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Mortality Investigation
Institute and Faculty of Actuaries

High Age Mortality Working Party

WORKING PAPER 106

A proposed approach to closing off CMI mortality tables

June 2018

Summary

In Working Paper 100, the High Age Mortality Working Party set out its approach to:

- Determining deaths and exposures data for the general population;
- Determining high age extensions for the general population; and
- Closing a mortality table at high ages, including a convergence approach from portfolio to population mortality.

In this paper, we set out a proposal for implementing the approach followed in Working Paper 100 into the CMI's graduation approach for tables determined by the mortality-related committees. This paper sets out:

- The proposed methodology for determining exposures data at high ages and the framework for setting mortality rates at high ages for portfolio graduations. The methodology is similar to that presented in Working Paper 100 with minor revisions to the process. We are releasing Excel VBA software to enable subscribers to test our proposed approach themselves.
- A proposal to set the reference general population portfolio at high ages using deaths and exposures from the UK as opposed to England & Wales.
- An assessment of the application of this process to the Self-Administered Pension Schemes (SAPS) proposed "S3" Series mortality tables.

This paper is issued alongside the publication of the proposed "S3" Series graduations in Working Paper 107.

The Working Party welcomes feedback on the proposed methodology so that this can be considered for the final version of the "S3" Series tables. Please send any feedback to HighAgeMortality@cmilimited.co.uk by 14 September 2018.

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1. Introduction

Estimating the population at high ages is important as a prerequisite for national population projections, estimating current mortality rates by age (and hence life expectancy) and estimating past patterns of mortality improvement. These have public policy implications for pension provision, social care and health care, and are of direct interest to pension providers and life insurance companies in the private sector.

For clarification, when discussing high age mortality we generally mean mortality at around age 85 and above (though this may vary from case to case according to the nature of the data under consideration).

The CMI has a particular interest in this field, as it feeds into:

- the calibration of the CMI Mortality Projections Model for estimating past (and future) improvements; and
- the determination of population mortality curves that may be used to close off portfolio mortality tables at high ages, where the underlying datasets lack credibility.

The High Age Mortality Working Party has produced two papers to date:

- In Working Paper 85, we summarised published analysis on very high age mortality, as well as indicative modelling on extinct cohort mortality and the potential impact of age mis-statement and delays in death reporting.
- In Working Paper 100, we reviewed the Kannisto-Thatcher method as currently used by the Office for National Statistics (ONS) for estimating high age population exposures for England & Wales, and proposed a set of modifications designed to address its limitations. We also concluded from an assessment of available studies into the shape of mortality at the 'oldest old' that a mortality curve which makes allowance for deceleration at advanced ages is appropriate for period mortality, i.e. mortality rates over the period analysed with no allowance for future improvements, at a population level. These elements were considered in the proposal of a framework to produce graduated mortality rates at high ages.

In Working Paper 100, we set out our intention to work with the CMI's mortality-related committees to evolve the framework set out in that paper to achieve a consistent approach to closing mortality rate tables across the CMI. This paper sets out this evolved framework for estimating the general population and the approach to graduating mortality rate tables at high ages. We are publishing our VBA tools which apply these approaches alongside this paper.

This paper has been prepared by the CMI High Age Mortality Working Party. The members of the Working Party involved in the preparation of this paper were Steve Bale (Chair), Mark Cooper, Andrew Gaches, Adrian Gallop, Andy Harding and Richard Lamb. We would be very pleased to receive any comments or questions on this paper; these can be sent via e-mail to HighAgeMortality@cmilimited.co.uk.

The Working Party wishes to acknowledge and thank Matthew Fletcher, Chair of the SAPS Committee, for his valuable review of a draft version of this paper.

The structure of this paper is as follows:

- Section 2 sets out our assessment of the UK population dataset as our choice of reference population table;
- Section 3 summarises the approach to determining population deaths and exposures under the methodology explored in Working Paper 100;
- Section 4 sets out the approach for blending from portfolio to population mortality rates;
- Section 5 summarises findings from applying the proposed framework to the SAPS "S3" mortality rate tables; and
- Section 6 sets out our intended next steps.

2. General population data

The Working Party has considered the choice of general population for use in extending graduated mortality rates to high ages, where the CMI's data is unreliable.

The proposed method extends the tables to high ages based on mortality in the general population, so the CMI would like to have an agreed version of general population mortality. This section considers the specific question of whether the general population used should be England & Wales (E&W) or the United Kingdom (UK).

2.1. Precedents

The Mortality Projections Committee (MPC) considered the same issue with regard to the calibration data for the CMI Model, as part of the consultation in 2016.

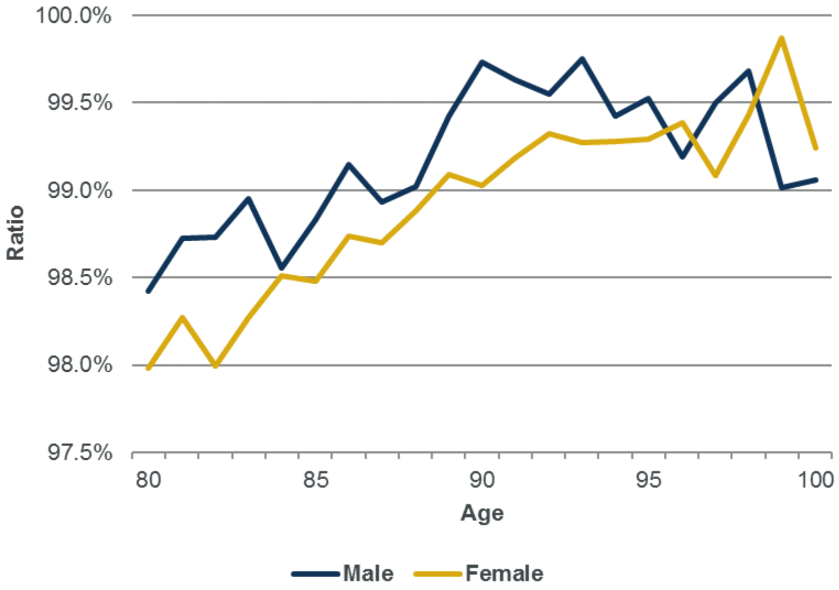
- In Working Paper 90 it said "if the timing and availability of data allows, the Model should be calibrated to the UK population (including England, Wales, Scotland and Northern Ireland). This would better reflect the populations that UK actuaries typically apply the Model to."
- In Working Paper 93 it said "somewhat reluctantly, the Committee has decided that future versions of the Model will continue to be calibrated to data for England & Wales, in order to ensure its timely and reliable publication." Data for Scotland and Northern Ireland has typically been available in March following the end of the previous year, i.e. after the recent versions of the Model have been published.
- The extension of the SAPS "S2" Series tables to young ages made use of mortality at age 16 for the UK population.

The MPC and SAPS committees have both demonstrated a preference for UK data, and it is only the practical timing constraint that has prevented the CMI Model being calibrated to UK data.

2.2. Materiality

Chart 2A plots England & Wales mortality divided by UK mortality. In each case mortality is crude mortality: total deaths in 2012-2016 inclusive divided by total mid-year populations in 2012-2016 inclusive.

Chart 2A: England & Wales mortality as a percentage of UK mortality – males and females



By age 95 – when an extension might typically start to be applied – the ratio in this period is between 99% and 100%. And at age 120 the ratio would be 100% by definition if we assume $\mu_{120} = 1$ for the extension to both populations.

The impact of applying the UK general population instead of the England & Wales general population is assessed in Section 5, and the difference is found to be negligible for the SAPS "S3" graduation.

2.3. Recommendation

We recommend that investigation committees use UK population data for the purpose of high age (and low age) extensions.

- It better reflects the population that CMI data relates to.
- It is consistent with previous preferences expressed by CMI committees.
- While consistency with the CMI Model would be desirable:
  - The MPC approach is a compromise due to timing, and there is no need to compromise elsewhere. The consultation summarised in Working Paper 93 reported that whilst there was support for UK data, there was a critical need that the publication of the Model was not delayed due to delays in other UK constituent data.
  - Consistency may be unachievable, or undesirable, due to MPC needing to use estimated data.
- It is more “inclusive”, reflecting the origins of the Institute and Faculty of Actuaries and being mindful of the Scottish constituency.

3. Deaths and exposures for the general population

Working Paper 100 included a section on population modelling for England & Wales (E&W). This covered

- a review of the Kannisto-Thatcher (KT) method as currently used by the Office for National Statistics (ONS) for estimating high age population exposures for England & Wales,
- analysis of variant methods designed to address limitations in the current approach, and
- for CMI use, a set of proposed modifications to the ONS methodology, based on this analysis.

The analysis reported in Working Paper 100 suggested that there was merit in adopting the following modifications to the current ONS methodology for estimating high age population exposures for E&W:

- refine the projection of survivor ratios in the KT method, by allowing for local mortality trends over time and correspondingly reducing the number of birth-year cohorts in the survivor ratio from 5 to 2;
- extend the high age population method down to a lower age than 90 (for example, age 85) to avoid placing undue reliance on the underlying census-based estimates in this age range;
- incorporate a more sophisticated approach to adjusting the input deaths data to a 1 January age definition (as required by the KT method when using deaths data by age at death) using Lexis triangles;
- smooth the final exposure estimates as a pragmatic solution to issues arising from uneven birth and death distributions during a year.

This section documents our proposed methodology in a concise and unambiguous mathematical format, so that subscribers can consider its implementation. Although originally tested on E&W data, the Working Party also considers this methodology suitable for application to the wider UK dataset described in Section 2. Section 3.1 sets out the details of the method in seven distinct steps, highlighting where our proposal ('HAMWP approach') differs from the current ONS methodology ('ONS current approach'); Section 3.2 explains one area in which the method set out in Section 3.1 has been simplified compared with the original proposals in Working Paper 100:

- this relates to the 'Lexis adjustments' used to convert the input deaths data to a 1 January age definition – a linear formula has been suggested in place of the full grid of adjustments by individual age and calendar year;
- this simplification has been introduced on pragmatic grounds, to make the method easier and more transparent to implement whilst capturing the main features of the original grid;
- we have tested the approximation to check that it is reasonable, with a focus on the mortality rates for calendar years 2009 to 2016 as used in the draft "S3" graduations;
- it is possible that the full grid approach may still be preferred if applying the method in another context, and we recommend that the CMI Secretariat keep this under review.

Section 3.3 explains the rationale for the choice of geometric decay factor 'r_x' proposed in Section 3.1:

- this is used for decomposing grouped high age deaths data into single years of age, before applying the KT method;
- this element of the process was not formally analysed in Working Paper 100, which treated the single year of age data as an 'input' rather than part of the method;
- however, users will need to take a view on this data processing step when they are applying the method in practice, so we have now provided a concrete proposal based on further analysis;
- the proposed approach is to use a decay factor which reduces linearly with age, based on the actual pattern of decay observed at lower ages.

Aside from the two changes noted above, we do not revisit the rationale for (or the impact of) our proposed changes to the current ONS methodology. This analysis was covered in detail in Working Paper 100.

3.1. Technical specification

1. Adjust input data

Suppose the input data for deaths comprises (for $t_0 \leq t < T$):

- $D^I_{x,t}$ $(x < h)$ – the number of deaths during year t of individuals aged x last birthday at date of death; and
- $D^I_{h+,t}$ – the number of deaths during year t of individuals aged h or greater last birthday at date of death.

(a) First, decompose the grouped death data for ages $h+$ into single years of age:

- $D^I_{h,t} = \frac{D^I_{h+,t}}{(1+\sum_{x=h+1}^{\omega} R_x)}$
- $D^I_{x,t} = D^I_{x-1,t} \times r_x \qquad\qquad h < x \leq \omega$

where $R_x = r_{h+1} \times r_{h+2} \times \ldots \times r_x$, with r_x an assumed geometric decay factor, and ω an assumed maximum age of survival.

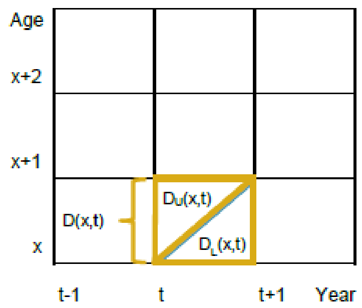
(b) Then convert the death data to the format needed for the KT method (age last birthday as at the *start* of the year):

- $D^K_{\omega,t} = D^I_{\omega,t} \times L_{\omega,t}$
- $D^K_{x,t} = D^I_{x,t} \times L_{x,t} + D^I_{x+1,t} \times \left(1 - L_{x+1,t}\right) \qquad x < \omega$

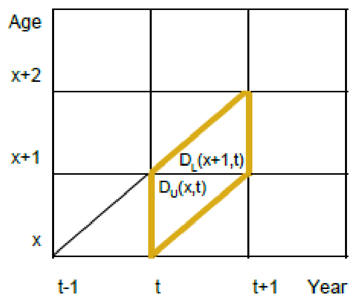
where $L_{x,t}$ is an assumed grid of 'Lexis adjustments'.

The ONS UK data which is readily available to the CMI has death format in line with $D^I_{x,t}$ above (age last birthday at date of death) and grouped death counts above age 104 (i.e. $h = 105$).

Available ONS data
Deaths aged x last birthday at death, over calendar year t



Required data format
Deaths aged x last birthday at the start of calendar year t



$$L_{x,t} = \frac{D_U(x,t)}{D(x,t)}$$

HAMWP approach

- $\omega = 125$
- $h = 105$
- $r_x = 0.5\eta_x + 0.1(1 - \eta_x)$ (males)
  $0.6\eta_x + 0.1(1 - \eta_x)$ (females)

where $\eta_x = \frac{1}{20}\max\{0, \min(20,125 - x)\}$

- $L_{x,t} = 0.51\gamma^m_x + 0.45(1 - \gamma^m_x)$ (males)
  $0.51\gamma^f_x + 0.46(1 - \gamma^f_x)$ (females)

where $\gamma^m_x = \frac{1}{25}\min(25,105 - x)$ and
$\gamma^f_x = \frac{1}{20}\min(20,105 - x)$

ONS current approach

- $\omega = 120$
- $h = 120$ (i.e. actual deaths data by single year of age used throughout)
- r_x – n/a (since input deaths data is ungrouped)
- $L_{x,t} = 0.5$

The formulae for $L_{x,t}$ and r_x under the HAMWP approach are explained in Sections 3.2 and 3.3 respectively.

2. Apply method of extinct generations

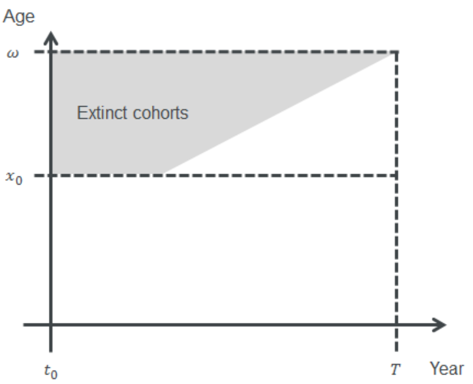
Let $P^K_{x,t}$ denote the estimated population aged x last birthday as at the start of year t.

Under the method of extinct generations, we set (for $t_0 \le t < T$):

- $P^K_{\omega,t} = D^K_{\omega,t}$
- $P^K_{\omega-i,t-i} = P^K_{\omega-i+1,t-i+1} + D^K_{\omega-i,t-i}$ $1 \le i \le \omega - x_0$

where x_0 is a chosen 'join age' for the KT method (that is, the age at and above which the method is applied.)

This procedure determines population estimates for all cohorts known to be 'extinct' ($x > \omega$) by the start of year T.



HAMWP approach

- $x_0 = 85$

ONS current approach

- $x_0 = 90$

The choice of join age x_0 is relevant to later steps in the technical specification. Reducing join age from 90 to 85 was a core element of the Working Party's proposal in Working Paper 100.

The analysis in Working Paper 100 used the time range defined by $t_0 = 1961$ and $T = 2016$, but this was not a fixed part of the proposal. We would expect T to advance as further years of data become available.

3. Apply survivor ratio method with correction factor

For $x = \omega$ down to x_0 inclusive:

(a) Define the 'survivor ratio' $S_{x,t}$ by

- $S_{x,t} = \frac{\sum_{j=1}^{m} P^K_{x,t-j}}{\sum_{j=1}^{m}\sum_{i=1}^{k} D^K_{x-i,t-j-i}}$ (or zero if $\sum_{j=1}^{m}\sum_{i=1}^{k} D^K_{x-i,t-j-i} = 0$)

where k and m are parameters controlling the size of the data 'window' over which the ratio is determined.

(b) Define the linear projection

- $\hat{S}_{x,t} = \alpha_x + \beta_x t$

where α_x and β_x are chosen functions of the past survivor ratio data $S_{x,t}$ for $t \le T$.

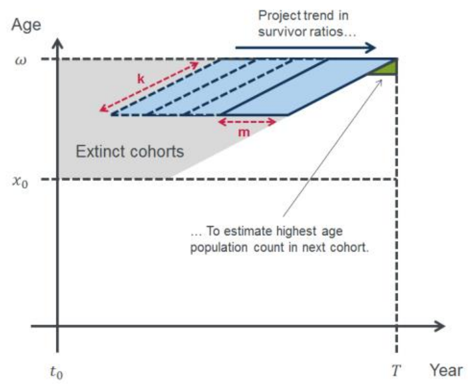
(c) Determine $P^K_{x,T}$ as

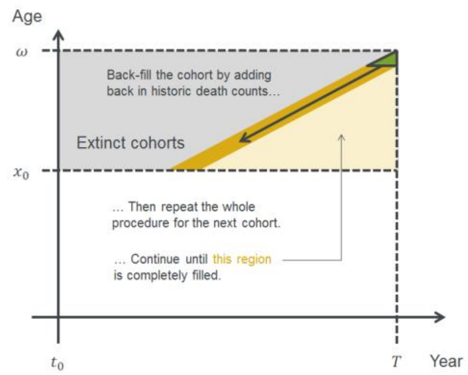
- $P^K_{x,T} = P^K_{x,T}(c) = c \times \max\left(0, \hat{S}_{x,T+\frac{1}{2}(m+1)}\right) \times \sum_{i=1}^{k} D^K_{x-i,T-i}$

where c is a global 'correction factor' under the KT method.

(d) Back-fill the cohort aged x at the start of year T as follows, based on the deaths data:

- $P^K_{x-i,T-i}(c) = P^K_{x-i+1,T-i+1}(c) + D^K_{x-i,T-i}$ $1 \le i \le x - x_0$





HAMWP approach

- $k = 5$
- $m = 2$
- $\hat{S}_{x,t}$ has
  - $\alpha_x = \overline{S_x} - \beta_x \bar{t}$
  - $\beta_x = \frac{\sum_{t=T-4}^{T}(S_{x,t}-\overline{S_x})(t-\bar{t})}{\sum_{t=T-4}^{T}(t-\bar{t})^2}$

  where
  - $\overline{S_x} = \frac{1}{5}\sum_{t=T-4}^{T} S_{x,t}$
  - $\bar{t} = \frac{1}{5}\sum_{t=T-4}^{T} t = T - 2$

This is straightforward linear regression on $S_{x,t}$, i.e. we are projecting the arithmetic trend in survivor ratios.

ONS current approach

- $k = 5$
- $m = 5$
- $\hat{S}_{x,t}$ has
  - $\alpha_x = S_{x,T}$
  - $\beta_x = 0$

This is equivalent to applying the latest known survivor ratio $S_{x,T}$ to the unknown region of data, i.e. without any projection for observed past trends.

The global correction factor c is not determined in advance. Instead, it is solved at the next step to meet a constraint on the final year's population total.

4. Solve for correction factor

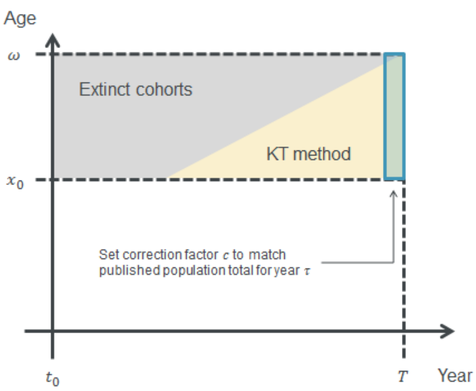
Set the global correction factor c so that the total estimated population at and above the join age matches the published total from the ONS (for the latest year of available data).

That is, solve the equation

- $\sum_{x=x_0}^{\omega} P_{x,\tau}^{K}(c) = \sum_{x=x_0}^{\omega} \frac{1}{2}\left(P_{x,\tau}^{I} + P_{x,\tau-1}^{I}\right)$

in c, where

- $P_{x,t}^{I}$ is the published population count aged x last birthday as at the mid-point of year t, and
- τ is the latest year for which we have the published population figures $P_{x,t}^{I}$ available.
- The mid-year format of $P_{x,t}^{I}$ here reflects the definition used in the published ONS population data. It requires adjustment to align with the start-year format of the KT method estimate $P_{x,t}^{K}$ – hence the interpolation between adjacent years in the formula.



HAMWP approach

The constraint is applied with join age $x_0 = 85$.

ONS current approach

The constraint is applied with join age $x_0 = 90$.

The different choice of join age here has a significant impact on the constrained population estimates.

For the analysis in Working Paper 100, population data were available up to mid-2015 (i.e. $\tau = 2015$).

5. Adjust back to mid-year timing

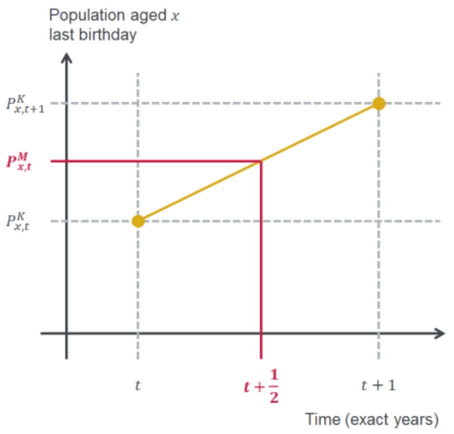
Convert the start-year KT estimates $P^K_{x,t}$ back to the mid-year format of the published ONS data.

If $P^M_{x,t}$ denotes the estimated population count aged x last birthday at the mid-point of year t, then we set

- $P^M_{x,t} = \frac{1}{2}\left(P^K_{x,t} + P^K_{x,t+1}\right)$

for each

- $x_0 \le x \le \omega$
- $t_0 \le t < T$



HAMWP approach	**ONS current approach**
Same as the ONS approach.	Simple linear interpolation as above.

6. Apply uniform annual scaling adjustments

Scale the population estimates for each year (at and above the join age) so that they match the published annual population totals from the ONS (i.e. $\sum_{x=x_0}^{\omega} P^I_{x,t}$, where $P^I_{x,t}$ is the mid-year population for age x and year t published by the ONS).

For $x_0 \le x \le \omega$ and $t_0 \le t < T$, set

- $P^S_{x,t} = \lambda_t P^M_{x,t}$

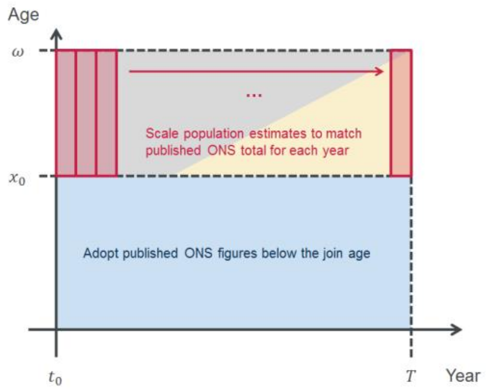
where

$$\lambda_t = \frac{\sum_{x=x_0}^{\omega} P^I_{x,t}}{\sum_{x=x_0}^{\omega} P^M_{x,t}}$$

For ages below the join age, simply adopt the published ONS figures, i.e. set

- $P^S_{x,t} = P^I_{x,t}$

for $x < x_0$ and $t_0 \le t < T$.



HAMWP approach	**ONS current approach**
The constraints are applied with join age $x_0 = 85$.	The constraints are applied with join age $x_0 = 90$.
The different choice of join age has a significant impact on the constrained population estimates.	

7. Smooth exposures

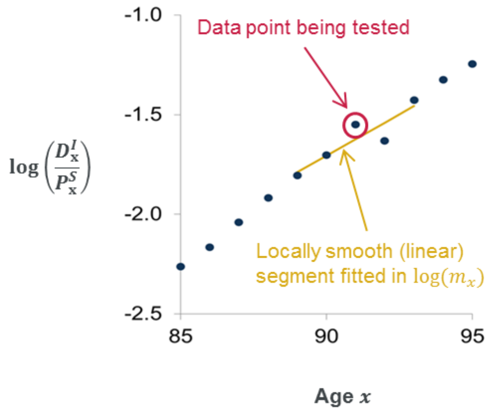
We now have

- death counts $D^I_{x,t}$ (during year t, aged x last birthday at date of death) – from the input data at step 1(a), and
- population estimates $P^S_{x,t}$ (aged x last birthday as at the middle of year t) – from the constrained KT method at step 6,

for the whole grid of ages and years under consideration,

- $x_{min} \le x \le x_{max}$
- $t_0 \le t < T$

Our final step is to smooth the population estimates, using the method specified in Section 5.9 of Working Paper 90.



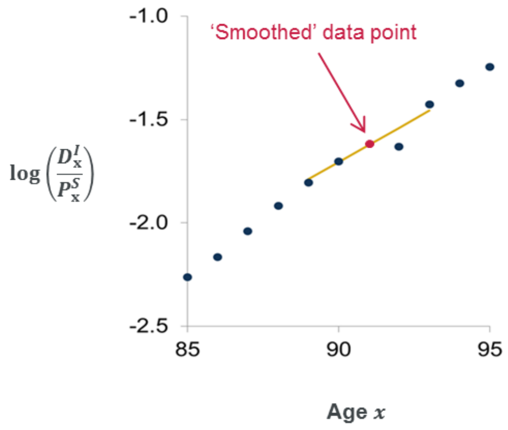
Define:

- $\log(m_{x,t}) = \frac{1}{(2n+1)} \sum_{y=x-n}^{x+n} \log\left(\frac{D^I_{y,t}}{P^S_{y,t}}\right)$
- $r_{x,t} = \sqrt{2\left(D^I_{x,t} \log\left(\frac{D^I_{x,t}}{P^S_{x,t} m_{x,t}}\right) - \left(D^I_{x,t} - P^S_{x,t} m_{x,t}\right)\right)}$

Then set

- $P_{x,t} = P^S_{x,t}$ if $r_{x,t} \le \Phi^{-1}\left(1 - \frac{p}{2}\right)$
- $P_{x,t} = \frac{D^I_{x,t}}{m_{x,t}}$ if $r_{x,t} > \Phi^{-1}\left(1 - \frac{p}{2}\right)$

to deliver final estimates $P_{x,t}$ of the population aged x last birthday as at the middle of year t.



HAMWP approach

The smoothing algorithm is applied with parameters

- $n = 2$
- $p = 1\%$

For ages at and near the edges of the data we need to use a lower value of n, e.g. for ages $x_{min} + 1$ and $x_{max} - 1$ we use $n = 1$, and for ages x_{min} and x_{max} we make no adjustment.

ONS current approach

This smoothing step is not applied under the current ONS approach (which is equivalent to using $p = 0$).

The final population estimates are simply left as

- $P_{x,t} = P^S_{x,t}$

for all x and t.

The application of smoothing under the HAMWP approach has a significant impact on the final population estimates in some regions of the data (e.g. the 'problem' birth cohort of 1919/20).

3.2. Simplification to Lexis adjustments

In order to apply the KT method to calendar year intervals, it is first necessary to convert the input deaths data from an 'age at death' definition to an 'age at 1 January' definition – see step 1(b) of the technical specification in Section 3.1.

Working Paper 100 proposed a grid of 'Lexis adjustments' $L_{x,t}$ by age x, year t and sex for this purpose, based on analysis of historic birth patterns, death seasonality and survival run-down for England & Wales. This contrasts with the current ONS methodology of assuming an even distribution of deaths between adjacent cohorts ($L_{x,t} = 0.5$).

For practical purposes, we recognise that it may be helpful to have a simplified form of these Lexis adjustments which can be implemented more easily (and transparently) by users.
The proposed grids from Working Paper 100 are included in Appendix B. They exhibit two key features:

- There is a systemic downward drift in the adjustment factors by age, for both males and females, towards a value *below* 0.5 at the highest ages.
  - The drift makes sense in light of reducing survival by age within each age/year cell.
  - This was identified in the Supplementary Technical Paper to Working Paper 100 as a key driver of the life expectancy reduction from using the grids compared with assuming $L_{x,t} = 0.5$.
- Overlaid on this, there are idiosyncratic cohort effects reflecting factors such as the historic variation in birth rates from year to year.

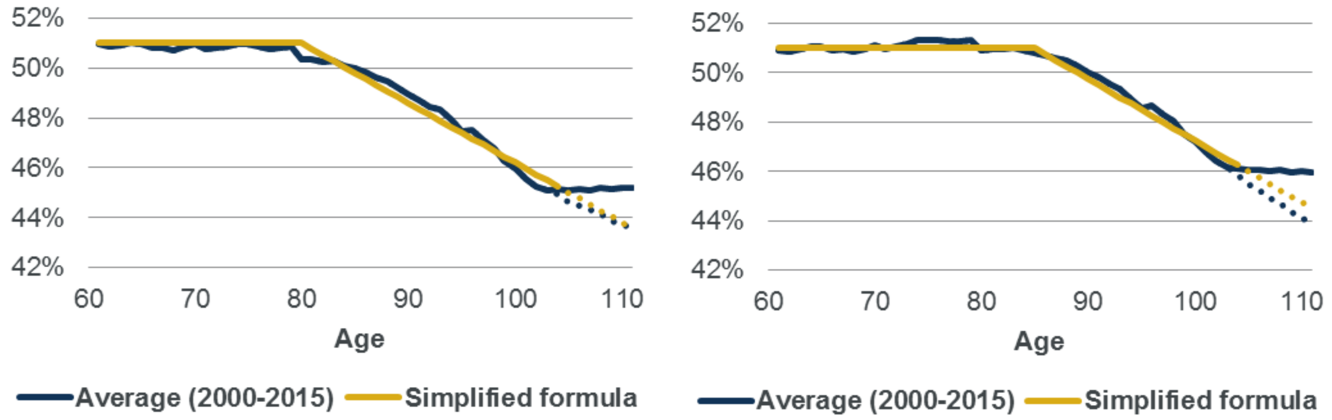
The second of these features is not expected to be material in the context of the wider method, noting that the final exposure smoothing step in Section 3.1 is designed to clean up data anomalies of this kind anyway.

The systemic drift is more important but this can be captured reasonably well using a linear formula for each gender. Chart 3A illustrates this, by plotting the average Lexis adjustment (by age) for calendar years 2000 to 2015 inclusive, alongside the simplified formula which we propose for each gender, namely

- $L_{x,t} = 0.51\gamma_x^m + 0.45(1-\gamma_x^m)$ where $\gamma_x^m = \frac{1}{25}\min(25, 105-x)$ (males)

  $0.51\gamma_x^f + 0.46(1-\gamma_x^f)$ where $\gamma_x^f = \frac{1}{20}\min(20, 105-x)$ (females)

Chart 3A: Average Lexis adjustments (2000-2015) and simplified formula
(a) Males **(b) Females**



On each chart, the solid blue line shows the average Lexis adjustment (for 2000-2015) from the full grid which was published alongside Working Paper 100. This was calibrated to data under age 105, with a simplified (broadly flat) extrapolation to higher ages on the grounds that

- the volume of data at the very highest ages (and the mortality curves used to allow for survival run-off) were less credible, leading to more uncertain values for the Lexis adjustments themselves,

- the input deaths data from the ONS used in the KT method were grouped for ages 105+ anyway (which meant that a complex Lexis split in this age range may have been spurious) and
- the impact on life expectancies was largely immaterial.

Since the analysis of Working Paper 100, we have been able to assess the pattern of 'actual' Lexis adjustments which would have resulted if the method had been extended a little further, up to age 111. This is shown by the dotted blue line for each sex in Chart 3A. Notwithstanding the reduced credibility at these ages, the results suggest it would be reasonable to assume continuation of the general downward trend in Lexis adjustments observed below age 105, rather than levelling off the adjustments from this age onward. This also makes sense in light of the main driver of the effect identified in Working Paper 100 – i.e. the steepening survival gradient within each age/year cell as a result of increasing mortality with age. We would naturally expect this steepening to continue above age 105.

The simplified formula proposed in this paper therefore projects the downward trend in Lexis adjustment by age beyond age 105 – this is shown by the gold lines (with dotted extensions) in Chart 3A. For simplicity, we suggest using the same linear gradient above age 105 as below age 105 (which was the age range used to *set* the gradient, being the more important for typical mortality and life expectancy estimation). The chart shows that this is a good fit to the extended data for males and, whilst it deviates slightly from the extended data for females, the difference is not material in the context of the uncertainty. In fact, the choice of Lexis adjustment extension above age 105 (flat vs linear continuation) has less than 0.2% impact on crude period life expectancy at all ages up to 105, and negligible impact after *graduating* the mortality curves under our proposed method.

Although the formula has been derived based on E&W data we propose it would also be used in deriving exposures at high ages for UK data given that E&W makes up a very high proportion of the UK dataset and there is no reason to suspect major differences in patterns.

Testing the simplified formula

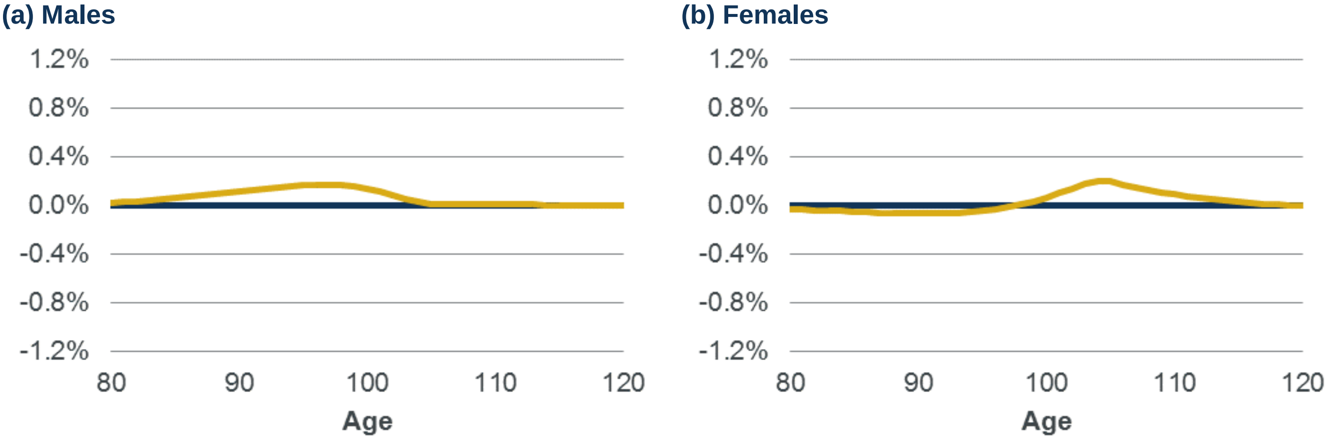
To check the validity of this simplification, we have re-assessed the diagnostic plots and life expectancy impacts from Working Paper 100, as they would have been under the simplified formula (rather than the full grid from the Appendix). The diagnostics are little changed, and still appear reasonable after application of the exposure smoothing step (including for idiosyncratic birth cohorts like 1919). The impact on graduated period life expectancy is also marginal, up to around 0.2% in size with no discernible bias or pattern by age.

A current focus for the CMI is its application of the Working Party's proposals to derive draft "S3" graduations for consultation (as described in Working Paper 107). In this context, the methodology of Section 3.1 would be applied to UK deaths and population data to derive exposure estimates for calendar years 2009 to 2016 inclusive. The crude deaths and exposure estimates would then be graduated to produce a smooth 'reference population' mortality curve for the UK, which is used to extend the SAPS mortality graduations as described in Sections 4 and 5.

Given its intended application, we have checked the impact of the proposed Lexis simplification on the reference population mortality curve graduated across UK data for 2009 to 2015 inclusive[1]. Chart 3B shows the impact on period life expectancy for the reference population across a range of ages, for each of males and females. The impact is again no more than 0.2%, which provides comfort that the linear approximations proposed in this section are fit for purpose for the draft "S3" graduations.

[1] The final year in the draft "S3" period 2009-2016 has been dropped for this comparison, because the original grid of Lexis adjustments from Working Paper 100 covered calendar years up to 2015 only.

Chart 3B: % difference in graduated period life expectancy (2009-2015) – full grid vs simplified formula



Although the proposed simplifications appear immaterial, it should be recognised that they have been calibrated and tested in the context of UK data with a focus on recent years. The full grid approach may still be preferred if applying the method in another context – indeed an *alternative* grid may be suitable if estimating exposures for a dataset other than the UK or England & Wales.

We recommend that the CMI Secretariat keep this under review to ensure that this simplified approach and its parameters remain appropriate for each future use.

3.3. Deaths data at ages 105+

When the ONS publishes counts of deaths, those for ages 105 and above are typically published for a combined '105+' category. These need to be assigned to single years of age in order to apply the KT method for estimating population exposures – see step 1(a) of the technical specification in Section 3.1.

This element of the process was not formally analysed in Working Paper 100, which treated the single year of age data as an 'input' rather than part of the method. For the purposes of comparing alternative methods we simply adopted a fixed dataset for which the grouped '105+' death counts had been decomposed into single years of age under a simple 'decay' assumption (as used by the CMI in its previous applications of the KT method):

- Let $D^I_{x,t}$ be the number of deaths, in the ONS input data, during year t of individuals aged x last birthday at date of death.
- The decay assumption in Working Paper 100 was that $D^I_{x,t} = D^I_{x-1,t} \times 0.5$ for $x > 105$ and each year t.

In other words, the number of deaths was assumed to halve from each age to the next.

Although it was not previously the focus of analysis, users will need to take a view on this data processing step when they are estimating population exposures in practice. We have therefore examined the issue in a little more detail.

In what follows we refer to $r_{x,t} = \frac{D^I_{x,t}}{D^I_{x-1,t}}$ as the 'decay ratio' for age x (in year t).

Chart 3C shows the average decay ratio for the UK deaths data across calendar years 2009 to 2016[2], for each of males and females, under two alternative decomposition methods:

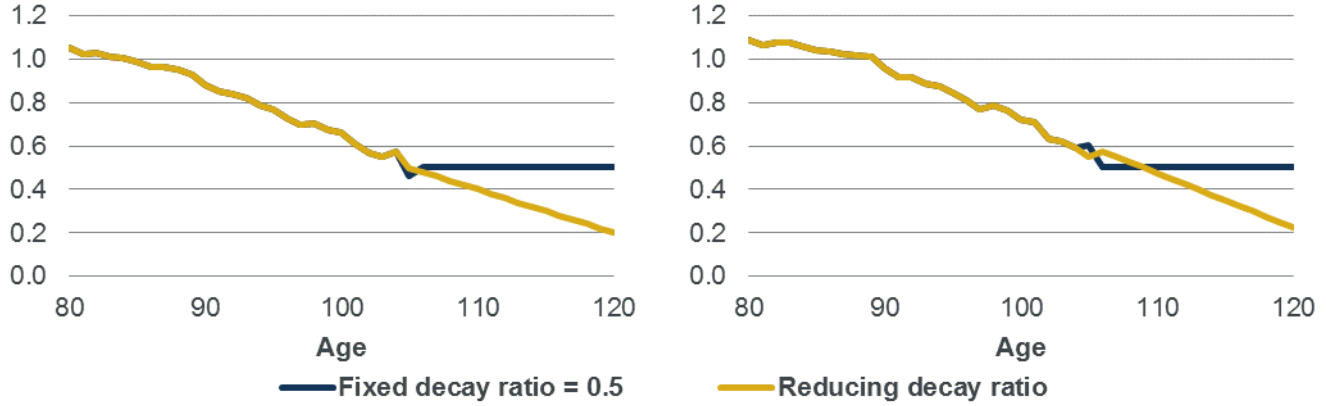
- the dark blue line shows the original method used in Working Paper 100, with a fixed decay ratio $r_{x,t} = 0.5$ above age 105;
- the gold line shows an alternative method, with reducing decay ratio $r_{x,t}$ above age 105, given by
  - $r_{x,t} = 0.5\eta_x + 0.1(1 - \eta_x)$ (males)
  - $0.6\eta_x + 0.1(1 - \eta_x)$ (females)

where

$$\eta_x = \frac{1}{20}\max\{0, \min(20, 125 - x)\}$$

In each case the charts show the *actual* decay ratio (from the deaths data) for ages up to and including 105, and the *estimated* decay ratio (from the relevant decomposition approach) for ages over 105.

Chart 3C: Average decay ratio for UK dataset (2009-2016) – actual ($x < 105$) and estimated ($x \geq 105$)
(a) Males **(b) Females**



The fixed decay ratio of 0.5 (dark blue line) does appear to be broadly representative of the observed data around age 105, but the charts show that there is a clear, and relatively stable, downward trend in the decay ratio leading up to that age.

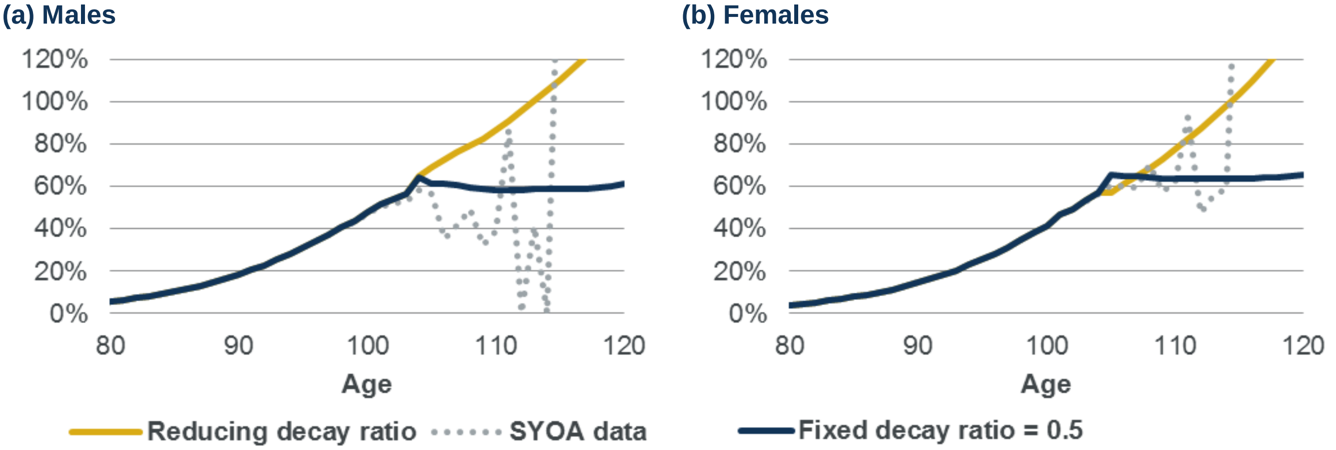
Our view is that it would be more appropriate to project a continuation of that downward trend, in line with the alternative approach specified above (gold line).

This also gives a more sensible shape to the crude mortality rates produced by the method in Section 3.1. Chart 3D shows the pattern of crude central mortality rates by age, for the UK data over the years 2009 to 2016 inclusive, under each of the alternative methods:

- The fixed decay ratio of 0.5 (dark blue line) produces a peak in mortality around age 105, which then flattens off towards higher ages.
- In contrast, the reducing decay ratio (gold line) produces a smoothly increasing mortality curve which better accords with evidence for the trajectory of mortality, and its limiting rate, at the very highest ages. (This evidence was discussed in Working Paper 100.)

[2] By the 'average' decay ratio for 2009-2016, we mean $\sum_{t=2009}^{2016} D_{x,t}^{I} / \sum_{t=2009}^{2016} D_{x-1,t}^{I}$.

Chart 3D: Average crude central mortality rates for UK dataset (2009-2016)



The dashed grey lines above show the estimated crude central mortality rates which would result from using the underlying single year of age data direct from the ONS (rather than decomposing a grouped count for ages 105+). Although not routinely available as part of the ONS's main annual deaths data publication, we have been able to obtain this granular data for these years for this analysis.

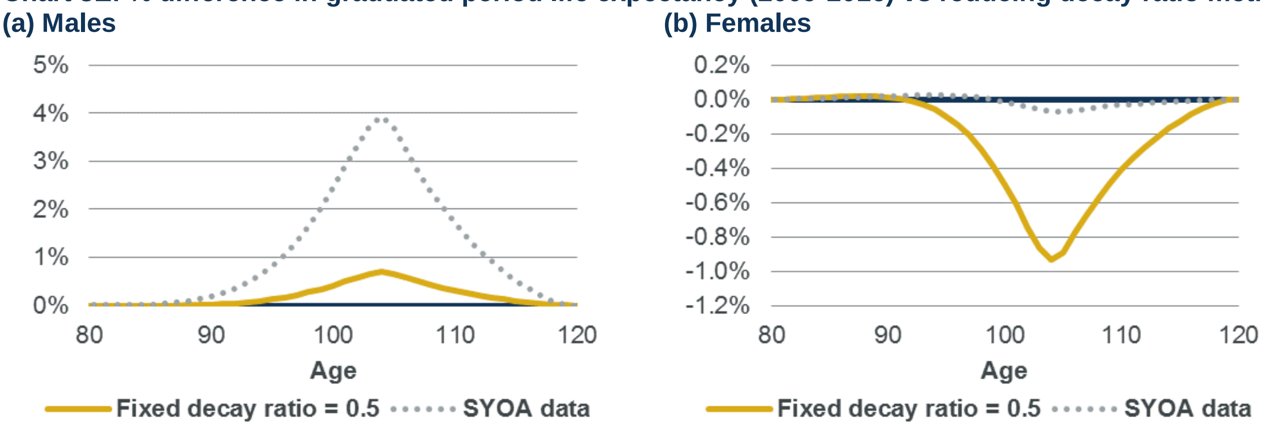
In principle, it would be possible to use the single year of age data *directly* in this way for estimating population exposures. However, we prefer to use the grouped data at ages 105+ (decomposed using the reducing decay ratio illustrated above). This is because:

- single year of age data is not part of the standard annual deaths data release from the ONS and may be harder to obtain (and potentially to compile across constituent UK countries) on a regular basis;
- using single year of age data above age 105 may expose the estimation methods to more noise than using grouped data projected in a sensible way; and
- data quality issues may be more prevalent at the very highest ages.

The last two points are, to some extent, visible in Chart 3D as there is significant noise in the underlying data for both males and females. For males, the reduction in mortality rates between ages 105 and 110 also conflicts with reasonable expectations (and independent evidence – covered in Working Paper 100) for the trajectory of mortality into the highest ages. This is suggestive of potential data quality issues.

Chart 3E shows how graduated period life expectancy for the UK reference population (over 2009-2016) would be impacted by using the fixed decay ratio of 0.5, or the underlying single year of age data, compared with our proposed approach (i.e. the reducing decay ratio described above).

Chart 3E: % difference in graduated period life expectancy (2009-2016) vs reducing decay ratio method



4. High age extensions for CMI table graduations

In this section we set out the approach for extending mortality rates to high ages for CMI graduations using mortality rates derived from the general population.

The formula used for the extensions does not appear in Working Paper 100, and thus this section sets out the formulaic implementation of the approach set out in Working Paper 100.

The rest of this section uses the following notation:

- $\mu_x^{(g)}$ graduated forces of mortality at age x
- $\mu_x^{(r)}$ forces of mortality in the reference population at age x
- μ_x combined/extended forces of mortality at age x
- x_0 age (in years) at which the extension starts
- N convergence interval (in years)
- c_N rate of convergence (per convergence interval)

The extended table is specified by:

$$\mu_x = \mu_x^{(g)} \qquad \text{if } x \le x_0$$

$$\mu_x = \mu_x^{(r)} \times \left(1 + \left(\frac{\mu_{x_0}^{(g)}}{\mu_{x_0}^{(r)}} - 1\right)(1 - c_N)^{\left(\frac{x - x_0}{N}\right)}\right) \qquad \text{if } x \ge x_0$$

so that μ_x tends asymptotically to $\mu_x^{(r)}$ as age increases.

For the Working Paper 100 calculations:

- Mortality for the reference population ($\mu_x^{(r)}$) was obtained by graduating England and Wales population data between ages 60 and 113 for females and 109 for males and applying the standard old-age extension in the CMI graduation software between ages 105 and 120 (using parameters $\mu_{120} = 1$ and $c = 1.25$). The s parameter in the G(s) formula was set to 5 following analysis of a suitable value using information criteria.
- The convergence interval was fixed at $N = 5$ years
- Rates of convergence c_N reflected the convergence below age x_0 for each dataset.

For future CMI graduations we are recommending an approach in line with Working Paper 100 but with the following modifications:

- The reference population is set to use UK general population data in line with the recommendation in Section 2;
- The reference population graduation uses ages 60 – 105, reflecting the greater uncertainty with deaths data above age 105 and the fact that the impact of increasing this age has very limited impact on the resulting graduations; and
- A value of $c = 1$ is used in the high age extension of the reference population, which provides a smoother path to $\mu_{120} = 1$. This point is illustrated further in Section 5.

We set out a case study of sensitivities as applied to the draft "S3" graduations in Section 5.

5. Case study

The CMI SAPS Committee has published draft "S3" graduations in Working Paper 107 which have adopted the recommended high age extension as proposed in this paper. The purpose of this section is to test sensitivities to some of the parameter choices made in the reference population graduation and assumptions made about convergence between the graduated and reference populations.

5.1. Sensitivities to reference population gradations

Table 5.1 sets out the sensitives we have considered.

Table 5.1: Description of sensitivity tests carried out on the reference population graduations

Sensitivity Name	Description
E&W	Use England & Wales population and deaths data instead of UK data
c=1.25	Change high age extension “c” parameter from 1 to 1.25
c=0.75	Change high age extension “c” parameter from 1 to 0.75
μ_{125}	Change limiting age x at which $\mu_x = 1$ from 120 to 125
AgeTo110	Graduate population data to age 110 instead of 105
G(7/8)	Change G(s) parameter from 5 to 7 for males and 8 for females (see below for rationale)

Most of the sensitivities above are self-explanatory in the context of Working Paper 100 and previous discussion in this paper.

Having changed some of the graduation parameters (like age range), the results of graduation tests which were used to support the choice of G(5) in Working Paper 100 have changed. In light of this, we have considered G(7) for male and G(8) for female graduations of the UK data, which perform better than G(5) under a variety of tests. In particular for males, G(7) is supported by the QBIC, BIC, AICc and serial correlation is significantly better than G(6). For females, G(8) is supported by the QBIC and performs well under BIC, AICc and serial correlation tests. We set out the results of these tests in Appendix 3.

Charts 5A to 5D set out the resulting reference population graduations for ages over 100 for males and females. We have split the sensitivities across two graphs to make comparisons easier.

Chart 5A: Sensitivities to Male Reference Graduation (c parameter)

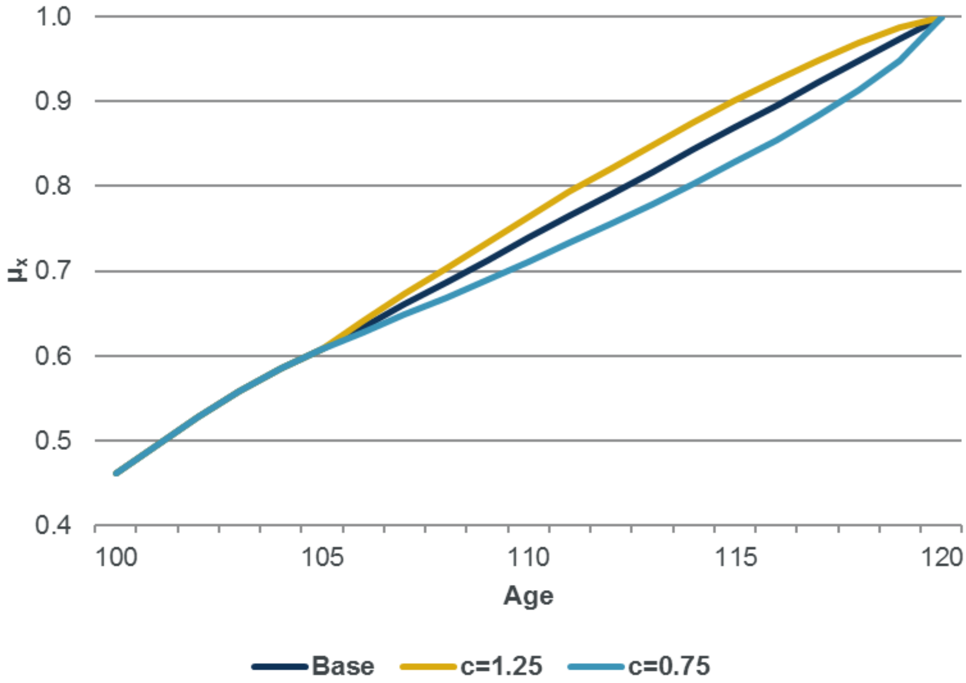
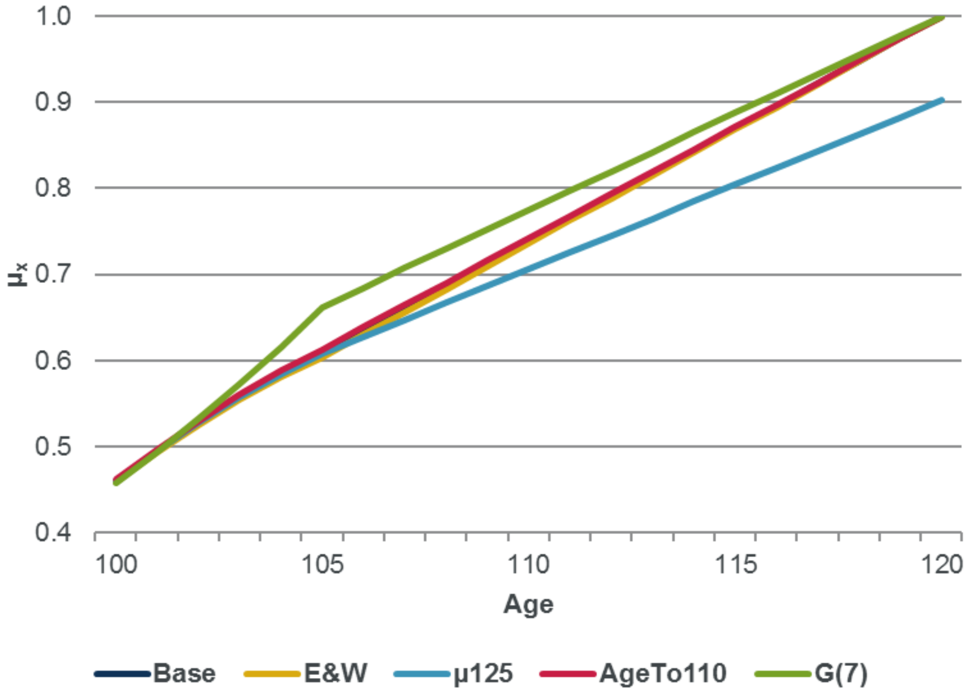


Chart 5B: Sensitivities to Male Reference Graduation (Other)



We can see for males that a value of c=1 seems to provide a smoother high age progression to $\mu_{120} = 1$ than the base case of 1.25, chosen in Working Paper 100 for the smoothness of progression on that data set so it would seem reasonable to adopt a value of 1 for this data.

We also note that although the G(7) graduation performs better in tests over the graduated age range 60-105, the slope and level at age 105 seems less compatible with a smooth progression to $\mu_{120} = 1$.

Chart 5C: Sensitivities to Female Reference Graduation (c parameter)

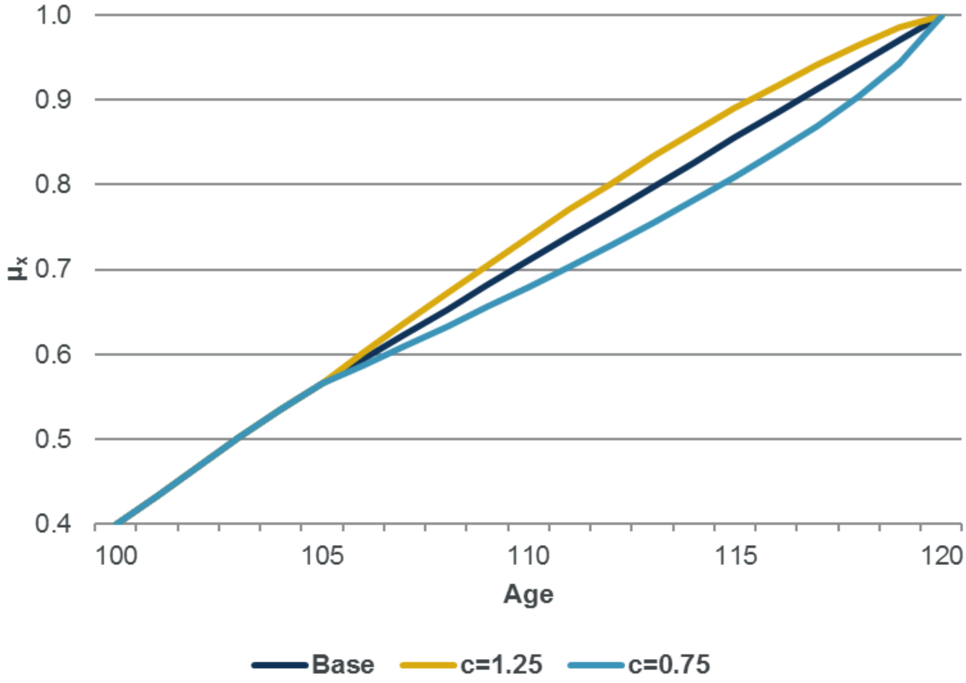
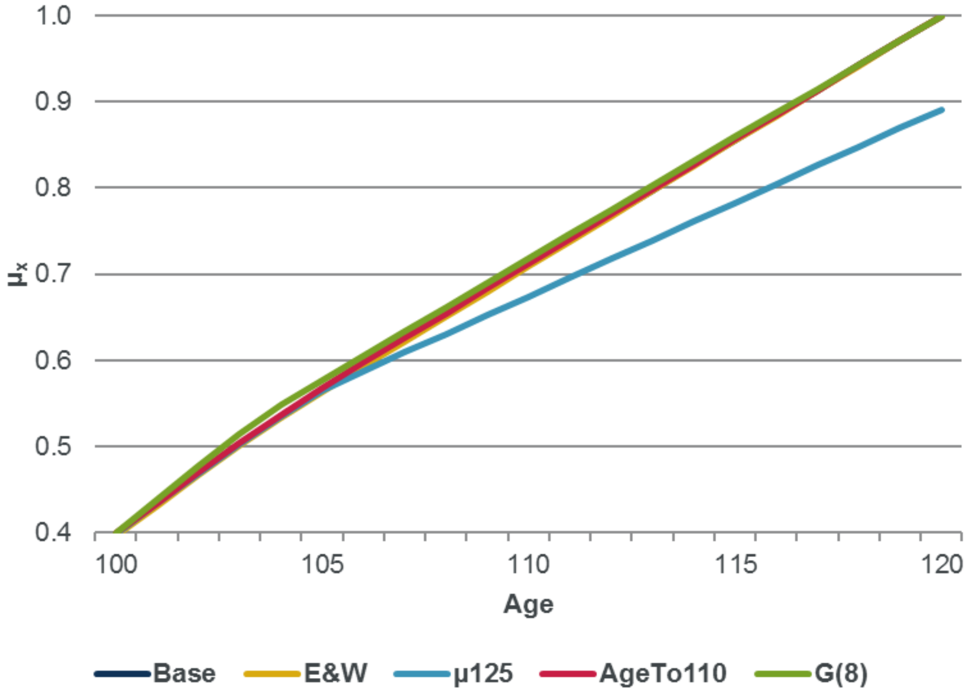


Chart 5D: Sensitivities to Female Reference Graduation (Other)



We see a similar observation regarding the c parameter for females, and the other sensitivities giving fairly similar shapes to the base graduation (apart from $\mu_{125} = 1$ as expected).

Table 5.2 sets out the sensitivities to period life expectancy both for the reference population and for the draft S3PXA mortality tables, having been extended using the proposed high age methodology.

It is worth noting that the reference population mortality table only affects the ages where the S3 mortality rates commence their convergence to population rates (the extension age is 95 for S3PMA and 90 for S3PFA) and that the impact is somewhat dampened given the mortality rates below age 95 are unchanged in each sensitivity.

The impact of changing the reference table on the S3 table above the extension age will be affected by the ratio of the S3 table to the reference table at the extension age and the level of the reference table for ages above the extension age (see the formula in Section 4). This can mean that changes to the reference table may have a bigger impact on life expectancy of the S3 table than the difference in life expectancy between the original and revised reference tables. (As an example, see the G(8) sensitivity below for females, albeit these are low materiality).

Table 5.2: Life Expectancy Sensitivities

(a) Male Reference Table Life Expectancy

Age	Base	E&W	c=1.25	c=0.75	μ_{125}	AgeTo110	G(7)
65	18.39	18.51	18.39	18.39	18.39	18.39	18.39
75	11.23	11.30	11.23	11.23	11.23	11.23	11.23
85	5.82	5.84	5.82	5.82	5.82	5.82	5.81
95	2.72	2.73	2.72	2.72	2.72	2.72	2.73
% Change							
65		0.68%	0.00%	0.00%	0.00%	0.00%	-0.01%
75		0.66%	0.00%	0.00%	0.00%	0.00%	0.03%
85		0.45%	0.00%	0.00%	0.00%	0.00%	-0.07%
95		0.26%	-0.01%	0.01%	0.01%	0.01%	0.36%

(b) S3PMA Table Life Expectancy

Age	Base	E&W	c=1.25	c=0.75	μ_{125}	AgeTo110	G(7)
65	20.49	20.49	20.49	20.49	20.49	20.49	20.50
75	12.48	12.48	12.48	12.48	12.48	12.48	12.48
85	6.30	6.30	6.30	6.30	6.30	6.30	6.30
95	2.79	2.80	2.79	2.79	2.79	2.79	2.80
% Change							
65		0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
75		0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
85		0.01%	0.00%	0.00%	0.00%	0.00%	0.01%
95		0.08%	-0.01%	0.01%	0.01%	0.01%	0.17%

(c) Female Reference Table Life Expectancy

Age	Base	E&W	c=1.25	c=0.75	μ_{125}	AgeTo110	G(8)
65	20.87	21.01	20.87	20.87	20.87	20.87	20.87
75	13.01	13.10	13.01	13.01	13.01	13.01	13.01
85	6.81	6.86	6.81	6.81	6.81	6.81	6.80
95	3.12	3.14	3.12	3.12	3.12	3.12	3.13
% Change							
65		0.67%	0.00%	0.00%	0.00%	0.00%	-0.02%
75		0.72%	0.00%	0.00%	0.00%	0.00%	0.07%
85		0.64%	0.00%	0.00%	0.00%	0.00%	-0.15%
95		0.39%	-0.01%	0.01%	0.02%	0.02%	0.32%

(d) S3PFA Table Life Expectancy

Age	Base	E&W	c=1.25	c=0.75	μ_{125}	AgeTo110	G(8)
65	22.70	22.70	22.70	22.70	22.70	22.70	22.70
75	14.16	14.16	14.16	14.16	14.16	14.16	14.17
85	7.20	7.20	7.20	7.20	7.20	7.20	7.21
95	3.18	3.19	3.18	3.19	3.19	3.19	3.20
% Change							
65		0.01%	0.00%	0.00%	0.00%	0.00%	0.03%
75		0.02%	0.00%	0.00%	0.00%	0.00%	0.05%
85		0.04%	0.00%	0.00%	0.00%	0.00%	0.15%
95		0.16%	-0.01%	0.02%	0.02%	0.02%	0.41%

Some observations from the analysis above are:

- Using England & Wales data leads to marginally higher life expectancies in the reference population table. The impact on the draft S3 graduations is minimal for lower ages (since the S3 graduated rates dominate) but does lead to an increase in life expectancy of 0.08% at age 95 for males and 0.16% for females.
- Given the low materiality of changing the c parameter and the desire to have a smooth progression of mortality rates, it seems reasonable that the S3 graduations use a value of c=1 in the reference population graduations.
- Although visually quite different for ages above 105, setting $\mu_{125} = 1$ has very little impact on life expectancy, even at age 95.
- Graduating the population data to 110 also has very little impact on the results and so the choice to graduate to 105 seems reasonable.
- Increasing the order of the s parameter for males to improve the graduation performance worsens the shape of the very high age mortality rates and makes very little impact on life expectancy. Therefore a graduation of G(5) seems like a preferable choice. For females, the G(8) graduation with extension looks reasonable at very high ages and leads to an increase in LE at age 95 greater than the increase seen in the reference population table (as noted previously). However, the order of magnitude of increase is small and so we are comfortable with retaining a G(5) graduation for the reference population to be consistent with males.

5.2. Sensitivities to convergence assumptions

In addition to the sensitivities to reference population graduation set out above, we have considered two types of sensitivity in relation to extending the S3 tables, specifically:

- Sensitivity to convergence rate parameters
- Sensitivity to choice of parent table

In the draft "S3" tables each table converges towards its “parent” table using a rate of convergence c_N of 15% (and a convergence interval N of 1 year). This convergence rate has been determined by observing the typical rate of convergence between the graduated rates and the parent table over the 9-year age range immediately below the age at which the extension starts. The observed convergence rate varied between -21% and 27% with an average of 12%, and 15% was chosen as a broadly representative rate.

This approach differs to that adopted in Working Paper 100, which considered the rate of convergence between the crude rates underlying the graduation, and the parent / reference table.

Each approach has its merits:

- Reasons to focus on graduated rates include:
  - The graduated rates will be smoother than the crude rates, and this may lead to observed convergence rates that are less subject to volatility.
  - It may also allow a relatively shorter age range to be considered (as the rates are smoothed by the graduation).
- On the other hand, reasons to focus on crude rates include:
  - Crude rates above the graduated range can be used. These may be informative for the convergence at high ages, even if not used in the graduation itself.
  - The graduation is designed to fit over the whole of the age range; it will not optimise the fit to the gradient at extreme ages and so convergence rates may be distorted by the need to fit well at younger ages.

We have therefore also considered the rates of convergence between the crude rates and parent tables, over the age range starting 10 years below the age from which the extension is applied, and finishing at age 100. In order to deal with the volatility arising from low data volumes at higher ages, the rate of convergence is weighted towards those ages where there is the most data. Under this approach the observed rate of convergence varies amongst the tables between 1% and 7%, with an average of 4%. We have therefore considered the sensitivity of adopting a rate of convergence of 5%.

The sensitivities in Table 5.3 show that the impact of the convergence parameter is relatively small at younger ages but more significant at higher ages, particularly for females. The convergence parameter has a larger impact on pension amount band tables (not shown) and a lower convergence parameter leads to greater variation between amount band tables.

Table 5.3: Sensitivity of life expectancy to different convergence rate assumptions

Table	S3PML			S3PMA			S3PFL			S3PMA		
Convergence	15%	5%	Change	15%	5%	Change	15%	5%	Change	15%	5%	Change
Age 65	19.252	19.253	+0.0%	20.494	20.497	+0.0%	22.029	22.049	+0.1%	22.696	22.725	+0.1%
Age 75	11.668	11.669	+0.0%	12.476	12.478	+0.0%	13.717	13.739	+0.2%	14.162	14.194	+0.2%
Age 85	5.981	5.982	+0.0%	6.300	6.304	+0.1%	7.077	7.109	+0.5%	7.201	7.244	+0.6%
Age 95	2.755	2.764	+0.3%	2.794	2.814	+0.7%	3.170	3.230	+1.9%	3.185	3.263	+2.5%

Having considered both approaches to determining the convergence parameter we have concluded that the approach followed by the SAPS Committee in Working Paper 107 is reasonable.

5.3. Sensitivity to choice of parent table

The draft “S3” Series tables use population tables as the parent tables of S3 tables without pension bands, the corresponding PMA, NMA, PFA or NFA tables as parents of Heavy, Middle and Light amount band tables, and the Light tables as the parent tables of Very Light tables. We have investigated the impact of using the corresponding PMA, NMA, PFA or NFA tables as the parent table for Very Light tables and found that the choice of parent table makes little difference in these cases, when using a convergence parameter of 15%

6. Next steps

The Working Party welcomes feedback on this paper and the proposed framework, whether or not you agree with our proposals. Please send your responses to HighAgeMortality@cmilimited.co.uk. The Working Party would appreciate feedback on our proposed framework by 14 September 2018. We will consider all responses prior to finalising our approach in time for the final version of the "S3" Series tables.

We are also intending to perform quantitative and qualitative analyses of mortality data for large pension schemes with the intention of producing a further Working Paper setting out our findings, before disbanding later this year.

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Appendix 1: Current ONS approach to modelling population exposures

We include a summary from Working Paper 100 on the current approach used by the ONS for estimating high age population exposures, to provide a starting reference point.

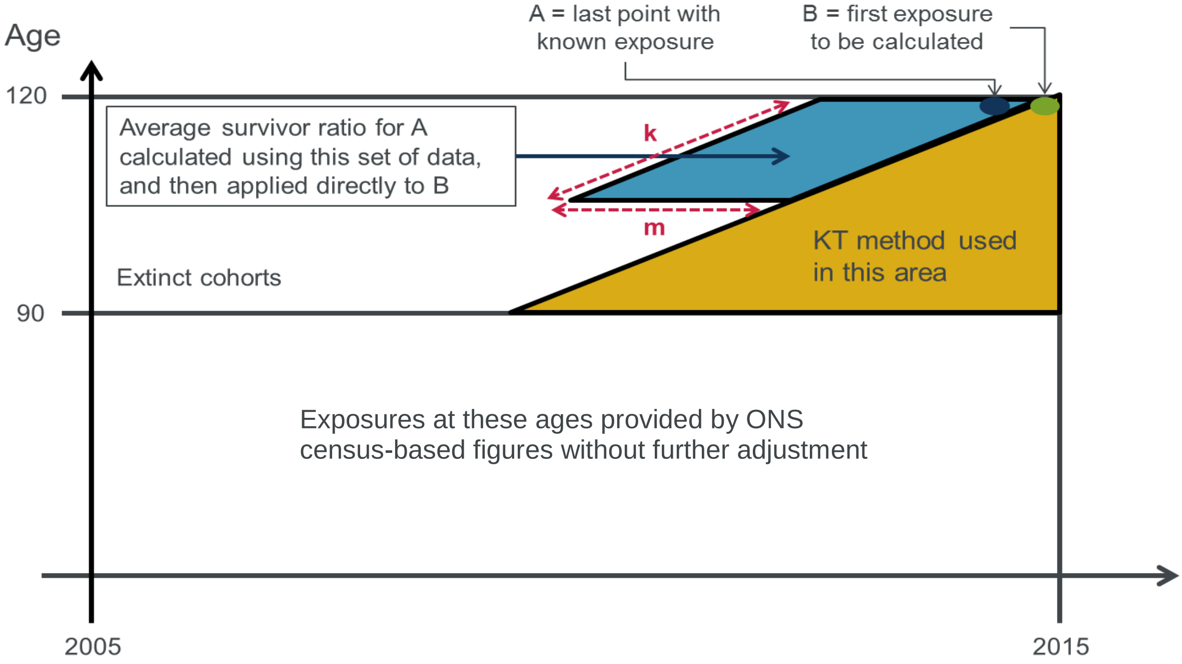
The ONS currently uses the KT method to estimate population numbers at the highest ages.

This method is often used where the official population data by single year of age is assumed to be unreliable at the highest ages but, in contrast, the reporting of deaths is assumed to be far more accurate. This can be considered the case in E&W as it is a legal requirement to register all deaths occurring, whereas census data is subject to a range of known recording and processing issues and in any case is gathered only at 10-year intervals.

The methodology used by the ONS to determine its high age population estimates for E&W is set out online. Chart A1A illustrates how the ONS's current implementation of the KT method works.

- For ages 89 and below, the census-based figures by single year of age are adopted without adjustment. These figures are mid-year estimates produced by rolling the decennial census counts forward allowing for ageing, births, deaths and migration (the cohort component method).
- For ages 90 and above, the KT method is applied using deaths data to estimate the single year of age population counts. Ideally, death occurrence data would always be used rather than death registration data, as the former records when deaths actually occurred, rather than when deaths were registered. However, in practice, deaths data here means registration data to 1992 (when occurrence data is not available) and occurrence data from 1993 onwards (except for the most recent year which is registration data, due to registration data being available more quickly, to allow an earlier release). We note that the ONS expect the difference between occurrence and registration data to be negligible in practice.
- In the diagram, it is the gold area for which we need to estimate population exposures. The population estimates in the rest of the figure are known, or have already been determined. Lives aged above 90 in the white and the blue areas are believed to be fully extinct (i.e. all lives are assumed to have died) and so the deaths information can be used to calculate historical exposures.
- The current ONS method takes a parallelogram of past data (the blue area) comprising m birth cohorts, each with k years of prior deaths data, and calculates the average survivor ratio (survivors ÷ deaths) for point A based on that set of cohorts. The current ONS methodology uses 5 years of past deaths data over 5 years of birth cohorts, i.e. k = 5 and m = 5.
- It then applies the calculated survivor ratio to the single birth cohort containing point B, multiplying up by the known deaths count over the prior k ages in the cohort to give the population estimate for point B. Once this first exposure estimate has been calculated, population estimates at previous ages along the cohort can be reconstructed by adding back in the recorded deaths. Then the completed figures for that cohort can be used to estimate the population of the next cohort, and subsequent cohorts until exposure estimates for the whole of the gold triangle in Chart A1A have been calculated.
- Two further adjustments are made to ensure that the total population estimates for ages 90 and above calculated by the KT methodology are the same as the official population estimates for each year:
    - The total population estimate for ages 90+ in the final year is constrained to the census-based figure for that year. This implies a correction factor c (in the language of the original KT method), which represents the assumed constant year-on-year adjustment factor that applies to the survivor ratios when projecting them, and should ideally be close to 1. This will have a knock-on impact to estimates calculated using the survivor ratio approach.
    - A uniform annual scaling to the resulting population estimates at ages 90+ in each previous year is then applied to constrain the 90+ population total to the census-based figure for that year.

Chart A1A: Current ONS implementation of KT method



The population estimates over age 104 are then grouped into a single 105+ estimate for publication.

In December 2016, the ONS released their paper "Accuracy of official high-age population estimates, in England and Wales: an evaluation". We consider these are consistent with the findings of our earlier work in Working Papers 85 and 100.

Appendix 2: Lexis adjustments

An illustration of Lexis adjustments $D_U(x,t)$ / $D(x,t)$ for age 60-120 and calendar years 2000-2015 (ONS E&W male/female data)

Key:
Orange indicates a Lexis adjustment greater than 50%;
White indicates a Lexis adjustment equal to 50%;
Blue indicates a Lexis adjustment less than 50%.

Males

Age \ Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
60	51.7%	51.7%	49.1%	50.4%	49.0%	53.7%	48.0%	49.9%	53.1%	52.0%	51.9%	51.2%	50.8%	50.5%	50.5%	51.3%
61	51.3%	51.7%	51.7%	49.1%	50.4%	49.0%	53.7%	48.0%	49.9%	53.1%	52.0%	51.9%	51.2%	50.8%	50.5%	51.3%
62	50.9%	51.2%	51.6%	51.6%	49.0%	50.3%	48.9%	53.6%	47.9%	49.8%	53.1%	51.9%	51.8%	51.2%	50.7%	50.4%
63	51.1%	50.9%	51.2%	51.6%	51.6%	49.0%	50.3%	48.9%	53.6%	47.9%	49.8%	53.1%	51.9%	51.8%	51.2%	50.7%
64	51.2%	51.2%	51.0%	51.3%	51.7%	51.7%	49.0%	50.3%	49.0%	53.6%	47.9%	49.9%	53.1%	51.9%	51.9%	51.2%
65	51.2%	51.1%	51.2%	50.9%	51.3%	51.7%	51.7%	49.0%	50.3%	49.0%	53.6%	47.9%	49.9%	53.1%	51.9%	51.9%
66	50.2%	51.1%	51.1%	51.1%	50.9%	51.2%	51.6%	51.6%	48.9%	50.3%	48.9%	53.6%	47.9%	49.8%	53.1%	51.9%
67	51.9%	50.1%	51.0%	51.0%	51.0%	50.8%	51.2%	51.6%	51.6%	48.9%	50.2%	48.9%	53.6%	47.9%	49.8%	53.1%
68	51.5%	51.9%	50.1%	51.1%	51.0%	51.0%	50.8%	51.2%	51.6%	51.6%	48.9%	50.3%	48.9%	53.6%	47.9%	49.9%
69	51.7%	51.5%	51.9%	50.1%	51.1%	51.0%	51.1%	50.9%	51.2%	51.6%	51.6%	49.0%	50.3%	49.0%	53.7%	48.0%
70	50.8%	51.6%	51.4%	51.9%	50.1%	51.0%	51.0%	51.0%	50.8%	51.2%	51.6%	51.6%	48.9%	50.3%	49.0%	53.6%
71	51.2%	50.7%	51.5%	51.3%	51.8%	50.0%	50.9%	50.9%	50.9%	50.7%	51.1%	51.5%	51.5%	48.9%	50.2%	48.9%
72	50.3%	51.2%	50.6%	51.4%	51.2%	51.7%	49.9%	50.9%	50.8%	50.9%	50.7%	51.0%	51.5%	51.5%	48.8%	50.2%
73	51.8%	50.2%	51.1%	50.6%	51.4%	51.2%	51.7%	49.9%	50.8%	50.8%	50.8%	50.7%	51.0%	51.4%	51.4%	48.8%
74	51.1%	51.8%	50.2%	51.1%	50.6%	51.3%	51.2%	51.6%	49.9%	50.8%	50.8%	50.8%	50.7%	51.0%	51.4%	51.4%
75	51.1%	51.1%	51.8%	50.1%	51.0%	50.5%	51.3%	51.1%	51.6%	49.8%	50.8%	50.8%	50.8%	50.7%	51.0%	51.4%
76	51.2%	51.0%	51.0%	51.7%	50.1%	51.0%	50.5%	51.3%	51.1%	51.6%	49.8%	50.8%	50.8%	50.8%	50.6%	51.0%
77	50.5%	51.1%	50.9%	50.9%	51.6%	50.0%	50.9%	50.4%	51.2%	51.0%	51.5%	49.7%	50.7%	50.7%	50.8%	50.6%
78	51.9%	50.4%	51.0%	50.8%	50.8%	51.5%	49.9%	50.8%	50.3%	51.1%	50.9%	51.5%	49.7%	50.7%	50.7%	50.7%
79	52.1%	51.8%	50.4%	50.9%	50.7%	50.7%	51.4%	49.8%	50.7%	50.2%	51.1%	50.9%	51.4%	49.7%	50.7%	50.7%
80	43.8%	52.1%	51.8%	50.4%	50.9%	50.7%	50.7%	51.4%	49.8%	50.7%	50.2%	51.0%	50.9%	51.4%	49.7%	50.7%
81	51.6%	43.7%	52.0%	51.7%	50.3%	50.8%	50.6%	50.6%	51.3%	49.7%	50.6%	50.1%	50.9%	50.8%	51.4%	49.6%
82	49.9%	51.5%	43.6%	51.9%	51.6%	50.2%	50.7%	50.5%	50.5%	51.2%	49.6%	50.5%	50.0%	50.8%	50.7%	51.3%
83	53.1%	49.8%	51.3%	43.5%	51.7%	51.5%	50.1%	50.6%	50.4%	50.3%	51.1%	49.4%	50.4%	49.9%	50.7%	50.6%
84	49.8%	52.9%	49.6%	51.2%	43.4%	51.6%	51.4%	49.9%	50.5%	50.2%	50.2%	50.9%	49.3%	50.3%	49.8%	50.6%
85	50.8%	49.6%	52.8%	49.5%	51.0%	43.2%	51.5%	51.2%	49.8%	50.3%	50.1%	50.1%	50.8%	49.2%	50.2%	49.7%
86	49.5%	50.6%	49.4%	52.6%	49.3%	50.9%	43.1%	51.3%	51.1%	49.7%	50.2%	50.0%	50.0%	50.7%	49.1%	50.0%
87	49.0%	49.4%	50.4%	49.2%	52.4%	49.1%	50.7%	42.9%	51.1%	50.9%	49.5%	50.0%	49.9%	49.9%	50.6%	49.0%
88	49.2%	48.8%	49.2%	50.3%	49.1%	52.3%	48.9%	50.5%	42.8%	51.0%	50.7%	49.4%	49.9%	49.7%	49.7%	50.4%
89	49.1%	49.0%	48.6%	49.0%	50.1%	48.9%	52.1%	48.7%	50.3%	42.6%	50.8%	50.5%	49.1%	49.7%	49.5%	49.5%
90	48.9%	48.9%	48.7%	48.4%	48.7%	49.8%	48.6%	51.8%	48.5%	50.0%	42.3%	50.5%	50.3%	48.9%	49.4%	49.3%
91	48.7%	48.7%	48.6%	48.5%	48.1%	48.5%	49.6%	48.4%	51.6%	48.2%	49.8%	42.1%	50.3%	50.1%	48.7%	49.2%
92	47.4%	48.6%	48.5%	48.5%	48.4%	48.0%	48.4%	49.5%	48.3%	51.5%	48.1%	49.7%	42.0%	50.1%	49.9%	48.5%
93	48.4%	47.3%	48.5%	48.4%	48.4%	48.2%	47.9%	48.2%	49.3%	48.1%	51.3%	48.0%	49.6%	41.9%	50.0%	49.8%
94	47.4%	48.1%	47.0%	48.2%	48.1%	48.1%	47.9%	47.6%	48.0%	49.1%	47.9%	51.1%	47.8%	49.3%	41.7%	49.8%
95	47.6%	47.1%	47.7%	46.6%	47.8%	47.8%	47.7%	47.6%	47.2%	47.6%	48.7%	47.5%	50.8%	47.4%	49.0%	41.5%
96	47.1%	47.2%	46.7%	47.4%	46.3%	47.5%	47.4%	47.4%	47.3%	46.9%	47.3%	48.4%	47.2%	50.5%	47.1%	48.7%
97	46.7%	46.8%	46.9%	46.4%	47.1%	46.0%	47.2%	47.1%	47.1%	46.9%	46.6%	47.0%	48.1%	46.9%	50.2%	46.8%
98	46.4%	46.4%	46.5%	46.6%	46.1%	46.8%	45.7%	46.9%	46.8%	46.8%	46.6%	46.3%	46.7%	47.8%	46.6%	49.9%
99	46.1%	46.1%	46.1%	46.2%	46.3%	45.8%	46.5%	45.4%	46.6%	46.5%	46.5%	46.3%	46.0%	46.4%	47.5%	46.3%
100	45.9%	45.7%	45.8%	45.8%	45.9%	46.0%	45.5%	46.1%	45.1%	46.3%	46.2%	46.2%	46.0%	45.6%	46.0%	47.1%
101	45.7%	45.6%	45.4%	45.4%	45.4%	45.5%	45.6%	45.2%	45.8%	44.7%	45.9%	45.9%	45.8%	45.7%	45.3%	45.6%
102	45.5%	45.4%	45.3%	45.1%	45.1%	45.1%	45.2%	45.3%	44.8%	45.5%	44.4%	45.6%	45.6%	45.5%	45.4%	45.0%
103	45.6%	45.4%	45.2%	45.1%	44.9%	45.0%	44.9%	45.0%	45.1%	44.6%	45.3%	44.2%	45.4%	45.4%	45.3%	45.2%
104	45.6%	45.6%	45.4%	45.2%	45.1%	44.9%	45.0%	44.9%	45.0%	45.1%	44.6%	45.3%	44.2%	45.4%	45.4%	45.3%
105	44.4%	45.6%	45.6%	45.4%	45.2%	45.1%	44.9%	45.0%	44.9%	45.0%	45.1%	44.6%	45.3%	44.2%	45.4%	45.4%
106	46.0%	44.4%	45.6%	45.6%	45.4%	45.2%	45.1%	44.9%	45.0%	44.9%	45.0%	45.1%	44.6%	45.3%	44.2%	45.4%
107	45.0%	46.0%	44.4%	45.6%	45.6%	45.4%	45.2%	45.1%	44.9%	45.0%	44.9%	45.0%	45.1%	44.6%	45.3%	44.2%
108	45.9%	45.0%	46.0%	44.4%	45.6%	45.6%	45.4%	45.2%	45.1%	44.9%	45.0%	44.9%	45.0%	45.1%	44.6%	45.3%
109	44.1%	45.9%	45.0%	46.0%	44.4%	45.6%	45.6%	45.4%	45.2%	45.1%	44.9%	45.0%	44.9%	45.0%	45.1%	44.6%
110	45.8%	44.1%	45.9%	45.0%	46.0%	44.4%	45.6%	45.6%	45.4%	45.2%	45.1%	44.9%	45.0%	44.9%	45.0%	45.1%
111	45.3%	45.8%	44.1%	45.9%	45.0%	46.0%	44.4%	45.6%	45.6%	45.4%	45.2%	45.1%	44.9%	45.0%	44.9%	45.0%
112	45.6%	45.3%	45.8%	44.1%	45.9%	45.0%	46.0%	44.4%	45.6%	45.6%	45.4%	45.2%	45.1%	44.9%	45.0%	44.9%
113	45.6%	45.6%	45.3%	45.8%	44.1%	45.9%	45.0%	46.0%	44.4%	45.6%	45.6%	45.4%	45.2%	45.1%	44.9%	45.0%
114	44.7%	45.6%	45.6%	45.3%	45.8%	44.1%	45.9%	45.0%	46.0%	44.4%	45.6%	45.6%	45.4%	45.2%	45.1%	44.9%
115	45.5%	44.7%	45.6%	45.6%	45.3%	45.8%	44.1%	45.9%	45.0%	46.0%	44.4%	45.6%	45.6%	45.4%	45.2%	45.1%
116	44.3%	45.5%	44.7%	45.6%	45.6%	45.3%	45.8%	44.1%	45.9%	45.0%	46.0%	44.4%	45.6%	45.6%	45.4%	45.2%
117	44.6%	44.3%	45.5%	44.7%	45.6%	45.6%	45.3%	45.8%	44.1%	45.9%	45.0%	46.0%	44.4%	45.6%	45.6%	45.4%
118	44.5%	44.6%	44.3%	45.5%	44.7%	45.6%	45.6%	45.3%	45.8%	44.1%	45.9%	45.0%	46.0%	44.4%	45.6%	45.6%
119	44.4%	44.5%	44.6%	44.3%	45.5%	44.7%	45.6%	45.6%	45.3%	45.8%	44.1%	45.9%	45.0%	46.0%	44.4%	45.6%
120	44.3%	44.4%	44.5%	44.6%	44.3%	45.5%	44.7%	45.6%	45.6%	45.3%	45.8%	44.1%	45.9%	45.0%	46.0%	44.4%

Females

Age \ Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
60	51.7%	51.7%	49.0%	50.3%	48.9%	53.6%	47.9%	49.8%	53.1%	51.9%	51.8%	51.2%	50.7%	50.4%	50.4%	51.2%
61	51.3%	51.7%	51.7%	49.0%	50.3%	48.9%	53.6%	47.9%	49.8%	53.1%	51.9%	51.8%	51.2%	50.7%	50.4%	51.2%
62	50.9%	51.3%	51.7%	51.6%	49.0%	50.3%	48.9%	53.6%	47.9%	49.8%	53.0%	51.9%	51.8%	51.2%	50.7%	50.4%
63	51.2%	51.0%	51.3%	51.7%	51.7%	49.0%	50.3%	49.0%	53.6%	47.9%	49.9%	53.1%	51.9%	51.8%	51.2%	50.8%
64	51.2%	51.2%	51.0%	51.4%	51.8%	51.7%	49.1%	50.4%	49.0%	53.7%	48.0%	49.9%	53.2%	52.0%	51.9%	51.3%
65	51.2%	51.2%	51.2%	51.0%	51.3%	51.8%	51.7%	49.1%	50.4%	49.0%	53.7%	48.0%	49.9%	53.2%	52.0%	51.9%
66	50.3%	51.2%	51.2%	51.2%	51.0%	51.3%	51.7%	51.7%	49.0%	50.3%	49.0%	53.7%	48.0%	49.9%	53.2%	52.0%
67	52.1%	50.3%	51.2%	51.2%	51.2%	51.0%	51.3%	51.7%	51.7%	49.1%	50.4%	49.0%	53.7%	48.0%	50.0%	53.2%
68	51.7%	52.1%	50.3%	51.2%	51.2%	51.2%	51.0%	51.3%	51.8%	51.7%	49.1%	50.4%	49.1%	53.7%	48.0%	50.0%
69	51.9%	51.7%	52.1%	50.3%	51.2%	51.2%	51.2%	51.0%	51.3%	51.8%	51.7%	49.1%	50.4%	49.1%	53.7%	48.0%
70	51.1%	51.8%	51.6%	52.1%	50.3%	51.2%	51.1%	51.2%	51.0%	51.3%	51.7%	51.7%	49.1%	50.4%	49.0%	53.7%
71	51.6%	51.0%	51.8%	51.6%	52.0%	50.2%	51.1%	51.1%	51.1%	50.9%	51.3%	51.7%	51.7%	49.0%	50.3%	49.0%
72	50.6%	51.5%	51.0%	51.8%	51.5%	52.0%	50.2%	51.1%	51.1%	51.1%	50.9%	51.3%	51.7%	51.7%	49.0%	50.3%
73	52.2%	50.6%	51.5%	51.0%	51.7%	51.5%	52.0%	50.2%	51.1%	51.1%	51.1%	50.9%	51.2%	51.7%	51.7%	49.0%
74	51.5%	52.2%	50.6%	51.5%	51.0%	51.7%	51.5%	52.0%	50.2%	51.1%	51.1%	51.1%	50.9%	51.3%	51.7%	51.7%
75	51.5%	51.5%	52.2%	50.6%	51.5%	51.0%	51.7%	51.5%	52.0%	50.2%	51.1%	51.1%	51.1%	50.9%	51.3%	51.7%
76	51.7%	51.5%	51.5%	52.2%	50.6%	51.4%	50.9%	51.7%	51.5%	52.0%	50.2%	51.1%	51.1%	51.1%	51.0%	51.3%
77	51.1%	51.6%	51.4%	51.4%	52.1%	50.5%	51.4%	50.9%	51.7%	51.5%	52.0%	50.2%	51.1%	51.1%	51.1%	50.9%
78	52.4%	51.0%	51.6%	51.4%	51.4%	52.1%	50.4%	51.3%	50.8%	51.6%	51.4%	51.9%	50.1%	51.1%	51.1%	51.1%
79	52.7%	52.4%	51.0%	51.5%	51.3%	51.3%	52.0%	50.4%	51.3%	50.8%	51.5%	51.4%	51.9%	50.1%	51.0%	51.0%
80	44.4%	52.7%	52.4%	51.0%	51.5%	51.3%	51.3%	52.0%	50.3%	51.2%	50.7%	51.5%	51.3%	51.8%	50.1%	51.0%
81	52.3%	44.4%	52.7%	52.4%	50.9%	51.4%	51.3%	51.2%	51.9%	50.3%	51.2%	50.7%	51.5%	51.3%	51.8%	50.1%
82	50.7%	52.3%	44.3%	52.6%	52.3%	50.9%	51.4%	51.2%	51.2%	51.9%	50.3%	51.2%	50.7%	51.5%	51.3%	51.8%
83	53.9%	50.6%	52.2%	44.3%	52.5%	52.2%	50.8%	51.3%	51.1%	51.1%	51.8%	50.2%	51.1%	50.6%	51.4%	51.2%
84	50.7%	53.8%	50.5%	52.1%	44.2%	52.4%	52.1%	50.7%	51.2%	51.0%	51.0%	51.7%	50.1%	51.0%	50.5%	51.3%
85	51.7%	50.5%	53.7%	50.4%	51.9%	44.0%	52.3%	52.0%	50.6%	51.1%	50.9%	50.9%	51.6%	50.0%	50.9%	50.4%
86	50.5%	51.6%	50.4%	53.6%	50.2%	51.8%	43.9%	52.2%	51.9%	50.5%	51.0%	50.8%	50.8%	51.5%	49.9%	50.8%
87	50.0%	50.4%	51.5%	50.3%	53.4%	50.1%	51.7%	43.8%	52.1%	51.8%	50.4%	50.9%	50.7%	50.7%	51.4%	49.8%
88	50.3%	49.9%	50.3%	51.3%	50.2%	53.3%	50.0%	51.6%	43.7%	52.0%	51.7%	50.3%	50.8%	50.6%	50.6%	51.3%
89	50.3%	50.1%	49.8%	50.1%	51.2%	50.0%	53.1%	49.8%	51.4%	43.5%	51.8%	51.5%	50.1%	50.7%	50.5%	50.5%
90	50.1%	50.0%	49.9%	49.5%	49.9%	51.0%	49.8%	52.9%	49.6%	51.1%	43.3%	51.6%	51.3%	49.9%	50.5%	50.3%
91	49.9%	49.9%	49.8%	49.7%	49.3%	49.7%	50.7%	49.5%	52.7%	49.4%	50.9%	43.2%	51.4%	51.1%	49.7%	50.3%
92	48.5%	49.7%	49.7%	49.6%	49.5%	49.1%	49.5%	50.5%	49.3%	52.5%	49.2%	50.7%	43.0%	51.2%	50.9%	49.5%
93	49.4%	48.3%	49.5%	49.4%	49.4%	49.2%	48.9%	49.3%	50.3%	49.1%	52.3%	49.0%	50.5%	42.8%	51.0%	50.8%
94	48.5%	49.1%	48.0%	49.2%	49.2%	49.1%	49.0%	48.6%	49.0%	50.1%	48.9%	52.1%	48.7%	50.3%	42.6%	50.8%
95	48.7%	48.2%	48.8%	47.8%	48.9%	48.9%	48.8%	48.7%	48.3%	48.7%	49.8%	48.6%	51.8%	48.5%	50.1%	42.4%
96	48.3%	48.5%	48.0%	48.6%	47.5%	48.7%	48.6%	48.6%	48.4%	48.0%	48.4%	49.5%	48.3%	51.6%	48.2%	49.8%
97	48.0%	48.1%	48.2%	47.7%	48.3%	47.2%	48.4%	48.3%	48.3%	48.1%	47.7%	48.1%	49.2%	48.1%	51.3%	48.0%
98	47.6%	47.7%	47.8%	47.9%	47.4%	48.0%	46.9%	48.1%	48.0%	48.0%	47.8%	47.5%	47.8%	49.0%	47.8%	51.0%
99	47.2%	47.4%	47.4%	47.6%	47.7%	47.1%	47.7%	46.6%	47.8%	47.7%	47.7%	47.5%	47.2%	47.6%	48.7%	47.5%
100	46.8%	46.9%	47.1%	47.1%	47.3%	47.3%	46.8%	47.4%	46.3%	47.5%	47.4%	47.4%	47.2%	46.9%	47.3%	48.4%
101	46.5%	46.5%	46.6%	46.7%	46.8%	46.9%	47.0%	46.5%	47.1%	46.0%	47.1%	47.1%	47.1%	46.9%	46.6%	46.9%
102	46.3%	46.2%	46.3%	46.3%	46.4%	46.5%	46.6%	46.7%	46.1%	46.8%	45.7%	46.8%	46.8%	46.7%	46.6%	46.2%
103	46.2%	46.0%	46.0%	46.0%	46.0%	46.1%	46.2%	46.3%	46.4%	45.8%	46.4%	45.3%	46.5%	46.4%	46.4%	46.3%
104	46.2%	46.2%	46.0%	46.0%	46.0%	46.0%	46.1%	46.2%	46.3%	46.4%	45.8%	46.4%	45.3%	46.5%	46.4%	46.4%
105	45.0%	46.2%	46.2%	46.0%	46.0%	46.0%	46.0%	46.1%	46.2%	46.3%	46.4%	45.8%	46.4%	45.3%	46.5%	46.4%
106	46.6%	45.0%	46.2%	46.2%	46.0%	46.0%	46.0%	46.0%	46.1%	46.2%	46.3%	46.4%	45.8%	46.4%	45.3%	46.5%
107	45.6%	46.6%	45.0%	46.2%	46.2%	46.0%	46.0%	46.0%	46.0%	46.1%	46.2%	46.3%	46.4%	45.8%	46.4%	45.3%
108	46.4%	45.6%	46.6%	45.0%	46.2%	46.2%	46.0%	46.0%	46.0%	46.0%	46.1%	46.2%	46.3%	46.4%	45.8%	46.4%
109	44.7%	46.4%	45.6%	46.6%	45.0%	46.2%	46.2%	46.0%	46.0%	46.0%	46.0%	46.1%	46.2%	46.3%	46.4%	45.8%
110	46.4%	44.7%	46.4%	45.6%	46.6%	45.0%	46.2%	46.2%	46.0%	46.0%	46.0%	46.0%	46.1%	46.2%	46.3%	46.4%
111	45.9%	46.4%	44.7%	46.4%	45.6%	46.6%	45.0%	46.2%	46.2%	46.0%	46.0%	46.0%	46.0%	46.1%	46.2%	46.3%
112	46.3%	45.9%	46.4%	44.7%	46.4%	45.6%	46.6%	45.0%	46.2%	46.2%	46.0%	46.0%	46.0%	46.0%	46.1%	46.2%
113	46.4%	46.3%	45.9%	46.4%	44.7%	46.4%	45.6%	46.6%	45.0%	46.2%	46.2%	46.0%	46.0%	46.0%	46.0%	46.1%
114	45.4%	46.4%	46.3%	45.9%	46.4%	44.7%	46.4%	45.6%	46.6%	45.0%	46.2%	46.2%	46.0%	46.0%	46.0%	46.0%
115	46.2%	45.4%	46.4%	46.3%	45.9%	46.4%	44.7%	46.4%	45.6%	46.6%	45.0%	46.2%	46.2%	46.0%	46.0%	46.0%
116	45.1%	46.2%	45.4%	46.4%	46.3%	45.9%	46.4%	44.7%	46.4%	45.6%	46.6%	45.0%	46.2%	46.2%	46.0%	46.0%
117	45.4%	45.1%	46.2%	45.4%	46.4%	46.3%	45.9%	46.4%	44.7%	46.4%	45.6%	46.6%	45.0%	46.2%	46.2%	46.0%
118	45.3%	45.4%	45.1%	46.2%	45.4%	46.4%	46.3%	45.9%	46.4%	44.7%	46.4%	45.6%	46.6%	45.0%	46.2%	46.2%
119	45.2%	45.3%	45.4%	45.1%	46.2%	45.4%	46.4%	46.3%	45.9%	46.4%	44.7%	46.4%	45.6%	46.6%	45.0%	46.2%
120	45.1%	45.2%	45.3%	45.4%	45.1%	46.2%	45.4%	46.4%	46.3%	45.9%	46.4%	44.7%	46.4%	45.6%	46.6%	45.0%

Source: Tabs 'Lexis - males' and ‘Lexis – females’ of Excel appendix to CMI Working Paper 100.

Appendix 3: Reference population graduation tests

Table A3.1: Statistics and parameter values for reference population tables

Table	UK_M	UK_M	UK_F	UK_F
Age range	60-105	60-105	60-105	60-105
Deaths	1,906,487	1,906,487	2,155,966	2,155,966
Exposures	53,992,581	53,992,581	63,915,622	63,915,622
Formula	G(5)	G(7)	G(5)	G(8)
Deviance	116.04	94.89	186.06	85.93
VIF	1.60	1.60	1.61	1.61
QBIC	91.72	86.15	134.60	83.95
P(runs)	0.0907	0.0546	0.0053	0.4859
P(signs)	0.4415	0.3294	0.4415	0.2307
P(correl)	0.0001	0.0012	0.0000	0.0498
Parameter b_1	−2.967321E+1	+5.173236E+2	−1.053196E+1	+4.455258E+3
Parameter b_2	+1.270617E+0	−4.071254E+1	+3.254125E−1	−3.960567E+2
Parameter b_3	−2.623988E−2	+1.308071E+0	−9.427325E−3	+1.496559E+1
Parameter b_4	+2.522779E−4	−2.222595E−2	+1.212226E−4	−3.120400E−1
Parameter b_5	−8.800587E−7	+2.108309E−4	−4.983567E−7	+3.877439E−3

Note that parameter values b_1 to b_5 are shown in scientific E-notation, where “E” denotes multiplying the numerical value to the left by ten to the power of the numerical value to the right, e.g. 3E-4 = 3×10^{-4} = 0.0003.

Appendix 4: Software

This Working Paper describes two methods that are not contained in previous CMI software. The CMI has released two pieces of software to address this:

- Adjusted deaths and exposure data for the general population – we have released a standalone spreadsheet “CMI WP106 exposure adjustment method v01 2018-06-05.xlsm” to allow users to do these calculations, and this is described in this appendix.
- Extending graduated mortality rates to high ages, with reference to another population – the facility to do this is included in a spreadsheet “CMI WP107 S3 graduation software v01 2018-06-05.xlsm” that accompanies Working Paper 107 and is described in Appendix 14 of that paper.

Overview

The software contains deaths and exposures data, before adjustment, published by the ONS. In order to adjust the data, first fill in the parameters on the “Parameters” sheet, and then press the “Run” button. The code should only take a second or so to do the calculations, and will then produce results in a separate workbook.

Unadjusted data

The deaths and exposures data, before adjustment, is taken from the file “Population estimates and deaths by single year of age for England and Wales and the UK, 1961 to 2016” produced by the ONS[3]. We expect to produce an updated version of the software each year, following the release of new data by the ONS.

Although we recommend the use of the United Kingdom as the reference population for CMI tables, we have included data for England & Wales to allow assessment of the materiality of this choice.

[3] Available from https://www.ons.gov.uk/peoplepopulationandcommunity/birthsdeathsandmarriages/lifeexpectancies/adhocs/007598populationestimatesanddeathsbysingleyearofageforenglandandwalesandtheuk1961to2016

Parameters

Table A4.1 lists the parameters on the sheet "Specification", describes them, and shows the current recommended values. The notation used in the table corresponds to Section 3 of this paper.

Table A4.1: Description of parameters in the software

Name	Description	Recommended value
Gender	Which gender to do calculations for: "Male" or "Female"	
Country	Which country's data to use: "UK" for United Kingdom, or "EW" for England & Wales	UK
Join age (x_0)	The age from which the Kannisto-Thatcher method is applied: "x_0"	85
Maximum age (ω)	The assumed maximum age of survival: "ω"	125
Lexis method	The method to use for Lexis adjustments "$L_{x,t}$": "Formula" means using the formula in Section 3 of this paper "Grid" means using the values specified on the sheet "Lexis_Male" or "Lexis_Female", according to the choice of "Gender" A numeric input will be used for all ages and years.	Formula
Ratio for assigning deaths (age 105)	The value of "r_x", used for assigning grouped deaths to single years of age, at age 105; i.e. "r_{105}"	0.5 (males) 0.6 (females)
Ratio for assigning deaths (age ω)	The value of "r_x" , used for assigning grouped deaths to single years of age, at age ω; i.e. "r_ω"	0.1
Number of ages (k)	The number of ages to use in the survivor ratio method: "k"	5
Number of cohorts (m)	The number of cohorts to use in the survivor ratio method: "m"	2
Number of years to assess trend (y)	The number of years to use to assess the trend in the survivor ratio method: "y"	5
Half window (n)	Used to specify the size of the window used for smoothing exposures: "n"	2
Threshold probability (p)	The threshold for adjusting data when smoothing exposures: "p"	1%
Minimum age (x_{min})	The minimum age at which to consider smoothing exposures: "x_{min}"	20
Maximum age (x_{max})	The maximum age at which to consider smoothing exposures: "x_{max}"	100

Outputs

Table A4.2 lists the sheets contained in the output workbook and describes them. The key outputs are:

- "Deaths_DI": deaths, after assigning to single years of age; and
- "Population_P": populations, after making the various adjustments.

Sheets with name starting with "Debug_" contain intermediate steps of the calculation and may be helpful in understanding the process.

Table A4.2: Description of output sheets produced by the software

Name	Description
Contents	A list of the worksheets in the output workbook
Audit	Description of when the calculations were done, by whom, and the software version used
Input_Parameters	A copy of the parameters used
Input_Lexis	A copy of the Lexis grid, if this was used
Input_Deaths	The unadjusted deaths
Input_Population	The unadjusted populations
Deaths_DI	Deaths, after assigning to single years of age (D^I)
Population_P	The adjusted populations (P)
Debug_DK	Deaths, after Lexis adjustments (D^K)
Debug_PKc1	Populations, applying survivor ratio method without using the global correction factor (P^K with $c = 1$)
Debug_PK	Populations, applying survivor ratio method and using the global correction factor (P^K)
Debug_PM	Populations, adjusted to mid-year timing (P^M)
Debug_PS	Populations, after applying uniform annual scaling adjustments (P^S)
Debug_Adj	Values of: • Global correction factor (c); and • Uniform annual scaling adjustments (λ)

Password protection

Worksheets and the VBA code are protected to guard against accidental changes. They can be unprotected by using the password "CMI" but any changes are made at the user's own risk.

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Continuous Mortality Investigation Limited ("CMI") is registered in England & Wales
Company number: 8373631
Registered Office: 7th floor, Holborn Gate, 326-330 High Holborn, London, WC1V 7PP

Correspondence address: Cheapside House, 138 Cheapside, London, EC2V 6BW
Email: info@cmilimited.co.uk
Tel: 020 7776 3820

Website: www.cmilimited.co.uk (redirects to www.actuaries.org.uk)

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