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PhD studentship output

Risk of Ignorance: Estimating Credit Rating Transition Matrices

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Motivation

- PhD mostly concerned with Liquidity Premium, hot topic under Solvency II
- Credit Spread = Difference in Yield between risky bond and (often) a Treasury of same maturity
- What drives a credit spread?

Credit Rating

- In this short piece, we are interested in the evolution of a bond's rating over time
- Credit Rating decided by Credit Rating Agencies (e.g. Moody's)
- *How do we estimate Credit Migration Matrices and to what extent are violations of the assumptions economically significant?*
    - Model Risk

Rating as a Continuous Time Markov Chain

Generator of Λ with $R(t) \in S = 1, 2, \cdots, K$;

$$\Lambda = \begin{pmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} & \cdots & \lambda_{1,K} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} & \cdots & \lambda_{2,K} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \lambda_{K-1,1} & \lambda_{K-1,2} & \lambda_{K-1,3} & \cdots & \lambda_{K-1,K} \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}$$

with instantaneous transition probabilities;

$$\lambda_{ij}(\Delta t) \equiv P[R(t + \Delta t) = j | R(t) = i] \geq 0, \forall i, j \in S$$

estimated by Maximum Likelihood;

$$\tilde{\lambda}_{ij} = \frac{N_{ij}(0, T)}{\int_0^T Y_i(s)ds}$$

Data Description

- Sample of 19060 corporate bond rating events (S&P) from 10439 issuers covers the period 1980 to 2002
- Disregard tranches, 8 rating classes
- Only transition events from North America, from 11 different sectors (74% of total)
- Sample includes 950 defaults, 3340 censored ratings and 3421 'NR' (Not Rated) assignments

Challenging the Markov Assumption

Is there a "risk" associated with ignoring potential violations of the Markov assumptions underlying the estimation procedure?

- Does rating history affect transition probabilities?
- Do transition probabilities vary over time?
- Are transition matrices subject to statistical error?
- Simulation exercise to check whether effects have economic significance

Rating Drift: Extending the State Space

	AAA	AA	AA*	A	A*	BBB	BBB*	BB	BB*	B	B*	CCC	CCC*	D
AAA	X		X		X		X		X		X		X	X
AA	X	X			X		X		X		X		X	X
AA*	X	X	X		X		X		X		X		X	X
A	X	X		X			X		X		X		X	X
A*	X	X		X	X		X		X		X		X	X
BBB	X	X		X		X			X		X		X	X
BBB*	X	X		X		X	X		X		X		X	X
BB	X	X		X		X		X			X		X	X
BB*	X	X		X		X		X	X		X		X	X
B	X	X		X		X		X		X			X	X
B*	X	X		X		X		X		X	X		X	X
CCC	X	X		X		X		X		X		X		X
CCC*	X	X		X		X		X		X		X	X	X
D														X

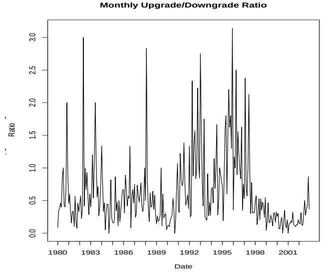
- Possible migrations under the extended state space are marked with ‘X’
- Both the AAA and default category do not have excited states

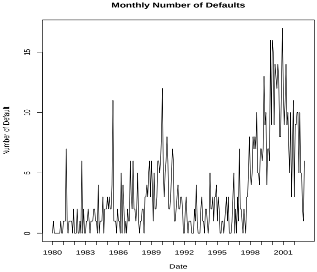
Rating Drift; Downward Momentum

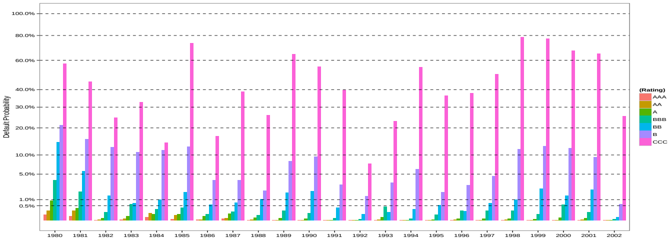
	Lower	Estimate	Upper	Significance
AAA	0.000000	0.000007	0.000013	NA
AA	0.000015	0.000086	0.000138	
AA*	0.000003	0.000048	0.000092	.
A	0.000099	0.000166	0.000224	
A*	0.000084	0.000158	0.000227	
BBB	0.000603	0.001045	0.001397	
BBB*	0.001080	0.002301	0.003330	***
BB	0.005247	0.006409	0.007491	
BB*	0.012532	0.016619	0.020753	***
B	0.036854	0.040490	0.044397	
B*	0.084314	0.095172	0.107225	***
CCC	0.109219	0.146456	0.190627	
CCC*	0.486243	0.521876	0.562614	***

Bootstrapped estimates of the default probability under the extended state space. Significance test for difference in default probability between excited and non-excited equivalent.

Time Inhomogeneity

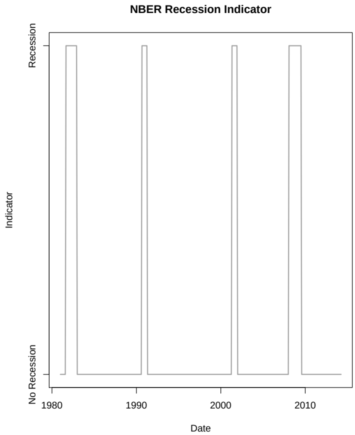


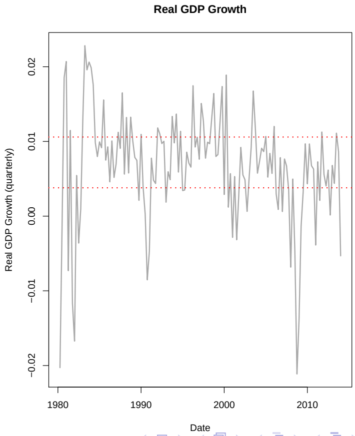




Time Inhomogeneity (II)

1. Condition on *Calendar Year*
2. Condition on *Recessions*
3. Condition on *GDP growth* states





Time Inhomogeneity (III)

(a) Non-Recession

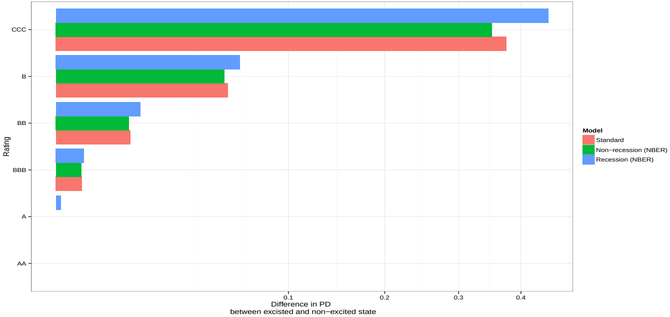
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	94.1	5.1	0.7	0.1	0.1	0.0	0.0	0.0
AA	0.6	92.1	6.6	0.6	0.1	0.1	0.0	0.0
A	0.1	1.6	92.9	4.8	0.4	0.2	0.0	0.0
BBB	0.0	0.2	3.8	90.6	4.5	0.7	0.1	0.1
BB	0.0	0.1	0.5	4.9	85.7	7.6	0.7	0.5
B	0.0	0.1	0.3	0.4	4.4	84.6	5.6	4.6
CCC	0.1	0.0	0.3	0.5	1.0	7.5	46.0	44.6

(b) Recession

AAA	AA	A	BBB	BB	B	CCC	D
83.3	15.2	1.4	0.1	0.0	0.0	0.0	0.0
0.3	87.0	11.0	1.1	0.5	0.1	0.0	0.0
0.3	3.4	83.0	11.5	1.6	0.2	0.0	0.0
0.1	1.0	5.7	79.0	11.5	2.2	0.2	0.3
0.2	0.6	1.1	4.4	70.1	19.7	2.0	1.9
0.0	0.1	0.2	0.8	4.5	78.3	8.4	7.6
0.0	0.0	0.1	0.7	2.6	11.7	35.1	49.8

	Trough		Normal		Peak	
	Persistence	Default	Persistence	Default	Persistence	Default
AAA	86.1	0.0	91.7	0.0	96.4	0.0
AA	87.1	0.0	89.5	0.0	95.1	0.0
A	86.4	0.0	88.4	0.0	96.4	0.0
BBB	82.9	0.2	84.9	0.2	95.1	0.0
BB	79.1	1.0	80.4	0.7	91.3	0.3
B	79.0	6.7	80.8	5.9	90.3	2.8
CCC	40.8	50.4	40.0	47.3	55.3	38.1

Interaction of Downward Momentum & Business Cycle



Difference in default probability between excited and non-excited state:

- CCC 27% higher
- B 19% higher
- BB 33% higher
- BBB 23% higher

VaR by Monte Carlo Simulation

Asset Value $$X_i = \sqrt{\rho}F + \sqrt{1-(\rho)}Z_i$$

Thresholds: $$d_{i,w} = \Phi^{-1}(\sum_{w=i}^{8} p_{i,w})$$

where $\Phi^{-1}(1) \equiv +\inf$ and therefore $d_{i,1} = +\inf$

Rating: $$S_i = j \quad \text{when} \quad d_{ij} \leq X_i \leq d_{i(j+1)})$$

Price : $$B_{i,R,t} = \sum_{t}^{T} \frac{C(1-PD_{R,t}) + C(RR_i \times PD_{R,t})}{(1+r)^t} + 100 \times \frac{(1-PD_{R,T}) + (RR_i \times PD_{R,T})}{(1+r)^T}$$

where $PD_{R,t}$ is the default probability of rating R over t years, RR is a stochastic recovery rate (beta distributed), r is a fixed risk-free rate, C are the annual coupon payments

Portfolio VaR Estimates & Sensitivities

Parameters:

- N = 100
- T = 6
- C = 5%
- r = 2%
- $\rho = 0.1998$
- RR $= \beta\ (\mu = .47,\ \sigma = .229)$
- Benchmark $\text{VaR}_{\alpha=99\%}$ = -4.01%

	Benchmark	'Trough'	'Normal'	'Peak'	'Recession'	'Non-Recession'	$\gamma = \infty$
Low ρ	+ 287	-102	-40	+175	-280	+38	-180
Standard ρ	0	-144	-58	+163	- 307	+ 24	- 200
High ρ	- 563	-178	-75	+145	-343	+13	- 278
Small sample (50)	- 185	-114	-30	+189	-256	+59	- 235
Standard sample (100)	0	-144	-58	+163	- 307	+ 24	- 200
Large sample (500)	+ 50	-180	-40	+134	-319	+34	- 156
Low risk	+ 210	-134	-12	+210	-288	+45	- 267
Standard risk	0	-144	-58	+163	- 307	+ 24	- 200
High risk	- 166	-189	-45	+240	-365	+13	- 362

Sensitivity of non-benchmark matrices are relative to benchmark equivalent

Conclusion and on-going work

- Evidence for Downward Momentum
- Evidence for link between real economy & transition probabilities
- Under simple simulation conditions these violations of Markovian behaviour seem economically significant

Can we arrive at more informative default probabilities than a 30-year long average on an aggregate level?
Can we further decompose the credit spread, and attempt to quantify the various risk premia?