agree with the naming convention “CMI_yyyy_x [a%;s]” where “s” is the period smoothing parameter, $S_x$?**

**Responses**
There is a range of views but a majority would be happy with using “CMI_yyyy_x [a%]” if the default value of $S_x$ is used, and only explicitly including it as CMI_yyyy_x [a%;s] if a non-Core value is used.

**Comment**
As noted in Section 2.7, the period smoothing parameter does not need to be included in the name if the Core value is used.
Q 14.1 Do you have any other comments or questions for the Committee?

<table>
<thead>
<tr>
<th>Responses</th>
<th>Several respondents express their appreciation.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>There are a wide variety of other responses. Individual respondents:</td>
</tr>
<tr>
<td></td>
<td>- would like to better understand the increase in the cohort effect compared to the existing Model and the sensitivity of the Model to age and calendar year range</td>
</tr>
<tr>
<td></td>
<td>- note that convergence periods have implicitly extended by two years, due to the removal of the step-back</td>
</tr>
<tr>
<td></td>
<td>- “particularly like the review of stochastic models provided in WP91 and would support further research in this area”</td>
</tr>
<tr>
<td></td>
<td>- ask for “confidence intervals around the initial rates and if possible some indication of certainty around the projected values” and another asks for confidence intervals</td>
</tr>
<tr>
<td></td>
<td>- make several comments</td>
</tr>
<tr>
<td></td>
<td>- related to their concerns over cohort improvements for young cohorts, they wonder if the “A3” model (with age-modulated cohort) might address this</td>
</tr>
<tr>
<td></td>
<td>- would be interested in references to papers that have been considered for state space modelling</td>
</tr>
<tr>
<td></td>
<td>- note the contrast in evaluations of M7 between Working Paper 91 and the IFoA basis risk report</td>
</tr>
<tr>
<td></td>
<td>- would be interested in a stochastic development of the Model</td>
</tr>
<tr>
<td></td>
<td>- refer to compliance with Technical Actuarial Standards (TASs); arguing that tables in the Working Paper(s) fall under the TASs and note that “it will be important that the CMI fully discloses all (implicit and explicit) assumption[s] made in the model”</td>
</tr>
<tr>
<td></td>
<td>- suggest the use of negative-binomial models for annual volatility</td>
</tr>
<tr>
<td></td>
<td>- ask about work with the ONS on population data</td>
</tr>
<tr>
<td></td>
<td>- note that different users can use alternative timing assumptions and suggest that the CMI could provide clarity</td>
</tr>
<tr>
<td></td>
<td>- ask if the CMI will provide more guidance on the long-term rate</td>
</tr>
<tr>
<td></td>
<td>- note the difference in results when fitting to different data periods. They ask the Committee to “rationalize that the results under the 40 year calibration are appropriate”</td>
</tr>
<tr>
<td></td>
<td>- would find it helpful if the CML 2016 Working Paper could illustrate the impact of the changes to the Core Model in terms of the LTR. e.g. how much would the LTR have to change to counteract changes in method. They also ask if the smoothing parameters are likely to change from year to year.</td>
</tr>
</tbody>
</table>

| Comment   | The Committee has noted these comments and questions, some of which have been addressed through this paper; we will consider which of the other areas should be explored further. |
References


Please note that while this Working Paper is being made publically available, versions of the Model and some other outputs issued since March 2013 are restricted to those organisations and individuals who register as CMI users. Information on how to register is available on the CMI’s website: https://www.actuaries.org.uk/learn-and-develop/continuous-mortality-investigation/about-cmi/how-access-cmi-research.

Note: Working Paper 90 was re-issued in August 2016
Appendix A – Implementation of cohort constraints in the APCI Model

In Section 3 we describe how cohort components of initial mortality improvements will be constrained in CMI_2016; in this appendix we provide technical detail of how this will be achieved.

For ease of explanation, we use numerical values that correspond to the data that will be used in CMI_2016. In later versions of the Model, these values will change according to the data used.

For CMI_2016 we will calibrate to data for ages 20-100, calendar years 1976-2016, and hence birth years 1876 to 1996. We will make cohort improvements zero for ages 30 and younger, and for ages 110 and older in 2016; i.e. for birth years 1906 and earlier and for 1986 and later.

The cohort improvements are derived from the cohort parameters, $\gamma_c$, which are fitted in the APCI model.

A cohort mortality improvement is described in Section 3.3 of Working Paper 91, in terms of age and period, as:

$$ M_{x,t}^{\text{Cohort}} = \gamma_t - x - 1 - \gamma_{t-x} $$

or, equivalently, defined in terms of cohort as:

$$ M_x^{\text{Cohort}} = \gamma_{c-1} - \gamma_c $$

If we want a cohort mortality improvement to be zero, this can be achieved by making consecutive values of $\gamma_c$ identical.

To make cohort improvements zero for birth years 1906 and earlier and for 1986 and later, we do the following for cohort parameters $\gamma_c$:

- for $1906 \leq c \leq 1985$ fit $\gamma_c$ directly, using the APCI model
- for $c < 1906$ set $\gamma_c = \gamma_{1906}$
- for $c > 1985$ set $\gamma_c = \gamma_{1985}$

For example, $M_{30,2016}^{\text{Cohort}} = \gamma_{1985} - \gamma_{1986}$ and this is zero as $\gamma_{1986} = \gamma_{1985}$.

Similarly, $M_{110,2016}^{\text{Cohort}} = \gamma_{1905} - \gamma_{1906}$ and this is zero as $\gamma_{1905} = \gamma_{1906}$.

Under this approach we only directly fit values of $\gamma_c$ for years 1906 to 1985 inclusive, and other values of $\gamma_c$ are derived from these. This means that we need to amend the third-order penalty function accordingly.

The penalty function in Working Paper 90 has the form:

$$ \lambda \Sigma (\gamma_c - 3\gamma_{c-1} + 3\gamma_{c-2} - \gamma_{c-3})^2 $$

This needs to be amended for those cohorts that are not fitted directly; e.g. one of the summands in the standard penalty function is:

$$ (\gamma_{1904} - 3\gamma_{1905} + 3\gamma_{1906} - \gamma_{1907})^2 $$

but since $\gamma_{1904}$ and $\gamma_{1905}$ are not fitted directly, and $\gamma_{1904} = \gamma_{1905} = \gamma_{1906}$, this is expressed in terms of the fitted parameters as:

$$ (\gamma_{1906} - 3\gamma_{1906} + 3\gamma_{1906} - \gamma_{1907})^2 $$

i.e.

$$ (\gamma_{1906} - \gamma_{1907})^2 $$
Consequently we need to amend the penalty matrix $D_\gamma$ referred to in Section 3.5 of Working Paper 91. We currently have a matrix of size $(N - 3) \times N$ where $N$ is the number of cohorts (121 from 1876-1996) and:

$$
D_\gamma = \begin{bmatrix}
+1 & -3 & +3 & -1 & 0 & 0 & 0 \\
0 & +1 & -3 & +3 & -1 & 0 & 0 \\
0 & 0 & +1 & -3 & +3 & -1 & 0
\end{bmatrix}
$$

Instead, this becomes a matrix of size $(N_F + 1) \times N_F$ where $N_F$ is the number of cohorts actually fitted (80 from 1906 to 1985) and:

$$
D_\gamma = \begin{bmatrix}
+1 & -1 & 0 & 0 & 0 & 0 & 0 \\
-2 & +3 & -1 & 0 & 0 & 0 & 0 \\
+1 & -3 & +3 & -1 & 0 & 0 & 0 \\
0 & +1 & -3 & +3 & -1 & 0 & 0 \\
0 & 0 & +1 & -3 & +3 & -1 & 0
\end{bmatrix}
$$
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