



High Age Mortality Working Party

Supplementary note to Working Paper 100

October 2017

NOTE: This document is being made available publicly and its use is not subject to CMI's Terms and Conditions for Subscribers or Terms and Conditions for Academics and CMI Committee Members.

1. Introduction

In June 2017, the CMI High Age Mortality Working Party released [Working Paper 100](#): "A second report on high age mortality". The paper includes a framework for extending graduated mortality rates to higher ages, where the underlying data lacks credibility. The framework is detailed in Section 4 of the paper; specifically:

- Section 4.1 describes the proposed framework;
- Section 4.2 contains the Working Party's estimate of population mortality over the period 2005-2010; and
- Section 4.3 describes case studies on recent Annuities and SAPS data; intended to illustrate the proposed approach. The results of these case studies were shown by means of charts and life expectancy values at various ages.

Since the paper was released, the CMI has received requests from Subscribers for the mortality rates themselves; these rates are contained in a spreadsheet, released alongside this document. These extensions are being issued with support from the Annuities and SAPS Mortality Committees for the convenience of Subscribers interested in the impact of the proposed framework. Whilst Working Paper 100 and this note are available publicly, access to the spreadsheet is restricted to Authorised Users¹.

The calculations in the spreadsheet are not identical to those in earlier work; further details are set out in Section 4, below. We have therefore included amended versions of Tables 4.7 and 4.9 in this note, consistent with the mortality rates in the spreadsheet. Those tables, together with the original values from Working Paper 100 are also included in the spreadsheet, to allow easy comparison.

Please note that these rates are illustrative and do not form part of, or replace, the Annuities and SAPS tables that have been published and should not form definitive expectations of future graduations by the mortality-related committees.

2. TAS compliance

This note and the accompanying spreadsheet are intended to provide actuaries with further information about the framework proposed in Working Paper 100. They comply with the principles in the Financial Reporting Council's Technical Actuarial Standard "TAS 100: Principles for Technical Actuarial Work". Any person using them should exercise judgement over their suitability and relevance.

¹ Most of the CMI's research is only available to "Authorised Users"; i.e. employees of subscribers and researchers, for non-commercial use. Details of how to access the full paper and the CMI's other research can be found on the [CMI's web pages](#).



3. Extension method

This section describes the method used in Working Paper 100 and in the spreadsheet to extend mortality tables. The method is applied to forces of mortality, which can later be converted to mortality rates.

We use the following notation:

$\mu_x^{(g)}$	graduated force of mortality at age x
$\mu_x^{(r)}$	force of mortality in the reference population at age x
μ_x	combined/extended force of mortality at age x
x_0	age (in years) at which the extension starts
N	convergence interval (in years)
c_N	rate of convergence (per convergence interval)

The extended table is specified by:

$$\begin{aligned} \mu_x &= \mu_x^{(g)} && \text{if } x \leq x_0 \\ \mu_x &= \mu_x^{(r)} \times \left(1 + \left(\frac{\mu_{x_0}^{(g)}}{\mu_{x_0}^{(r)}} - 1 \right) (1 - c_N)^{\left(\frac{x-x_0}{N} \right)} \right) && \text{if } x \geq x_0 \end{aligned}$$

so that μ_x tends asymptotically to $\mu_x^{(r)}$ as age increases.

In Working Paper 100, mortality in the reference population ($\mu_x^{(r)}$) was estimated by the Working Party by graduating data for England & Wales published by the Office for National Statistics (ONS), after making a number of extensions and adjustments to the data.

4. Differences from earlier work

There are three areas in which the mortality rates contained in the accompanying spreadsheet differ from earlier work. The numerical impact of these differences is small and does not affect the conclusions of Working Paper 100.

Conversion from forces of mortality to mortality rates

The CMI's recent graduations have used forces of mortality, which are then converted to initial mortality rates for the convenience of Subscribers. However, in Working Paper 100, a simpler method was used for this conversion than the CMI's conventional approach, used in the "08" and "S2" graduations. The calculations in the spreadsheet use the conventional method, and so are not identical to those in Working Paper 100. The difference is negligible/insignificant, so we have not re-issued Working Paper 100.

By definition, the forces of mortality and the initial mortality rates are related by:

$$q_x = 1 - \exp\left(-\int_0^1 \mu_{x+t} dt\right)$$

Typically μ_x cannot be integrated analytically, so must be integrated numerically. The CMI's conventional practice (used for the "08" and "S2" graduations) is to use Boole's rule:

$$\int_0^1 f(t) dt \approx \frac{1}{90} (7f(0) + 32f(0.25) + 12f(0.5) + 32f(0.75) + 7f(1))$$

which leads to:

$$q_x \approx 1 - \exp\left(-\frac{1}{90} (7\mu_x + 32\mu_{x+0.25} + 12\mu_{x+0.5} + 32\mu_{x+0.75} + 7\mu_{x+1})\right)$$



The extensions in Working Paper 100 used the simpler approximation:

$$q_x \approx 1 - \exp\left(-\frac{1}{2}(\mu_x + \mu_{x+1})\right)$$

which is equivalent to using the trapezium rule:

$$\int_0^1 f(t) dt \approx \frac{1}{2}(f(0) + f(1))$$

Boole's rule and the trapezium rule are both examples of Newton-Cotes formulae, with the CMI's conventional practice being more accurate (fourth rather than first degree), but requiring values of μ at non-integer ages.

Approximation of $\mu_x^{(r)}$

In Working Paper 100, the values of $\mu_x^{(r)}$ were approximated using:

$$\mu_x^{(r)} \approx \frac{1}{2}(m_{x-1}^{(r)} + m_x^{(r)})$$

rather than working directly with values of $\mu_x^{(r)}$ produced by the CMI graduation software. The reference tables in the spreadsheet released alongside this note calculate $\mu_x^{(r)}$ directly from the CMI graduation software.

Rounding of mortality rates

The CMI's conventional practice is to round published mortality rates to six decimal places. This was not done in the spreadsheet containing the "08" Series Annuities tables, issued alongside [Working Paper 81](#). Again, the impact of this should be insignificant and the Annuities Committee has decided not to re-issue the rates as this could create confusion. Rounding has been applied to the rates in the spreadsheet that accompanies this paper.

5. Restatement of Tables 4.7 and 4.9

This section has restated versions of Tables 4.7 and 4.9 of Working Paper 100, using the methodology described in Section 4, above. Note that life expectancies for the "published table" columns also differ from those in Working Paper 100, as those figures also used the simpler method, described in Section 4, to convert from μ to q .

Table 4.7: Impact of alternative extension methodology on period life expectancy for selected "08" Series tables

Table	Period life expectancy (published table)				Period life expectancy (alternative extension)				Impact			
	65	75	85	95	65	75	85	95	65	75	85	95
PFL08	21.950	13.648	7.025	3.182	21.997	13.700	7.100	3.261	+0.21%	+0.38%	+1.07%	+2.48%
PFA08	22.555	14.223	7.430	3.214	22.656	14.335	7.584	3.432	+0.45%	+0.79%	+2.07%	+6.68%
LFL08	22.252	13.728	7.053	3.082	22.385	13.872	7.258	3.317	+0.60%	+1.05%	+2.91%	+7.62%
PML08	19.350	11.737	5.910	2.765	19.386	11.779	5.982	2.829	+0.19%	+0.36%	+1.22%	+2.31%
PMA08	20.234	12.325	6.190	2.827	20.287	12.386	6.286	2.927	+0.26%	+0.49%	+1.55%	+3.54%
LML08	20.202	12.084	6.056	2.838	20.230	12.117	6.109	2.852	+0.14%	+0.27%	+0.88%	+0.49%



Table 4.9: Impact of alternative extension methodology on period life expectancy for selected “S2” Series tables

Table	Period life expectancy (published table)				Period life expectancy (alternative extension)				Impact			
	65	75	85	95	65	75	85	95	65	75	85	95
S2PFL	20.951	12.986	6.889	3.267	20.948	12.983	6.884	3.247	-0.01%	-0.02%	-0.07%	-0.61%
S2PFA	21.236	13.188	6.945	3.232	21.235	13.187	6.945	3.230	+0.00%	-0.01%	+0.00%	-0.06%
S2NFA	18.080	10.908	5.737	2.866	18.077	10.903	5.729	2.817	-0.02%	-0.05%	-0.14%	-1.71%
S2IFA	19.202	11.599	5.980	2.759	19.204	11.601	5.983	2.776	+0.01%	+0.02%	+0.05%	+0.62%
S2DFL	20.357	12.818	6.928	3.209	20.360	12.823	6.935	3.237	+0.01%	+0.04%	+0.10%	+0.87%
S2DFA	21.152	13.333	7.144	3.243	21.158	13.339	7.153	3.277	+0.03%	+0.05%	+0.13%	+1.05%
S2PML	21.591	13.378	7.010	3.251	21.590	13.378	7.010	3.248	+0.00%	+0.00%	+0.00%	-0.09%
S2PMA	19.420	11.692	6.013	2.857	19.418	11.690	6.009	2.839	-0.01%	-0.02%	-0.07%	-0.63%
S2NMA	19.207	11.888	6.331	3.348	19.193	11.872	6.303	3.208	-0.07%	-0.13%	-0.44%	-4.18%
S2IMA	15.939	9.660	5.184	2.680	15.937	9.657	5.178	2.632	-0.01%	-0.03%	-0.12%	-1.79%

6. Notes

As noted above, these rates are illustrative and do not form part of, or replace, the Annuities and SAPS tables that have been published. In particular:

- Unqualified references to (for example) “S2PMA” or “PFL08” should refer to the table published alongside Working Paper 71 or 81; not to a table that has been modified using these extensions (or otherwise).
- The Annuities and SAPS Committees may choose to apply the proposed framework to future tables, or they may modify it or consider a different approach to be appropriate. Other than for a portfolio with a very high average age, we would expect the differences in future tables arising from the framework to be considerably lower than the differences arising from the underlying data (and, for annuities, from allowing for product type, for example).
- The start age and the rate of convergence for each table were selected to illustrate the approach and they were not expected to be used as part of a formal table.
- The “population mortality rates” used in Working Paper 100 incorporated a number of adjustments to the data published by the ONS, on which they were based. Fewer adjustments have been applied to the dataset underlying recent versions of the CMI Mortality Projections Committee. The CMI will therefore consider whether a consistent approach can be used for both, before the framework is applied to formal base mortality tables.
- Neither Working Paper 100 nor the current calculations adjust the timing of the population mortality rates, to align them with the effective dates of the Annuities and SAPS tables. The date applicable to each set of rates is stated in the accompanying spreadsheet.
- The approach and the parameter values have not been subject to consultation with prospective users, as would normally be undertaken for formal tables.

Disclaimer: This Document has been prepared by and/or on behalf of Continuous Mortality Investigation Limited (CMI). The CMI does not accept any responsibility and/or liability whatsoever for the content or use of this Document. Whilst care has been taken during the development of the Document, CMI does not (i) warrant its accuracy; or (ii) guarantee any outcome or result from the application of this Document or of any of CMI's work (whether contained in or arising from the application of this Document or otherwise). You assume sole responsibility for your use of this Document, and for any and all conclusions drawn from its use. CMI hereby excludes all warranties, representations, conditions and all other terms of any kind whatsoever implied by statute or common law in relation to this Document, to the fullest extent permitted by applicable law. If you are in any doubt as to using anything produced by CMI, please seek independent advice.

Copyright: You may reproduce the contents of this Document free of charge in any format or medium provided it is:

1. reproduced accurately and is unaltered;
2. not used in a misleading context; and
3. correctly referenced and includes both CMI's Disclaimer notice set out above and CMI's copyright notice, as follows:

© Continuous Mortality Investigation Limited.

Continuous Mortality Investigation Limited ("CMI") is registered in England & Wales
Company number: 8373631
Registered Office: 7th floor, Holborn Gate, 326-330 High Holborn, London, WC1V 7PP

Correspondence address: Cheapside House, 138 Cheapside, London, EC2V 6BW
Email: info@cmilimited.co.uk
Tel: 020 7776 3820

Website: www.cmilimited.co.uk (redirects to the CMI pages of www.actuaries.org.uk)

Continuous Mortality Investigation Limited is wholly owned by the Institute and Faculty of Actuaries.