Key insights in decumulation strategies

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Overview

I. Introduction
II. Optimal investment strategies
III. Pooling retirement funds
IV. Questions and comments
Introduction

Since 2015, pension freedom

- Sharp decline in annuities

Battocchio et al. (2007)

- Like annuity
  - Income for life
  - Actuarial fair price

- Unlike annuity
  - One customer
  - Free to invest to create profit (Black Scholes model)

- Ruin in only 0.01% of scenarios
Introduction

State of the art, a good retirement product looks like …

Value

1\textsuperscript{st} half: drawdown/investment

2\textsuperscript{nd} half: fund pooling against longevity risk

High number

Stable income

Lifetime protection

Retirement

Time
Optimal investment strategies

Black-Scholes model

\[ dS = S \cdot (\mu dt + \sigma dW) \]
\[ dB = B \cdot r dt \]

Mathematical description

- Max life consumption
  \[ \mathbb{E}\left[ \int_0^T U(t, c)dt + V(T, X) \right] \]
- Max above level
  \[ \mathbb{E}\left[ \int_0^T U(t, c - h)dt + V(T, X - H) \right] \]
- Max expectation min variance
  \[ \mathbb{E}[X(T)] - \gamma \text{Var}[X(T)] \]
- Min distance from a target
  \[ \mathbb{E}\left[ \int_0^T a(t) \cdot (c(t) - f(t))^2 dt + b(t) \cdot (X(t) - F(t))^2 \right] \]
- Min ruin probability
  \[ \mathbb{P}[\tau < T], \quad \tau = \text{first time when } X \text{ hits } 0 \]

\( S \) Stock, \( \mu \) drift, \( \sigma \) volatility, \( W \) noise, \( B \) Bond, \( r \) interest, \( \mathbb{E} \) expectation, \( T \) maturity/lifespan, \( U \) and \( V \) utilities, \( c \) consumption, \( X \) wealth, \( h \) and \( H \) minimal levels, \( \gamma \) “risk aversion”, \( \text{Var} \) Variance, \( a \) and \( b \) time preferences, \( f \) and \( F \) targets, \( \mathbb{P} \) probability
Optimal investment strategies

Intuitive results, quantifiable answers

- Max life consumption (e.g. Merton, 1971), min ruin probability
  - Mutual fund separation ✓ Presenting equity as one thing
  - Constant mixed strategy ✓ How insurance companies invest
  - Equity ↓ then Longevity risk ↑ ✓ ~50% in equity for lowest lifetime ruin
  - Changing consumption ❌ Unstable income
  - Deplete savings ✓ Bequest is 2nd degree
  - Savings don’t last forever ✓ Annuity

- 4% rule for a stable income (Bengen, 1994)
  - Varying success (how long? how much left?)

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Optimal investment strategies

Intuitive results, quantifiable answers

• Max above level, max expectation min variance, min distance from a target
  – Similar to max life consumption
  – Variance increases over time
  – Varying percentage
  – Stable profit

Optimal solutions are robust
Control
How investment firms invest
Predictable outcome

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Optimal investment strategies

Drawdown today, the 4% rule

- 50% in equity
- Inflation adjusted percentage from initial savings
- Probability to last at least …

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Simulated data using a Black Scholes model
## Optimal investment strategies

Max expectation min variance

- Annual optimization problem
- Inflation adjusted percentage from initial savings
- Probability to last at least …

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Simulated data using a Black Scholes model

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Optimal investment strategies

Undesirable features

- Difficult to communicate
  - Car mechanic analogy
- Sensitive to parameters
  - Indication for wrong set-up
- Non-explicit
  - Explicit in idealistic situation, indication for outcome
- No constraints
  - Numerical solutions
Pooling retirement funds

Annuity

- Guaranteed income, in return for savings
- Actuarial fair cost
  - No investment
  - Mortality driven price
  - Not at all times favourable
- Low value at retirement
- Age ~80 longevity credits outweigh investments
- Not at all times favourable
- Optimal stopping

- State of the art
  - Investment/drawdown opposite to annuity
  - Annuity best option at high ages
  - Delay full annuitization (phase transition, delayed annuities)
Pooling retirement funds

Modern Tontine

• No guaranteed income, irreversible decision
• No cost (besides fees, taxes, …)
  – Investment High value from the beginning
  – Performance/experienced mortality driven Fluctuation

• Main ideas
  – Investment in addition to longevity credits
  – Beneficial at all ages (ignoring bequest motives)
Pooling retirement funds

Implicit Tontine

• Features
  – One pool account
  – Influenced by experienced investment (changing fund value)
  – Influenced by experienced mortality (changing income)

• Group Self-Annuitization by Piggott et al. (2005)
  – Same aged group
  – Income calculated like annuity

\[ c_x = \frac{1}{l_x^*} \frac{F(x - 65)}{\ddot{a}_x} \]

\( c_x \) income at age \( x \), \( F(x - 65) \) fund value after \( x - 65 \) years, \( l_x^* \) count of survivors of age \( x \), \( \ddot{a}_x \) annuity factor age \( x \)
Pooling retirement funds

Explicit Tontine

- Features
  - Individual member accounts
  - Explicit sharing rule (actuarial gain zero)
  - In general tend to $\lambda_i X_i$ (when pool big)

- Sabin (2010)
  - Only survivors earn longevity credits
  - Implicit equations
    \[ 0 = \sum_{d \neq i} \lambda_d \alpha_{i,d} X_d - \lambda_i X_i, \quad \sum_{i \neq j} \alpha_{ij} = 1 \]

- Donnelly et al. (2014)
  - Survivors and deceased member earn longevity credits
  - Explicit equation
    \[ \beta_i = \frac{\lambda_i X_i}{\sum_{d \in \text{Group}} \lambda_d X_d} \]

\[ \lambda_i \text{ force of mortality of } i\text{-th member}, \ X_i \text{ account value of } i\text{-th member}, \alpha_{i,d} \text{ share of deceased } d\text{'s fund value to } i\text{-th member}, \beta_i \text{ share of deceased member's fund value to } i\text{-th member} \]
Pooling retirement funds

Longevity credit, current work on explicit Tontines

• Longevity credits based on investment (ruin is possible)

• Extreme sensitivity of longevity credits with respect to reasonable consumption rates
  – 80% in explicit Tontine
  – Mortality table S1PMA
  – Monetary amounts, no inflation or investment risk / value amounts, investment for exact inflation exactly
  – Constant / inflation adjusted withdrawals
  – 100,000 initial wealth

• From example
  – No ruin with 4.7% initial withdrawal percentage
  – Ruin with 5% at age 94

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Key Insights

• Varying percentage in equity for a stable income
• Tontines combine investment returns with longevity credits

Key Questions

• Is there an investment puzzle? Would we benefit from target driven investment?
• Are Tontines the new annuities? How could we make it work? Maybe in a CDC framework?
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