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Pooling longevity for a better retirement income: how many people are needed?

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Joint work with Thomas Bernhardt, U. Michigan.

The '**Minimising Longevity and Investment Risk while Optimising Future Pension Plans**' research programme is funded by the Actuarial Research Centre.

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What do people want in retirement from a financial perspective?

- A stable income for life?

- Access to their money at all times?
- A “rainy day” fund?
- Money for the kids/grandkids?

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Imagine yourself…

What to do?

Seeking advice…

Age 70 with $100K pot

	Pooled annuity fund		
Annual income	$6,650		
Age at which out-live savings	120 years		
Money left to heirs	Nothing		
(Mortality basis, Investment return basis),	(S1PMA, UK yield curve)		

Age 70 with $100K pot

	Pooled annuity fund	Life annuity	
Annual income	$6,650	$5,650	
Age at which out-live savings	120 years	Never	
Money left to heirs	Nothing	Nothing	
(Mortality basis, Investment return basis),	(S1PMA, UK yield curve)	(S1PMA-5 [chosen to match current annuity rate], UK yield curve)	

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Age 70 with $100K pot

	Pooled annuity fund	Life annuity	Income drawdown
Annual income	$6,650	$5,650	$6,650
Age at which out-live savings	120 years	Never	85 years
Money left to heirs	Nothing	Nothing	Whatever left in pot at death
(Mortality basis, Investment return basis),	(S1PMA, UK yield curve)	(S1PMA-5 [chosen to match current annuity rate], UK yield curve)	(N/A, UK yield curve)

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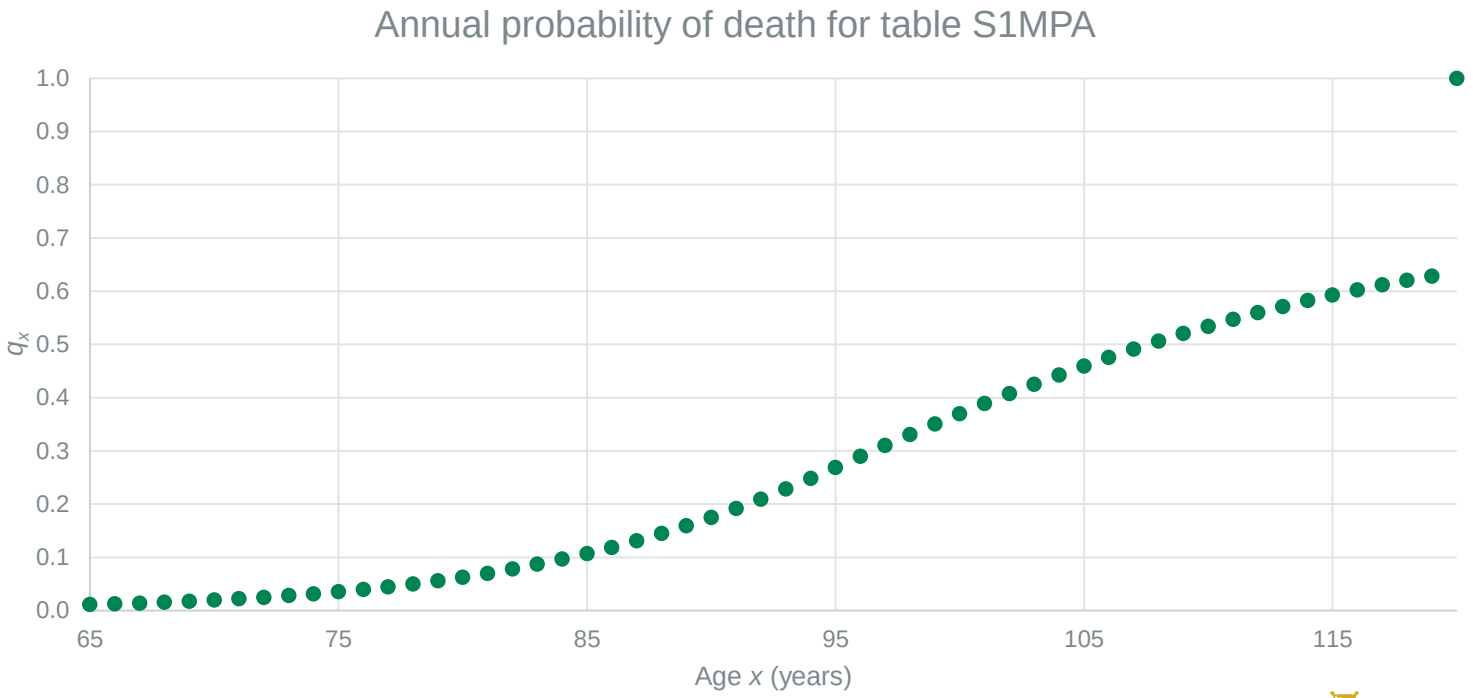
Calculations based on UK mortality table S1PMA



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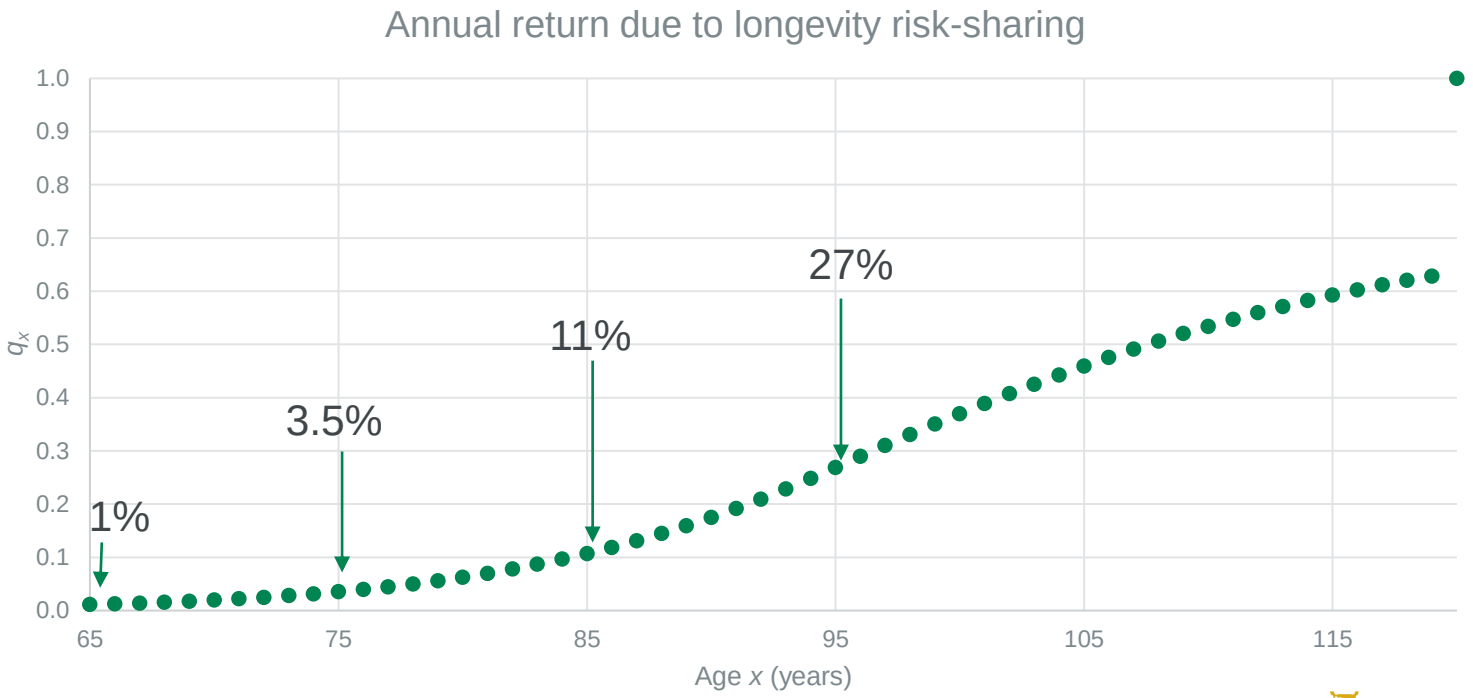

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A pooled annuity fund composition

- At time 0,
  - Known number of participants in the fund who are all the same age,
  - Future lifetimes are i.i.d. random variables,
  - Everyone brings the same amount of money to the fund.

- No new people join after time 0 (closed fund).

- No-one leaves after time 0 except through death.

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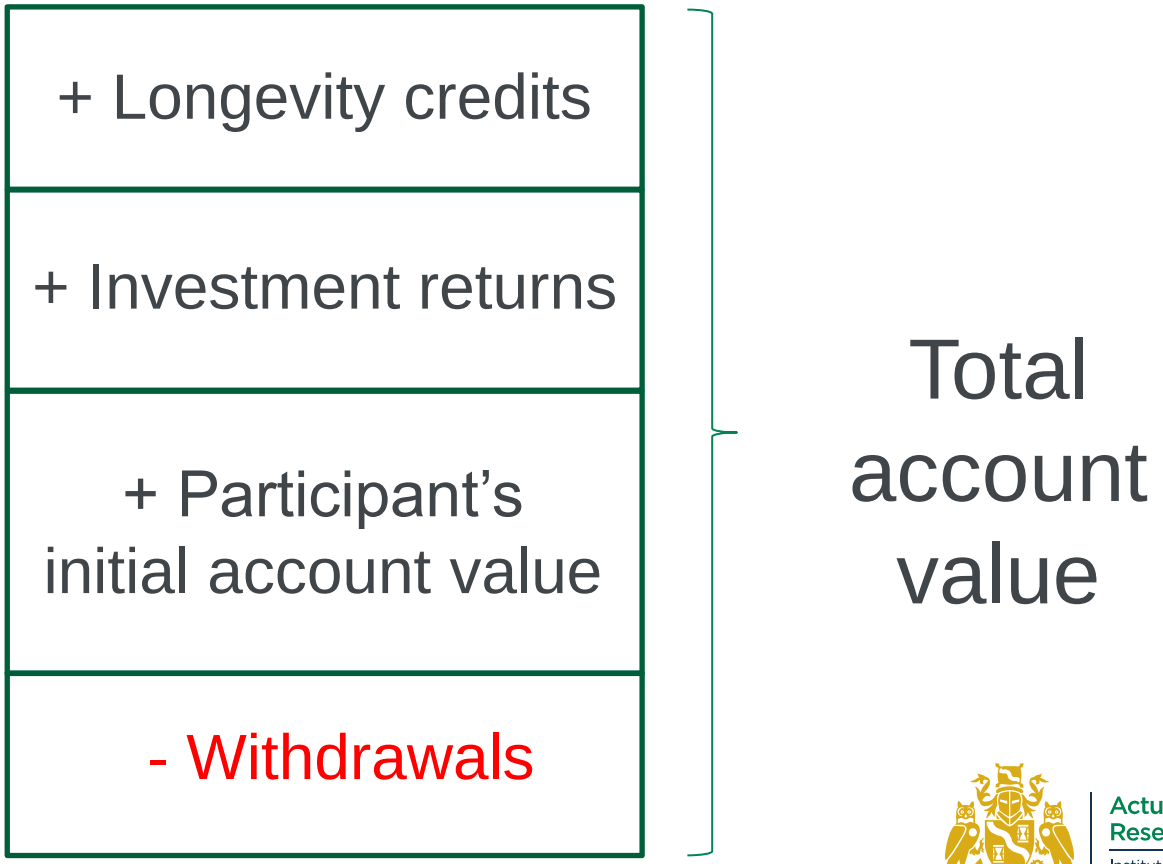
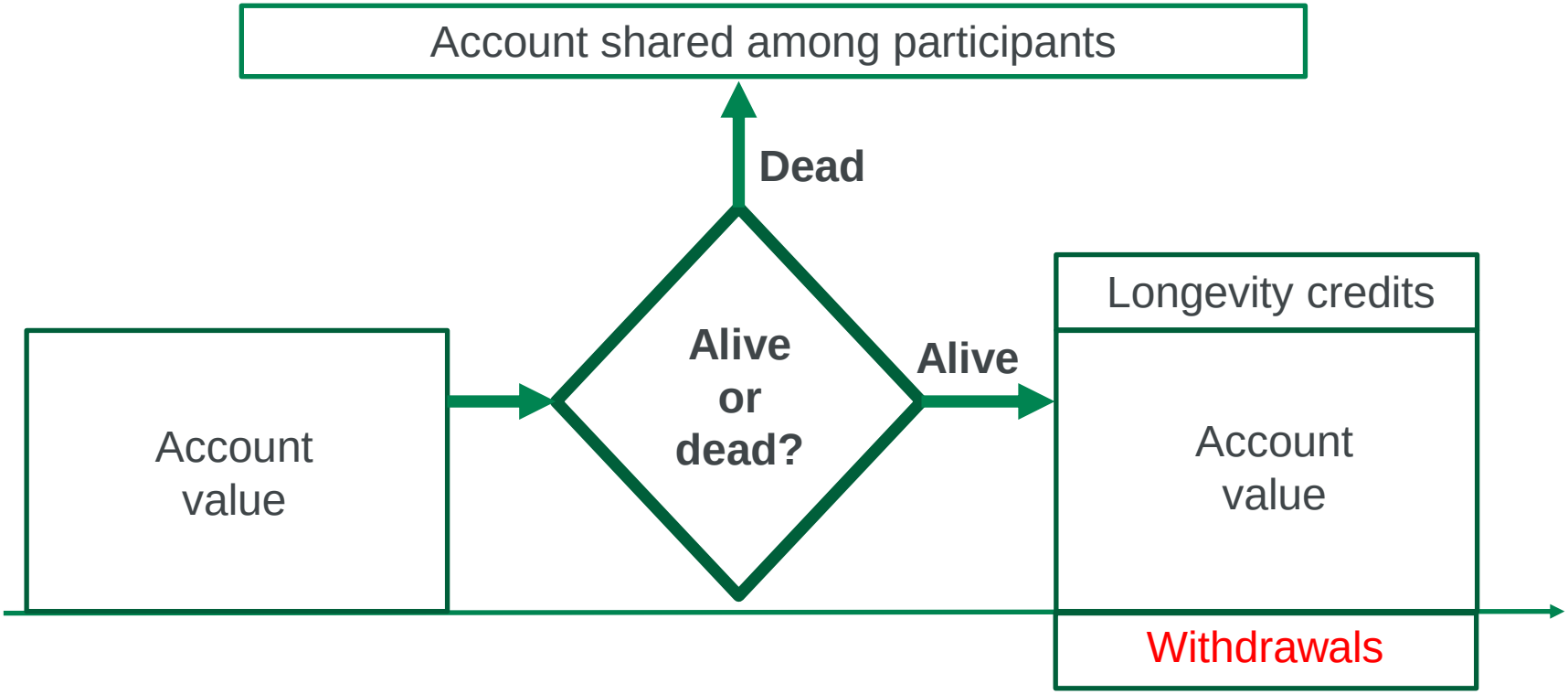
Individual account structure



Individual account structure



Investment returns + Longevity credits

- Investment returns are constant.

- Longevity credit paid to each surviving member at time *n* is

$$\frac{Total\ account\ value\ of\ those\ who\ died\ over\ (n-1,n]}{Number\ of\ survivors\ at\ time\ n}.$$

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Withdrawals

- Assume time is measured in months.

- The monthly income paid to each surviving participant is

$$Income(n) = \frac{Account\ value\ of\ the\ surviving\ participant\ at\ time\ n}{a(n)},$$

where $a(n)$ is the expected present value of a single life annuity paid monthly, starting at time n.

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Withdrawals

- Then

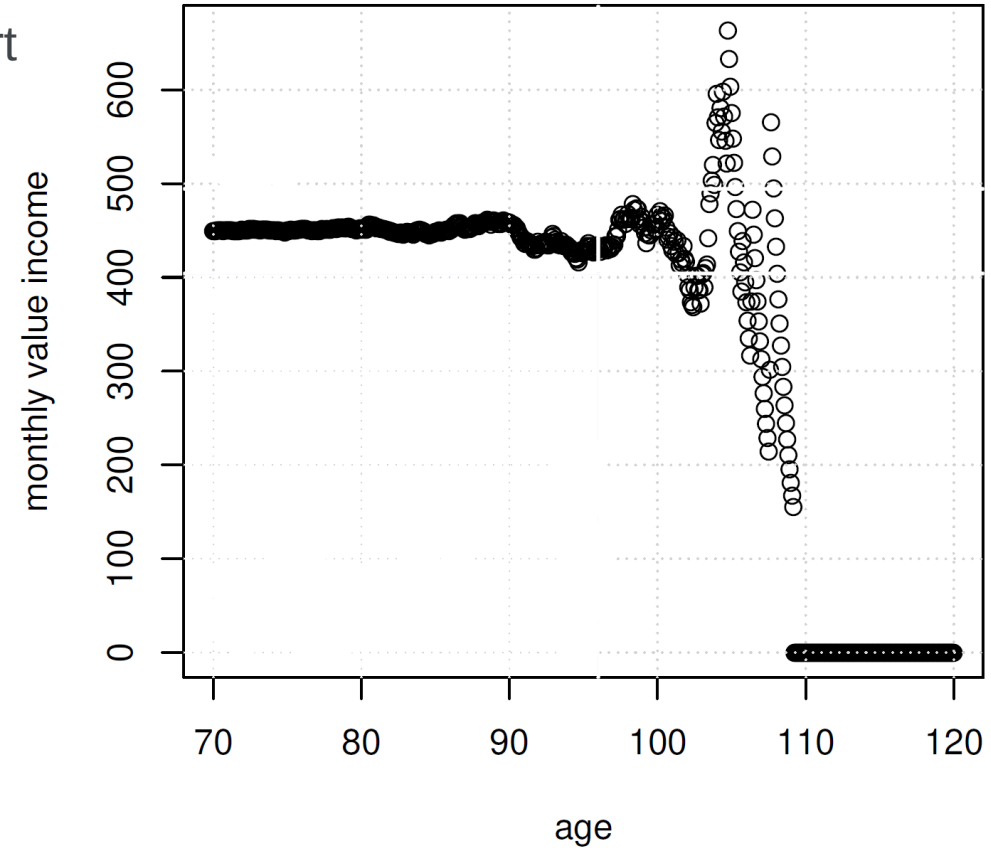
$$Income(n) = Income(n-1) \times \frac{true\ \mathbb{P}(Survive\ from\ time\ n-1\ to\ time\ n | Alive\ at\ time\ n-1)}{empirical\ \widehat{\mathbb{P}}(Survive\ from\ time\ n-1\ to\ time\ n | Alive\ at\ time\ n-1)}$$

- Study changes in income: focus on idiosyncratic longevity risk.
- There is no systematic longevity risk in our model.

A typical sample path of the income process

Closed, single-cohort pooled annuity fund.

Members are independent and identical copies.



Study the income volatility caused by random fluctuations in deaths.

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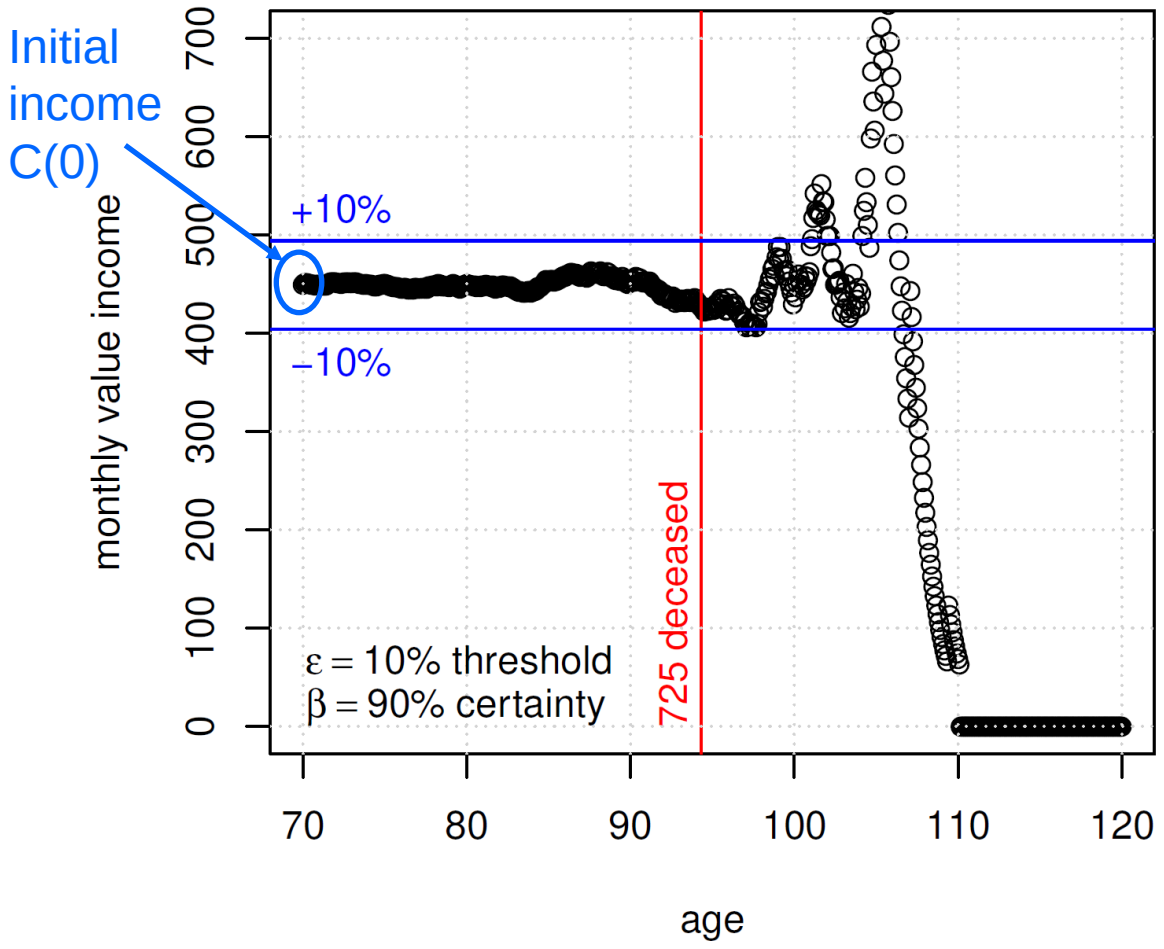
The problem



For how long is the income process **stable**?

Solve this indirectly.

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The problem

- Determine how many people get a stable lifetime income, in a specified proportion β of future scenarios.
- $C(n)$ = income paid at time n.
- $T_{(k)}$ = ordered random time of death of kth participant, $k = 1,..,N$.
- Goal: find the maximal k solving

$$\mathbb{P}\left[1+\varepsilon \geq \frac{C(m)}{C(0)} \geq 1-\varepsilon \ \ \forall m \in \left\{1,2,\ldots,\left\lfloor T_{(k)} \right\rfloor\right\}\right] \geq \beta.$$

- Difficult to solve (polynomial of order = number of initial participants).
- Instead, find a close, lower bound on k.

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The solution method

- Suppose for some $k \in \{1, \dots, N\}$

$$\mathbb{P}\left[(1-\varepsilon)\frac{i-1}{N} + \varepsilon \geq U_{(i)} \geq (1+\varepsilon)\frac{i \wedge (N-1)}{N} - \varepsilon \ \ \forall n \in \{1,2, \dots, k\}\right] \geq \beta.$$

- Then

$$\mathbb{P}\left[1+\varepsilon \geq \frac{C(m)}{C(0)} \geq 1-\varepsilon \ \ \forall m \in \left\{1,2, \dots, \lfloor T_{(k)} \rfloor\right\}\right] \geq \beta.$$

- Idea: Calculate the maximal integer k_U satisfying the first inequality.
- Then k_U will also satisfy the second inequality.
- Is k_U close to the maximal k satisfying the second inequality? Yes

(few percent ≡ 2-4 months).

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Percentage of membership whose income is ±10% of the initial income, for their life…

Number of members initially	In 90% of future scenarios	
100	21%	
500	57%	
2,000	84%	
5,000	93%	
10,000	96%	

These values are independent of the mortality table chosen.

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Percentage of membership whose income is ±10% of the initial income, for their life…

Number of members initially	In 90% of future scenarios	In 99% of future scenarios
100	21%	9%
500	57%	39%
2,000	84%	72%
5,000	93%	86%
10,000	96%	93%

These values are independent of the mortality table chosen.

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Percentage of membership whose income is ±10% **±5%** of the initial income, for their life…

Number of members initially	In 90% of future scenarios	In 99% of future scenarios
100	21% **6%**	9% **1%**
500	57% **25%**	39% **13%**
2,000	84% **57%**	72% **39%**
5,000	93% **77%**	86% **61%**
10,000	96% **87%**	93% **76%**

These values are independent of the mortality table chosen.

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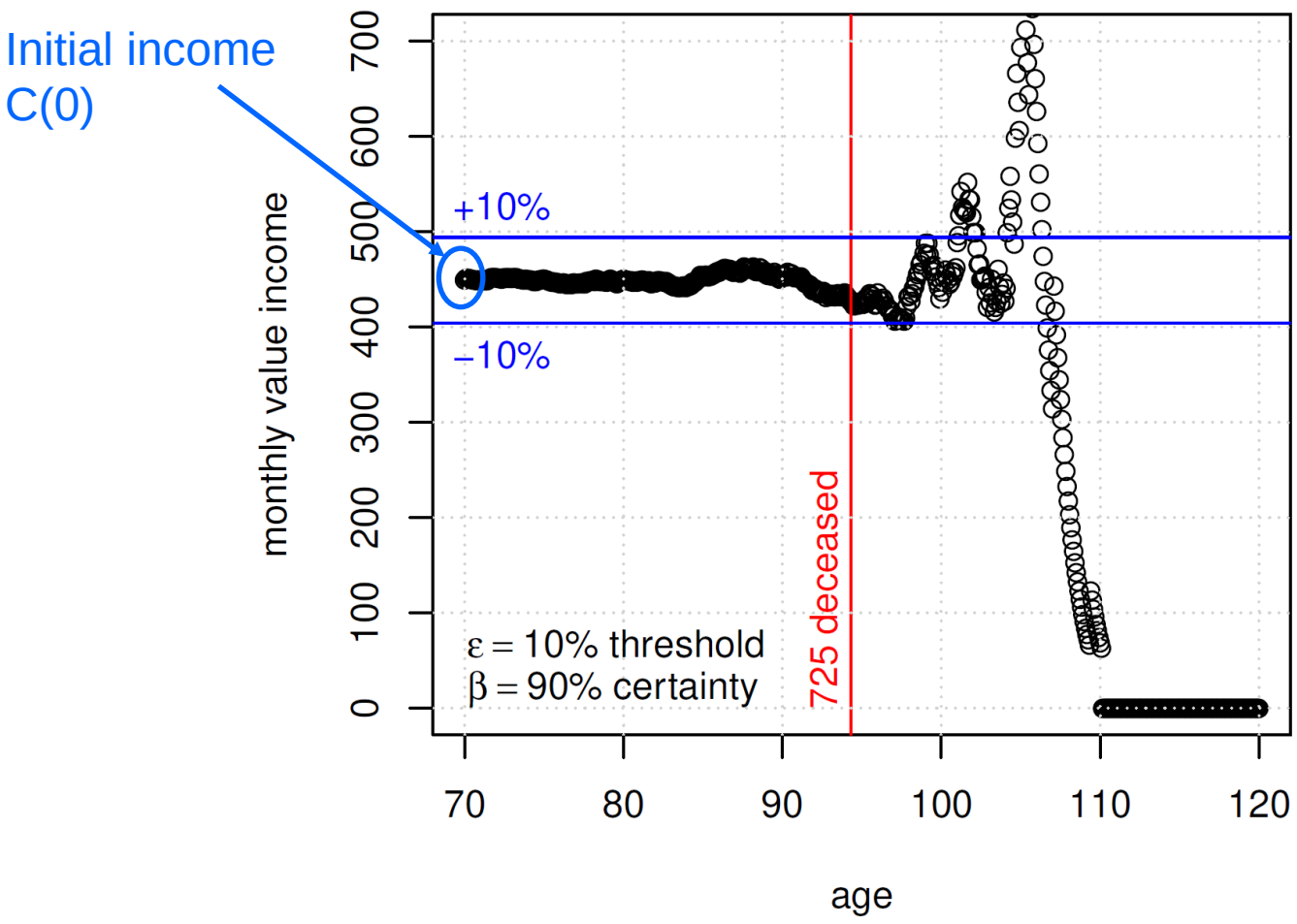
For how long is the income process stable?



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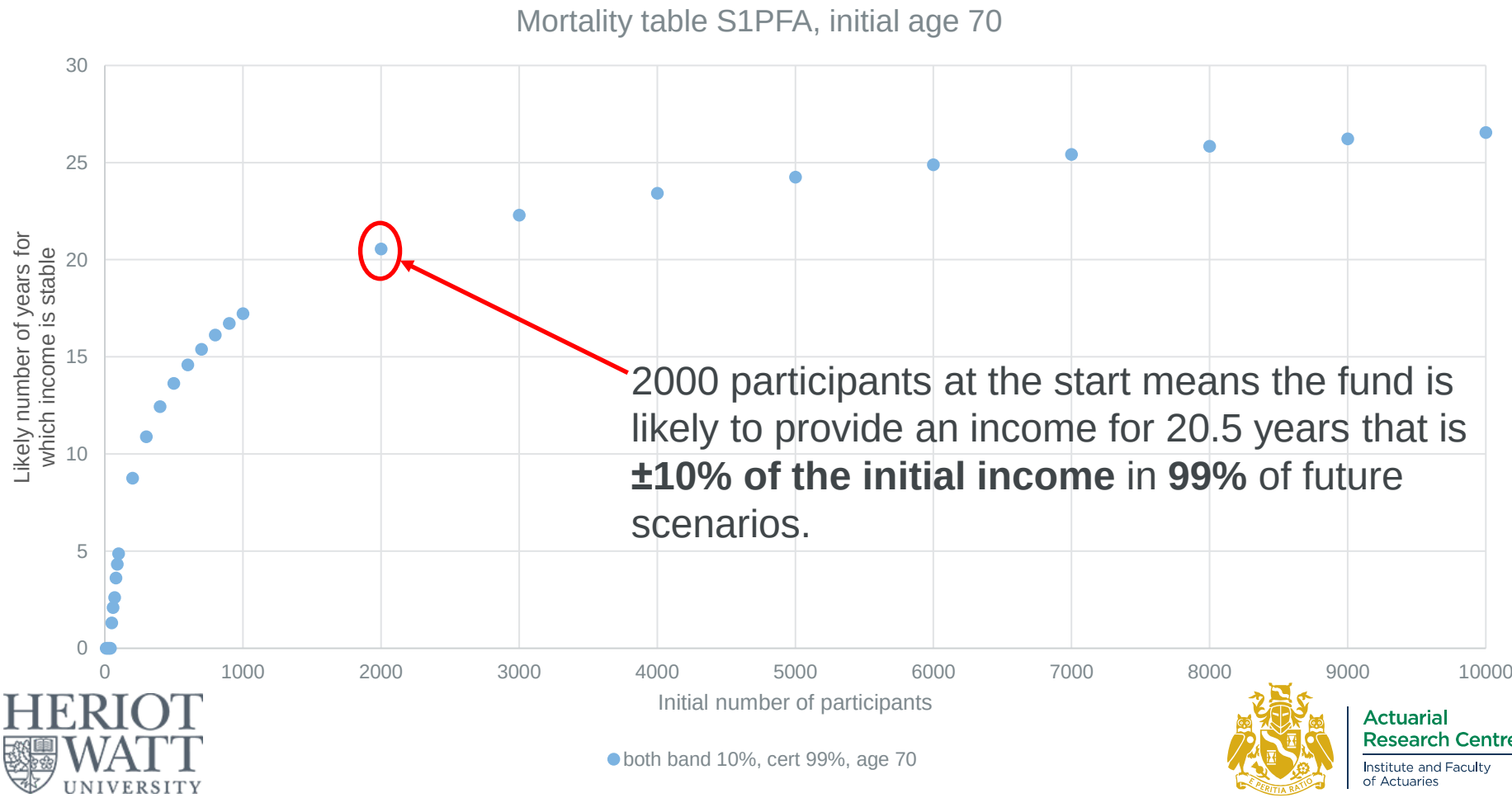
To calculate number of years, choose a mortality table S1PFA ($e_{70} = 15$ years)



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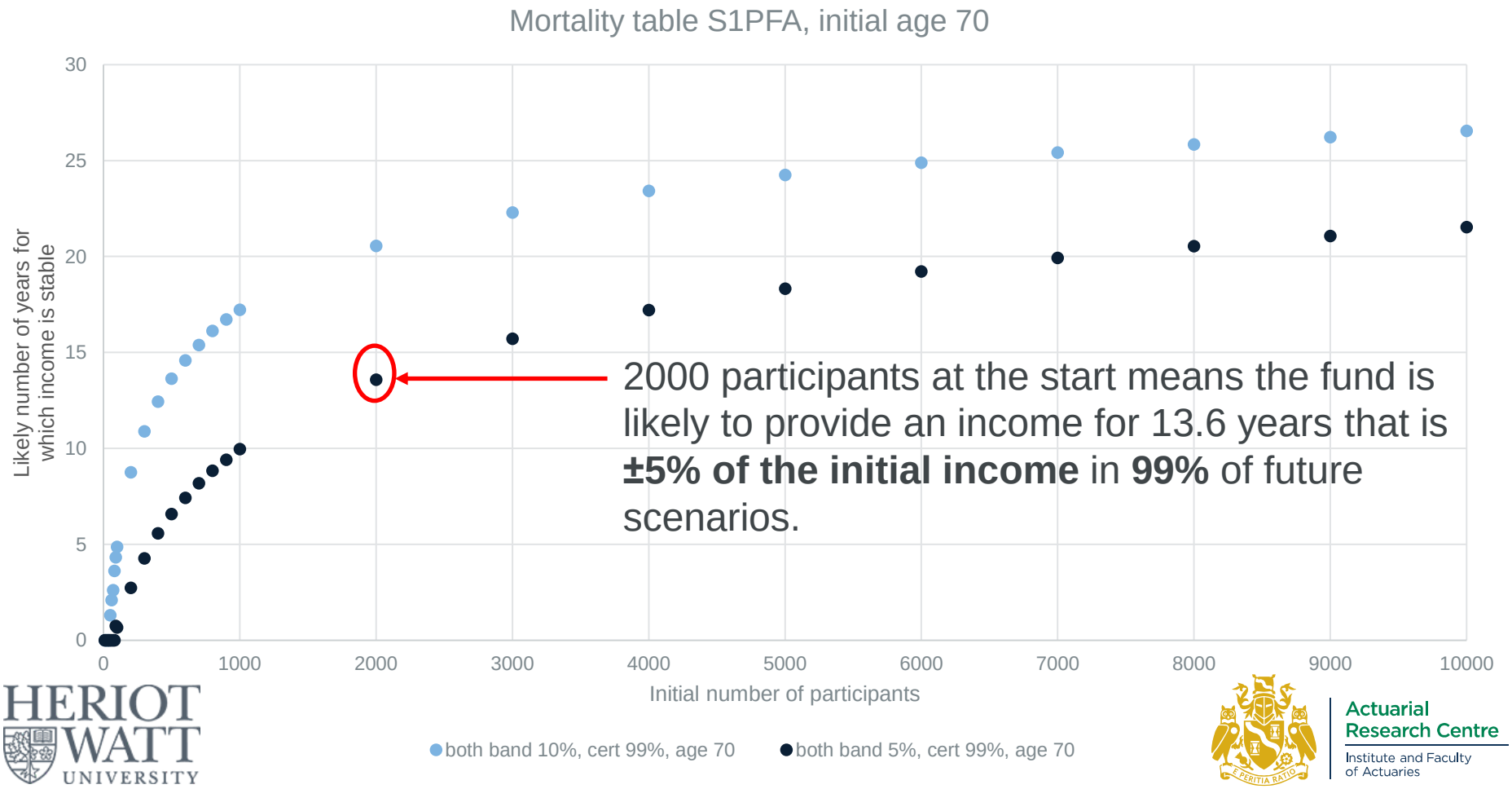

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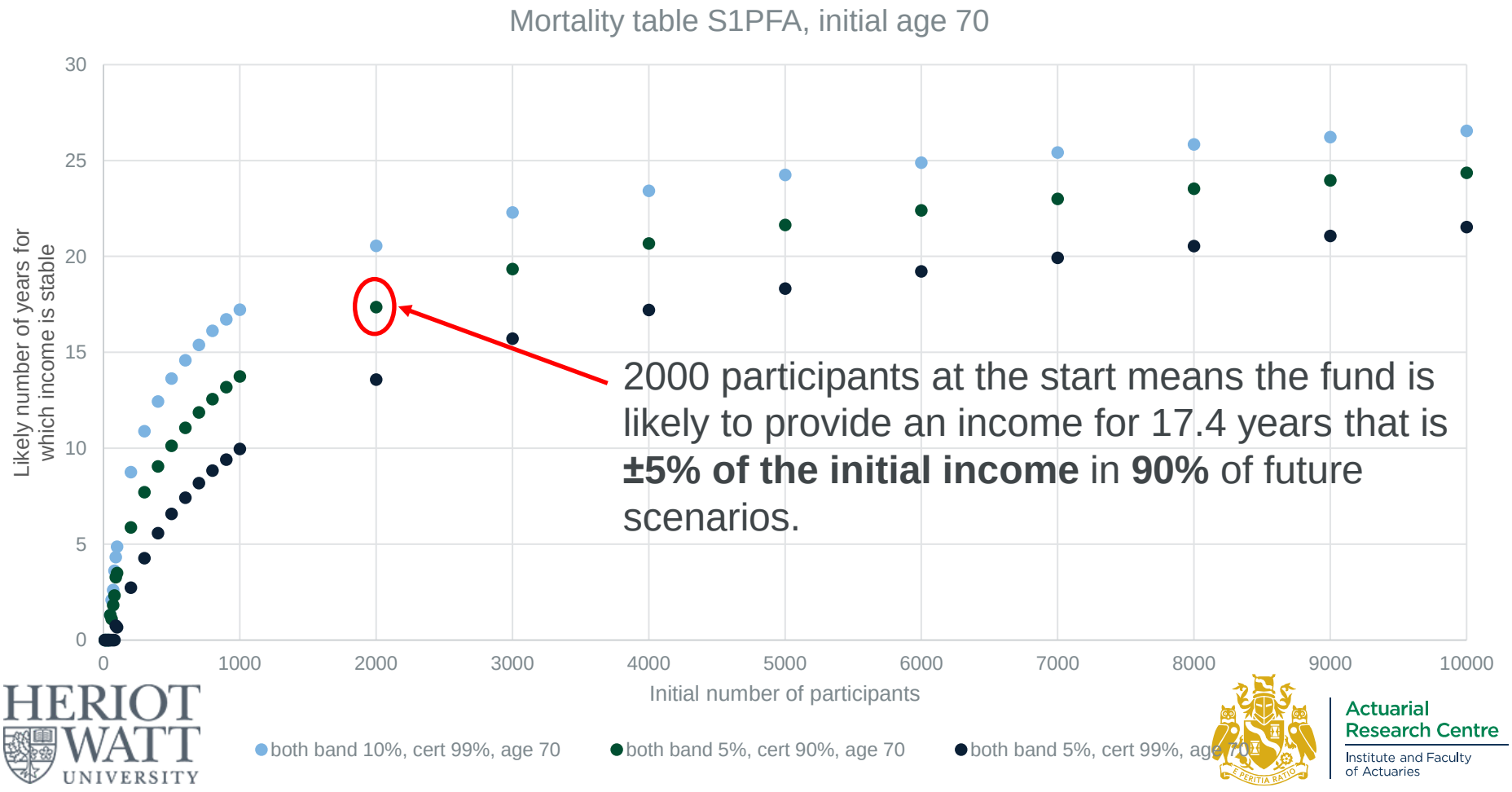

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Summary of our results

- Income from pooled annuity funds is stable for many years.
- Longest lived participants experience:
  - Small income fluctuations for first part of their retirement,
  - Wilder income fluctuations for last part.
- Risk management is essential for the last part of longest-lived's lifetime.

Summary of retirement options

- Compared to income drawdown, pooled annuities give:
  - A higher income, everything being equal,
  - A significantly reduced chance of running out of money in retirement,
  - No bequest.
- Unlike life annuities,
  - Pooled annuities are not risk-free,
  - Although income is stable for many years.

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Questions

Comments

The views expressed in this presentation are those of the presenter.

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