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How many people does it take to run a tontine?

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What is a tontine/pooled annuity fund?

- A tontine is a structure to pool longevity risk.
- A pure tontine has no guarantees – the pool of people bear the longevity risk.
- The purpose of modern tontines is to pay an income for life.
- Also called pooled annuity funds.

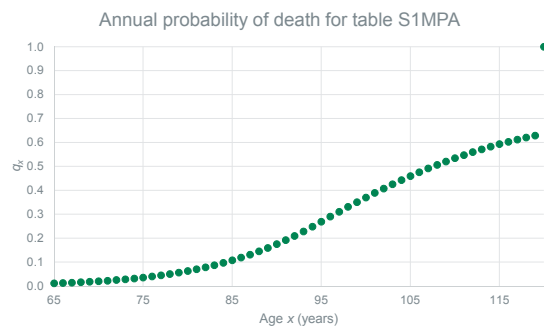


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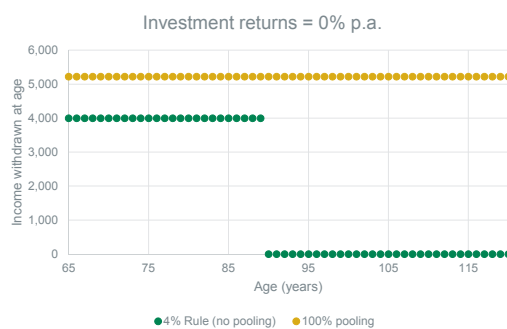


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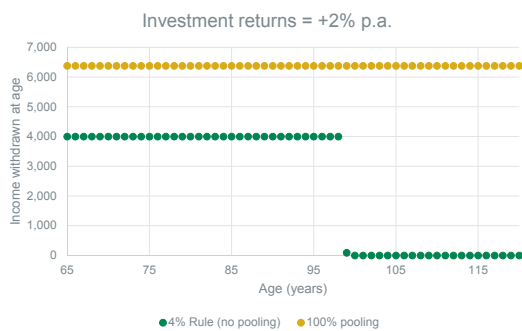
UK mortality table S1PMA



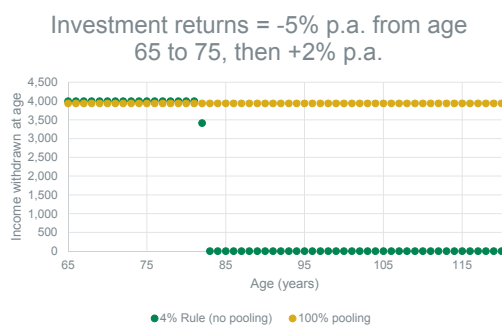
Example: 0% investment returns



Example: +2% p.a. investment returns



Example: Investment returns are -5% p.a. from age 65 to 75, then +2% p.a.



How are tontines related to conventional life annuities?

In a world with

- Certain investment returns,
- Known distribution of future lifetimes,
- No costs, fees, taxation differences, etc,

Then a sufficiently pooled tontine gives the same income as a life annuity.

But we don't live in such a world...



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Modern tontines

- Aim: retirement income, not a life-death gamble.
- Various tontines structures have been proposed, either
 - Explicit tontine: longevity credits and investment returns are explicitly credited to individual members' individual accounts. Income withdrawals can be tailored to the individual's needs.
 - Implicit tontine: longevity credits and investment returns are pooled. The members need only know the income they are getting.

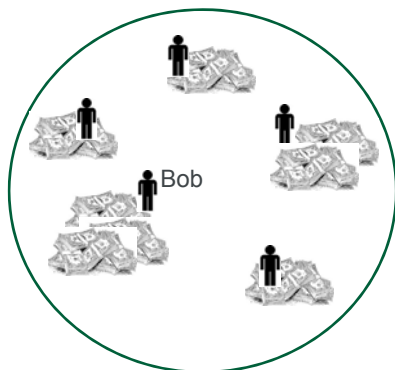


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Longevity risk pooling – an explicit tontine

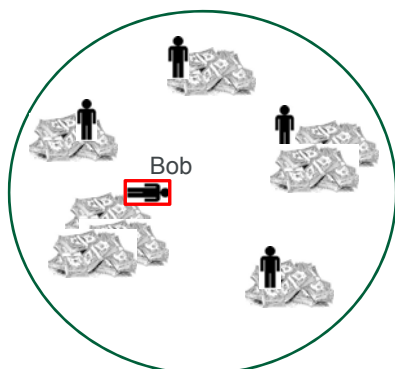


Pool risk over lifetime

Individuals withdraw income from their own funds

However, when someone (Bob) dies at time T ...

Longevity risk pooling – an explicit tontine



Share out remaining funds of Bob.

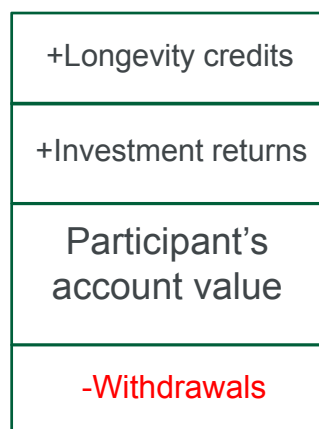
Longevity risk pooling rule [DGN]

- $\lambda^{(i)}$ = Force of mortality of i^{th} member at time T .
- $W^{(i)}$ = Fund value of i^{th} member at time T .
- Payment (longevity credit) to i^{th} member:

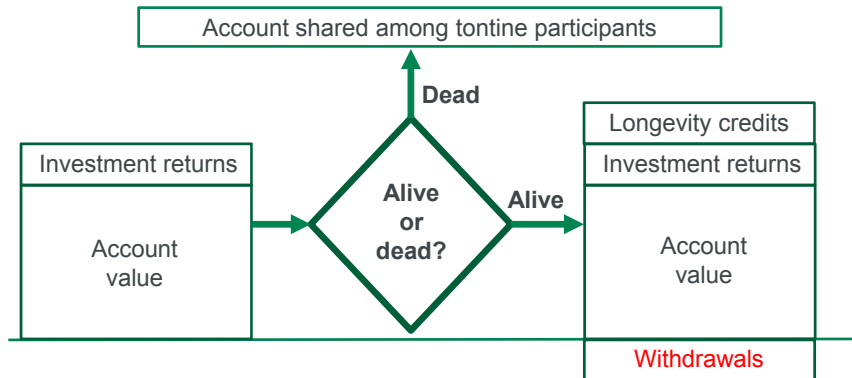
$$\frac{\lambda^{(i)} \times W^{(i)}}{\sum_{k \in \text{Group}} \lambda^{(k)} \times W^{(k)}} \times \{\text{Bob's remaining fund value}\}$$



Pure modern tontine – individual account structure



Pure modern tontine



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How many people in a tontine?

- How many people is enough to pool longevity risk?
- Define a measure of income stability.
- Set up
 - There are N people in the tontine with the same: account values and age x at time 0 and attitude to risk.
 - All future lifetimes are independent and identically distributed random variables.
 - Tontine members withdraw an annual income from their own individual accounts.
 - Want to keep annual income withdrawals stable.



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How many people in a tontine?

- Tontine member withdraws $I(0)$ from their individual account at time 0 for the first year.
- At time $n=1,2,\dots$, they withdraw $I(n)$.
- Determine the maximum time T at which

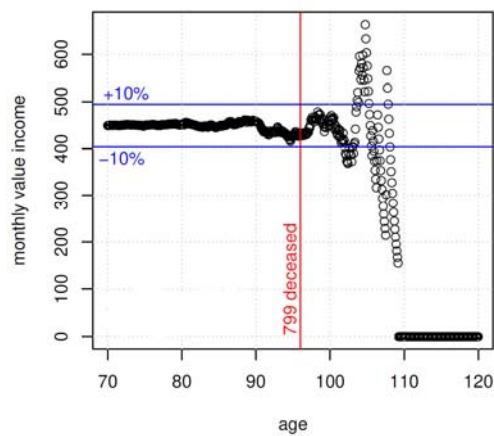
$$\mathbb{P} \left[\frac{I(n)}{I(0)} \geq 0.95 \text{ for all } n \leq T \right] \geq 0.90$$

i.e. there is at least a 90% chance that each withdrawal up to time T is at least 95% of $C(0)$.



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How many people in a tontine?



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How many people in a tontine?

- Individual account value F evolves as

$$F(n+1) = (F(n) - I(n)) \times (1+R) + LC(n+1)$$

Annual constant return over (n-1,n) ↓
Withdrawal ↑ Longevity credit ↑

in which the longevity credit is calculated as

$$LC(n+1) = \frac{(F(n) - I(n)) \times (1+R) \times \text{Number of deaths over } (n, n+1)}{\text{Number left alive at time } n+1}$$



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How many people in a tontine?

- Withdrawal at time n is

$$I(n) = \frac{F(n)}{\ddot{a}_{x+n}}$$

in which

$$\ddot{a}_{x+n} = 1 + \sum_{k=1}^{\infty} (1+R)^{-k} {}_k p_{x+n}$$

- In this set-up,

$$\frac{I(n)}{I(0)} = \frac{{}_n p_x}{{}_n \tilde{p}_x}$$

in which $\{ {}_n \tilde{p}_x; n = 0, 1, 2, \dots \}$ are the empirical probabilities of survival in the tontine group.



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How many people in a tontine?

Recap:

- N members at time 0, all independent copies of each other.
- Income withdrawal $I(n) = \frac{F(n)}{\ddot{a}_{x+n}}$ at time $n=0,1,2,\dots$
- Assume that ${}_n\tilde{p}_x$ and ${}_np_x$ arise from the same distribution.
- How effective is a tontine at keeping $I(n)$ close to $I(0)$?



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How many people in a tontine?

- How effective is a tontine at keeping $I(n)$ close to $I(0)$?
- Re-express

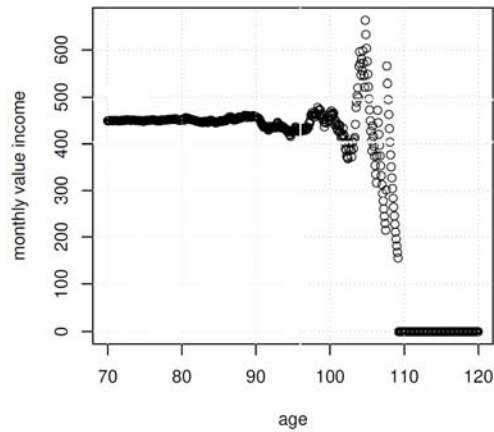
$$\mathbb{P} \left[\frac{I(n)}{I(0)} \geq 1 - \epsilon, \text{ for all } n \leq T \right] \geq 0.90$$

in a form independent of the underlying mortality distribution
(after doing some mathematics...).

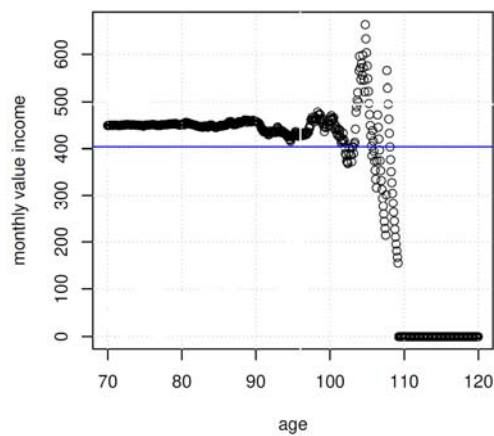


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How many people in a tontine?



How many people in a tontine?



Lower bound is
either 5% or 10%
below initial
income level

Percentage of membership who get a lifelong income with 90% certainty

Number of members in the tontine at time 0 N	When withdrawals are $\geq 95\%$ of initial withdrawal $\epsilon = 0.05$	When withdrawals are $\geq 90\%$ of initial withdrawal $\epsilon = 0.1$
100	6%	25%
500	31%	66%
1,000	48%	80%
5,000	83%	95%
10,000	91%	98%



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Percentage of membership who get a lifelong income with 90% certainty (in years starting from age 65 using S1FPL)

Number of members in the tontine at time 0 N	When withdrawals are $\geq 95\%$ of initial withdrawal $\epsilon = 0.05$	When withdrawals are $\geq 90\%$ of initial withdrawal $\epsilon = 0.1$
100	6% (5.1 years)	25% (13.3 years)
500	31% (15.0 years)	66% (23.2 years)
1,000	48% (19.2 years)	80% (26.5 years)
5,000	83% (27.4 years)	95% (31.9 years)
10,000	91% (30.0 years)	98% (34.4 years)



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Percentage of membership who get a lifelong income with 90% certainty (in years starting from age 65 using S1MPL)

Number of members in the tontine at time 0 N	When withdrawals are $\geq 95\%$ of initial withdrawal $\epsilon = 0.05$	When withdrawals are $\geq 90\%$ of initial withdrawal $\epsilon = 0.1$
100	6% (3.4 years)	25% (10.1 years)
500	31% (11.6 years)	66% (19.6 years)
1,000	48% (15.6 years)	80% (23.1 years)
5,000	83% (23.9 years)	95% (28.9 years)
10,000	91% (26.8 years)	98% (31.5 years)



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Percentage of membership who get a lifelong income with 90% certainty **99% certainty**

Number of members in the tontine at time 0 N	When withdrawals are $\geq 95\%$ of initial withdrawal $\epsilon = 0.05$	When withdrawals are $\geq 90\%$ of initial withdrawal $\epsilon = 0.1$
100	6% 1%	25% 9%
500	31% 14%	66% 43%
1,000	48% 26%	80% 61%
5,000	83% 66%	95% 89%
10,000	91% 80%	98% 94%



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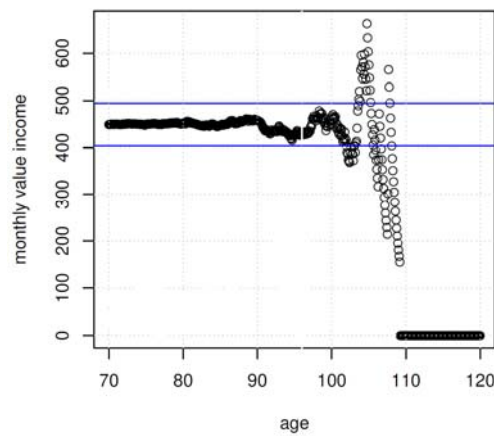
Years to get a lifelong income with 90% certainty **99% certainty** starting from age 65 using S1FPL

Number of members in the tontine at time 0 N	When withdrawals are $\geq 95\%$ of initial withdrawal $\epsilon = 0.05$	When withdrawals are $\geq 90\%$ of initial withdrawal $\epsilon = 0.1$
100	5.1 years 1.1 years	13.3 years 6.9 years
500	15.0 years 9.3 years	23.2 years 18.0 years
1,000	19.2 years 13.6 years	26.5 years 22.1 years
5,000	27.4 years 23.2 years	31.9 years 29.2 years
10,000	30.0 years 26.5 years	34.4 years 31.4 years



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How many people in a tontine?



Bounds are
either 5% or 10%
around initial
income level



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Percentage of membership who get a lifelong income with 90% certainty (change from lower bound only)

Number of members in the tontine at time 0 N	When withdrawals are $\pm 5\%$ of initial withdrawal $\epsilon = 0.05$	When withdrawals are $\pm 10\%$ of initial withdrawal $\epsilon = 0.1$
100	6% (+0%)	21% (-4%)
500	25% (-6%)	57% (-9%)
1,000	39% (-9%)	73% (-7%)
5,000	77% (-6%)	93% (-2%)
10,000	87% (-4%)	97% (-1%)



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Summary

- Analysed ability of tontine/pooled annuity fund to provide an income for life.
- Developed a closed-form expression (not shown here) to determine how many members get a lifetime income of a bounded variation.
- Longest-lived members get a lifetime income, but with greater variation.
- Equivalent number of years can be read off from any life table.



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Summary

Results assumed:

- Everyone independent and identical copies of each other,
- No uncertainty about the underlying mortality distribution,
- Investment returns are constant.

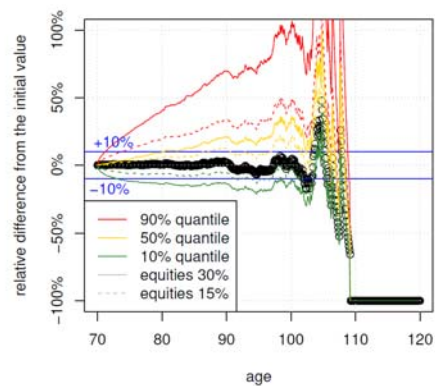


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Extensions



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Questions



Comments

The views expressed in this presentation are those of the presenter.



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The '**Minimising Longevity and Investment Risk while Optimising Future Pension Plans**' research programme is being funded by the ARC.

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