Forecasting Socio-Economic Differences in the Mortality of Danish Males

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Joint work with D. Blake, K. Dowd, M. Kallestrup-Lamb, C. Rosenskjold

International Mortality and Longevity Symposium 2016

Modelling, Measurement and Management of Longevity and Morbidity Risk

- New/improved models for modelling longevity
- Management of longevity risk
- Underlying drivers of mortality
- Modelling morbidity risk for critical illness insurance

Outline

- Introduction and motivation for multi-population modelling
- Constructing a new dataset
- Modelling Danish sub-population mortality
- Forecast correlations and basis risk
- Applications

1. Motivation: Stochastic Mortality

n lives, probability p of survival, N survivors

- Unsystematic mortality risk:
  $\Rightarrow N|p \sim$ Binomial(n, p)
  $\Rightarrow$ risk is diversifiable, $N/n \to p$ as $n \to \infty$
- Systematic mortality risk:
  $\Rightarrow p$ is uncertain
  $\Rightarrow$ risk associated with p is not diversifiable

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Motivation: Longevity Risk

Interested in *longevity risk*:

The risk that in aggregate people live longer than anticipated.

$\Rightarrow$ pension plan has insufficient cash to pay promised pensions

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Multi-Population Challenges

- Data availability
- Data quality and depth
- Model complexity
  - single population models can be complex
  - 2-population versions are more complex
  - multi-pop ......
- Multi-population modelling requires
  - (fairly) simple single-population models
  - simple dependencies between populations

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2. A New Case Study and a New Model

- Sub-populations differ from national population
  - socio-economic factors
  - other factors
- Denmark
  - High quality data on ALL residents
  - 1981-2012 available
  - Can subdivide population using covariates on the database

Danish Data

- *What can we learn from Danish data that will inform us about other populations?*

- Key covariates (amongst others):
  - Wealth
  - Income

Problem

- High income $\Rightarrow$ "affluent" *and low mortality* BUT
- Low income $\not\Rightarrow$ not affluent, high mortality
- High wealth $\Rightarrow$ "affluent" *and low mortality* BUT
- Low wealth $\not\Rightarrow$ not affluent, high mortality

Empirical solution: use a combination

- Affluence, $A =$ wealth $+K\times$ income
- $K = 15$ seems to work well *statistically* as a predictor
- Low affluence, A, predicts poor mortality

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Subdividing Data (after much experimentation!)

- Males resident in Denmark for the previous 12 months
- Divide population in year t
  - into 10 equal sized Groups (approx)
  - using *affluence*, A
- Individuals can change groups up to age 67
- Group allocations are locked down at age 67

(better than not locking down at age 67)

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Subdivided Data

- Ages 55-94;   Years 1985-2012
- Exposures $E^{(i)}(t,x)$ for groups $i = 1, \ldots, 10$
  range from over 4250 down to 13
- Deaths $D^{(i)}(t,x)$
  range from 151 down to 4
- Crude death rates
  $\hat{m}^{(i)}(t,x) = D^{(i)}(t,x)/E^{(i)}(t,x)$
- Small groups $\Rightarrow$ Poisson risk is important

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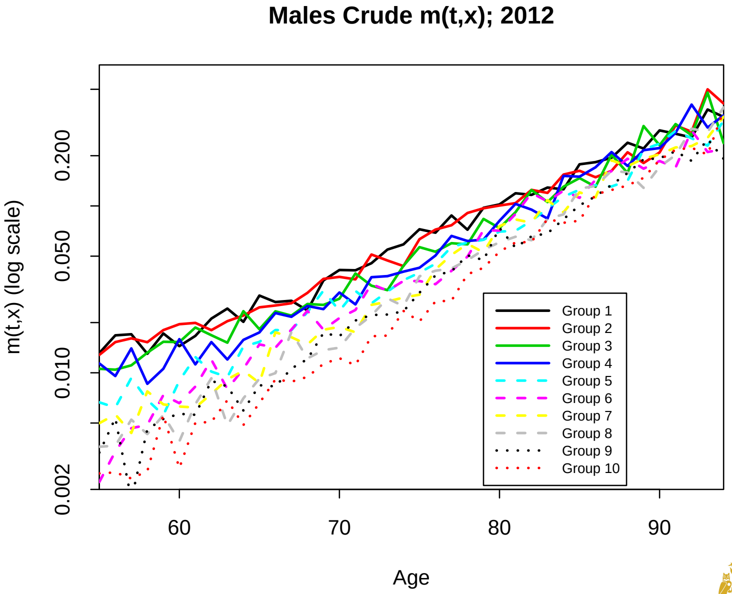
Crude death rates 2012



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Modelling the underlying death rates, $m^{(k)}(t,x)$

$m^{(k)}(t,x) =$ pop. k death rate in year t at age x

Population k, year t, age x

$$\log m^{(k)}(t,x) = \beta^{(k)}(x) + \kappa_1^{(k)}(t) + \kappa_2^{(k)}(t)(x-\bar{x})$$

(Extended CBD with a non-parametric base table, $\beta^{(k)}(x)$)

- 10 groups, $k = 1, \ldots, 10$ (low to high affluence)
- 28 years, $t = 1985, \ldots, 2012$
- 40 ages, $x = 55, \ldots, 94$

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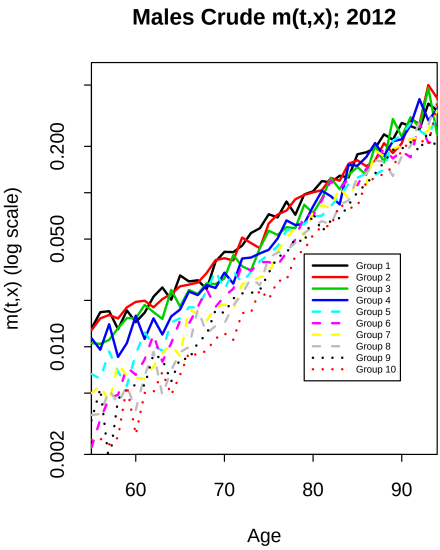
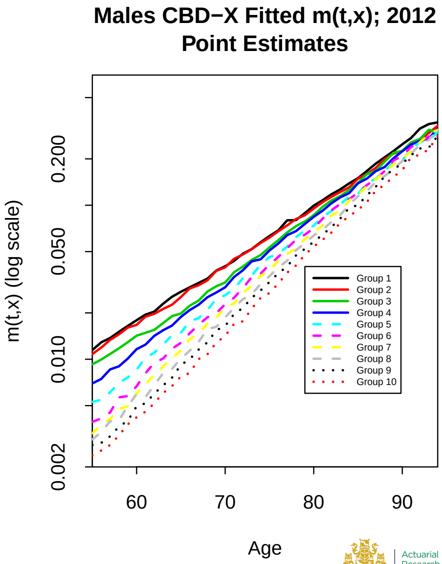
Model-Inferred Underlying Death Rates 2012





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Modelling the death rates, $m_k(t, x)$

$$\log m^{(k)}(t,x) = \beta^{(k)}(x) + \kappa_1^{(k)}(t) + \kappa_2^{(k)}(t)(x - \bar{x})$$

- Model fits the 10 groups well without a cohort effect
- Non-parametric $\beta^{(k)}(x)$ is essential to preserve group rankings
  - Rankings are evident in crude data
  - *"Bio-demographical reasonableness"*:

    more affluent $\Rightarrow$ healthier

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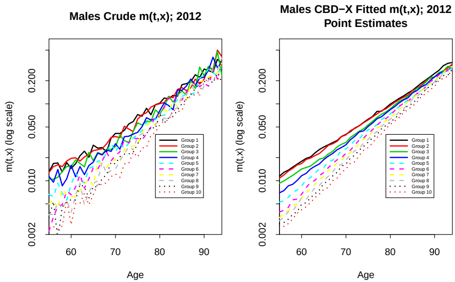
Model-Inferred Underlying Death Rates 2012



- Gap reduces from over 5× to 1.3×
- Or +14 years difference for Group 1→10, age 55; +9 at 67.
- Convergence ⇒ way ahead for modelling very high ages???

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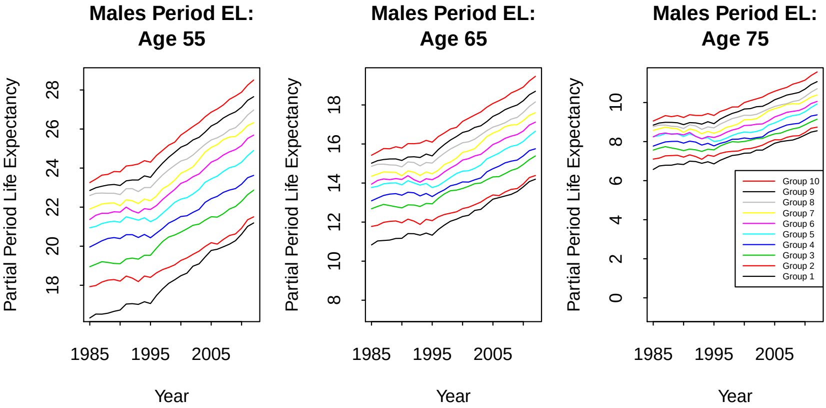
Partial Period Life Expectancy for Groups 1-10



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- $t \to t + 1$: Allow for correlation
  - between $\kappa_1^{(k)}(t + 1)$ and $\kappa_2^{(k)}(t + 1)$
  - between groups $k = 1, \ldots, 10$

- Medium/long term, $t \to t + T$:
  group specific period effects *gravitate* towards the national trend (*coherence*)
  $\Rightarrow$ Bio-demographical reasonableness:
  groups should not diverge

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A specific model

$$\begin{aligned}
\kappa_1^{(i)}(t) &= \kappa_1^{(i)}(t-1) + \mu_1 + Z_{1i}(t) && \text{(random walk)} \\
&\quad -\psi\left(\kappa_1^{(i)}(t-1) - \bar{\kappa}_1(t-1)\right) && \text{(gravity between groups)} \\
\kappa_2^{(i)}(t) &= \kappa_2^{(i)}(t-1) + \mu_2 + Z_{2i}(t) \\
&\quad -\psi\left(\kappa_2^{(i)}(t-1) - \bar{\kappa}_2(t-1)\right)
\end{aligned}$$

where

$$\bar{\kappa}_1(t) = \frac{1}{n}\sum_{i=1}^{n} \kappa_1^{(i)}(t) \quad \text{and} \quad \bar{\kappa}_2(t) = \frac{1}{n}\sum_{i=1}^{n} \kappa_2^{(i)}(t)$$

A specific model

$$\kappa_1^{(i)}(t) = \kappa_1^{(i)}(t-1) + \mu_1 + Z_{1i}(t) - \psi\left(\kappa_1^{(i)}(t-1) - \bar{\kappa}_1(t-1)\right)$$

$$\kappa_2^{(i)}(t) = \kappa_2^{(i)}(t-1) + \mu_2 + Z_{2i}(t) - \psi\left(\kappa_2^{(i)}(t-1) - \bar{\kappa}_2(t-1)\right)$$

Model structure $\Rightarrow$

- $(\bar{\kappa}_1(t), \bar{\kappa}_2(t)) \sim$ bivariate random walk
- Each $\kappa_1^{(i)}(t) - \bar{\kappa}_1(t) \sim AR(1)$ reverting to 0
- Each $\kappa_2^{(i)}(t) - \bar{\kappa}_2(t) \sim AR(1)$ reverting to 0
- $\beta^{(i)}(x)$ vs $\beta^{(j)}(x) \Rightarrow$ intrinsic group differences

Non-trivial correlation structure: between different ages and groups

$$\begin{aligned}
\kappa_1^{(i)}(t) &= \kappa_1^{(i)}(t-1) + \mu_1 + Z_{1i}(t) - \psi\left(\kappa_1^{(i)}(t-1) - \bar{\kappa}_1(t-1)\right) \\
\kappa_2^{(i)}(t) &= \kappa_2^{(i)}(t-1) + \mu_2 + Z_{2i}(t) - \psi\left(\kappa_2^{(i)}(t-1) - \bar{\kappa}_2(t-1)\right)
\end{aligned}$$

The Z_{ki} are multivariate normal, mean 0 and

$$Cov(Z_{ki}, Z_{lj}) = \begin{cases} v_{kl} & \text{for } i = j \\ \rho v_{kl} & \text{for } i \neq j \end{cases}$$

$\rho =$ cond. correlation between $\kappa_1^{(i)}(t)$ and $\kappa_1^{(j)}(t)$ etc.

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Comments

- Model is very simple
  - One gravity parameter, $0 < \psi < 1$
  - One between-group correlation parameter, $0 < \rho < 1$
- Many generalisations are possible
- But more parameters + more complex computing
- This simple model seems to fit quite well.
- Nevertheless $\Rightarrow$ potential for further work

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Prior distributions

- As uninformative as possible
- $\mu_1,\ \mu_2 \sim$ improper uniform prior
- $\{v_{ij}\} \sim$ Inverse Wishart
- $\rho \sim \text{Beta}(2,2)$
- $\psi \sim \text{Beta}(2,2)$

State variables and process parameters estimated using MCMC (Gibbs + Metropolis-Hastings)

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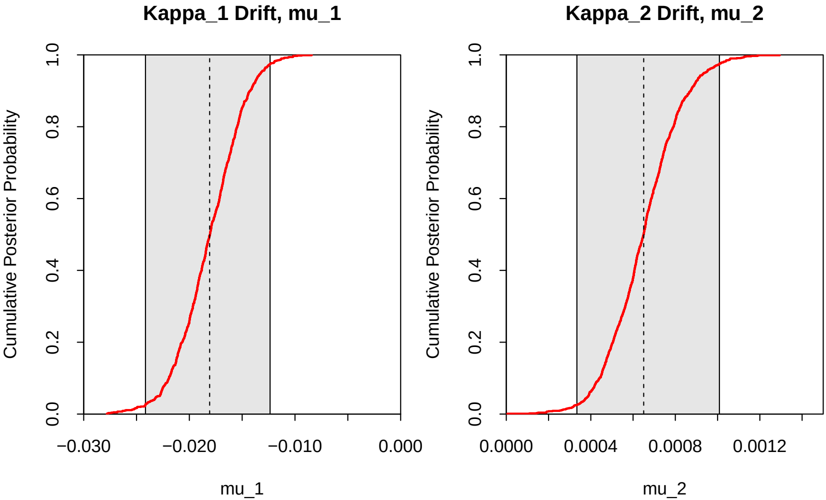
Posterior Distributions and 95% Credibility Intervals



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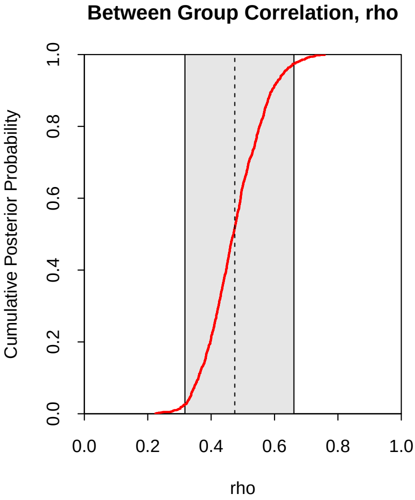
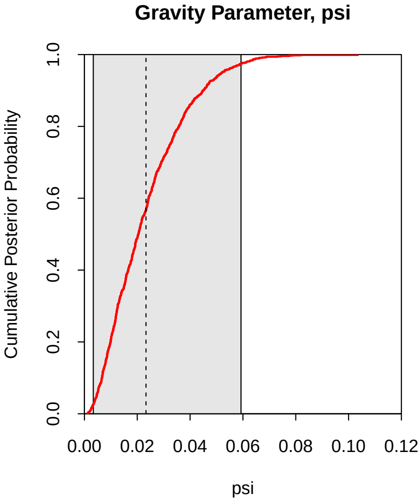
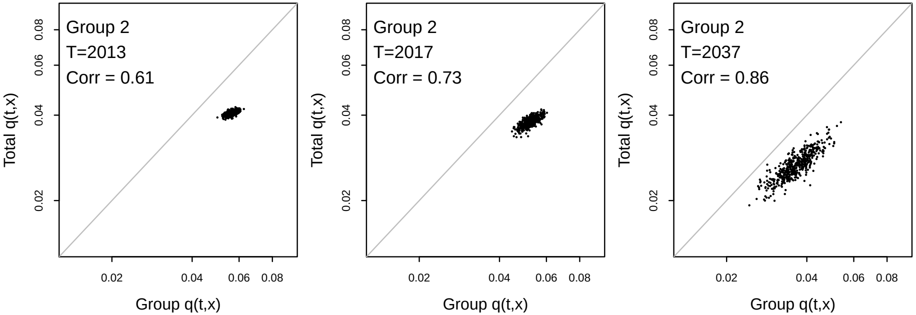
Posterior Distributions and 95% Credibility Intervals





Simulated Group versus Population Mortality, $q(t,x)$



As T increases: $+1$ year; $+5$ years; $+25$years

- Scatterplots become more dispersed
- Shift down and to the left
- Correlation increasess

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Forecast Correlations

Deciles are quite narrow subgroups

More diversified e.g.

- Blue collar pension plan
  $\Rightarrow$ equal proportions of groups 2, 3, 4
- White collar pension plan
  $\Rightarrow$ equal proportions of groups 8, 9, 10
- Mixed plan
  $\Rightarrow$ proportions $(0, 0, 1, 2, \ldots, 7, 8)/36$ (e.g. amounts)

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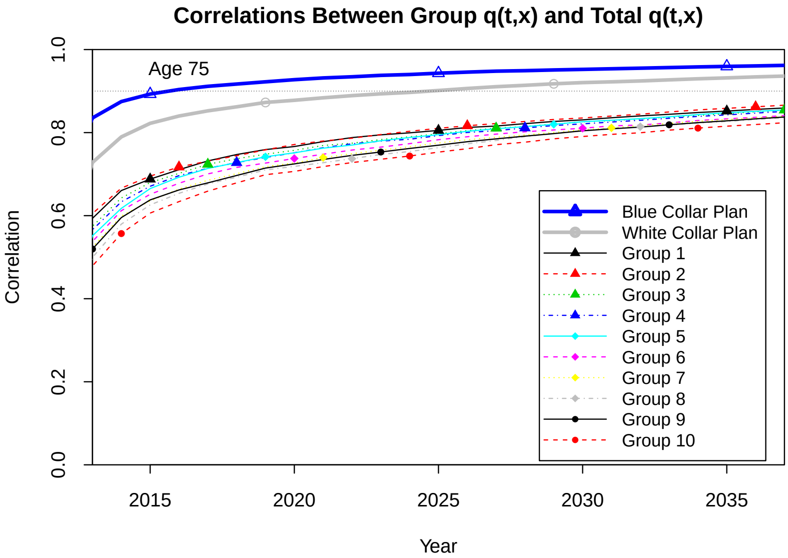
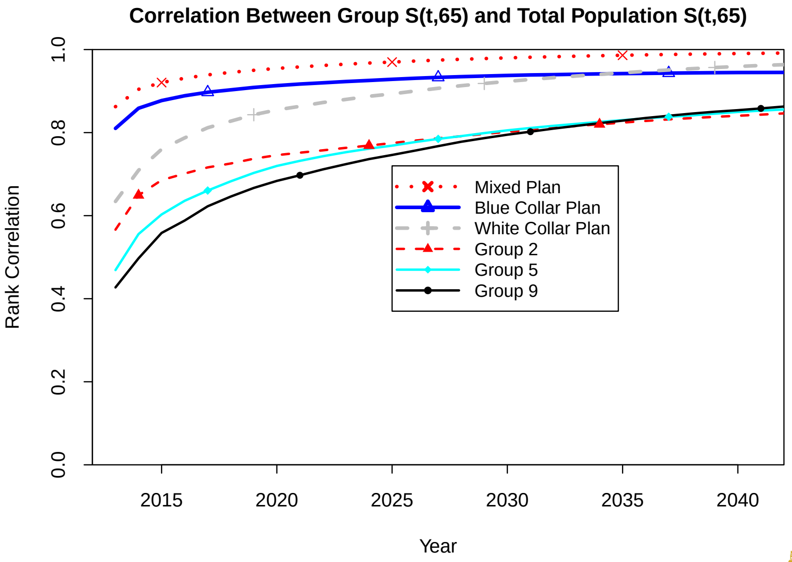
Forecast Correlations: Mortality Rates at Age 75



Forecast Correlations: Cohort Survivorship from 65



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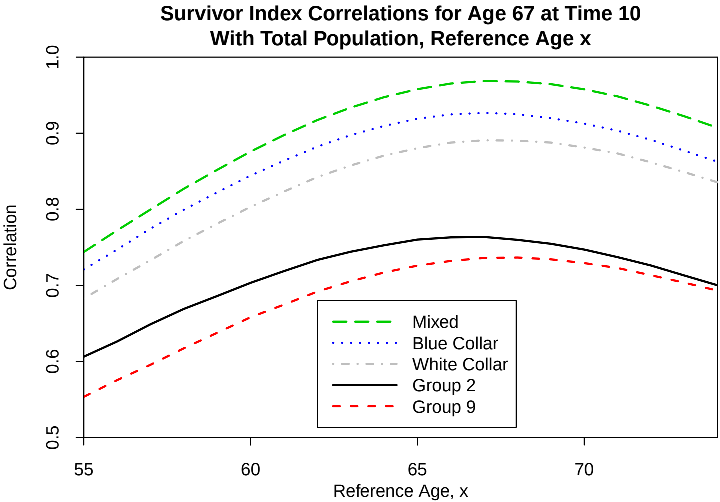
Forecast Correlations: Cohort Survivorship

Different reference ages: $cor\left(S_X(10, 67),\ S_{TOT}(10, x)\right)$



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Modelling Conclusions

- Development of a new multi-population dataset for Denmark

  strong bio-demographically reasonable group rankings based on a new measure of affluence
- Unlike multi-country data

  a priori ranking of affluence-related groups
- Proposal for a simple new multi-population model
- Mortality rates converge at high ages
- Strong correlations over medium to long term

  even allowing for parameter uncertainty
- Correlations depend strongly on diversity of sub-population

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3. Postscript: Education as an Alternative Covariate

- Level of Highest Education also known to be a good predictor
  - Various US studies
  - Mackenbach et al. (2003) including Denmark: Std. Mortality Rates
  - Bronnum-Hansen and Baadsgaard (2012) Denmark: $LE(x = 30)$
- As close as possible on a *like for like* basis:

  Crude death rates; age 30+; matching years.

  Affluence $\Rightarrow$
  - Wider spread of SMR's than M. et al. (2003)
  - Wider spread of $LE(30)$ than BHB (2012)
- More to be done.

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Choices

- No hedging
- Hedge using own experience
- Hedge using standardised instrument: national mortality

Basis Risk

Two sources of basis risk considered here

- Population basis risk
- Sub-optimal choice of hedging instrument

  tradeoff: price *vs* basis risk

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Economic capital relief using longevity options

- Population 1: national population; reference for hedge

  notional portfolio of males aged 65: A_1 =P.V. pension payments

- Population 2: hedger's own population

  portfolio of males aged 65: A_2 =P.V. pension payments

Economic capital relief using longevity options

- Three choices:
  - No hedging of A_2
  - Hedge A_2 with population 1 longevity swap $A_1 - \hat{A}_1$
  - Hedge A_2 with out-of-the-money option on $A_1(T)$
    Payoff at $T = 20$; underlying $A_1(T)$ includes estimated $t > T$ cashflows

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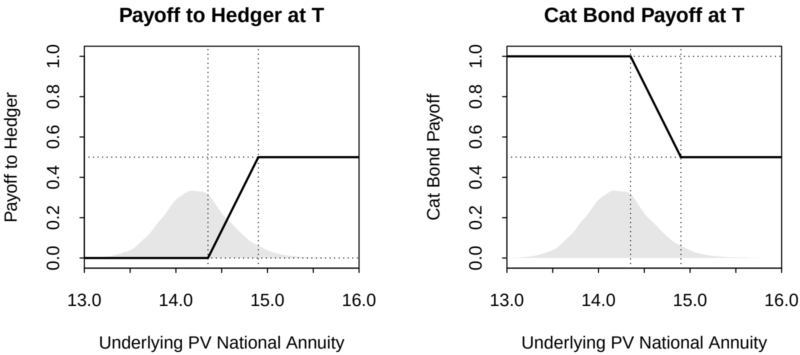
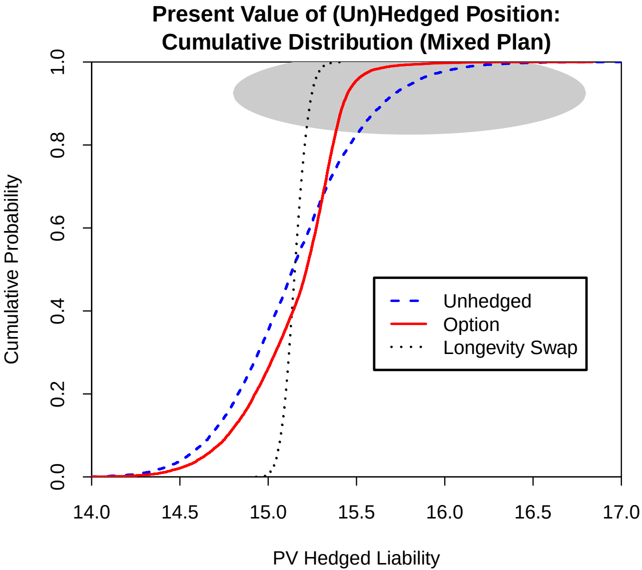
Index Based Hedge: Payoffs



Attachment/Detachment at approx 60% / 95%

Impact of Hedging with $T = 20$ Option or Index Swap



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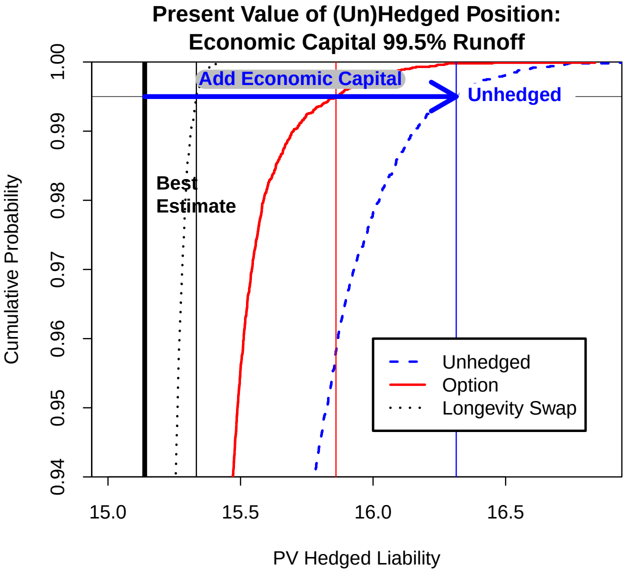

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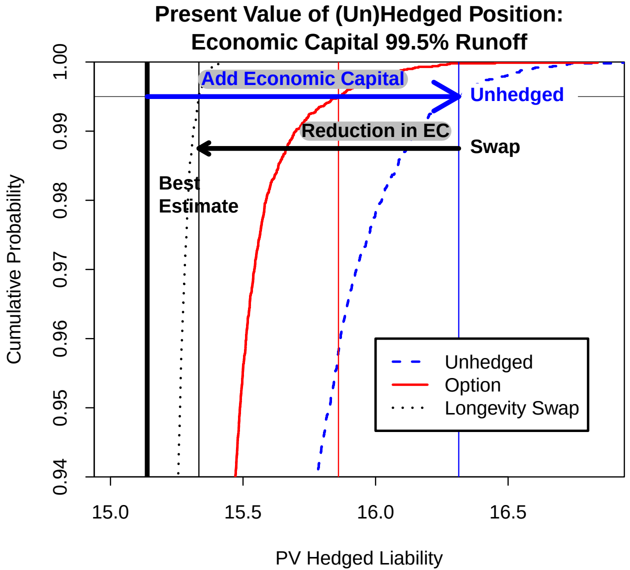

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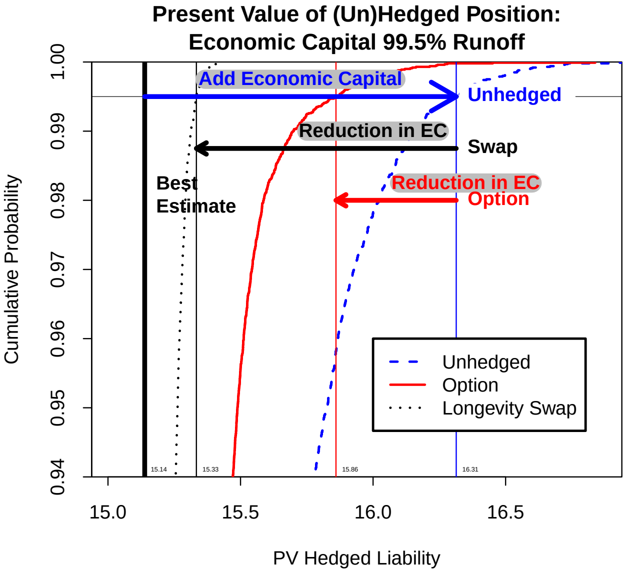

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Challenges 1

- Simulation example assumes swaps and options priced at actuarially fair value
- But swap and option premiums might be more expensive
- Compare premium versus
value of reduction in Economic Capital
over multiple time periods

Challenges 2

- Bull spread option:
  - Choice of attachment/detachment points $K_1,\ K_2$
  - Maximum cat bond loss
  - Capital markets capacity $\leftrightarrow$ annuity liabilities
  - Risk premiums
  - Sub-optimal instrument basis risk

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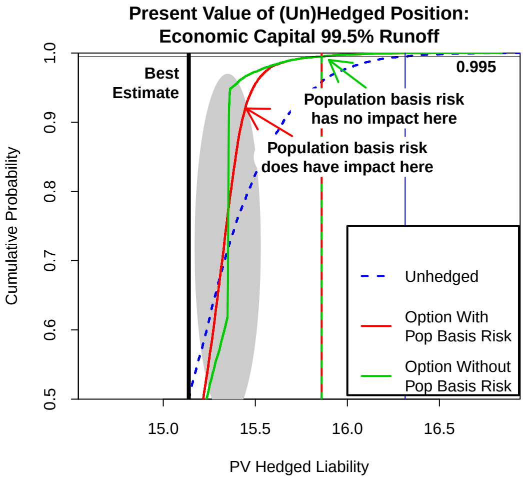
What is the impact of population basis risk?



Conclusion depends on $K_2 << 99.5\%$ quantile.

Basis risk from "sub-optimal" choice of hedging instrument.

5. Summary

- Danish data allows insight into relative mortality dynamics between socio-economic sub-populations
- Conclusions for other countries likely to be similar
- Economic capital example is one of many potential risk management applications

Working paper available on website.

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Thank You!

Questions?

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