Assessing the Economic Impact of Longevity Hedges

Andrew J.G. Cairns

Heriot-Watt University, Edinburgh

and

Director, Actuarial Research Centre,

Institute and Faculty of Actuaries

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Outline

- Introduction and motivation
- Hedging longevity risk with an index-based call-spread option contract
- Anatomy of a hedging calculation
- Numerical example
- Discussion
Motivation

- Longevity risk
- Measurement
  - e.g. Capital Requirement
  - Best estimate + extra for risk
- Longevity risk management
  - customised hedges
  - index-based hedges
Motivation

Why use General Population Longevity Index based risk transfer instruments?

→ Capacity and Price

Pros/cons

- Transferred risk is efficiently priced
- But hedger left with basis risk

Thus we need

- a clear and rigorous approach to quantify basis risk
- hedger and regulator agreement on approach
- to quantify properly the Capital Relief
Life insurer
Aim 1: measure mortality/longevity risk
Aim 2: manage mortality/longevity risk
  e.g. to *reduce* regulatory capital
  e.g. to *reduce* economic capital
  e.g. to *increase* economic value
Solvency II options:

- Solvency Capital Requirement, \( SCR = \) difference between
  Best estimate of annuity liabilities (BE) and
  Annuity liabilities following an immediate 20% reduction in mortality

- or \( SCR = \) extra capital required at time 0 to
  ensure solvency at time 1 with 99.5% probability

- or \( SCR = \) extra capital at time 0 to ensure
  solvency at time \( T \) with \( x\% \) probability
Liability to be Hedged

- \( L \) = random PV at time 0 of liabilities

- \( L(0) \) = point estimate of \( L \) based on time 0 info

- \( L(T) \) = point estimate of \( L \) based on info at \( T \)
  = PV of actual cashflows up to \( T \)
  + PV of estimated cashflows after \( T \)

- Risk \( \Rightarrow \) capital requirements

What type of hedge to modify capital requirements and manage risk?
Index-based hedge (derivative)

- Synthetic $\tilde{L}(T) \approx$ true $L(T)$
- Call spread derived from underlying $\tilde{L}(T)$

Payoff at $T$, *per unit*

\[
H(T) = \begin{cases} 
0 & \text{if } \tilde{L}(T) < AP \\
\tilde{L}(T) - AP & \text{if } AP \leq \tilde{L}(T) < EP \\
EP - AP & \text{if } EP \leq \tilde{L}(T)
\end{cases}
\] (Attachment Point)

(Exhaustion Point)
The Synthetic $\tilde{L}(T)$

- $\tilde{L}$ = random PV at time 0 of a portfolio of synthetic liabilities
- Synthetic mortality experience
  - based on general population mortality
  - adjusted using experience ratios

- $\tilde{L}(T)$ = point estimate of $\tilde{L}$ based on info at $T$
  $= PV$ of actual synthetic cashflows up to $T$
  $+ PV$ of estimated synthetic cashflows after $T$
Questions and Observations

- What is the impact of the hedge: \( L(T) \rightarrow L(T) - H(T) \)?

- Need a two population mortality model

- Practical reality: calculation is more complex than academic ‘ideal world’

- What are good choices of \( AP, EP, T \)?
Anatomy of a Hedging Calculation: Looks Complex!

\[ \text{General (National) Population} \]

\[ \text{Specific (Hedger's) Population} \]

\[ m_G(x, t) \]

\[ m_P(x, t) \]

\[ L(T) \]

\[ H(T) \]

\[ \tilde{L}(T) \]

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Longevity Risk Hedging
Historical Data

General (National) Population

Population G

\[ E_G(x, t) \]
\[ D_G(x, t) \]
\[ m_G^c(x, t) \]

Specific (Hedger’s) Population

Population S

\[ E_P(x, t) \]
\[ D_P(x, t) \]
\[ m_P^c(x, t) \]
Modelling Based on Data Up To Time 0

\[
\begin{align*}
& t \leq 0 \\
& 0 \leq t \leq T \\
& t > T
\end{align*}
\]

General (National) Population

Population G
\[
\begin{align*}
E_G(x, t) \\
D_G(x, t) \\
m_G(x, t)
\end{align*}
\]

Specific (Hedger’s) Population

Population S
\[
\begin{align*}
E_P(x, t) \\
D_P(x, t) \\
m_P(x, t)
\end{align*}
\]

\[
\begin{align*}
L(T) & \rightarrow H(T) \\
\tilde{L}(T) & \rightarrow \mu(T)
\end{align*}
\]
Generate Stochastic Scenarios Up To Time $T$

$t \leq 0$

$\geq 91$

90

Population G
$E_G(x, t)$
$D_G(x, t)$
$m_G(x, t)$

$C(0)$ $\mu(0)$

$\mu(0)$

$\mu(0)$

$C(T)$ $\mu(T)$ $\mu(0)$

$D_G(x, t)$

$m_G(x, t)$

$E_R(0)$

$M155\times(0)$

$t \geq 0$

$\geq 90$

89

Population S
$E_P(x, t)$
$D_P(x, t)$
$m_P(x, t)$

$C(0)$ $\mu(0)$

$\mu(0)$

$C(T)$ $\mu(T)$ $\mu(0)$

$D_P(x, t)$

$m_P(x, t)$

$t \leq T$

$1 \leq t \leq T$

$t > T$

General (National) Population

Specific (Hedger’s) Population

$\tilde{L}(T)$ $H(T)$

$E_R(T)$

$E_R(0)$

$M155\times(0)$

$E_R(T)$

$E_R(0)$

$E_R(T)$

$\tilde{L}(T)$ $H(T)$

$L(T)$
Modelling Based on Data Up To Time $T$

General (National) Population

Specific (Hedger’s) Population

$t \leq 0$

$1 \leq t \leq T$

$t > T$

$E_G(x, t)$
$D_G(x, t)$
$m_G(x, t)$

$E_P(x, t)$
$D_P(x, t)$
$m_P(x, t)$

$C(0)$
$\mu(0)$

$C(T)$
$\mu(T)$

$\hat{L}(T)$
$H(T)$

$\hat{H}(T)$

$\hat{M}(0)$

$\hat{M}(T)$

$\hat{L}(T)$

$L(T)$

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Longevity Risk Hedging
Central Forecast After $T$ For Each Scenario Up To $T$

$E_G(x, t)$
$D_G(x, t)$
$m_G(x, t)$

$E_P(x, t)$
$D_P(x, t)$
$m_P(x, t)$

$t \leq 0$

$1 \leq t \leq T$

$t > T$

General (National) Population

Specific (Hedger’s) Population

$\bar{L}(T) \rightarrow H(T)$

$L(T)$
Extract $m_G/P(x, t)$: Calculate $L(T)$, $\tilde{L}(T)$, $H(T)$
How many models do you need?

Academic ‘ideal’: One model

In practice:

- **Time 0:**
  - Liability valuation model (BE + SCR)
  - Simulation model (0 $\rightarrow$ $T$)

- **Time $T$:**
  - Hedge instrument valuation model
  - Liability valuation model

- ‘Models’ for extrapolating to high (and low) ages
Time 0 Models

- **Unhedged Liabilities:**
  Deterministic BE + 20% stress

- **Simulation:** (by way of example)
  - General population: (Lee-Carter/M1)
    \[
    \ln m_{gen}(x, t) = A(x) + B(x)K(t) \quad \text{(Lee-Carter/M1)}
    \]
  - Hedger’s own population: (M1-M5X)
    \[
    \ln m_{pop}(x, t) = \ln m_{gen}(x, t) + a(x) + k_1(t) + k_2(t)(x - \bar{x})
    \]
Hedge instrument:
- Lee-Carter (M1) for general population
- Recalibration: *on basis specified at time 0*

\[ q_{\text{pop}}^H(x, t) = q_{\text{gen}}^H(x, t) \times ER(x, 0) \rightarrow \tilde{L}(T) \rightarrow H(T) \]

Liability: specific (hedger’s) population
- Lee-Carter (M1) for general population
- Possibly different calibration from the hedge instrument

\[ q_{\text{pop}}^L(x, t) = q_{\text{gen}}^L(x, t) \times ER(x, T) \rightarrow L(T) \]
- Approach must mimic local practice
Hedging Example

- Data: Netherlands
  - CBS national data
  - CVS insurance data (Dutch aggregated industry experience data)

- Hedge instrument maturity: $T = 10$
- Attachment and exhaustion points at 60% and 95% quantiles of $\tilde{L}(T)$
- Key point: $EP << 99.5\%$ quantile of $\tilde{L}(T)$
Hedging Example

- Portfolio of deferred and immediate annuities
- Current ages 40 to 89
- Weights (≡ pension amounts):

![Pension Weights (Amounts)](image)

- Before and after: Compare $L(T)$ with $L(T) - H(T)$
- $SCR = 99.5\%$ quantile $-$ mean
Hedging Example \((n = 10,000\) scenarios\)

Simulated Annuity Portfolio Present Values

Correlation=0.978

Note: Population basis risk typically increases SCR (without hedge) as a percentage of BE.
What is the Impact of Population Basis Risk?

With $EP = 95\%$ quantile

At the much higher 99.5% level: $H(T)$ pays off in full with or without population basis risk.
Hedging Example: Higher AP (0.65) and EP (0.995)

Liability Distribution Functions

Cumulative Probability

Liability With/Without Hedge

No Hedge
Hedge with Basis Risk
Hedge with NO Basis Risk

Liability Distribution Functions

Cumulative Probability

Liability With/Without Hedge

<EP−AP = EP−AP
99.5%
### Numerical Example: AP, EP = 60% and 95% quantiles

<table>
<thead>
<tr>
<th></th>
<th>SCR&lt;sub&gt;20%stress&lt;/sub&gt;</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>L(0)</strong></td>
<td>840</td>
<td></td>
</tr>
<tr>
<td><strong>L(T):</strong></td>
<td>SCR&lt;sub&gt;10&lt;/sub&gt;</td>
<td>840</td>
</tr>
<tr>
<td><strong>L(T) - H(T):</strong></td>
<td>SCR&lt;sub&gt;11&lt;/sub&gt;</td>
<td>478</td>
</tr>
<tr>
<td><strong>L(T):</strong></td>
<td>SCR&lt;sub&gt;20&lt;/sub&gt;</td>
<td>960</td>
</tr>
<tr>
<td><strong>L(T) - H(T):</strong></td>
<td>SCR&lt;sub&gt;21&lt;/sub&gt;</td>
<td>598</td>
</tr>
</tbody>
</table>

(Pop 1; no hedge) (Pop 1; with \(\tilde{L}(T)\) hedge) (Pop 2; no hedge) (Pop 2; with \(\tilde{L}(T)\) hedge)

**Table:** SCR values in excess of the mean liability. For the hedging instrument \(AP = 10779\) (60% quantile) and \(EP = 11228\) (95% quantile). Pop 1: synthetic \(\tilde{L}(T)\). Pop 2: true \(L(T)\).
How good is the hedge? Issues:

- “Good” → price and risk reduction
- “Good” ↔ Types of basis risk
  - Structural (e.g. non-linear payoff)
  - Population basis risk
    - Within population (e.g. linkage to different cohort)
    - Different population
- Hedge effectiveness → % reduction in required capital
- Haircut → impact on capital relief as a result of population basis risk
- EIOPA Solvency II guidelines → regulatory approval should focus on the haircut
### Numerical Example: AP, EP = 60% and 95% quantiles

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<th>$L(0)$:</th>
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<td>$SCR_{21}$</td>
<td>598 (Pop 2; with $\tilde{L}(T)$ hedge)</td>
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**Table:** SCR values in excess of the mean liability. For the hedging instrument $AP = 10779$ (60% quantile) and $EP = 11228$ (95% quantile). Pop 1: synthetic $\tilde{L}(T)$. Pop 2: true $L(T)$.

What is the impact of Population basis risk on hedge effectiveness?

Haircut $HC = 1 - \frac{SCR_{20} - SCR_{21}}{SCR_{10} - SCR_{11}} = 0.000$. 
Haircut $\approx 0$: Interpretation

- Here $EP \ll 99.5\%$ quantile
- Above the 99.5\% quantile the call spread (almost) always pays off in full
- So population basis risk $\Rightarrow$ little impact
- Structural basis risk prevails

- More detailed analysis $\Rightarrow$
  Haircut is worst (highest) when EP is close to the 99.5\% quantile.
Haircut: Dependence on AP and EP

Haircut as a Function of the Attachment and Exhaustion Quantiles

Attachment Point Quantile

Exhaustion Point Quantile

Haircut

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Longevity Risk Hedging
Reduction in SCR: Dependence on AP and EP

Reduction in SCR with Hedge as a Percentage of SCR without Hedge

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Longevity Risk Hedging
Economic Benefits

Purpose of hedge:

- To manage and reduce risk
- To reduce statutory or economic capital requirements \( (t = 0) \)
- To enhance *economic/shareholder value*
Economic Value (work in progress)

Payments:
- Fixed $P_t$ payable at $t = 0, \ldots, T - 1$
- Contracted at time 0
- Time 0 value, $V_P = \sum_{t=0}^{T-1} P_t \exp(-rt)$

Benefits:
- $H(T)$ at time $T$
- Capital reduction, $CR_t$, at $t = 0, \ldots, T - 1$
- Time 0 value

\[
V_B = \text{value of } H(T) + \tilde{C}oC \times \text{‘value’ of } CR_0, \ldots, CR_{T-1}
\]

Compare $V_B$ with $V_P$. 
Discussion

- Rigorous approach: practical assessment of the impact of a longevity hedge
- Call spread: choice of EP $\Rightarrow$ impact on haircut $\Rightarrow$ impact on regulatory approval
- Choice of AP and EP $\Rightarrow$ impact on SCR reduction
- Interaction: SCR reduction $\leftrightarrow$ price $\Rightarrow$ tradeoff
- Applies equally well to economic capital
Thank You!

Questions?

Paper online at:

www.macs.hw.ac.uk/~andrewc/ARCresources
Tradeoffs and Other Considerations

How to choose Maturity, AP and EP?

- Reduction in SCR
- Cat Bond nominal
- Bull spread price
- Shareholder value added
- Insurer risk appetite, hedging objectives etc.
Sensitivity to Hedge Maturity, $T$

- e.g. $T = 20$

- % reduction in SCR is *slightly* higher
- Haircut is *slightly* worse
- Haircut is still $\approx 0$ for $EP \leq 99.5\%$ quantile

- The longer the maturity:
  - less liquid market
  - less confidence in future reserving method
  - more future capital relief (everything else held constant)
The Actuarial Research Centre (ARC) is the Institute and Faculty of Actuaries’ (IFoA) network of actuarial researchers around the world. The ARC seeks to deliver cutting-edge research programmes that address some of the significant, global challenges in actuarial science, through a partnership of the actuarial profession, the academic community and practitioners. The ’Modelling, Measurement and Management of Longevity and Morbidity Risk’ research programme is being funded by the ARC, the SoA and the CIA.

www.actuaries.org.uk/arc
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Three major programmes started in 2016, including

Modelling, Measurement and Management of Longevity and Morbidity Risk

- New/improved models for modelling longevity
- Management of longevity risk
- Underlying drivers of mortality
- Modelling morbidity risk for critical illness insurance