The devil is in the tails: actuarial mathematics and the subprime mortgage crisis

by Catherine Donnelly and Paul Embrechts

Heriot-Watt University, Edinburgh and Risklab, ETH Zurich, Switzerland

Winner of The Bob Alting von Geusau Memorial Prize (2010-2011)

Published in the ASTIN Bulletin (2010), 40(1), pp1-33.

2 June 2016

The 2007-2008 Financial Crisis

Origin: Subprime mortgage crisis

- Banks' "originate-to-hold" $\mapsto$ "originate-to-distribute" model.
- Securitization, failure to hold risky tranche.
- Ratings agencies.
- Blindness to risk in the competitive race.
- Systemic interdependence.
- $\vdots$

Popular press

The formula that felled Wall Street
by Sam Jones, Financial Times, Apr 2009.

- In the autumn of 1987, the man who would become the world's most influential actuary landed in Canada on a flight from China.
- ..., he attempted to solve one of Wall Street quants' most intractable problems: default correlation.
- "Default is like the death of a company." *David Li*.

Popular press

- **Formula from hell**,
  by Susan Lee, Forbes.com, Aug 2009.
- Was David Li **the guy who 'blew up Wall Street?'**,
  by Mike Hornbrook, CBC News, Apr 2009.
- Recipe for disaster: **the formula that killed Wall Street**,
  by Felix Salmon, Wired, Feb 2009.

Formal response to the crisis

The Turner Review: A regulatory response to the global banking crisis
by Adair Turner, Chairman of the FSA, Mar 2009.

- The very complexity of the mathematics used to measure and manage risk, moreover, made it increasingly difficult for top management and boards to assess and exercise judgement over the risks being taken. Mathematical sophistication ended up not containing risk, but providing false assurance that other prima facie indicators of increasing risk (e.g. rapid credit extension and balance sheet growth) could be safely ignored.
- The agenda for regulatory reform ... needs to address ...The complexity and opacity of the structured credit and derivatives system, built upon a misplaced reliance on sophisticated mathematics,...

Responses to the Turner Review

Letter to Lord Turner in response to the Turner Review
by David Wallace, The Council for Mathematical Sciences, Jun 2009.

- Mathematics is surely the only medium capable of describing quantitatively the complex nature of the products that traders, risk managers, etc are handling...
- Another aspect on which we would welcome dialogue concerns the reference to a "misplaced reliance on sophisticated maths" and the possible interpretation that mathematics *per se* has a negative effect in the city. You can imagine that we strongly disagree with this interpretation!

Responses to the Turner Review

Letter to Lord Turner in response to the Turner Review (cont'd)
by David Wallace, The Council for Mathematical Sciences, Jun 2009.

- The financial mathematics community also sees a role for itself in engaging the public in how mathematics is used in the financial services industry.
- ... we believe that the FSA and the research community share an objective to enhance public appreciation of the uncertainties in modelling future behaviour.

Responses to the Turner Review

Maths and markets

Editorial, Financial Times, Mar 2009.

- Markets + Maths = Mayhem. ... sums up an erroneous view of the role played by mathematics in the banking crisis, which is gaining currency in financial and regulatory circles.
- Contrary to Lord Turner's assertion, the banks' sums were not sophisticated enough. They over-simplified, ...
- ... we need more - and better - maths to underpin individual banks and the enhanced regulatory regime that will oversee them.
- ...the true equation: Markets minus Maths mean Mayhem.

The formula

What is all the fuss about 'the formula'?

$$\boxed{\mathbb{P}\left[T_A \leq 1, T_B \leq 1\right] = \Phi_2\left(\Phi^{-1}\left(F_A(1)\right), \Phi^{-1}\left(F_B(1)\right); \gamma\right),}$$

where

- Φ is the univariate standard normal distribution function,
- Φ_2 is the bivariate normal distribution function with correlation parameter ρ, and
- $F_A(1)$, $F_B(1)$ are the probabilities that companies A and B default within a year.

Copulas

Definition. A 2-dimensional copula $C : [0,1]^2 \to [0,1]$ is a distribution function with standard uniform marginal distributions.

Example 1. The independence copula

$$C^{\perp\!\!\!\perp}(u, v) := uv$$

Example 2. The Gaussian copula

$$C_\rho^{\mathrm{gau}}(u, v) := \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi(1-\rho^2)^{1/2}} \exp\left(-\frac{s^2 - 2\rho st + t^2}{2(1-\rho^2)}\right) ds\, dt.$$

Copulas

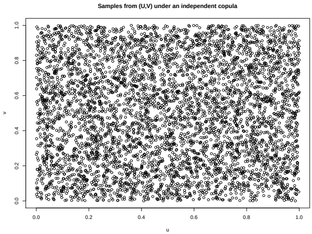

Figure 1: Sampling under the independent copula.

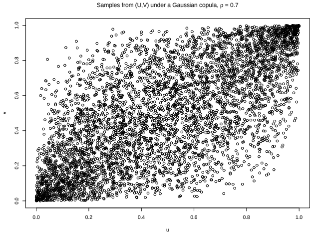

Figure 2: Sampling under the Gaussian copula.

Copulas

Theorem (Sklar). Let C be a copula and F_1, F_2 be univariate distribution functions. Define

$$H(x, y) := C\left(F_1(x), F_2(y)\right), \quad \forall (x, y) \in \mathbb{R}^2.$$

Then H is a joint distribution function with margins F_1, F_2.

Example 3. Set $F_1 = \Phi$, $F_2 = \Phi$ and $C = C_\rho^{\mathrm{gau}}$.

$$C_\rho^{\mathrm{gau}}(\Phi(x), \Phi(y)) = \int_{-\infty}^{x} \int_{-\infty}^{y} \frac{1}{2\pi(1-\rho^2)^{1/2}} \exp\left(-\frac{s^2 - 2\rho st + t^2}{2(1-\rho^2)}\right) dsdt,$$

i.e. $C_\rho^{\mathrm{gau}}(\Phi(x), \Phi(y)) = \Phi_2(x, y)$.

Copulas

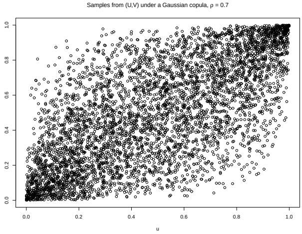

Figure 3: Sampling under the Gaussian copula with uniform marginals.

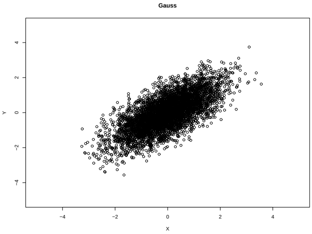

Figure 4: Sampling under the Gaussian copula with Gaussian marginals.

Copulas: same linear correlation

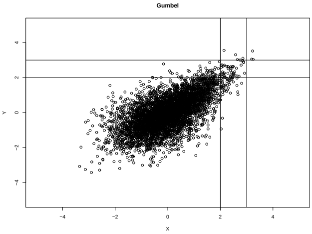

Figure 5: Sampling under the Gumbel copula with Gaussian marginals.

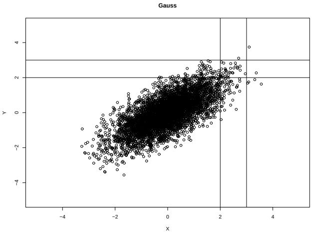

Figure 6: Sampling under the Gaussian copula with Gaussian marginals.

Copulas: same linear correlation

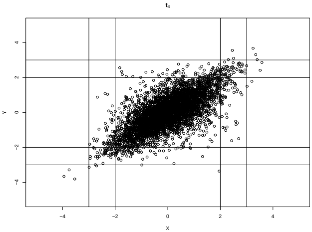

Figure 7: Sampling under the t_4 copula with Gaussian marginals.

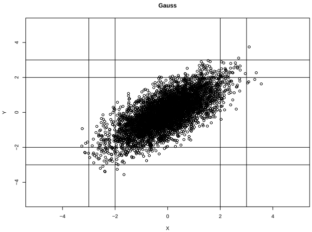

Figure 8: Sampling under the Gaussian copula with Gaussian marginals.

The formula again

$$\boxed{\mathbb{P}\left[T_A \leq 1, T_B \leq 1\right] = \Phi_2\left(\Phi^{-1}\left(F_A(1)\right), \Phi^{-1}\left(F_B(1)\right); \gamma\right),}$$

Advantages:

- simple to understand,
- computationally fast, and
- easy to calibrate.

Drawbacks of the Gaussian copula

Disadvantages:

- inadequate modelling of the default clustering,
- inconsistent pricing of CDO tranches, and
- stress-testing: no modelling of economic factors.

Inadequate modelling of the default clustering

In a crisis, corporate defaults tend to cluster.

But under a Gaussian copula model, with $\gamma < 1$, company defaults become independent.

Definition. Let X, Y be random variables with dfs F, G. The coefficient of upper tail dependence of X and Y is

$$\lambda_u := \lim_{q \to 1^-} \mathbb{P}\left[Y > G^{\leftarrow}(q) \,|\, X > F^{\leftarrow}(q)\right].$$

If $\lambda_u = 0$ then X and Y are asymptotically independent in the upper tail. Otherwise, they show upper tail dependence.

Popular press

- Formula from hell,
  by Susan Lee, Forbes.com, Aug 2009.
- Was David Li the guy who 'blew up Wall Street?',
  by Mike Hornbrook, CBC News, Apr 2009.
- Recipe for disaster: the formula that killed Wall Street,
  by Felix Salmon, Wired, Feb 2009.
- **How a formula ignited market that burned some big investors,**
  by Mark Whitehouse, Wall Street Journal, Sep 2005.

AIG

AIG is engaged in insurance and related activities in more than 130 countries.

In 2007, AIG had:

- 100 000 employees worldwide,
- $1 000 billion in assets, and
- a London-based subsidiary, AIGFP, with around 400 employees.

AIGFP sold credit default swaps (CDSs), with notional value around $527 billion in 2007.

AIGFP

AIG 2006 Annual Report:
"The likelihood of any payment obligation by AIGFP under each transaction is remote, even in severe recessionary market scenarios."

Joseph Cassano, the head of AIGFP, quoted by the New York Times, in August 2007:
"It is hard for us, without being flippant, to even see a scenario within any kind of realm of reason that would see us losing one dollar in any of those transactions."

AIGFP

What went wrong?

- Collateral postings on CDSs.
- Made worse by AIG's Securities Lending Program.
- AIG's losses were $99 billion for 2008.
- US government gave $145 billion to AIG by August 2009.

Conclusion

- Risk management shortcomings $\rightarrow$ huge financial losses.
- Education: model assumptions and applicability.
- Communication.
- Human weaknesses: temptation will come.

Thank you!