Regression-Based Approaches in Solvency Capital Forecasting

Karthik Tumuluru
MetLife
Dubai, UAE
karthik.tumuluru@metlife.ae

Navarun Jain
Lux Actuaries & Consultants
Dubai, UAE
navarun.jain@luxactuaries.com
Agenda

Introduction

Review of Existing Research

One-Year Reserve Risk
  • Overview
  • Using Tweedie GLM Modelling To Project Reserve Risk
  • Interpreting Results

One-Year Premium Risk
  • Overview
  • State Space Models - An Introduction
  • Projecting Premium Risk Components

Other Risks

Takeaways and Conclusions

Q&A
Introduction
Introduction

• Accuracy in projection/risk capital estimation key for Solvency II
• More dynamic approaches
• Challenges to solve
• Capturing complexity of underlying projection
• Approximating judgment
• Results might offer additional useful insights
• Focus on non-life underwriting (premium + reserve) risk
Review of existing literature

- GLM-based reserve risk estimation tools available in R
- Markov chain and state space models discussed to understand purchase dynamics (e.g. Bozzetto)
- Machine Learning-based approaches tested to forecast yield curve (Sambasivan/Das)
- Counterparty default probability estimation through rate construction of credit default swaps using various techniques like neural networks, support vector machines (Brummelhuis/ Luo)
One-Year Reserve Risk
Overview

• Sufficiency of existing reserves to cover outstanding and incurred-but-not-paid claims
• Unfolding existing reserves into cash flows
• Assuming run-off
• Projections based on Chain-Ladder model
  – Mack CL
  – Bootstrap CL
• Extending method to GLMs
  – Tweedie Model
Development Factors

- Calculate cumulative % developed
- Assume development pattern will remain constant
# Development Factors

<table>
<thead>
<tr>
<th>Loss Origin</th>
<th>Ratio of Cumulative Paid/Ult at t</th>
<th>Ratio of Cumulative Paid/Ult at t+1</th>
<th>Incremental % Paid at t+1</th>
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<tbody>
<tr>
<td>1978</td>
<td>100.00%</td>
<td>100.00%</td>
<td>0.00%</td>
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<tr>
<td>1979</td>
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<tr>
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<td>99.79%</td>
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<tr>
<td>1981</td>
<td>99.64%</td>
<td>99.79%</td>
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<tr>
<td>1982</td>
<td>98.79%</td>
<td>99.64%</td>
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<td>98.79%</td>
<td>38.43%</td>
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<td>1984</td>
<td>97.19%</td>
<td>98.04%</td>
<td>30.17%</td>
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<td>1985</td>
<td>95.85%</td>
<td>97.19%</td>
<td>32.41%</td>
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<tr>
<td>1986</td>
<td>93.24%</td>
<td>95.85%</td>
<td>38.63%</td>
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<tr>
<td>1987</td>
<td>89.92%</td>
<td>93.24%</td>
<td>32.89%</td>
</tr>
<tr>
<td>1988</td>
<td>84.60%</td>
<td>89.92%</td>
<td>34.52%</td>
</tr>
<tr>
<td>1989</td>
<td>77.50%</td>
<td>84.60%</td>
<td>31.57%</td>
</tr>
<tr>
<td>1990</td>
<td>66.63%</td>
<td>77.50%</td>
<td>32.58%</td>
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<tr>
<td>1991</td>
<td>54.20%</td>
<td>66.63%</td>
<td>27.14%</td>
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<tr>
<td>1992</td>
<td>39.89%</td>
<td>54.20%</td>
<td>23.80%</td>
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<tr>
<td>1993</td>
<td>24.50%</td>
<td>39.89%</td>
<td>20.38%</td>
</tr>
<tr>
<td>1994</td>
<td>12.95%</td>
<td>24.50%</td>
<td>13.27%</td>
</tr>
<tr>
<td>1995</td>
<td>4.01%</td>
<td>12.95%</td>
<td>9.32%</td>
</tr>
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</table>
Mack CL

- Stochastic model that estimates standard error of reserve estimates using the chain ladder method
- Assumes constant conditional mean and variance of loss development factors, independent accident years
- Estimated age-to-age factors are unbiased and uncorrelated
- Linearly regressing on claims at development year $t + 1$ with claims at development year $t$
Bootstrap CL

- Two-stage approach
- CL fitted to cumulative claims triangle, Pearson residuals calculated using incremental claims \((q)\) and their expected value:

\[
\tau_{w,d} = \frac{q_{w,d} - \mathbb{E}[q_{w,d}]}{\sqrt{q_{w,d}^z}}
\]

where \(w\) and \(d\) are the accident and development years, and \(z\) specifies the error distribution

- Bootstrap residuals and generate bootstrapped triangle
- Fit CL to each bootstrapped triangle
- Results provide full predictive distribution of reserves

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Histogram of Bootstrapped Total Reserves

Total Reserve

Frequency

Bootstrap CL
Tweedie GLMs - Overview

- Tweedie distribution: Exponential dispersion models ($\sim \mu, \sigma^2$) where:
  - Mean = $\mu$
  - Variance = $\mu^p\sigma^2$
  - $p$ is called the power parameter of the distribution, $\sigma^2$ is the dispersion parameter

- Why is the Tweedie distribution useful for GLMs?
  - Wide variety of distribution families
  - Therefore extremely flexible
Tweedie GLMs - The Process

- Tweedie model fit to incremental claims
- Regression structure used: \( \text{value} \sim \text{factor}(AY) + \text{factor}(DY) \)
- Predictions generate reserve as at current valuation
- Input values bootstrapped, next diagonal extrapolated using this
- Tweedie model is refit on new values with same regression structure to generate expected reserve at the next valuation
Let’s look at some results!!
One-Year Premium Risk
Overview

• One-year premium risk view
  – Evolution of portfolio
• State-space models
• Testing
• Using the results
What Makes Up Premium Risk

- Unearned premium
- Renewals
- Future Premiums
- Resulting adequacy for future claims
State Space Models - Overview and Uses

\[ X_0 \rightarrow X_1 \rightarrow X_2 \rightarrow \ldots \rightarrow X_T \]
\[ Y_1 \rightarrow Y_2 \rightarrow \ldots \rightarrow Y_T \]
Warning - Too many equations ahead!
State Space Models - Overview

• A time series with state and observation equations
  \[ y_t = ax_t + e_t \]
  \[ x_t = bx_{t-1} + e'_{t-1} \]

• Can be extended to multiple input variables
  \[ X_t = [x_{1,t}, x_{2,t}, x_{3,t}, \ldots, x_{m,t}] \]
  \[ \text{So } X_t = BX_{t-1} + \varepsilon_{t-1} \]
  • Where B is a transition matrix, and \( \varepsilon \) is an error matrix
State Space Models - Overview

\[ \begin{align*}
\dot{q} &= Aq + Bu \\
y &= Cq + Du
\end{align*} \]
State Space Models - Overview
State Space Models - Overview and Uses

- Dynamic approach to longer-term problem
- Capturing underlying evolution
- Can predict multiple steps ahead
Using SSMs for Premium Projection

• Multivariate SSMs
• Factors involved (nature of portfolio)
• Change of demographic figures in the next year, and coming years
• Estimating possible future premium growth
  – Claim growth as well
Using SSMs for Premium Projection

1. Time-varying variables taken as inputs
2. Transition matrices obtained from inputs
3. Dependent variable estimated at $t$
4. Dependent variable projected
Data

- Anonymous Australian insurer
- Quarterly data over 7 years
- Premiums, claims, reinsurance recoveries
- Demographic information
  - Age
  - Gender
And now for some results
Results

Claim Volume Increase by Age Band

<table>
<thead>
<tr>
<th>Age_Band</th>
<th>Volume Increase</th>
</tr>
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<tbody>
<tr>
<td>Under30</td>
<td>0</td>
</tr>
<tr>
<td>30To45</td>
<td>160k</td>
</tr>
<tr>
<td>Over45</td>
<td>140k</td>
</tr>
</tbody>
</table>

Premium Volume Increase by Age Band

<table>
<thead>
<tr>
<th>Age_Band</th>
<th>Volume Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under30</td>
<td>18k</td>
</tr>
<tr>
<td>30To45</td>
<td>16k</td>
</tr>
<tr>
<td>Over45</td>
<td>14k</td>
</tr>
</tbody>
</table>

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Results - Interpretation and Other Uses

• Obtain projection at 99.5% (1 in 200)
• Find possible sources of future premium risk

• Alternatives?
  – LSTM neural networks
  – Deep Markov models
  – Stochastic RNNs
Other Risks Considered

• Counterparty default
  – Rate estimation from Credit Default Swaps
  – LDA to classify solvency/insolvency
  – LGD estimation
  – XGBoost proven effective tool

• Market risk
  – IR Risk: Fitting LSTMs to the yield curve

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Takeaways and Conclusions
Takeaways and Conclusions

• Regression-based approaches:
  – Accurately capture patterns
  – Have potential for use in longer-term forecasts
  – Can offer valuable business insights
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