Efficient Sensitivity Analysis via Scenario Weighting

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Motivation

Complex quantitative models

- Capital modelling and beyond
- Granularity v opaqueness
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Questions

- Which parts of the portfolio drive performance?
- Where do model-risk vulnerabilities lie?
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Questions

- Which parts of the portfolio drive performance?
- Where do model-risk vulnerabilities lie?

Sensitivity analysis

- Repeated model runs
- What to do with the results?
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Complex quantitative models

- Capital modelling and beyond
- Granularity v opaqueness

Questions

- Which parts of the portfolio drive performance?
- Where do model-risk vulnerabilities lie?

Sensitivity analysis

- Repeated model runs
- Single model run
- What to do with the results? Consistent sensitivity measurement
Example
A non-linear insurance portfolio

Portfolio consisting of

- Two lines of business
- Same multiplicative factor, e.g. inflation
- Reinsurance layer on the portfolio
- Reinsurance company can default

Sensitivity Analysis with Scenario Weights
A non-linear insurance portfolio

Portfolio consisting of

- Two lines of business
- Same multiplicative factor, e.g. inflation
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<table>
<thead>
<tr>
<th>Input risk factors</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$</td>
<td>Claims from 1st LoB</td>
</tr>
<tr>
<td>$X_2$</td>
<td>Claims from 2nd LoB</td>
</tr>
<tr>
<td>$X_3$</td>
<td>Multiplicative factor</td>
</tr>
<tr>
<td>$X_4$</td>
<td>% of RI recovery lost</td>
</tr>
</tbody>
</table>
Risk assessment of the portfolio loss

Sensitivity Analysis with Scenario Weights
1. Which risk factor is most important?
1. Which risk factor is most important?

2. Which is the most plausible model that gives a 5% higher portfolio VaR?
Distribution of portfolio loss

Sensitivity Analysis with Scenario Weights
Distribution of portfolio loss

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Sensitivity Analysis with Scenario Weights
How to chose a model stress?

Scenario weighting!
Scenario Weights
Constructing scenario weights

1. Define a **stress** on the output as an increase of VaR or/and TVaR

2. Derive **scenario weights** (change of measure) such that
   - re-weighted output fulfils the required stress
   - **most plausible / least distorting** (minimal entropy)
   - mathematically consistent
Constructing scenario weights

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▷ Typically we have a Monte Carlo sample and work with the empirical distribution.
Monte Carlo sample: \( Y = \{311, 330, 362, 422, 522\} \) with equal probability = \( \{\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}\} \), that is equal weights
Constructing scenario weights

Re-weighting subject to constraints (e.g. increase in VaR)
Constructing scenario weights

▷ do NOT change the data points: $Y = \{311, 330, 362, 422, 522\}$

▷ change height between points: scenario probabilities

$= \{0.25, 0.2, 0.1, 0.15, 0.3\}$, that is different weights
Scenario weights

Before re-weighting

▷ Every scenario has equal probability of occurring

After re-weighting

▷ data points stay the same
▷ we change the probability that a scenario occurs
▷ such that the constraints (e.g. increase in VaR) are fulfilled
▷ scenarios are re-weighted in the most plausible way
Before re-weighting

▷ Every scenario has equal probability of occurring

After re-weighting

▷ data points stay the same
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▷ such that the constraints (e.g. increase in VaR) are fulfilled
▷ scenarios are re-weighted in the most plausible way

An increase in VaR means that scenarios where portfolio loss is high are given more weight: they are now more likely to occur.
Scenario weights for a stress on VaR

Scenario probabilities = 0.92 * \( \frac{1}{10^6} \), for low \( Y \)
Scenario probabilities = 3.77 * \( \frac{1}{10^6} \), for high \( Y \)
Scenario weights for a stress on VaR and TVaR

10% increase in VaR, 13% increase in TVaR
Back to the example
Recall:

- $X_1, X_2$ are claims from different LoB
- $X_3$ is positive multiplicative factor
- $X_4$ is % of RI lost to default
Stress VaR by 10% and TVaR by 13%, at level 0.95
Which input factor is most important?
Which input factor is most important?

Weighting applies to simulated scenarios, including inputs!
Sensitivity Analysis with Scenario Weights
## Insurance portfolio

<table>
<thead>
<tr>
<th></th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_3$</th>
<th>$X_4$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>150</td>
<td>200</td>
<td>1.05</td>
<td>0.10</td>
<td>362</td>
</tr>
<tr>
<td><strong>Mean, stressed</strong></td>
<td>157</td>
<td>202</td>
<td>1.05</td>
<td>0.14</td>
<td>371</td>
</tr>
<tr>
<td><strong>Relative increase</strong></td>
<td>5%</td>
<td>1%</td>
<td>0%</td>
<td>44%</td>
<td>3%</td>
</tr>
<tr>
<td><strong>Standard deviation</strong></td>
<td>35</td>
<td>20</td>
<td>0.02</td>
<td>0.20</td>
<td>36</td>
</tr>
<tr>
<td><strong>Standard deviation, stressed</strong></td>
<td>43</td>
<td>21</td>
<td>0.02</td>
<td>0.26</td>
<td>50</td>
</tr>
<tr>
<td><strong>Relative increase</strong></td>
<td>25%</td>
<td>5%</td>
<td>1%</td>
<td>30%</td>
<td>38%</td>
</tr>
</tbody>
</table>
Stressing the inputs
Stressing the inputs

Stress input risk factor by a 10% increase of its VaR, at level 0.9.

<table>
<thead>
<tr>
<th>Stress on input</th>
<th>Change in output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
</tr>
<tr>
<td>1st LoB</td>
<td>1.3%</td>
</tr>
<tr>
<td>2nd LoB</td>
<td>1.2%</td>
</tr>
<tr>
<td>Multiplicative factor (3% VaR stress)</td>
<td>0.4%</td>
</tr>
<tr>
<td>Loss to RI default</td>
<td>0.1%</td>
</tr>
</tbody>
</table>
Sensitivity measures
Sensitivity measure

Sensitivity measure for input risk factor $X_i$

$$
\Gamma_i = \frac{E^{\text{stressed}}(X_i) - E(X_i)}{\text{normalised}}
$$

- depends on the output through the scenario weights.
Proprietary model of a London insurance market portfolio

\[ Y = \sum_{i=1}^{72} a_i X_i \]

with exposures \( a_1, \ldots, a_{72} \).

Facts

- 500,000 Monte Carlo simulations of input and output
- no knowledge about distributional assumptions
Proprietary model of a London insurance market portfolio

\[ Y = \sum_{i=1}^{72} a_i X_i \]

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Facts

- 500,000 Monte Carlo simulations of input and output
- no knowledge about distributional assumptions

**Stress:** increase VaR by 8% and TVaR by 10%, at level 0.95
Real-data example

Distribution of the portfolio loss (blue) and after re-weighting (red).
Real-data example

Sensitivity Analysis with Scenario Weights
Summary

Sensitivity Analysis with Scenario Weights:

1. Define a stress on the output
2. Calculate the scenario weights
3. Compare the distribution before and after re-weighting

Variations:

- Stressing output or inputs
- Different stresses: VaR, TVaR, mean, standard deviation, higher moments
- Decrease or increase of VaR, TVaR
Outlook and discussion

- Coming soon: the SWIM package in R
- Applicability and business benefits
- Academia ↔ industry feedback loop
- If you are interested in using our approach, let us know!
Thank you!
Appendix
Non-linear insurance portfolio

\[ Y = L - (1 - X_4) \min \{(L - d)_+, l\} \]
\[ L = X_3(X_1 + X_2), \]

where

- \( X_1, X_2 \) different lines of business
- \( X_3 \) positive multiplicative risk factor, e.g. inflation
- \( X_4 \) percentage lost due to default of the reinsurance company
- reinsurance limit \( l \) and deductible \( d \)
Assumptions:

- $X_1 \sim \text{(truncated) LogNormal}$ with mean 150 and sd 35.
- $X_2 \sim \text{Gamma}$ with mean 200 and sd 20.
- $X_3 \sim \text{(truncated) LogNormal}$ with mean 1.05 and sd 0.02.
- $X_4 \sim \text{Beta}$ with mean 0.1 and sd 0.2.
- $X_1, X_2, X_3$ are independent.
- $X_4$ dependent on $L$ through a Gaussian copula with correlation 0.6.
- $d = 380$, $l = 30$. 

Sensitivity Analysis with Scenario Weights