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Key insights in decumulation strategies

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Overview

I. Introduction
II. Optimal investment strategies
III. Pooling retirement funds
IV. Questions and comments

Introduction

Since 2015, pension freedom

- Sharp decline in annuities

Battocchio et al. (2007)

- Like annuity
  - Income for life
  - Actuarial fair price
- Unlike annuity
  - One customer
  - Free to invest to create profit (Black Scholes model)
- Ruin in only 0.01% of scenarios

Investment

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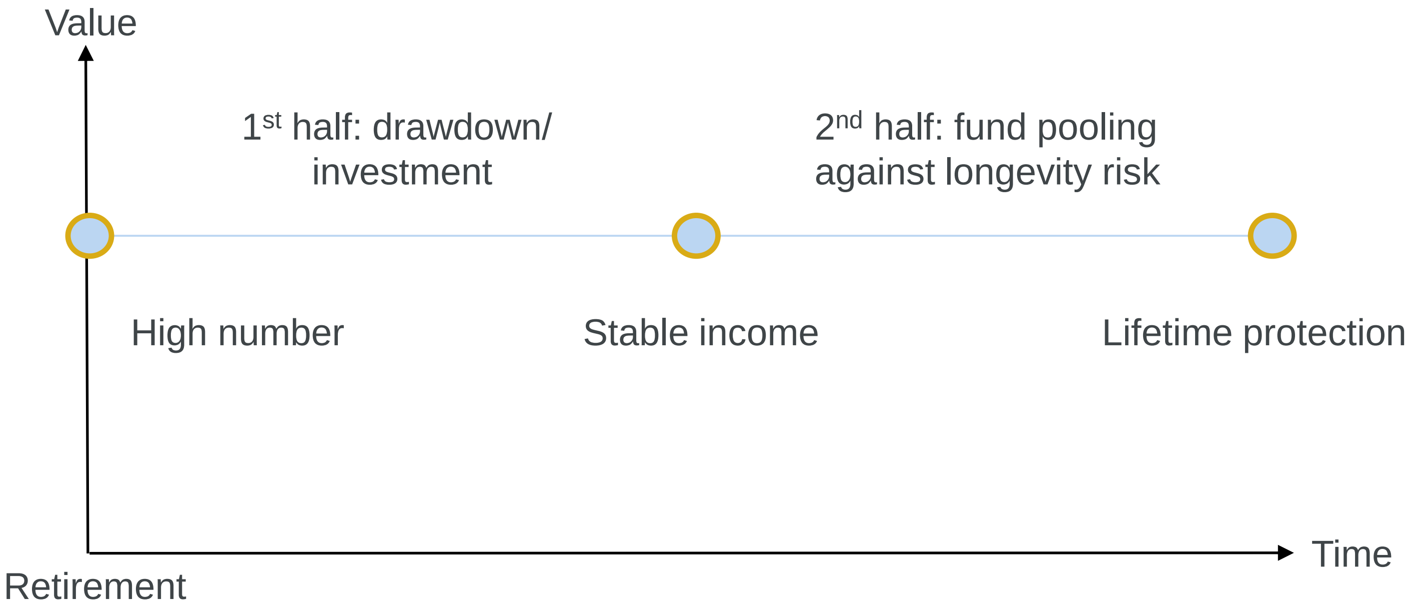
Introduction

State of the art, a good retirement product looks like …



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Optimal investment strategies

Black-Scholes model

$$dS = S * (\mu dt + \sigma dW)$$
$$dB = B * rdt$$

Mathematical description

- Max life consumption $\mathbb{E}[\int_0^T U(t,c)dt + V(T,X)]$
- Max above level $\mathbb{E}[\int_0^T U(t,c-h)dt + V(T,X-H)]$
- Max expectation min variance $\mathbb{E}[X(T)] - \gamma\, Var[X(T)]$
- Min distance from a target $\mathbb{E}[\int_0^T a(t) * (c(t) - f(t))^2 dt + b(t) * (X(t) - F(t))^2]$
- Min ruin probability $\mathbb{P}[\tau < T], \quad \tau = first\ time\ when\ X\ hits\ 0$

S Stock, μ drift, σ volatility, W noise, B Bond, r interest, $\mathbb{E}$ expectation, T maturity/lifespan, U and V utilities, c consumption, X wealth, h and H minimal levels, γ "risk aversion", Var Variance, a and b time preferences, f and F targets, $\mathbb{P}$ probability

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Optimal investment strategies

Intuitive results, quantifiable answers

- Max life consumption (e.g. Merton, 1971), min ruin probability
  - Mutual fund separation ✓Presenting equity as one thing
  - Constant mixed strategy ✓How insurance companies invest
  - Equity ↓ then Longevity risk ↑ ✓~50% in equity for lowest lifetime ruin
  - Changing consumption ×Unstable income
  - Deplete savings ✓Bequest is 2nd degree
  - Savings don't last forever ✓Annuity

GRIP 3 (Royal London, data sheet 31.07.2018)	
Equity	30%
Gilts	10%
Corporate Bonds	10%
Index Linked	10%
Property	7.5%
Absolute Return (Cash)	15%
High Yield	12.5%
Commodities	5%

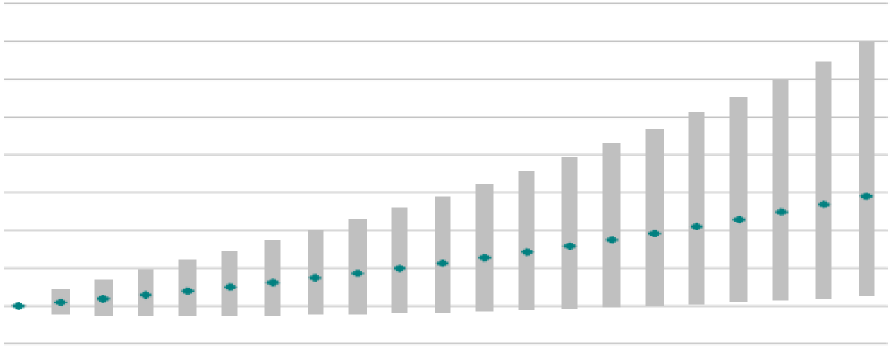
- 4% rule for a stable income (Bengen, 1994)
  - Varying success (how long? how much left?)

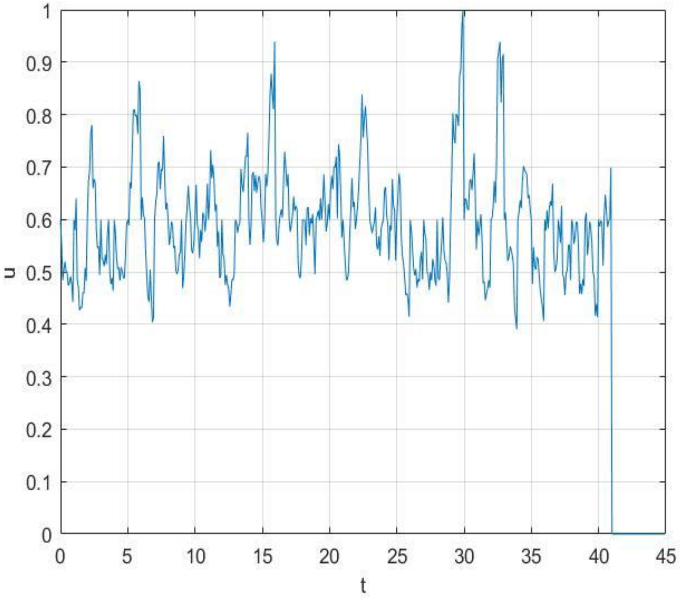
Years	3%	3.5%	4%	4.5%	5%	5.5%	6%
15	100%	100%	100%	100%	99%	97%	91%
20	100%	100%	98%	95%	85%	66%	41%
25	100%	97%	92%	77%	51%	28%	12%
30	97%	92%	75%	49%	27%	12%	5%
35	94%	81%	57%	33%	14%	6%	3%

Optimal investment strategies

Intuitive results, quantifiable answers

- Max above level, max expectation min variance, min distance from a target
  - Similar to max life consumption — Optimal solutions are robust
  - Variance increases over time — Control
  - Varying percentage — How investment firms invest
  - Stable profit — Predictable outcome





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Optimal investment strategies

Drawdown today, the 4% rule

- 50% in equity
- Inflation adjusted percentage from initial savings
- Probability to last at least …

Years	3.00%	3.50%	4.00%	4.50%	5.00%	5.50%	6.00%
15	99.98%	99.83%	99.20%	97.30%	93.14%	87.00%	77.50%
20	98.53%	95.00%	87.70%	76.47%	63.24%	49.28%	36.48%
25	91.05%	79.27%	65.48%	48.87%	34.60%	23.52%	14.92%
30	77.37%	60.04%	43.44%	29.39%	18.63%	11.13%	6.33%
35	62.14%	44.17%	28.23%	18.16%	10.53%	5.65%	2.98%

Simulated data using a Black Scholes model

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Optimal investment strategies

Max expectation min variance

- Annual optimization problem
- Inflation adjusted percentage from initial savings
- Probability to last at least …

Years	3.00%	3.50%	4.00%	4.50%	5.00%	5.50%	6.00%
15	98.95%	96.63%	94.17% -5.03	91.10%	89.60%	85.48%	77.82%
20	96.03%	90.07%	85.34% -2.36	80.35%	74.84%	63.14%	49.02%
25	91.99%	82.90%	75.49% +10.01	66.26%	46.09%	23.35%	12.63%
30	87.19% +9.82	75.03% +14.99	61.94% +18.50	37.28% +7.89	7.67% -10.96	1.48% -9.65	0.48% -5.85
35	78.75%	59.83%	30.65% +2.42	3.93%	0.15%	0%	0%

Simulated data using a Black Scholes model

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Optimal investment strategies

Max expectation min variance

- Annual optimization problem
- Inflation adjusted percentage from initial savings
- Probability to last at least …

Years	3.00%	3.50%	4.00%
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20	96.03%	90.07%	85.34% -2.36
25	91.99%	82.90%	75.49% +10.01
30	87.19% +9.82	75.03% +14.99	61.94% +18.50
35	78.75%	59.83%	30.65% +2.42

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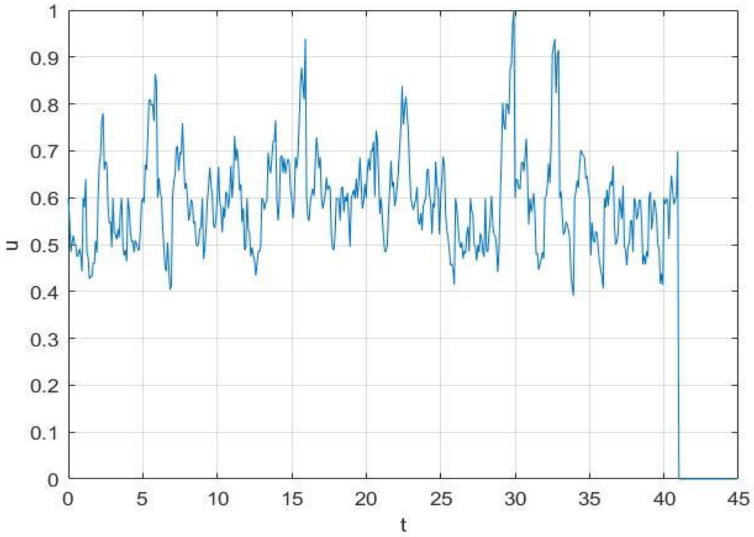

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Optimal investment strategies

Undesirable features

– Difficult to communicate	Car mechanic analogy
– Sensitive to parameters	Indication for wrong set-up
– Non-explicit	Explicit in idealistic situation, indication for outcome
– No constraints	Numerical solutions

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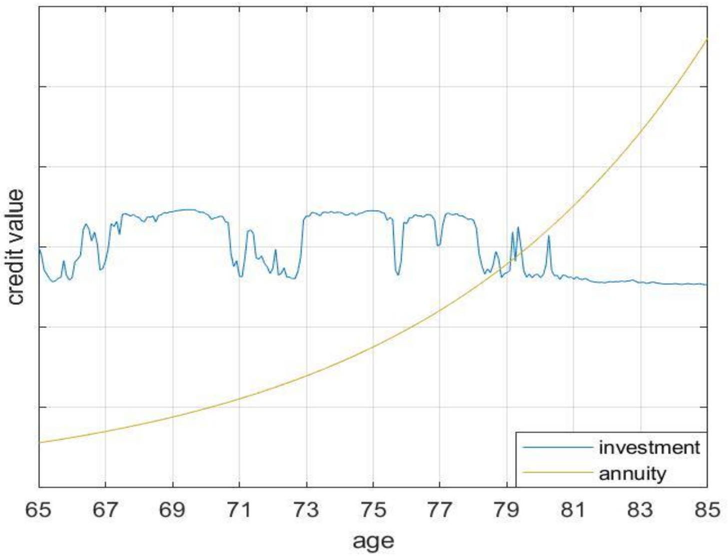
Pooling retirement funds

Annuity

- Guaranteed income, in return for savings
- Actuarial fair cost

– No investment	Low value at retirement
– Mortality driven price	Age ~80 longevity credits outweigh investments
– Not at all times favourable	Optimal stopping

- State of the art
  - Investment/drawdown opposite to annuity
  - Annuity best option at high ages
  - Delay full annuitization (phase transition, delayed annuities)



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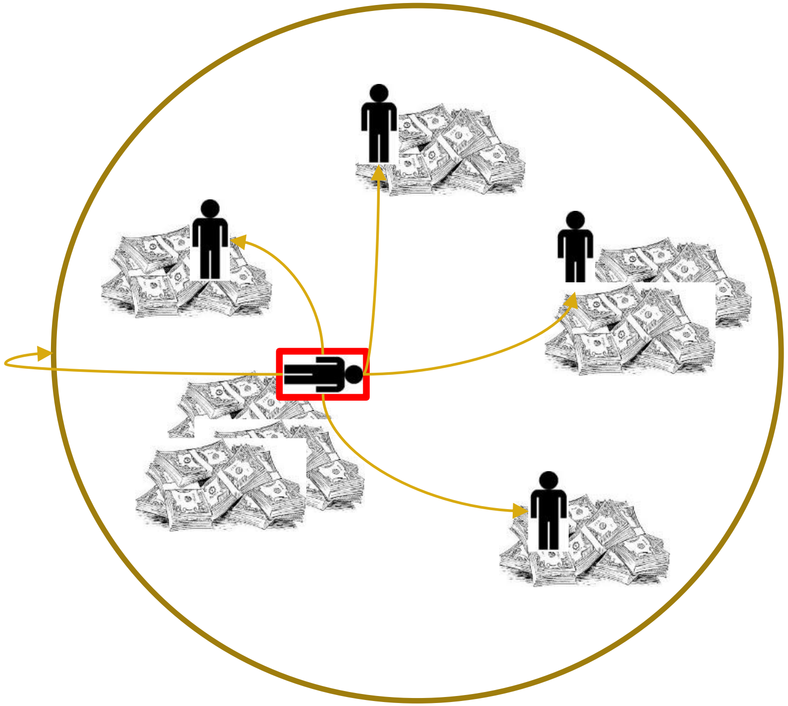
Pooling retirement funds

Modern Tontine

- No guaranteed income, irreversible decision
- No cost (besides fees, taxes, …)
  - Investment — High value from the beginning
  - Performance/experienced mortality driven — Fluctuation
- Main ideas
  - Investment in addition to longevity credits
  - Beneficial at all ages (ignoring bequest motives)



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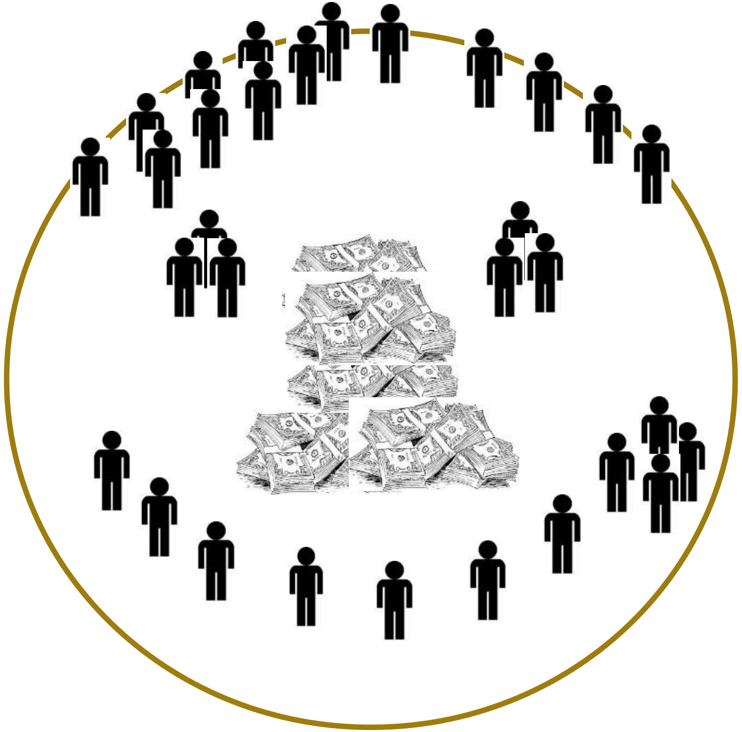
Pooling retirement funds

Implicit Tontine

- Features
  - One pool account
  - Influenced by experienced investment (changing fund value)
  - Influenced by experienced mortality (changing income)
- Group Self-Annuitization by Piggott et al. (2005)
  - Same aged group
  - Income calculated like annuity $$c_x = \frac{1}{l_x^*} \frac{F(x-65)}{\ddot{a}_x}$$

c_x income at age x, $F(x-65)$ fund value after $x-65$ years, l_x^* count of survivors of age x, $\ddot{a}_x$ annuity factor age x



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Pooling retirement funds

Explicit Tontine

- Features
  - Individual member accounts
  - Explicit sharing rule (actuarial gain zero)
  - In general tend to $\lambda_i X_i$ (when pool big)

- Sabin (2010)
  - Only survivors earn longevity credits
  - Implicit equations

$$0 = \sum_{d \neq i} \lambda_d \alpha_{i,d} X_d - \lambda_i X_i, \quad \sum_{i \neq j} \alpha_{ij} = 1$$

- Donnelly et al. (2014)
  - Survivors and deceased member earn longevity credits
  - Explicit equation $\beta_i = \dfrac{\lambda_i X_i}{\sum_{d \in Group} \lambda_d X_d}$

λ_i force of mortality of i-th member, X_i account value of i-th member, $\alpha_{i,d}$ share of deceased d's fund value to i-th member, β_i share of deceased member's fund value to i-th member

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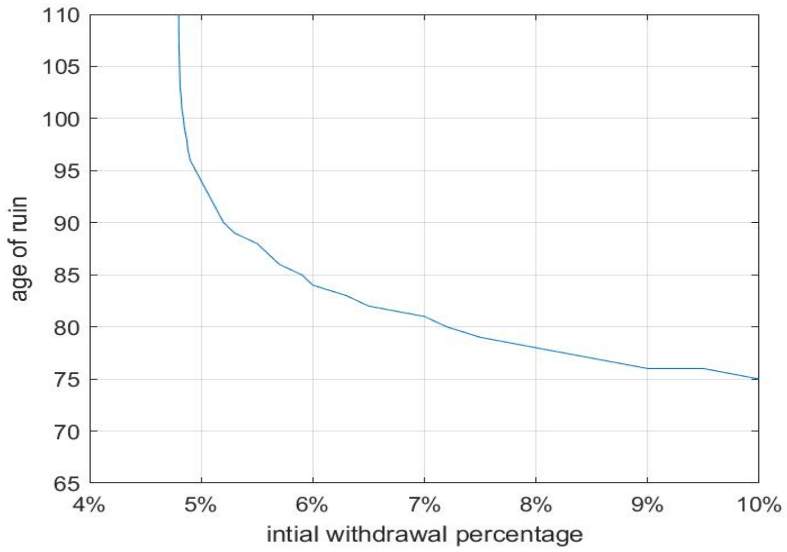
Pooling retirement funds

Longevity credit, current work on explicit Tontines

- Longevity credits based on investment (ruin is possible)
- Extreme sensitivity of longevity credits with respect to reasonable consumption rates
  - 80% in explicit Tontine
  - Mortality table S1PMA
  - Monetary amounts, no inflation or investment risk / value amounts, investment for exact inflation exactly
  - Constant / inflation adjusted withdrawals
  - 100,000 initial wealth
- From example
  - No ruin with 4.7% initial withdrawal percentage
  - Ruin with 5% at age 94



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Key Insights

- Varying percentage in equity for a stable income
- Tontines combine investment returns with longevity credits

Key Questions

- Is there an investment puzzle? Would we benefit from target driven investment?
- Are Tontines the new annuities? How could we make it work? Maybe in a CDC framework?

Questions

Comments

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