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Regression-Based Approaches in Solvency Capital Forecasting

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Agenda

Introduction

Review of Existing Research

One-Year Reserve Risk

- Overview
- Using Tweedie GLM Modelling To Project Reserve Risk
- Interpreting Results

One-Year Premium Risk

- Overview
- State Space Models - An Introduction
- Projecting Premium Risk Components

Other Risks

Takeaways and Conclusions

Q&A

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Introduction

Introduction

- Accuracy in projection/risk capital estimation key for Solvency II
- More dynamic approaches
- Challenges to solve
- Capturing complexity of underlying projection
- Approximating judgment
- Results might offer additional useful insights
- Focus on non-life underwriting (premium + reserve) risk

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Review of existing literature

- GLM-based reserve risk estimation tools available in R
- Markov chain and state space models discussed to understand purchase dynamics (e.g. Bozzetto)
- Machine Learning-based approaches tested to forecast yield curve (Sambasivan/Das)
- Counterparty default probability estimation through rate construction of credit default swaps using various techniques like neural networks, support vector machines (Brummelhuis/ Luo)

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One-Year Reserve Risk

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Overview

- Sufficiency of existing reserves to cover outstanding and incurred-but-not-paid claims
- Unfolding existing reserves into cash flows
- Assuming run-off
- Projections based on Chain-Ladder model
  - Mack CL
  - Bootstrap CL
- Extending method to GLMs
  - Tweedie Model

Development Factors

- Calculate cumulative % developed
- Assume development pattern will remain constant

Development Factors

Loss Origin	Ratio of Cumulative Paid/Ult at t	Ratio of Cumulative Paid/Ult at t+1	Incremental % Paid at t+1
1978	100.00%	100.00%	0.00%
1979	99.95%	100.00%	100.00%
1980	99.79%	99.95%	75.43%
1981	99.64%	99.79%	41.50%
1982	98.79%	99.64%	70.57%
1983	98.04%	98.79%	38.43%
1984	97.19%	98.04%	30.17%
1985	95.85%	97.19%	32.41%
1986	93.24%	95.85%	38.63%
1987	89.92%	93.24%	32.89%
1988	84.60%	89.92%	34.52%
1989	77.50%	84.60%	31.57%
1990	66.63%	77.50%	32.58%
1991	54.20%	66.63%	27.14%
1992	39.89%	54.20%	23.80%
1993	24.50%	39.89%	20.38%
1994	12.95%	24.50%	13.27%
1995	4.01%	12.95%	9.32%

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Mack CL

- Stochastic model that estimates standard error of reserve estimates using the chain ladder method
- Assumes constant conditional mean and variance of loss development factors, independent accident years
- Estimated age-to-age factors are unbiased and uncorrelated
- Linearly regressing on claims at development year t + 1 with claims at development year t

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Bootstrap CL

- Two-stage approach
- CL fitted to cumulative claims triangle, Pearson residuals calculated using incremental claims (q) and their expected value:

$$r_{w,d} = \frac{q_{w,d} - \mathrm{E}[q_{w,d}]}{\sqrt{q_{w,d}^{z}}}$$

where *w* and *d* are the accident and development years, and *z* specifies the error distribution

- Bootstrap residuals and generate bootstrapped triangle
- Fit CL to each bootstrapped triangle
- Results provide full predictive distribution of reserves

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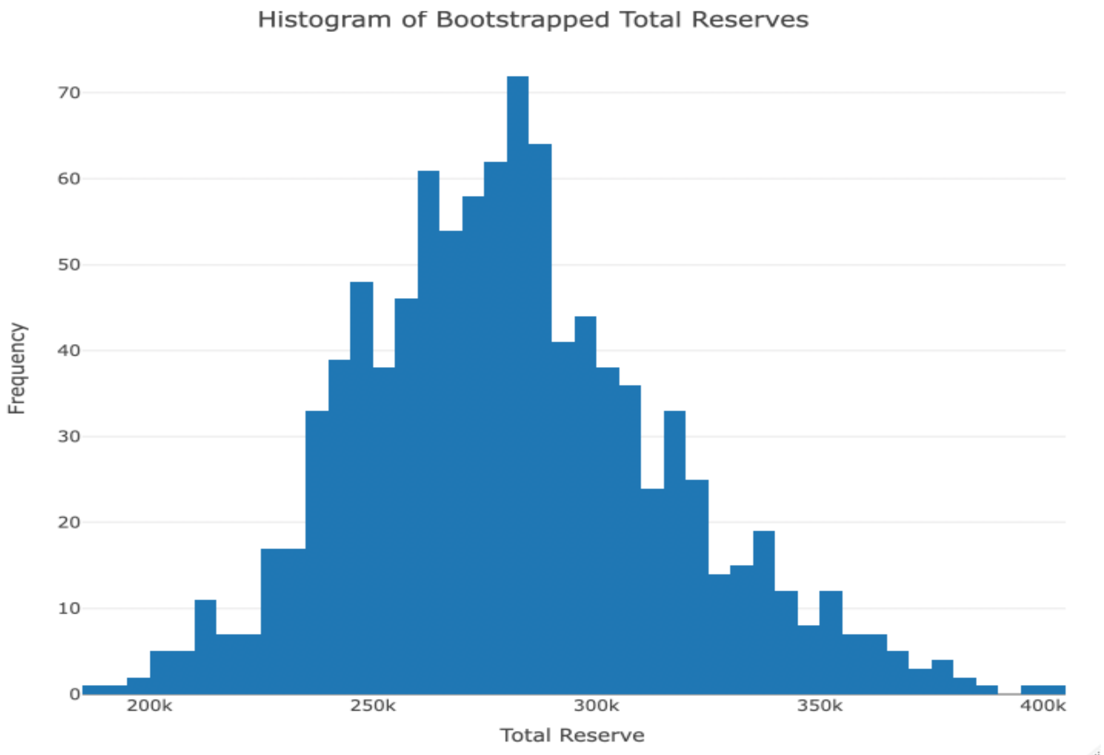
Bootstrap CL



Tweedie GLMs - Overview

- Tweedie distribution: Exponential dispersion models ($\sim\mu, \sigma^2$) where:
  - Mean = μ
  - Variance = $\mu^p\sigma^2$
  - *p* is called the power parameter of the distribution, σ^2 is the dispersion parameter
- Why is the Tweedie distribution useful for GLMs?
  - Wide variety of distribution families
  - Therefore extremely flexible

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Tweedie GLMs - The Process

- Tweedie model fit to incremental claims
- Regression structure used: *value ~ factor(AY) + factor(DY)*
- Predictions generate reserve as at current valuation
- Input values bootstrapped, next diagonal extrapolated using this
- Tweedie model is refit on new values with same regression structure to generate expected reserve at the next valuation

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Let’s look at some results!!

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One-Year Premium Risk

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Overview

- One-year premium risk view
  - Evolution of portfolio
- State-space models
- Testing
- Using the results

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What Makes Up Premium Risk

- Unearned premium
- Renewals
- Future Premiums
- Resulting adequacy for future claims

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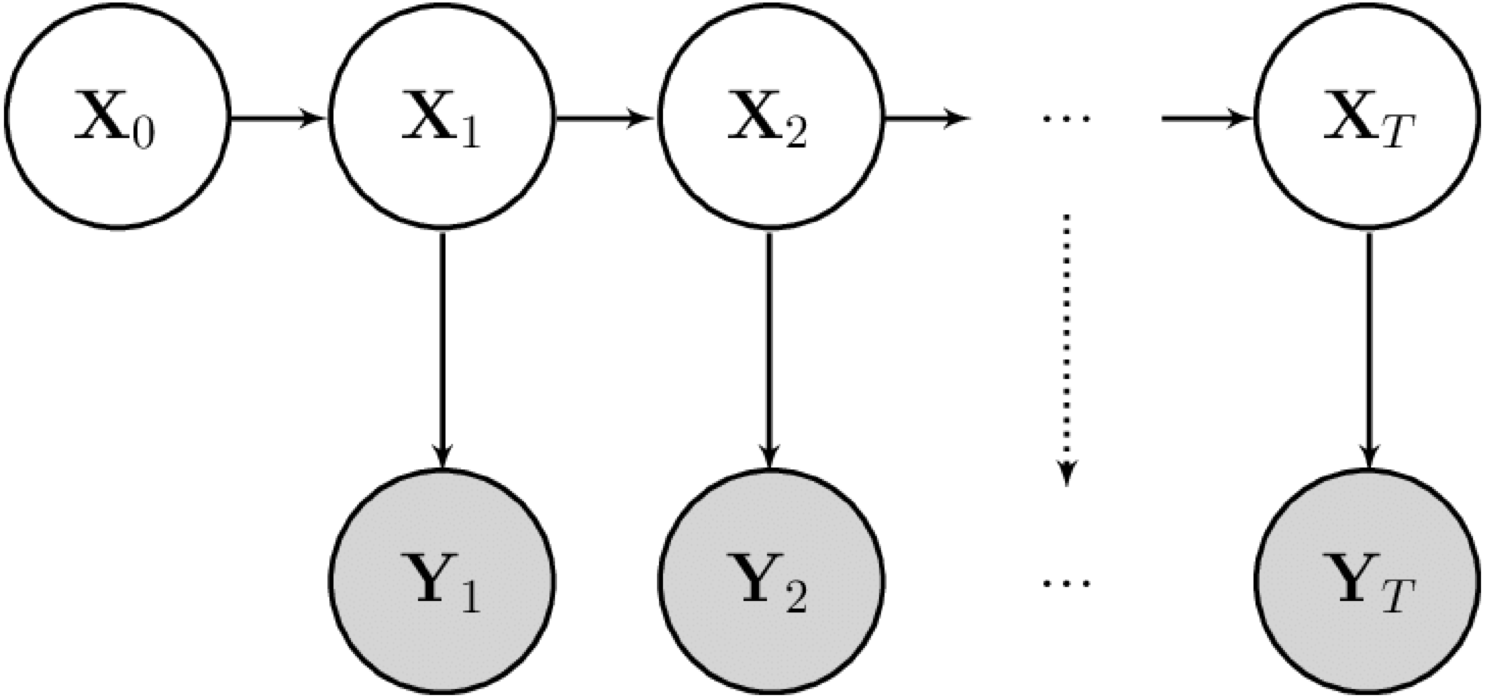
State Space Models - Overview and Uses



Warning - Too many equations ahead!

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State Space Models - Overview

- A time series with state and observation equations

  $y_t = ax_t + e_t$ $x_t = bx_{t-1} + e'_{t-1}$

- Can be extended to multiple input variables
  - $\mathbf{X}_t = [x_{1,t}, x_{2,t}, x_{3,t}, \ldots x_{m,t}]$
  - So $\mathbf{X}_t = B\mathbf{X}_{t-1} + \varepsilon_{t-1}$
    - Where B is a transition matrix, and ε is an error matrix

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State Space Models - Overview

$$\dot{\mathbf{q}} = \mathbf{Aq} + \mathbf{B}u \qquad \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_4 & -a_3 & -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ b_0 \end{bmatrix} x$$

$$y = \mathbf{Cq} + Du \qquad y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

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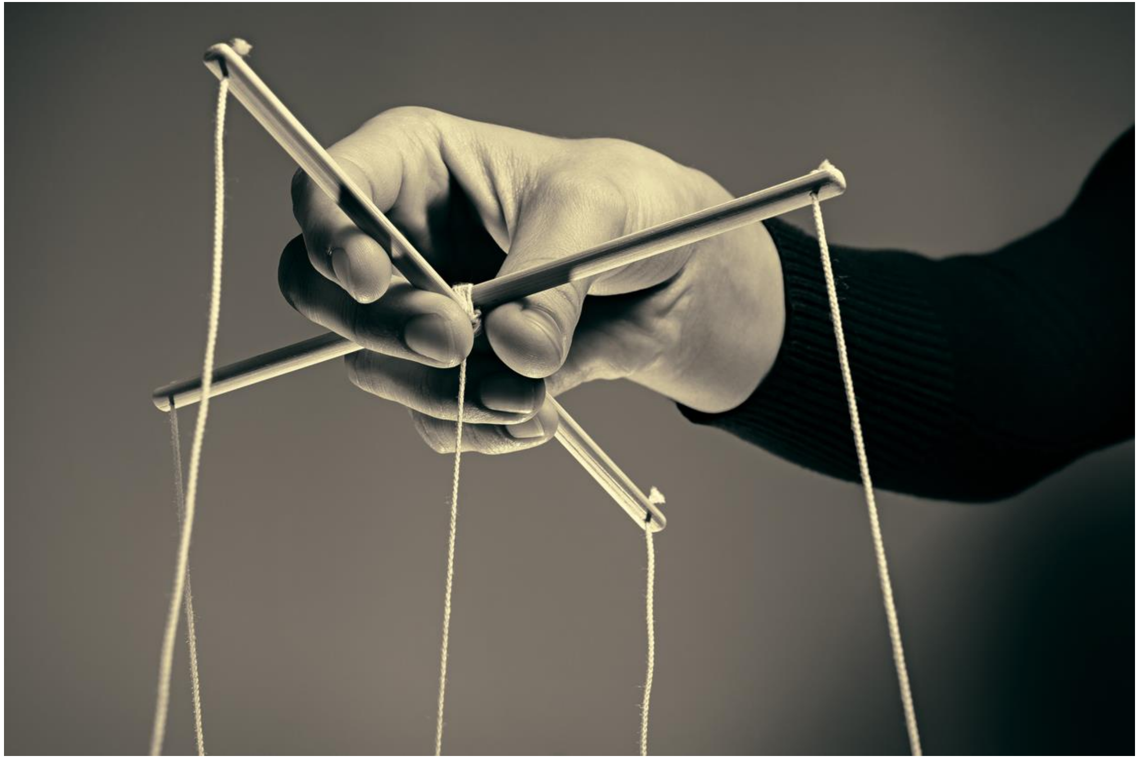
State Space Models - Overview



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State Space Models - Overview and Uses

- Dynamic approach to longer-term problem
- Capturing underlying evolution
- Can predict multiple steps ahead

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Using SSMs for Premium Projection

- Multivariate SSMs
- Factors involved (nature of portfolio)
- Change of demographic figures in the next year, and coming years
- Estimating possible future premium growth
  - Claim growth as well

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Using SSMs for Premium Projection

1. Time-varying variables taken as inputs
2. Transition matrices obtained from inputs
3. Dependent variable estimated at t
4. Dependent variable projected

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Data

- Anonymous Australian insurer
- Quarterly data over 7 years
- Premiums, claims, reinsurance recoveries
- Demographic information
  - Age
  - Gender

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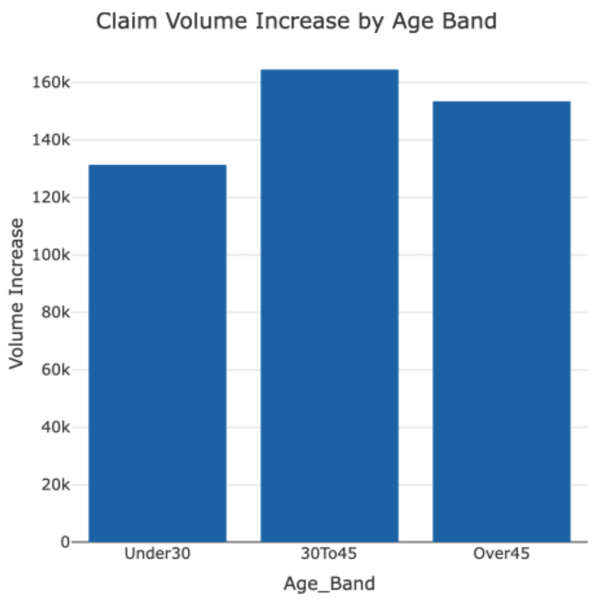
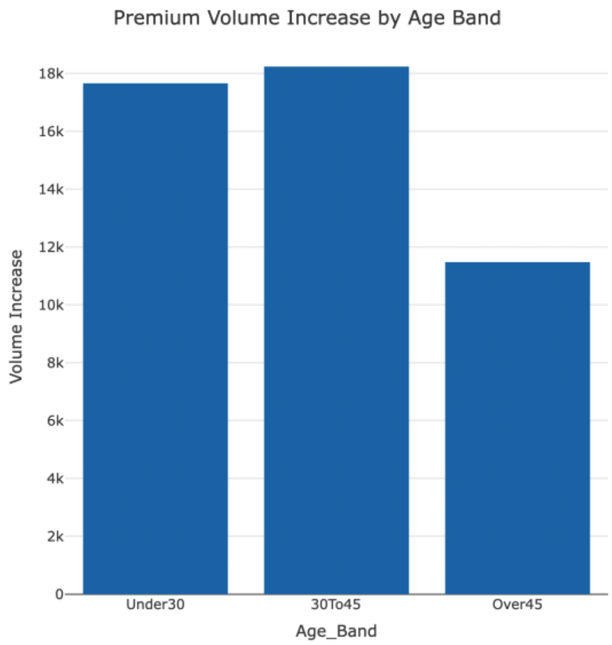
And now for some results

Results





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Results - Interpretation and Other Uses

- Obtain projection at 99.5% (1 in 200)
- Find possible sources of future premium risk

- Alternatives?
  - LSTM neural networks
  - Deep Markov models
  - Stochastic RNNs

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Other Risks Considered

- Counterparty default
  - Rate estimation from Credit Default Swaps
  - LDA to classify solvency/insolvency
  - LGD estimation
  - XGBoost proven effective tool
- Market risk
  - IR Risk: Fitting LSTMs to the yield curve

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Takeaways and Conclusions

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Takeaways and Conclusions

- Regression-based approaches:
  - Accurately capture patterns
  - Have potential for use in longer-term forecasts
  - Can offer valuable business insights

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Questions

Comments

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