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Section des risques financiers et GRE de l'AAI
IAA Financial Risks and ERM Section
AFIR/ERM

# Pension Design and Risk Sharing : New Mix Solutions between DB and DC

Professor Pierre DEVOLDER
University of Louvain ( UCL ) *Belgium*

Joint research with Dr.Sébastien de Valeriola ( UCL)

18 May 2016

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# 1. Motivation

# 2. Automatic Adjustment and Risk Sharing

# 3. Numerical illustration

# 4. Stochastic model

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# 1. Motivation

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- Problems of financial viability of classical Pay As You Go ( PAYG) social security pension schemes

- Most of them using a Defined Benefit (DB) structure

- Important risk factors :
  - Ageing
  - Volatility of the financial markets
  - 0% interest rates

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- Parametric reforms ( *retirement age, early retirement , indexation,…*)

- Move from DB schemes to DC schemes ( *Notional Accounts* , NDC )

- Introduction of Automatic Balance Mechanisms as an answer to risk exposure ( DB and DC ) to avoid any form of " Pension Populism"

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- …But Pension reform is not just a matter of financial stability

- Mission of the social security : social sustainability

- Fairness between generations and between categories of workers

- How to develop Social Security Pension Schemes in PAYG with fair risk sharing between contributors and retirees ?

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# 2. Automatic Adjustment and Risk Sharing

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# Equilibrium Equation in PAYG

## Incomes :

$A(t)$ = number of contributors at time t

$W(t)$ = mean wage

$\pi(t)$ = contribution rate

$$IN(t) = A(t).\pi(t).W(t)$$

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# Equilibrium Equation in PAYG

## Outcomes :

$B(t) =$ number of retirees at time $t$

$P(t) =$ mean pension

$\delta(t) =$ replacement rate

$$OUT(t) = B(t).P(t) = B(t).\delta(t).W(t)$$

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# Equilibrium Equation in PAYG

## Actuarial equilibrium :

$$IN(t) = OUT(t)$$

$$A(t).\pi(t).W(t) = B(t).P(t)$$

$$\pi(t) = \frac{B(t)}{A(t)} . \frac{P(t)}{W(t)}$$

$$D(t) = \frac{B(t)}{A(t)} = \text{dependence ratio}$$

$$\delta(t) = \text{replacement rate}$$

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# Automatic Adjustment

$$\pi(t) = \frac{B(t)}{A(t)} \cdot \frac{P(t)}{W(t)} = D(t) \cdot \frac{P(t)}{W(t)} = D(t) \cdot \delta(t)$$

Risk factor

**Automatic Adjustment :**

How to maintain automatically this equilibrium in case of change of D(t) ( *! Increase !*)

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# Automatic Adjustment

$$\pi(t) = \frac{B(t)}{A(t)} \cdot \frac{P(t)}{W(t)} = D(t) \cdot \frac{P(t)}{W(t)} = D(t) \cdot \delta(t)$$

Risk factor

Constant in pure DC

Constant in pure DB

Adjustment of δ → **Social threat**

Adjustment of π → **Financial threat**

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# Risk Sharing in a deterministic model

$$\pi(t) = D(t).\frac{P(t)}{W(t)}$$

$$\ln(\pi(t)) = \ln(D(t)) + \ln(P(t)) - \ln(W(t))$$

$$\frac{d\pi(t)}{\pi(t)} = (\frac{dP(t)}{P(t)} - \frac{dW(t)}{W(t)}) + \frac{dD(t)}{D(t)} = \frac{d\delta(t)}{\delta(t)} + \frac{dD(t)}{D(t)}$$

Spread of dynamic evolution between pension and wage

Ageing Effect

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# Risk Sharing

$$\frac{d\pi(t)}{\pi(t)} = \left(\frac{dP(t)}{P(t)} - \frac{dW(t)}{W(t)}\right) + \frac{dD(t)}{D(t)}$$

## CASE 1 : DB / Defined Benefit

$$\frac{dP(t)}{P(t)} = \frac{dW(t)}{W(t)}$$ → Full indexation of pensions on wages

$$\frac{d\pi(t)}{\pi(t)} = \frac{dD(t)}{D(t)}$$ → Full impact of the Ageing effect on the active generation

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# Risk Sharing

$$\frac{d\pi(t)}{\pi(t)} = \left(\frac{dP(t)}{P(t)} - \frac{dW(t)}{W(t)}\right) + \frac{dD(t)}{D(t)}$$

## CASE 2 : DC / Defined Contribution

$$\frac{d\pi(t)}{\pi(t)} = 0$$

→ Full stability of the cost

$$\frac{dP(t)}{P(t)} = \frac{dW(t)}{W(t)} - \frac{dD(t)}{D(t)}$$

→ Full impact of the Ageing effect on the retirees

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# Risk Sharing

$$\frac{d\pi(t)}{\pi(t)} - \left(\frac{dP(t)}{P(t)} - \frac{dW(t)}{W(t)}\right) = \frac{dD(t)}{D(t)}$$

## Fair risk sharing between generations :

$$\frac{d\pi(t)}{\pi(t)} = (1-\alpha(t)).\frac{dD(t)}{D(t)}$$

→ Ageing impact on the contribution rate

$$\frac{dP(t)}{P(t)} = \frac{dW(t)}{W(t)} - \alpha(t).\frac{dD(t)}{D(t)}$$

→ Ageing impact on the benefits

$0 \leq \alpha(t) \leq 1$: automatic adjuster

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# Replacement rate

$$\frac{d\pi(t)}{\pi(t)} = \frac{d\delta(t)}{\delta(t)} + \frac{dD(t)}{D(t)}$$

**DB**

$$d\delta(t) = 0$$

$$\frac{d\pi(t)}{\pi(t)} = \frac{dD(t)}{D(t)}$$

**DC**

$$\frac{d\delta(t)}{\delta(t)} = -\frac{dD(t)}{D(t)}$$

$$d\pi(t) = 0$$

**Risk Sharing**

$$\frac{d\delta(t)}{\delta(t)} = -\alpha(t).\frac{dD(t)}{D(t)}$$

$$\frac{d\pi(t)}{\pi(t)} = (1-\alpha(t)).\frac{dD(t)}{D(t)}$$

$\alpha(t) = 0$ (DB)

$\alpha(t) = 1$ (DC)

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# Example 1 : the Musgrave rule

$$\frac{d\pi(t)}{\pi(t)} - \left(\frac{dP(t)}{P(t)} - \frac{dW(t)}{W(t)}\right) = \frac{dD(t)}{D(t)}$$

## EXAMPLE : MUSGRAVE rule

Goal:

To keep constant the replacement rate but net of contributions

$$\delta(t) = \frac{P(t)}{W(t)}$$

→

$$M = \frac{P(t)}{W(t).(1-\pi(t))}$$

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# Example 1 : the Musgrave rule

$$M = \frac{P(t)}{W(t).(1-\pi(t))} = \frac{\delta(t)}{1-\pi(t)}$$

Musgrave Condition

$$\frac{dP(t)}{P(t)} = \frac{dW(t)}{W(t)} + \frac{d(1-\pi(t))}{1-\pi(t)} = \frac{dW(t)}{W(t)} - \frac{\pi(t)}{1-\pi(t)} . \frac{d\pi(t)}{\pi(t)}$$

Equilibrium Condition

$$\frac{d\pi(t)}{\pi(t)} = (\frac{dP(t)}{P(t)} - \frac{dW(t)}{W(t)}) + \frac{dD(t)}{D(t)}$$

$$\frac{d\pi(t)}{\pi(t)} = (1-\pi(t)).\frac{dD(t)}{D(t)}$$

$$\alpha(t) = \pi(t)$$

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# Example 1 : the Musgrave rule

$$\frac{d\pi(t)}{\pi(t)} = \frac{d\delta(t)}{\delta(t)} + \frac{dD(t)}{D(t)}$$

Musgrave

$$\frac{d\pi(t)}{\pi(t)} = (1-\pi(t)).\frac{dD(t)}{D(t)}$$

$$\frac{d\delta(t)}{\delta(t)} = -\pi(t).\frac{dD(t)}{D(t)}$$

Solution :

$$\pi(t) = \frac{K.D(t)}{1+K.D(t)}$$

$$\delta(t) = \frac{K}{1+K.D(t)}$$

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# Example 2 : the constant proportion

**Constant risk sharing between generations :**

$$\alpha(t) = \alpha \quad (\text{with} \quad 0 < \alpha < 1)$$

$$\frac{d\pi(t)}{\pi(t)} = (1-\alpha).\frac{dD(t)}{D(t)}$$

$$\frac{d\delta(t)}{\delta(t)} = -\alpha.\frac{dD(t)}{D(t)}$$

Solution :

$$\pi(t) = A.D(t)^{1-\alpha}$$

$$\delta(t) = A.D(t)^{-\alpha}$$

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# Summary

| | DB | Musgrave | Constant proportion | DC |
|---|---|---|---|---|
| Replacement Rate | $\delta(t) = \delta_0$ | $\delta(t) = \dfrac{K}{1 + K.D(t)}$ | $\delta(t) = A.D(t)^{-\alpha}$ | $\delta(t) = \pi_0.D(t)^{-1}$ |
| Contribution Rate | $\pi(t) = \delta_0.D(t)$ | $\pi(t) = \dfrac{K.D(t)}{1 + K.D(t)}$ | $\pi(t) = A.D(t)^{1-\alpha}$ | $\pi(t) = \pi_0$ |

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# 3. Numerical illustration

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# Example

## Mean reverting dependence ratio

$$D(t) = D_0.e^{-\beta t} + \bar{D}.(1 - e^{-\beta t}) \qquad (D_0 < \bar{D})$$

$$\delta(0) = \delta_0$$

$$\pi(0) = \pi_0 = D_0.\delta_0$$

$$D_0 = 40\% \qquad \bar{D} = 66\% \qquad \beta = 5\%$$

$$\delta(0) = 50\% \qquad \alpha = 50\%$$

$$\pi(0) = 20\%$$

$$\delta(t) = ? \qquad \pi(t) = ?$$

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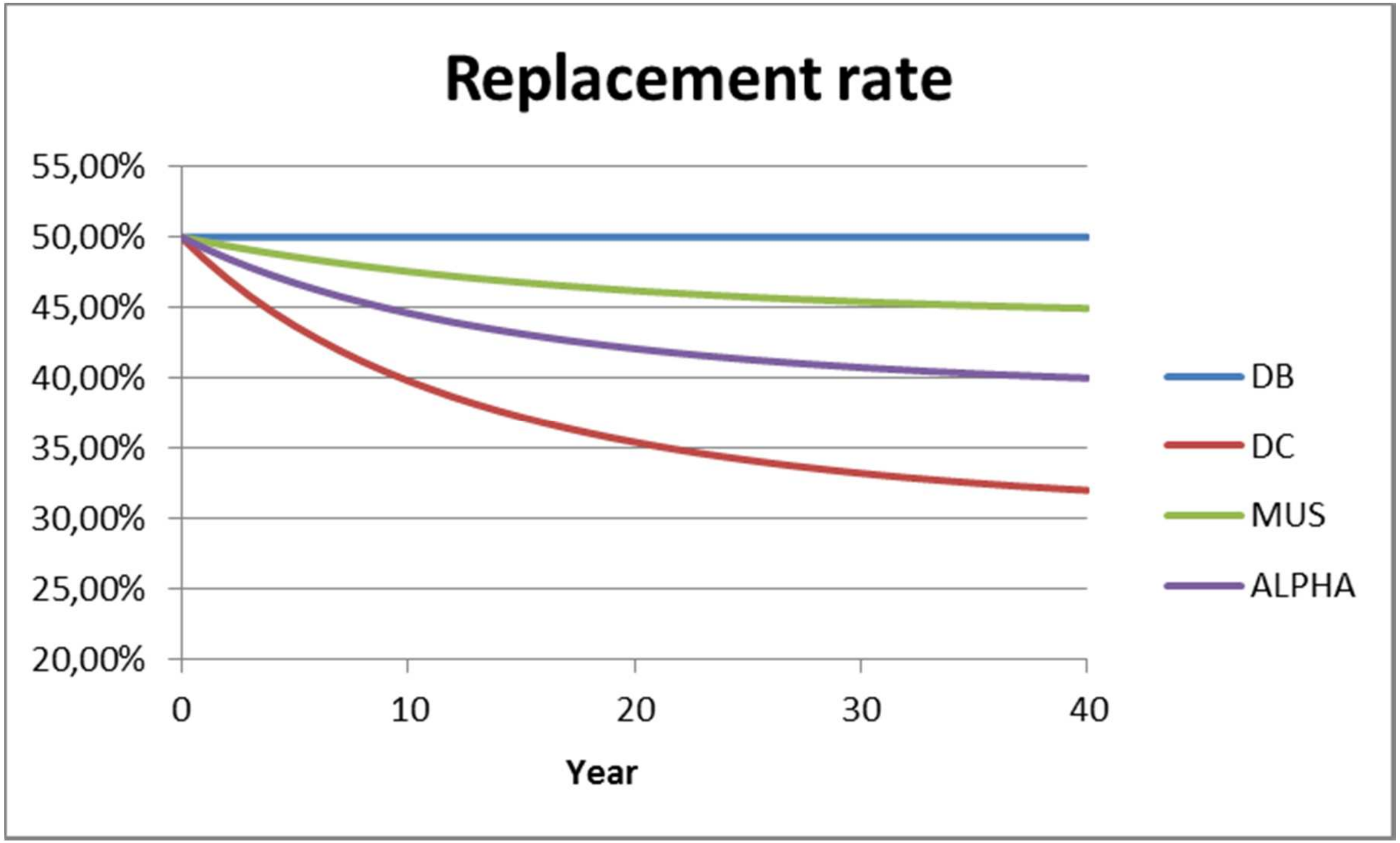
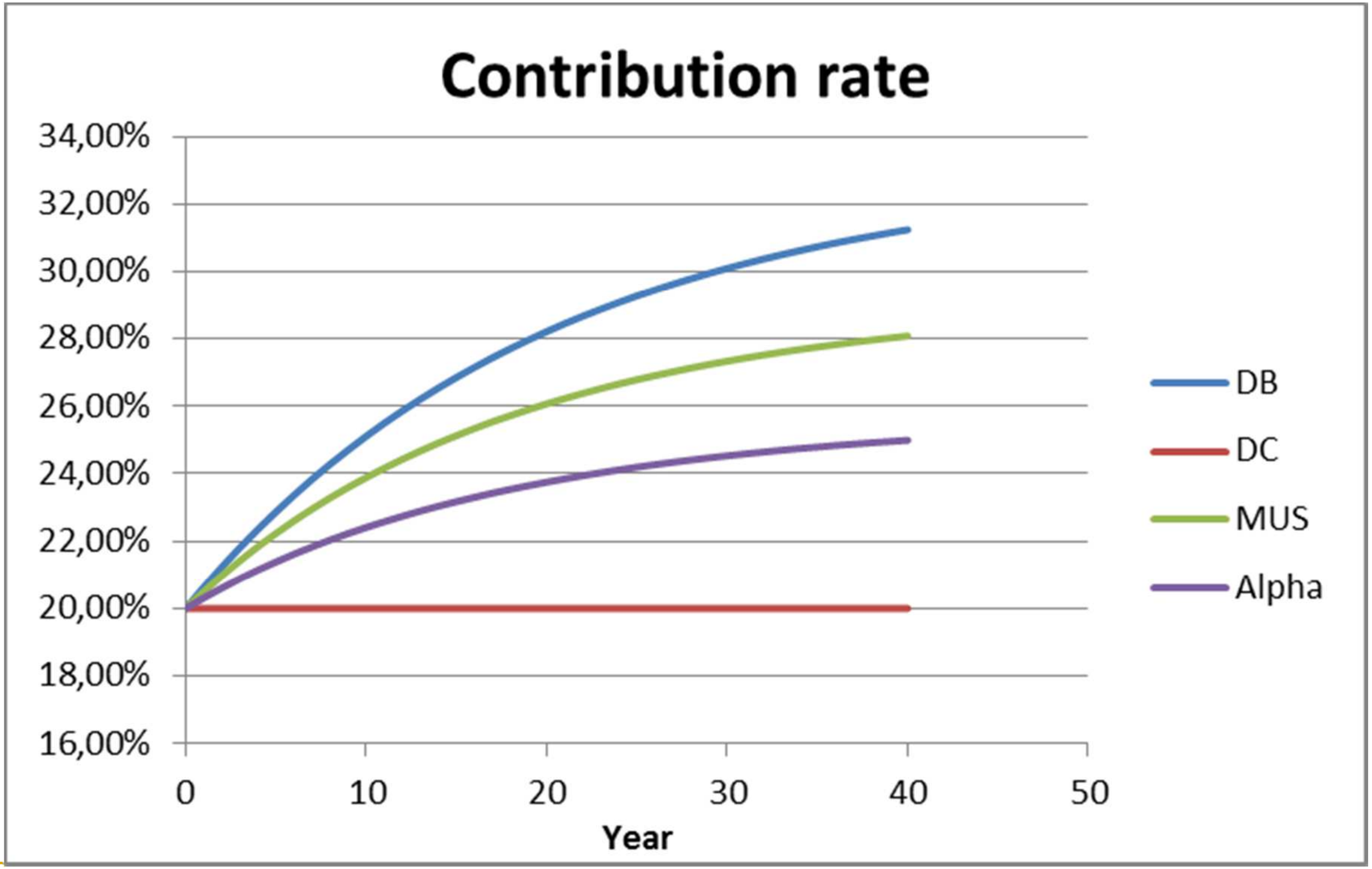
# Numerical illustration



# Numerical illustration



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# 4. Stochastic Model

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# Log normal model

**Geometric Brownian Motion for the replacement rate**

$$dD(t) = \gamma(t).D(t)\,dt + \sigma(t).D(t)\,dw(t)$$

With : $w(.) = \text{standard Brownian motion}$

$\gamma(.)$ and $\sigma(.) = \text{deterministic functions}$

**Adjustment :**

$$d\ln\pi(t) = (1-\alpha(t))\,d\ln D(t)$$

$$d\ln\delta(t) = -\alpha(t)d\ln D(t)$$

$$\alpha(.) = \text{adapted process}$$

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# Log normal model / constant proportion

## Geometric Brownian Motion for the replacement rate

$$dD(t) = \gamma.D(t)\,dt + \sigma.D(t)\,dw(t)$$

## Adjustment :

$$d\ln\pi(t) = (1-\alpha)\,d\ln D(t)$$

$$d\ln\delta(t) = -\alpha\;d\ln D(t)$$

## Solution : contribution and replacement = log normal

$$\pi(t) = \pi(0).\exp((1-\alpha).((\gamma-\sigma^2/2)t + \sigma.w(t)))$$

$$\delta(t) = \delta(0).\exp(-\alpha.((\gamma-\sigma^2/2)t + \sigma.w(t)))$$

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# Next steps

- Risk analysis / stochastic demography
  ( D = stochastic process )

- Optimal choice for the risk sharing parameter
  ( ? Optimal process α(t) ? )

- NDC with risk sharing

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# Questions

# Comments



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