Institute and Faculty of Actuaries

CHANGES TO THE SYLLABUS AND CORE READING FOR SUBJECT ST5 FOR THE 2018 EXAMINATIONS

Changes to the Syllabus and their impact on Core Reading

There have been no changes to the Syllabus.

Changes to Core Reading

UNIT 1

The detail has been updated and a revised Unit is attached.

UNIT 6

Amendments have been made to this Unit and a revised Unit is attached.

Attachments: Unit 1 and 6

UNIT 1 – INTRODUCTION TO FINANCE AND INVESTMENT SPECIALIST TECHNICAL A

1 Structure of professional examinations

Earlier parts of the actuarial qualification have covered accounting, discount rates, economics and financial economics in the CT subjects. CA1 – Actuarial Risk Management brings all the core technical skills together and shows how actuaries use them in risk management, which is the fundamental skill of an actuary. All of these form an important background to this subject and you are encouraged to refresh your knowledge of them before embarking on the study of this subject. Examination questions may include sections based on this prior knowledge in the formation of an answer.

2 Scope of ST5

The course is broken down into a number of component parts. Having learned in earlier courses about the use of risk free rates in discounting income streams, this course focuses on how these returns can be achieved and what other returns may be available dependent upon the level of risk that one is prepared to accept. It looks into the factors that affect the returns including political and regulatory aspects. It gives an appreciation of the different forms that investments may take, their key characteristics, how they should be assessed, including analysis of securities, and how they can be used to build portfolios for different investment mandates. Evaluating securitisations and credit derivatives now receives more emphasis. A key outcome from the course is the ability to develop appropriate portfolios for different types of investor with individual financial planning being as important as that for institutional mandates.

Risk management is a feature of the course emphasising both actuarial and investment aspects. Liquidity risk, counterparty risk, operational risk and credit risk also feature. In the wake of the 2007–08 financial crisis these have taken on more significance than has previously been the case. Value at risk and risk budgets are important aspects of the actuarial approach as are asset/liability modelling and liability hedging.

A key component of this course is performance measurement. This covers not only the analysis of returns for securities and portfolios but also performance attribution. There is a strong emphasis on risk adjusted returns and on the interpretation of the results from such analysis. Benchmarking and the construction and use of indices form important parts of this section.

Although not covered by many guidance notes or actuarial standards, it is important to understand the regulatory and legislative environment which applies to the investment management and securities industry. It is key that actuaries understand this because more and more we will find ourselves working with other professions. In particular, although it only applies to actuaries working in the UK, or for UK regulated entities, the Financial Reporting Council’s high level standard, TAS 100 – Principles for Actuarial Work (which replaces the Generic Technical Actuarial Standards TAS R, TAS D, and TAS M from July

2017) is a good foundation for all actuarial work and are covered in the CA subjects, particularly CA2.

Behavioural finance has become important in assessing investment outcomes. Although in most cases it cannot be quantified it is an input that has to be understood in evaluating market activity or, for example, the actions of trustees. It tends to explain why an alternative outcome happens to the one that looks most sensible in valuation terms.

E N D

UNIT 6 – VALUATION OF INVESTMENTS

Syllabus objectives

(e) Apply appropriate methods for the valuation of individual investments and demonstrate an understanding of their appropriateness in different situations.

- fixed income analytics and valuation (including interest rate swaps and futures)
- arbitrage pricing and the concept of hedging
- empirical characteristics of asset prices
- introduction into fixed income option pricing
- evaluating a securitisation (including CBO's and MBS's)
- evaluation of a credit derivative

Students will be expected to have prior knowledge from CT1 and CT2. In particular, they will be expected to be able to understand and manipulate accounting information as covered in subject CT2.

1 Fixed income analytics and valuation

The use of financial mathematics for the analysis and valuation of fixed income securities will be familiar. We now need to consider how these techniques may be extended to the valuation of more complex instruments.

The "clean" price of bond per 100 face value is given by:

$$\text{Price} = \left[\frac{\text{redemption}}{\left(1+\frac{\text{yld}}{\text{freq}}\right)^{\left(N-1+\frac{DSC}{E}\right)}}\right] + \left[\sum_{n=1}^{N}\frac{100*\frac{\text{rate}}{\text{freq}}}{\left(1+\frac{\text{yld}}{\text{freq}}\right)^{\left(n-1+\frac{DSC}{E}\right)}}\right] - \left(100*\frac{\text{rate}}{\text{freq}}*\frac{A}{E}\right)$$

Where:

rate = coupon rate p.a.
yld = redemption yield p.a.
redemption = redemption value
freq = number of coupon payments p.a.
DSC = number of days from settlement to next coupon date
E = number of days in coupon period in which the settlement date falls
N = number of coupons payable between settlement date and redemption date
A = number of days from beginning of coupon period to settlement date

The day-count convention may vary, as may the convention of quoting yields in nominal or effective form.

1.1 Yields

When we discount the individual payments under a bond by the appropriate zero-coupon spot yields, we obtain the theoretical price of the bond. If we discount the same sequence of payments at a single interest rate to give the same theoretical market value, this single rate is the *bond yield*. Also, we can determine the coupon rate that would be required to make the theoretical value of the bond equal to its nominal value under the prevailing pattern of zero-coupon interest rates – this is the *par yield*.

The zero-coupon yield curve can be constructed from the observed prices of coupon-bearing bonds by the technique of *bootstrapping* – that is, we build up the pattern of zero-coupon yields that is consistent with the market prices of the conventional bonds, using linear interpolation between observed values, where necessary.

1.2 Forward rates

We can also use the zero-coupon yields to determine the pattern of *forward* rates that are consistent with them. (A forward interest rate is the interest rate implied by current zero-coupon rates for a specified future time period.)

If R_1 and R_2 are the zero-coupon rates for maturities T_1 and T_2 respectively, and R_F is the forward interest rate for the period between T_1 and T_2, then

$$R_F = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

Rearranging this, we have

$$R_F = R_2 + (R_2 - R_1) \times \frac{T_1}{(T_2 - T_1)}$$

Thus, if the zero curve is upward sloping (with $R_1 < R_2$), then $R_F > R_2$. If the zero curve is downward sloping (with $R_1 > R_2$), then $R_F < R_2$.

In the limit, as T_2 approaches T_1 with a common value of T,

$$R_F = R + T\,\frac{\partial R}{\partial T}$$

where R is the zero rate for a maturity of T. The value of R_F obtained in this way is known as the *instantaneous forward* rate for a maturity of T.

1.3 Forward rate agreements

The nature of Forward Rate Agreements (FRAs) was discussed in Unit 5 Section 3.4. The value, V, of a FRA where it is specified that an interest rate R_K will be earned for the period of time between T_1 and T_2 on a specified principal of L can be evaluated as:

$$V = L(R_K - R_F)\,(T_2 - T_1)\; e^{-R_2 T_2}$$

i.e. the present value of the difference between the interest payments.

1.4 Interest rate futures

Three-month interest rate future contracts are typically available in a wide range of currencies. The contracts trade with specified delivery months (for example March, June, September and December) up to ten years into the future (as well as short-maturity contracts with other delivery months). The variable underlying the contract is the relevant market interest rate applicable to a specified period (e.g. a 90-day period beginning on the third Wednesday of the delivery month).

If Z is the quoted price for a futures contract, the contract price is

$$10{,}000\,[100 - 0.25(100 - Z)]$$

A change of one basis point, 0.01, in a Eurodollar futures quote corresponds to a contract price change of $25.

When the third Wednesday of the delivery month is reached and the actual interest rate for the 90-day period is known, the contract is settled in cash.

1.5 Relationship between forward and futures prices

One difference between many (OTC) forwards and (exchange-traded) futures is that forwards often have no cash flow until the maturity. For a future, there are daily marking-to-market and settlement of margin requirements. Nowadays, many forwards do require to be cleared through a clearing house, which will involve margin agreements and margin payment cashflows in the same way as futures. In such cases, the difference between forwards and futures becomes blurred and the discussion below would no longer be valid.

If interest rates are constant then the values of the cash flows are equal and, hence, the prices must also be equal. When interest rates vary unpredictably, (un-margined) forward and futures prices are no longer the same because of the daily cash flows from settlement and the interest earned on cash received (or paid on borrowing). When the price of the underlying asset is strongly positively correlated with interest rates, a long futures contract will be more attractive than a similar long forward contract and futures prices will tend to be higher than forward prices. The reverse holds true when the asset price is strongly negatively correlated with interest rates.

The theoretical differences between (un-margined) forward and futures prices for contracts that last only a few months are, in most circumstances, sufficiently small to be ignored. However, for long-term futures contracts, the differences between (un-margined) forward and futures rates are likely to become significant. To convert futures to forward interest rates, a *convexity adjustment* is applied:

$$\text{Forward rate} = \text{Futures rate} - \tfrac{1}{2}\sigma^2 t_1 t_2$$

where t_1 is the time to maturity of the futures contract, t_2 is the time to maturity of the rate underlying the futures contract and σ is the standard deviation of the change in the short-term interest rate in one year. (A typical value for σ is 1.2%). [Note that the forward and futures rates in this expression are expressed in continuously compounded form δ.]

1.6 Interest rate swaps

As we saw in Unit 5, section 3.1.1, an interest rate swap can be valued as a long position in one bond compared to a short position in another bond. The cash flows are usually discounted using LIBOR zero-coupon interest rates (since LIBOR is the cost of funds for a financial institution). Thus

$$V_{\text{swap}} = B_{\text{fl}} - B_{\text{fix}}$$

where B_{fix} is the value of the fixed-rate bond underlying the swap and B_{fl} is the value of the floating-rate bond underlying the swap. The fixed rate bond is valued in the usual way:

$$B_{\text{fix}} = \sum_{i=1}^{n} k e^{-r_i t_i} + L e^{-r_n t_n}$$

where the cash flows are k at time t_i ($1 \le i < n$) and L at time t_n.

The value of the floating rate bond will be L immediately after a payment date. Immediately before a payment date, its value will be $L + k^*$, where k^* is the floating-rate payment that will be made on the next payment date due at time t_1. Thus the value of the bond today is its value just before the next payment date discounted at rate r_1 for time t_1:

$$B_{\text{fl}} = (L + k^*)\ e^{-r_1 t_1}$$

Alternatively, the swap can be valued as a series of forward rate agreements. The procedure is:

1. Calculate forward rates for each of the LIBOR rates that will determine swap cash flows.

2. Calculate swap cash flows on the assumption that the LIBOR rates will equal the forward rates.

3. Set the swap value equal to the present value of these cash flows.

2 Arbitrage pricing and the concept of hedging

2.1 Introduction

Any two assets (or asset portfolios) that provide identical payoffs in all future times and conditions *must* have the same price. If this was not the case, then there would be opportunities for *arbitrage* – by selling the more expensive portfolio and buying the cheaper portfolio with the proceeds, an investor could produce an unlimited return without capital expenditure. As investors attempt to take advantage of this position, the demand for the cheaper portfolio will increase (causing its price to rise) and demand for the more expensive portfolio will fall. Equilibrium will be restored when the prices of the two portfolios are equal. (Note that we are making many assumptions here about tax, transaction costs, access to borrowing, divisibility of assets, investors' knowledge, etc.)

We can apply the concept to derive the price, F_0, of a forward contract in terms of the spot price S_0. No arbitrage requires that $F_0 = S_0e^{rT}$ where T is the time when the forward contract matures and r is the risk-free rate of interest (for an investment maturing at time T). If this equality did not hold, arbitrage possibilities would exist. If $F_0 < S_0e^{rT}$ the investor can sell the asset short at the current spot price S_0, invest the sale proceeds risk-free (to accumulate a sum S_0e^{rT}), and, at the same time, enter into a long forward contract to buy the asset at time T at price F_0. This will generate a risk-free profit of $S_0e^{rT} - F_0$ for no initial outlay.

Similarly, if $F_0 > S_0e^{rT}$ unlimited profit can be made from a strategy of borrowing S_0 now to buy the asset and entering into a short forward contract to sell the asset at time T for F_0. At that time the loan and accumulated interest of S_0e^{rT} will be repayable, leaving the investor with a risk-free profit of $F_0 - S_0e^{rT}$. The only price for the forward, F_0, that eliminates the arbitrage opportunities is S_0e^{rT}.

Where an asset will provide income with a present value of I during the lifetime of a forward contract, then

$$F_0 = (S_0 - I)\, e^{rT}.$$

If the income is in the form of a known (continuous) yield, q, then

$$F_0 = S_0e^{(r-q)T}.$$

2.2 Hedging

A *hedge* is defined as a trade to reduce market risk. Thus, an investor that knows she is due to sell an asset at a particular time can hedge by taking a short futures position (known as a *short hedge*). If the price of the asset goes down, the investor does not fare well on the sale of the asset, but makes a gain on the short futures position. If the price of the asset goes up, the investor gains from the sale of the asset but takes a loss on the futures position. Similarly, an investor that knows she is due to buy an asset in the future can hedge by taking a long futures position (known as a *long hedge*).

It is important to recognise that futures hedging does not necessarily improve the overall financial outcome. What it does is reduce the risk by making the outcome more certain. *Basis risk* may arise if:

- The asset whose price is to be hedged is not exactly the same as the asset underlying the futures contract.
- The hedger is uncertain as to the exact date when the asset will be bought or sold.
- The hedge requires the futures contract to be closed out well before its expiration date.

The optimal *hedge ratio*, h, (ratio of the size of the position taken in futures contracts to the size of the exposure) is given by:

$$h = \rho\sigma_S / \sigma_F$$

where σ_S is the standard deviation of ΔS, the change in spot prices, σ_F is the standard deviation in ΔF, the change in futures price and ρ is the correlation coefficient between ΔS and ΔF.

3 Empirical characteristics of asset prices

Empirical studies of asset prices and interest rates have identified departures in price and returns data from the assumptions commonly used in asset models. While there has been some debate about this empirical evidence, the statistical validity of the tests and the economic meaning of the results, the issues raised need to be understood if the standard modelling assumptions are used.

Most studies of equity return distributions and other financial data find them to be lepto-kurtic as compared with the normal distribution. This means that the return distributions are peaked at the mean and fat-tailed compared with the normal distribution. The distribution of price changes becomes more normal as the horizon over which these are measured is increased. Thus, annual returns are closer to normal than monthly returns. This “fat-tailed” feature of returns can be modelled by using a hetero-scedastic model.

Other studies of equity returns have found that the variance of price changes for longer horizons does not increase linearly (as would be expected if equity prices follow a random walk). The variance was found to grow at a slower-than-linear rate, suggesting that equity values exhibit long-run mean reversion. Studies of equity returns, interest rates and foreign exchange rates have also found significant non-linear dependence, with absolute values of returns and squared returns significantly auto-correlated. These studies generally do not find evidence of serial correlation in the prices themselves.

The non-linear dependence of squared returns suggests that the returns series could be hetero-scedastic, and not homo-scedastic as the random walk model assumes. Studies of stock market volatility over long periods of time in a number of markets suggest that financial market volatility varies in certain systematic ways. For example, volatility of equity markets has tended to increase in recessions and financial crises and to be lower in bull markets than in bear markets. This latter phenomenon is usually explained by the effect of leverage, where negative shocks result in proportionally higher volatility. Markets also exhibit volatility "clustering", where large and small "shocks" tend to be clustered together. This feature of asset returns is indicative of non-linear dependence in the returns series.

A contentious issue is the relationship between equity values, dividend growth rates and inflation. Time series analysis of equity returns suggest that there is little direct correlation between rates of equity growth for share price indices and the rate of inflation. Indeed, there is some evidence that equity share returns are *negatively* related to inflation, using year-on-year data. These studies need to be interpreted with care to determine whether equity values and inflation indices are "co-integrated" (i.e. move together) even though the rates of growth in these indices exhibit no significant cross-correlation.

Since equity assets are often used as an inflation "hedge", the nature of this relationship is fundamental for ALM and strategic asset allocation.

4 Introduction into fixed income option pricing

The use of the Black-Scholes formula for valuing options on shares was considered in Subject CT8. Here, we consider the extension of this approach to bond options, interest rate caps and floors and swaptions – derivatives whose payoffs are dependent in some way on the level of interest rates. These are more difficult to value than equity derivatives, since:

- The behaviour of an individual interest rate is more complicated than that of a stock price.
- For the valuation of many products, it is necessary to develop a model describing the behaviour of the entire yield curve.
- The volatilities of different points on the yield curve are different.

- Interest rates are used for discounting as well as for determining payoffs from the derivative.

The main classes of yield curve models were also introduced in Subject CT8.

We consider a European call option on a variable whose value is V. Defining:

T:	Maturity date of the option
F:	Forward price of V (for a contract with maturity T)
F_0:	Value of F at time 0
X:	Strike price of the option
$P(t,T)$:	Price at time t of a zero-coupon bond paying 1 at time T
V_T:	Value of V at time T
σ:	Volatility of F

Then, assuming V_T has a lognormal distribution with standard deviation of $\ln V_T$ equal to $\sigma\sqrt{T}$ and expected value of V_T equal to F_0, the value of the option, c is given by:

$$c = P(0, T)[F_0\,\Phi(d_1) - X\Phi(d_2)]$$

where $\Phi(x)$ is the standard cumulative Normal distribution function,

$$d_1 = (\ln(F_0/X) + (\sigma^2 T/2)\,)\,/\,\sigma\sqrt{T}$$
$$d_2 = (\ln(F_0/X) - (\sigma^2 T/2))\,/\,\sigma\sqrt{T}\;.$$

The value, p, of the corresponding put option is given by $P(0, T)[X\Phi(-d_2) - F_0\Phi(-d_1)]$.

Where the payoff is made at some later time T^* than the time at which the variable V is evaluated, we simply allow for this in the discount factor $P(0, T^*)$.

This model appears to involve two approximations:

1. The expected value of V_T is assumed equal to its forward price F_0.

2. The stochastic behaviour of interest rates is not taken into account in the way the discounting is done.

It can be shown, however, that these two assumptions have exactly offsetting effects.

4.1 Bond options

Assuming that bond prices at the maturity of the option are lognormally distributed, the equations in Section 4 above can be used to price European options on bonds with F_0 equal to the forward bond price. $\sigma\sqrt{T}$ is the standard deviation of the logarithm of the bond price at the maturity of the option.

To allow for income, F_0 can be calculated as

$$F_0 = (B_0 - I) / P(0, T)$$

where B_0 is the bond price at time zero
I is the present value of the coupons that will be paid during the life of the option.

Note that F_0, B_0 and X should all be expressed as "dirty" prices i.e. allowing for accrued income.

Volatilities are often expressed as yield volatilities. To convert these to price volatilities we use the expression

$$\sigma = D y_0 \sigma_y$$

where σ is the forward price volatility,
σ_y the corresponding forward yield volatility,
y_0 is the initial forward yield on the bond
D is the (modified) duration of the forward bond underlying the option which is given by D = Duration / $(1 + y/m)$ where m is the frequency per annum with which y is compounded.

4.2 Interest rate caps

A popular over-the-counter interest rate option is the *interest rate cap*, which is designed to provide insurance against the rate of interest on an underlying floating-rate note rising above a certain level. This level is known as the *cap rate*, R_X.

Suppose the interest rate R_K on the floating-rate note is reset every three months to equal LIBOR. (The time between resets is known as the *tenor*). The payoff provided by the cap will be $0.25 \times \max(R_K - R_X, 0) \times L$ where L is the principal amount specified for the contract.

This payoff calculation will occur every three months during the life of the cap, T. Note that payment of the payoff occurs at the end of the tenor period (here three months) not at the time of calculation (the beginning of the period). Thus the cap leads to a payoff at time t_{k+1} ($k = 1, 2, ..., n$) of $L\delta_k \max(R_K - R_X, 0)$ where t_{n+1} is defined as T and the reset dates are $t_1, t_2, ... t_n$. Note that R_K and R_X are expressed with a compounding frequency equal to the frequency of resets.

Each payoff is a call option on the LIBOR rate observed at time t_k (with the payoff occurring at time t_{k+1}) and is known as a *caplet*. The cap is a portfolio of n such options. If the rate R_K is assumed to be lognormally distributed with volatility σ_K, then we can use the expression in 4 above (substituting R_X for X, F_K (the forward rate for the period between time t_k and $t_{k=1}$) for F_0, t_K for T and σ_K for σ) to value each caplet. (Each caplet must be valued separately.) One issue to be considered is whether to use a different "spot" volatility for each caplet or to assume the same "flat" volatility for all the caplets comprising any particular cap (but to vary this volatility according to the life of the cap).

An equivalent approach is used for an *interest rate floor* contract providing payoff when the interest rate on an underlying floating-rate note falls below a certain rate. In this case, we are valuing a portfolio of put options. Note that the cap and floor prices satisfy the put / call parity relationship

cap price = floor price + value of swap

where the swap is an agreement to receive floating and pay a fixed rate R_X.

Caps and floors are often combined into a single instrument, the *interest rate collar* (or "floor-ceiling" agreement). This is designed to guarantee that the interest rate on the underlying floating-rate note always lies between two levels. It is usually constructed so that the price of the long position in the cap is initially equal to the price of the short position in the floor, so that the cost of entering into the collar is then zero.

Note also that an interest rate cap can be characterised as a portfolio of put options on zero-coupon bonds with payoffs on the put occurring at the time they are calculated. In the same way, a floor can be seen as a portfolio of call options on similar bonds.

4.3 European swaptions

The nature of swaption contracts was considered in Unit 5, section 3.2. We have seen that an interest rate swap can be regarded as an agreement to exchange a fixed-rate bond for a floating-rate bond. Also, at the start of a swap, the value of a floating-rate bond always equals the principal amount of the swap. Thus, a swaption can be regarded as an option to exchange a fixed-rate bond for the principal amount of the swap. If the swaption gives the holder the right to pay fixed and receive floating, it is a put option on a fixed-rate bond with a strike price equal to the principal. If the right is to receive fixed and pay floating, it is a call option on a similar bond.

Assuming that the swap rate at the maturity of the option is lognormally distributed, we consider a swaption with the right to pay R_X and receive LIBOR on a swap that will last n years starting in T years. We suppose there are m payments per year under the swap and that the principal is L.

The payoff from the swaption consists of a series of cash flows equal to $L/m \max(R - R_X, 0)$ where R is the swap rate for an n-year swap at the maturity of the swap option. The value of the swaption is

$$\Sigma L/m\, P(0, t_i)[F_0\Phi(d_1) - R_X \Phi(d_2)]$$

where F_0 is the forward swap rate. This expression can be simplified by substituting an annuity A where $A = 1/m\ \Sigma P(0, t_i)$. Thus, if the swaption gives the holder the right to receive a fixed rate R_X, the value of the swaption is

$$LA[R_X\,\Phi(-d_2) - F_0\,\Phi(-d_1)]$$

5 Evaluating a securitisation

Evaluation of assets for securitisation concentrates more on the predictability and sustainability of adequate cashflow than the asset's loan to value ratio. Thus factors such as:

- lease terms
- rental prospects
- degree of potential competition, and
- barriers to competitive entry

will be considered. Agreements may include a debt service-to-EBITDA ratio covenant. Outside the UK, the reduction of currency exchange risk has removed a barrier which previously inhibited pan-European securitisation (although differences in legal systems can still make the transfer of assets located in different jurisdictions a relatively complicated process).

The borrower's auditors are likely to be called upon not only to undertake audits of the assets, but also to provide various certificates concerning the assets, cash flows, tax positions and contingent liabilities. The complexity of the securitisation structure requires a much higher degree of reporting than under more usual banking facilities.

The focus of attention on cash flow volatility means that the analysis of credit risk is a key feature when evaluating a securitisation. Techniques for evaluating credit risk are considered in the next section.

Evaluation of a securitisation therefore follows the standard approach of modelling the anticipated cash flows and discounting these at an appropriate interest rate. The modelling of the cashflows will involve key features such as:

- statistics (early repayment experience)
- probability (default / recovery and timing)
- treasury management (payments in / payments out)
- structuring and security issuance

A more intractable problem relates to the choice of a suitable discount rate for each tranche of security. A common approach is to compare the risk characteristics of each tranche with similar securities already in existence in order to assess an appropriate credit rating for the tranche. From this, a market interest rate can be established.

6 Evaluating a Credit Default Swap

The price of a "plain vanilla" credit default swap (CDS) might, in theory, be derived from the yield on an associated bond, from the same issuer and for the same maturity, in excess of the risk-free rate. However, in practice the "basis", equal to the CDS price less the yield in excess of risk-free on the bond is not zero, and can in fact be quite volatile.

This reflects a number of factors:

- The package of a bond and a credit default swap, to protect against default, is not in practice risk-free due to:
  - counterparty credit risk on the credit default swap
  - the no-default value of the bond may be higher or lower than face value due to changes in interest rate
  - documentation differences
- The package of a bond plus CDS is illiquid and requires funding, and so typically basis will be negative when funding is expensive.
- Different supply and demand dynamics in different markets.

Another approach to directly estimating the price of credit risk, and hence the credit default swap premium, is to use structural models based on information from equity derivative markets. One such approach - the Merton model – was covered in CT8. Suppose, for example, that the company has a zero coupon bond outstanding, and that the bond matures at time T. Let:

V_t = the value of the company's assets at time t
E_t = the value of equity at time t
D = Debt (principal and interest) due to be paid at time T
σ_V, σ_E = volatility of V and E respectively

If $V_T < D$, it is rational for the company to default in its debt at T. The value of the equity is then zero. If $V_T > D$, the company should make the debt repayment and the value of the equity at this time is $V_T - D$. The value of the firm's equity at time T, E_T, is therefore $\max(V_T - D, 0)$. This shows that the equity is a call option on the value of the assets with a strike price of the amount of the debt, D.

The Black-Scholes formula can then be used in the usual way to give the current value of the equity, E_0, since $E_0 = V_0\Phi(d_1) - De^{-r_T}\,\Phi(d_2)$, where

$$d_1 = (\ln V_0/D + (r + \sigma^2_V/2)T)/\sigma_V\sqrt{T} \text{ and}$$
$$d_2 = d_1 - \sigma_V\sqrt{T}\ .$$

The value of the debt today is $V_0 - E_0$.

The risk neutral probability that the company will default is $\Phi(-d_2)$. This requires knowledge of both V_0 and σ_V, neither of which are directly observable. However, E_0 is observable, giving one equation in two unknowns. Also, from Ito's lemma, we have $\sigma_E E_0 = \delta E/\delta V\ \sigma_V\ V_0 = \Phi(d_1)\ \sigma_V\ V_0$. By estimating σ_E and solving this second equation, the risk neutral probability of default can be estimated. This provides an alternative figure for the theoretical price of a CDS contract, before allowing for the factors listed above.

7 Evaluation of investment alternatives

Actuaries are expected to be able to evaluate various types of investments. A good example of a practical problem faced by a company was set in the exam paper for ST5 September 2006, Question 4, as follows:

Question

ABC plc has a share price of 57 pence and paid a dividend of 2 pence per share in the previous twelve months. It is considering issuing:

- a convertible debenture, with a zero coupon, which is convertible into ordinary shares on the basis of 150 shares per £100 nominal at any time over the next five years; or
- a zero dividend preference share at £100 that will be redeemed in five years time at £138

(i) Evaluate the returns that might be achieved from each of these investments stating any assumptions. [5]

(ii) Explain which investors may prefer each investment giving reasons. [7]
[Total 12]

Solution

(i) The convertible has a minimum return of £100 per £100 invested assuming company remains solvent. ZDP has maximum of £138, convertible depends on share price.

Convertible is ZDP plus call option. Option price = 100 – 100/138 = 27.5.

Convertible return will be better than ZDP if share price above 92 pence.
ZDP compounds annually at 6.65% p.a.

Share has dividend yield of 3.5% p.a. assuming no change in rate.

Capital growth of 3.15% required to match ZDP over 5 years.

(ii) The equity could be attractive to any investor. It has a reasonable yield.

The convertible requires a very strong price rise to be attractive to equity orientated investors. Convertible needs 10.0% to be equivalent to the ZDP. It might be of interest to a hedge fund wishing to carry out arbitrage investment.

It could be of interest to an absolute return manager if market volatility was expected to increase.

The ZDP is of interest to risk averse investors with minimum return criteria. It might be of interest to structured product providers as collateral. (Other examples of possible investors received equivalent credit.)

It could be of interest to hedge funds in conjunction with the ordinary and convertible to create an absolute/arbitrage return.

E N D