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Simplifying retirement: Aligning retirement goals and outcomes

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Joint work with William Lim (ANU, Canberra) and Gaurav Khemka (ANU).

The '**Minimising Longevity and Investment Risk while Optimising Future Pension Plans**' research programme is being funded by the Actuarial Research Centre.

5 February 2019

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Overview

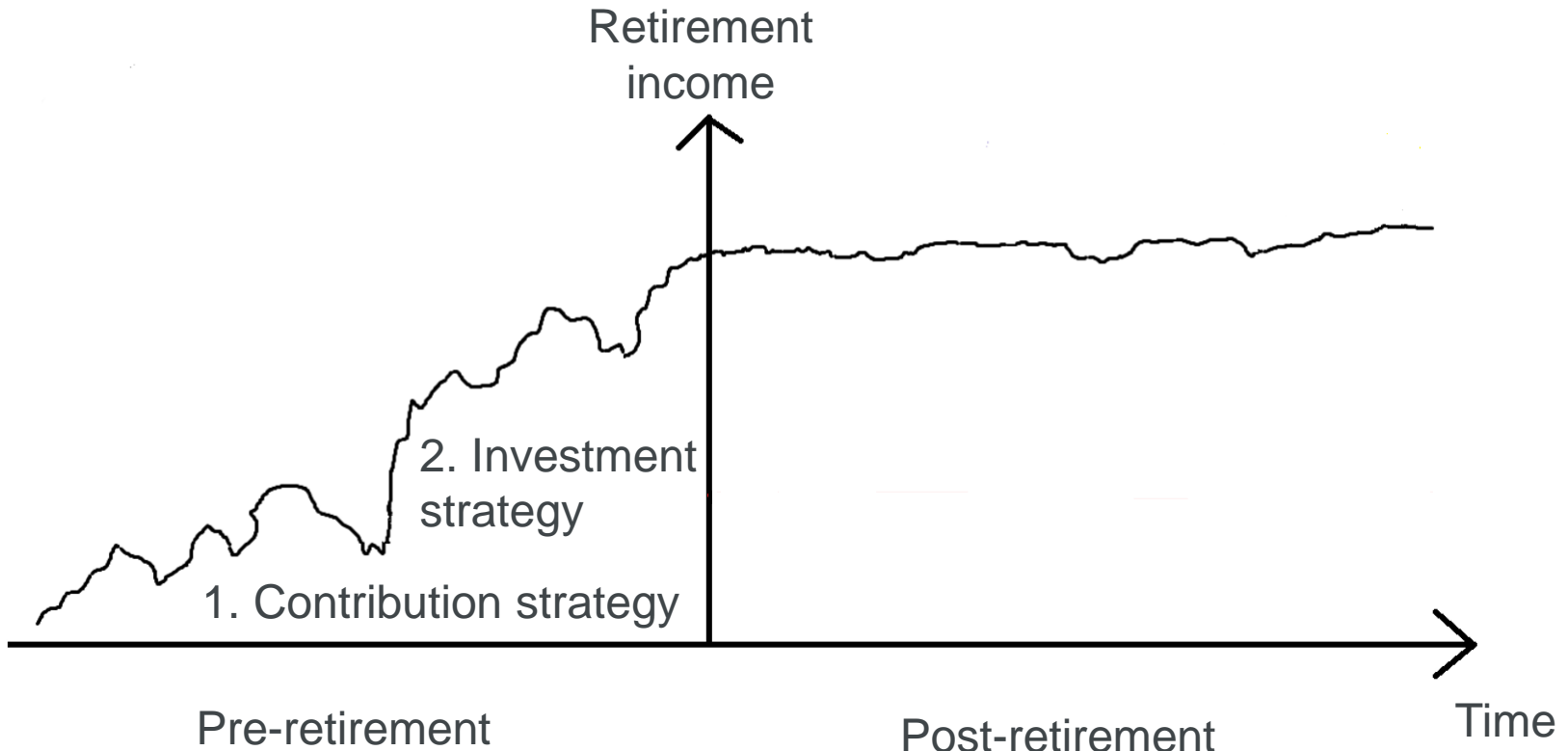
- Motivation
- Main conclusion
- Set-up
- Investment strategies considered and results
- Summary

Focus on defined contribution (DC) plans

- A lot of people affected, very important!
- Automatic enrolment into a “workplace pension” (DC plan) for most UK employees since Jan 2017.
- 84% of UK staff (9.5m) in workplace pension at March 2018.
- £90bn saved during 2017.
- Estimated 14.5m workers with total savings £682bn by 2035.
- Total minimum contribution rate 8% of salary from April 2019.
- Vast majority of savers are in default DC investment funds (>90%).



Typical current DC situation



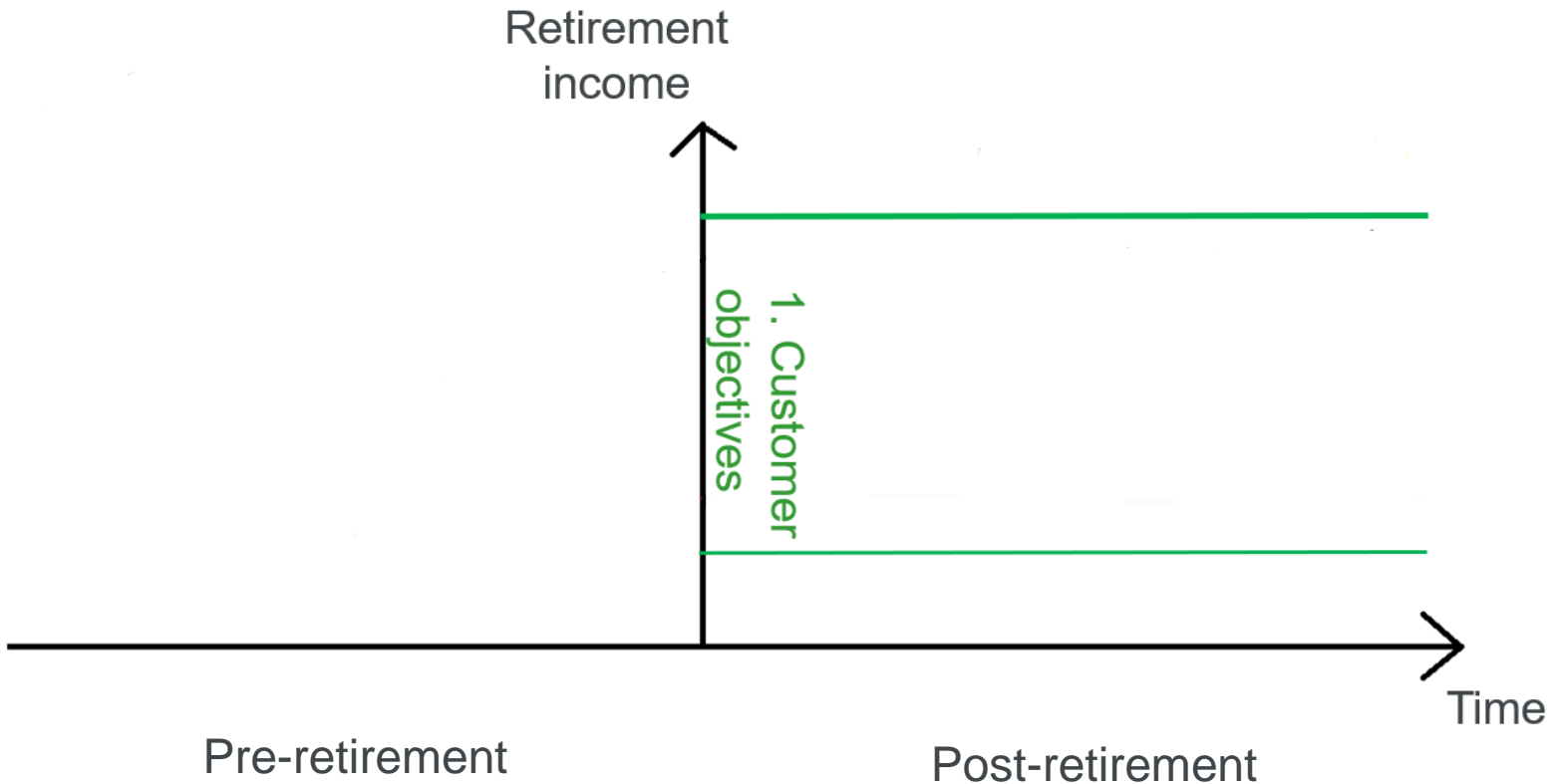
Default DC investment funds

- Typically, lifestyle funds are the default DC fund.
 - e.g. allocate 70% to equities for 20 years,
 - Then e.g. switch to 20% equities gradually over 10 years to retirement, investing released funds in bonds/cash.
- Some Diversified Growth Funds.
 - e.g. lower equity allocation, but
 - Allocation to real estate, commodities, infrastructure, etc.

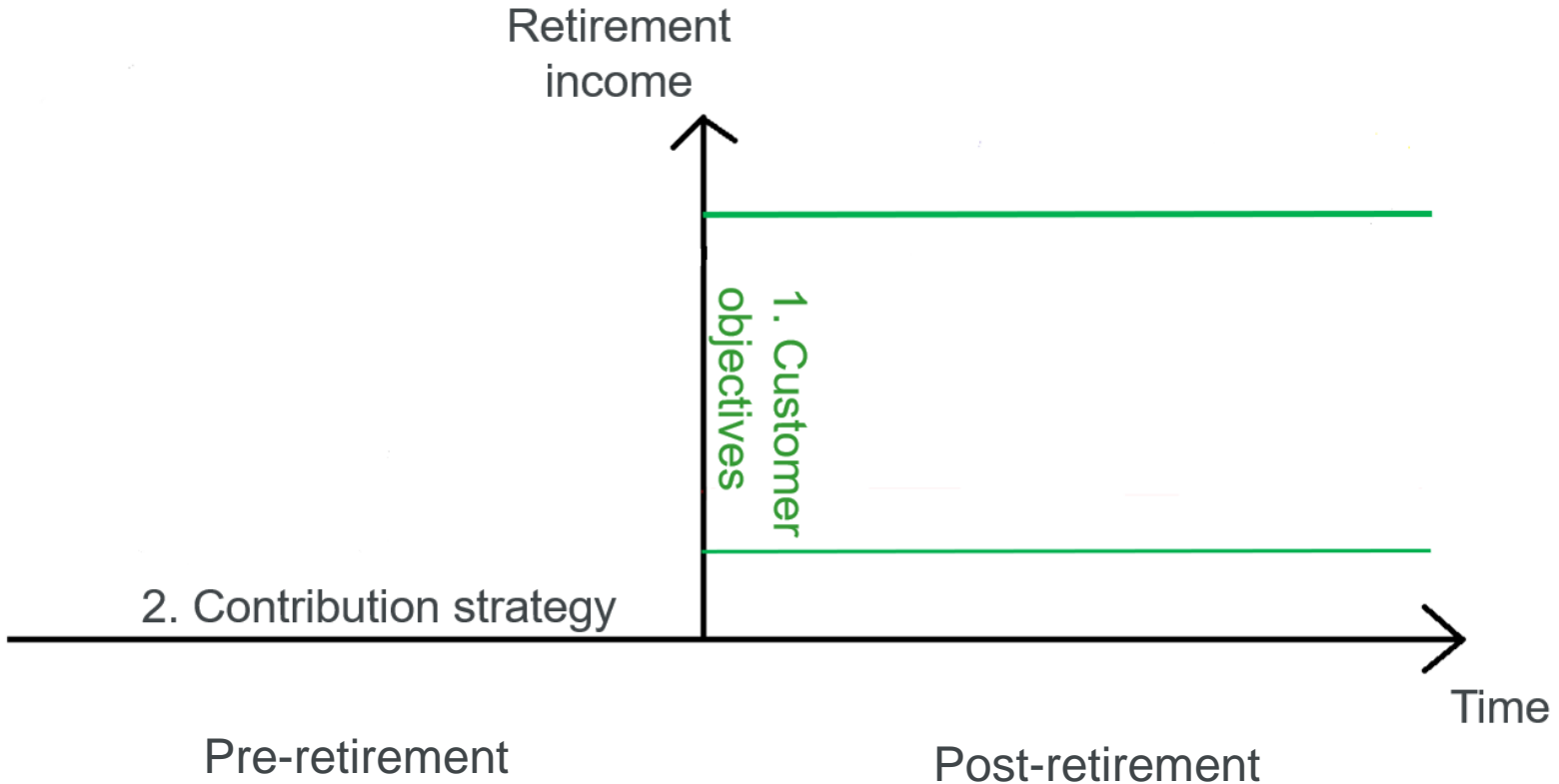
Put retirement objectives at the centre

- Idea: investor sets their retirement goals, and
- Investor gets more certainty about retirement income.
- Motivation from Robert C. Merton (2014) “The Crisis in Retirement Planning”, *Harvard Business Review*.

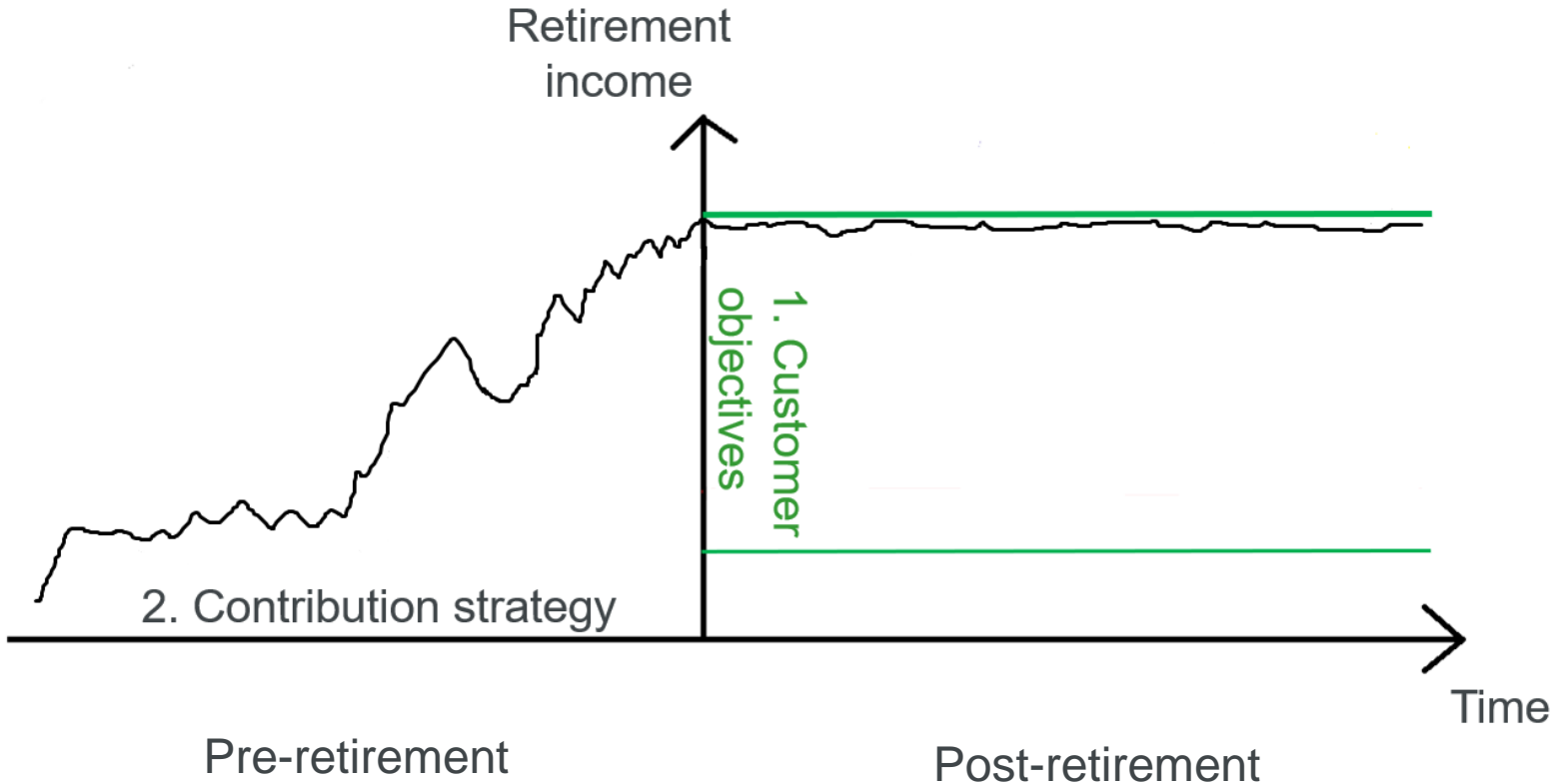
Put customer objectives at the centre



Put customer objectives at the centre



Put customer objectives at the centre



What did we do?

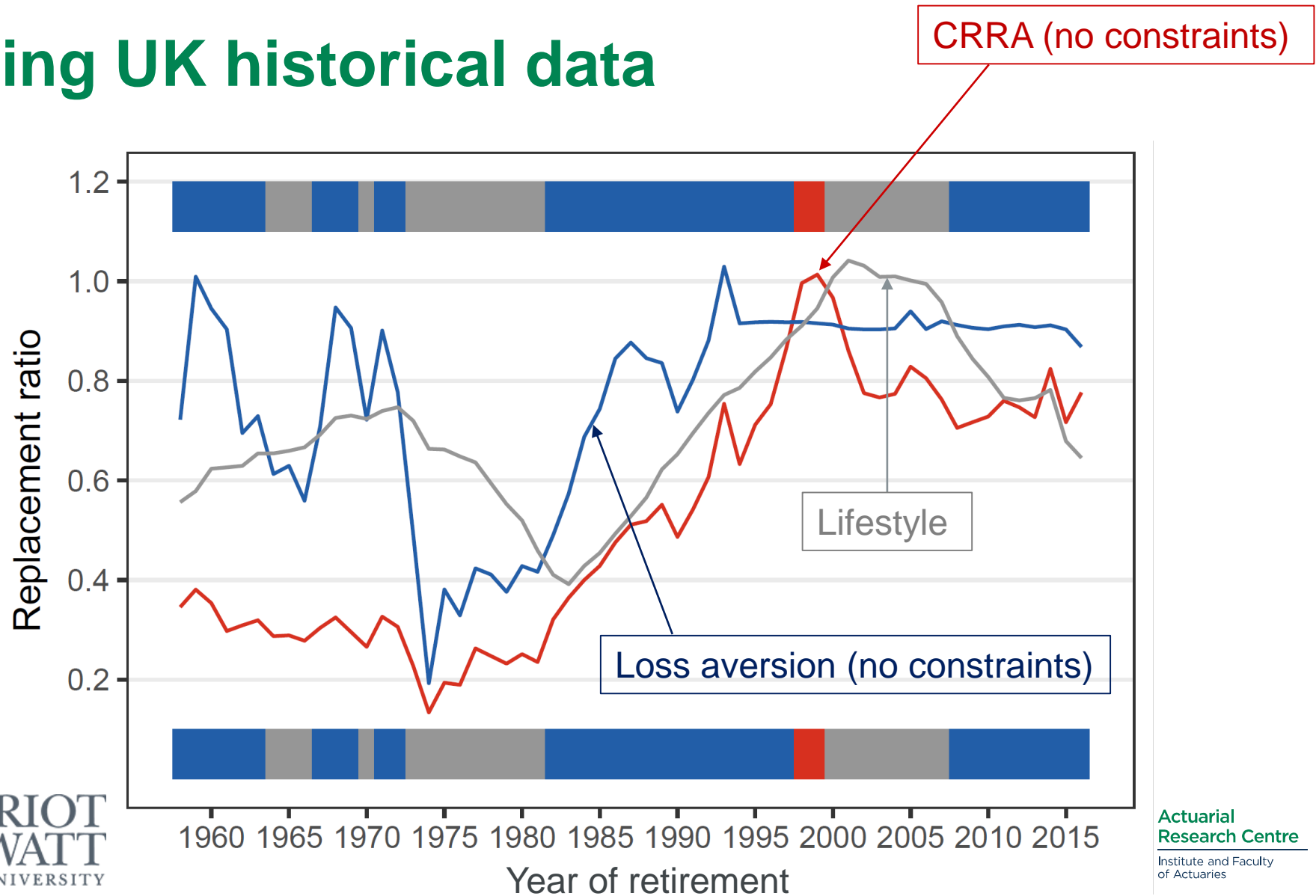
- Aim: give investor more certainty about final retirement outcome.
- Which optimization problem? Should you impose terminal wealth constraints?
- Looked at CRRA (power utility) and loss aversion utility,
- Found: terminal distribution more appealing under loss aversion with no constraints.
- Also looked at a lifestyle strategy, switching from 100% equity to 100% IL bonds over 10 years to retirement.



What did we do?

- Investor with
 - 40 years until retirement;
 - Investing 18.5% of salary each year;
 - Buys an inflation-indexed life annuity at retirement;
 - No short-selling, annual re-balancing.
- Can invest in FI bonds, IL bonds and equities.
- Replacement ratio at retirement
 - = real annuity income/salary in year before retirement.

Using UK historical data



Financial market

Either calibrate to UK data:

- Inflation index $\frac{dI(t)}{I(t)} = \mu_I dt + \sigma_I dW_1(t)$
- Nominal bond (FI) price $\frac{dS_0(t)}{S_0(t)} = r_N dt$
- Inflation-linked bond (ILB) price $\frac{dS_1(t)}{S_1(t)} = r_R dt + \frac{dI(t)}{I(t)}$
- Risky stock price $\frac{dS_2(t)}{S_2(t)} = \mu_2 dt + \sigma_{21} dW_1(t) + \sigma_{22} dW_2(t)$

Or use historical UK real return data

(for ILB: pre-1981, use r_R value from above model).



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1. CRRA utility

- Maximize the expected utility of real wealth at retirement.
- Power utility function, $U(x) = x^\gamma / \gamma$.
- Determine investment strategy that maximizes $\mathbb{E}U(\textit{Real wealth at retirement})$.
- Not the same solution as maximizing nominal wealth, contrary to Zhang (2012) claim (we prove analytical solution).
- But we do a numerical implementation via dynamic programming (indeed, for all our results).

1. CRRA utility

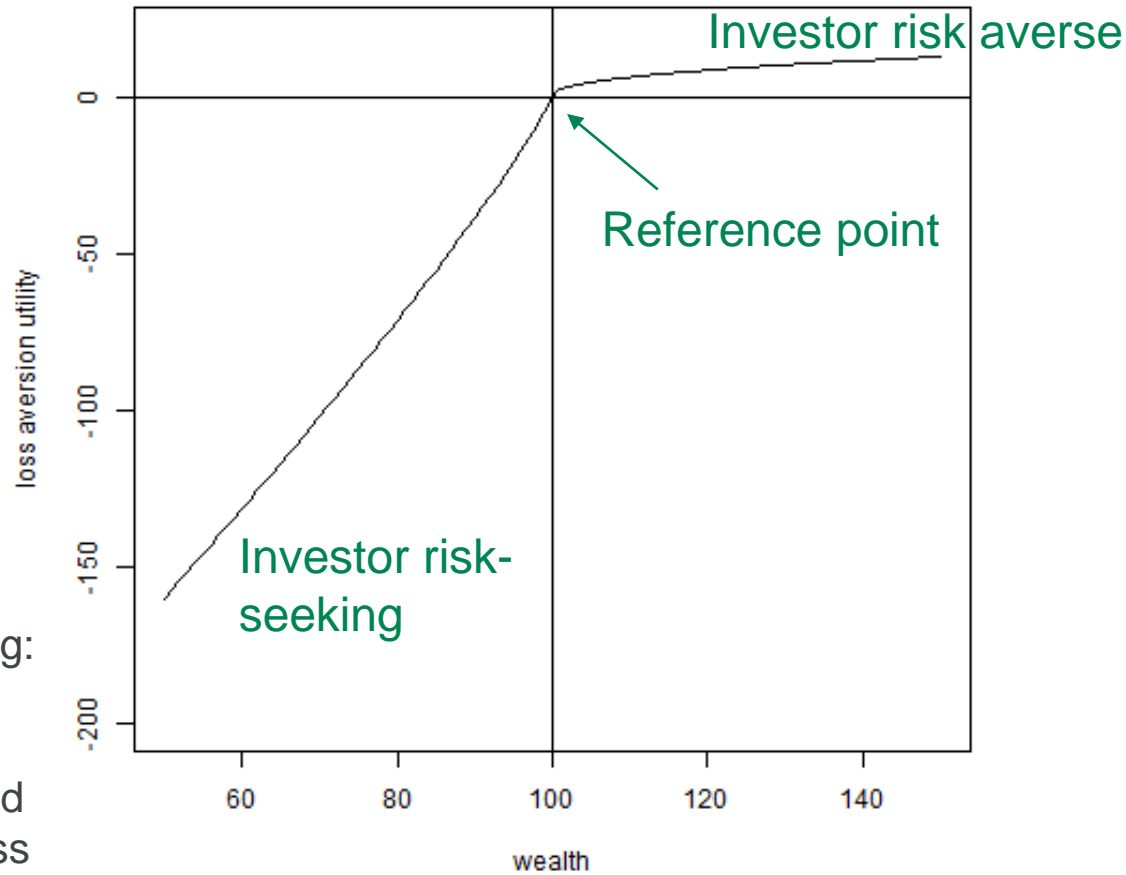


2. Loss aversion utility background

- Maximize the expected utility of real wealth at retirement.
- Loss aversion approach: people don't like losses.
- Reference point (target), e.g. 100, about which to measure gains/losses.
 - e.g. utility **gain** from obtaining 101 is **2**,
 - But utility **loss** from obtaining 99 is **5**.

2. Loss aversion utility plot

Loss aversion utility with reference point 100



Risk-seeking:
will risk a
bigger loss
when offered
a certain loss

Risk averse:
will not risk a
bigger gain
when offered
a certain gain



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2. Loss aversion utility

- Loss aversion utility approach.
- Extension of Blake et al. (2013) – annually updated target-based approach – to include inflation and 3 assets.
- Aiming for 90% replacement ratio.
- Numerical solution.

2. Loss aversion utility

- Annual interim targets + Retirement day target.

$$U_t(F_t) = \left\{ \begin{array}{ll} \frac{(F_t - Target(t))^{0.44}}{0.44} & \text{if } F_t \geq Target(t) \\ -2.25 \times \frac{(Target(t) - F_t)^{0.88}}{0.44} & \text{otherwise} \end{array} \right\}$$

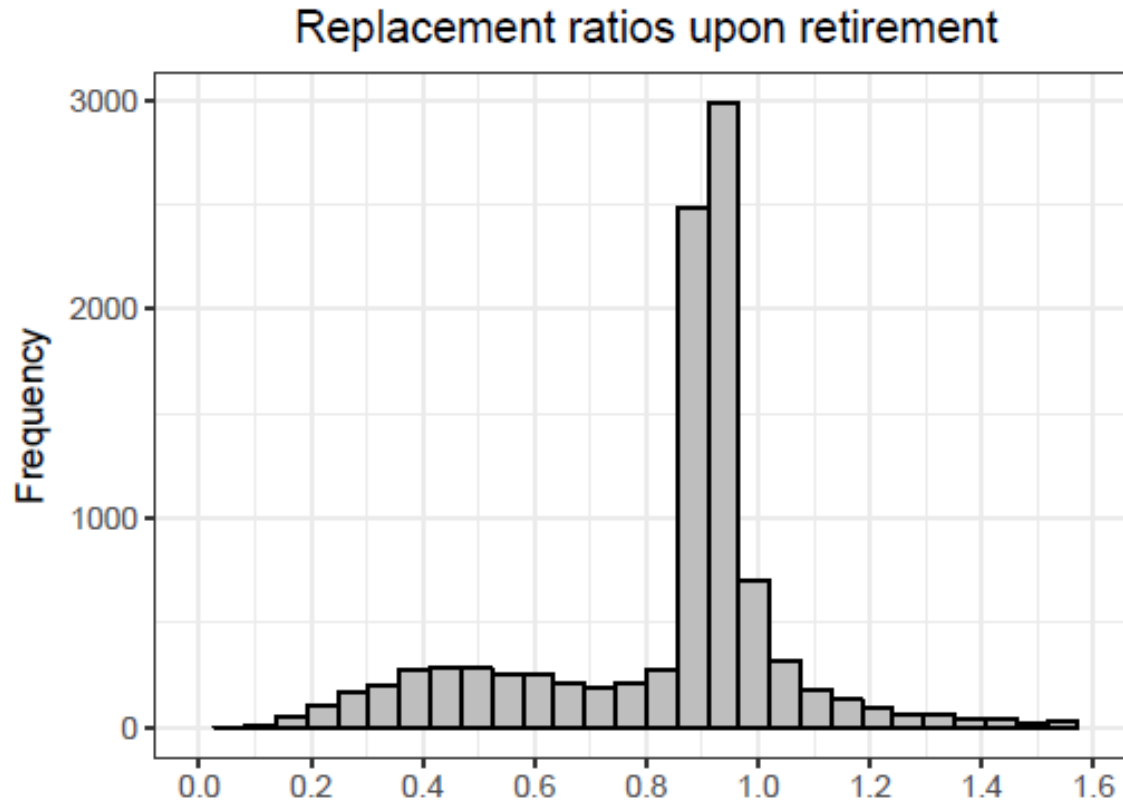
- Backward recursion to maximize for one-year buy-and-hold strategies:

$$0.5 \times U_t(F_t) + \beta \mathbb{E}_t(V_{t+1}(F_{t+1}))$$

- β^t is weight applied to utility of year t target
- 0.5 applies additionally to utility of interim targets: increase importance of retirement day target's utility.

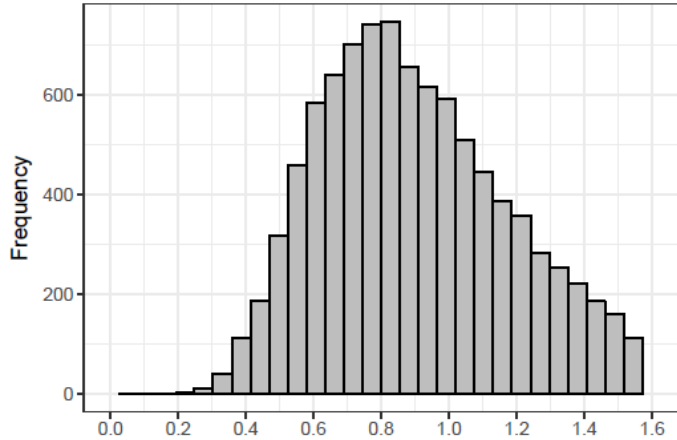


2. Loss aversion utility



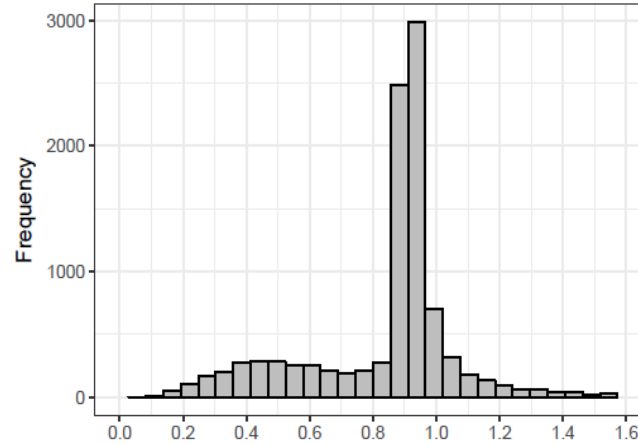
CRRA vs Loss aversion vs Lifestyle

Replacement ratios upon retirement



CRRA utility

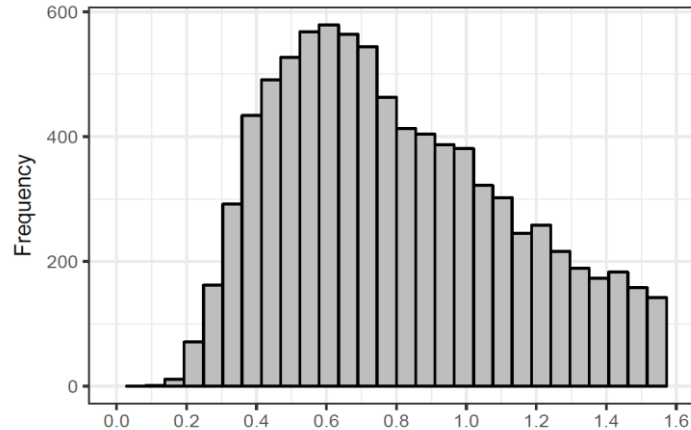
Replacement ratios upon retirement



Loss aversion utility

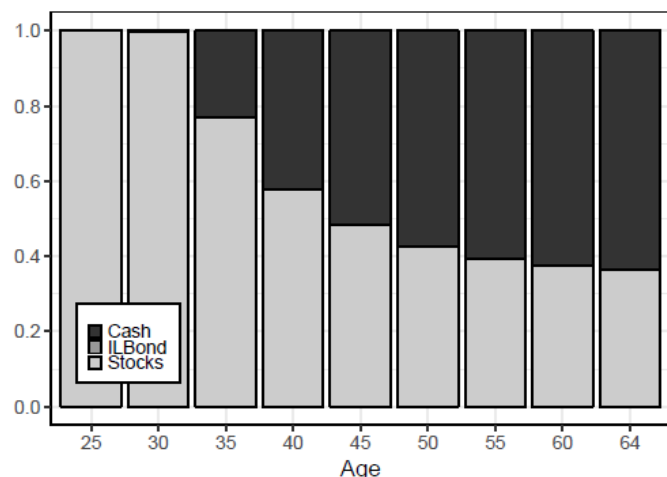
Replacement ratios upon retirement

Lifestyle:



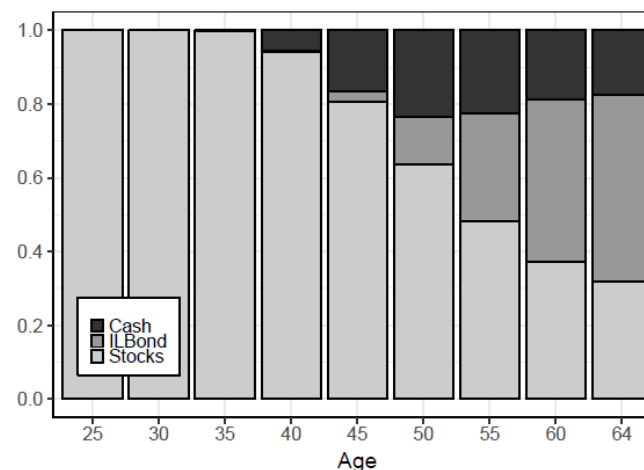
CRRA vs Loss aversion mean optimal investment strategy

Optimal asset allocation



CRRA utility

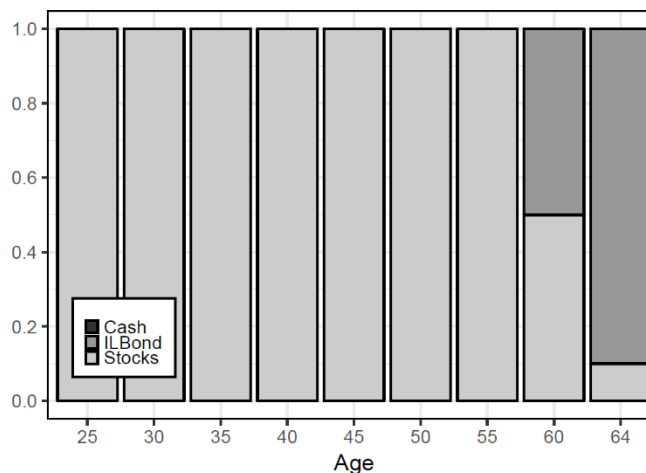
Optimal asset allocation



Loss aversion utility

Lifestyle:

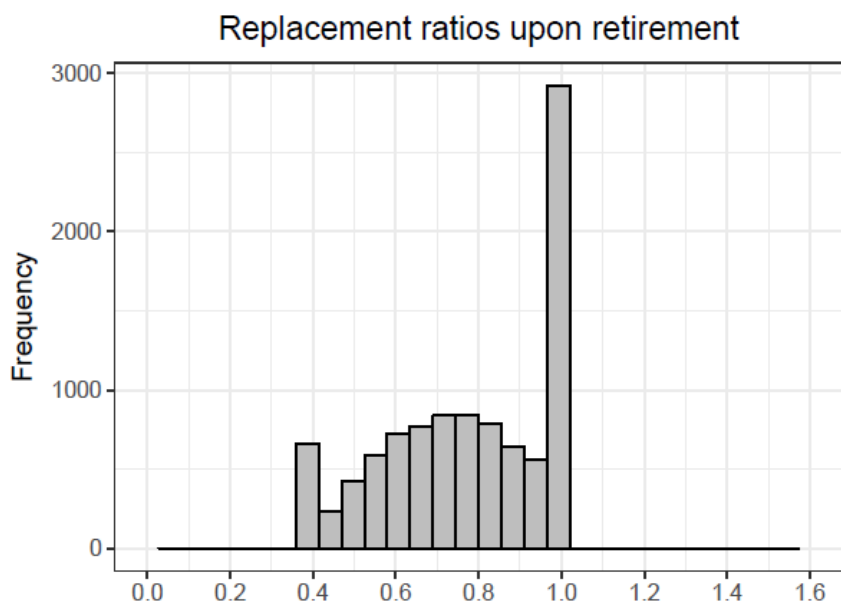
Optimal asset allocation



Added in terminal (i.e. retirement day) wealth constraints

- Constrain wealth at retirement to lie between:
 - Lower constraint \equiv 40% replacement ratio,
 - Upper constraint \equiv 100% replacement ratio,
 - For CRRA and loss aversion utility functions
- For CRRA, extension of Donnelly et al (2018) to include inflation and 3 assets.

3. Utility with terminal constraints (synthetic options available)



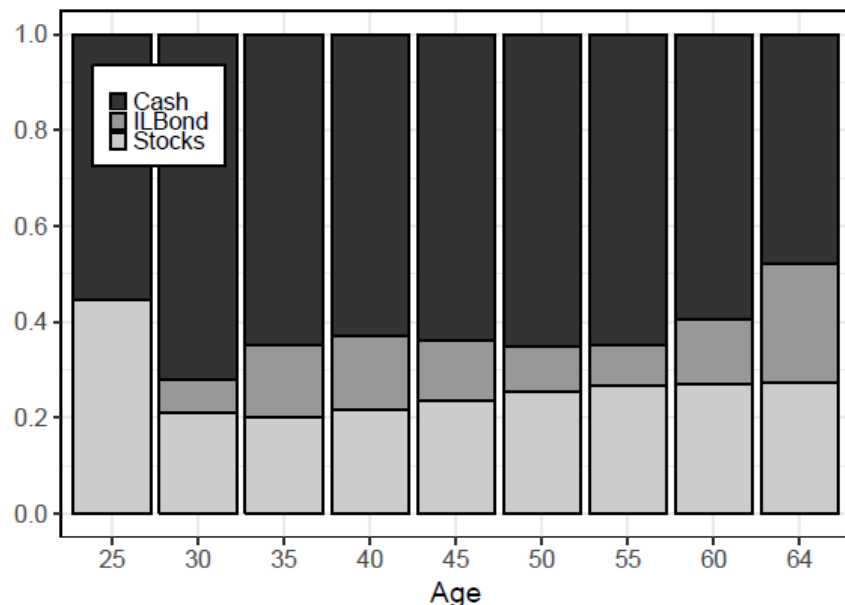
CRRA utility with terminal constraints



Loss aversion utility with terminal constraints

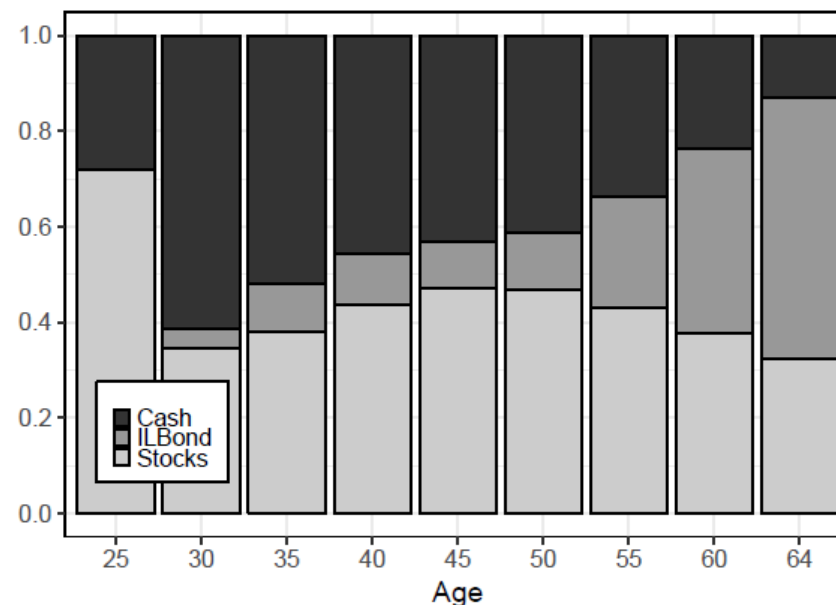
3. Utility with terminal constraints (synthetic options available)

Optimal asset allocation



CRRA utility with terminal constraints

Optimal asset allocation



Loss aversion utility with terminal constraints

Overall comparison

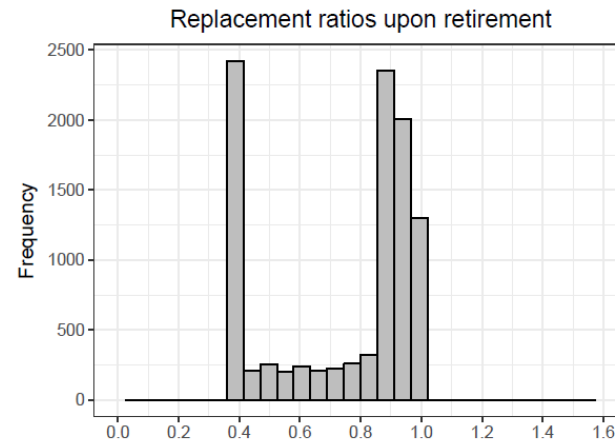
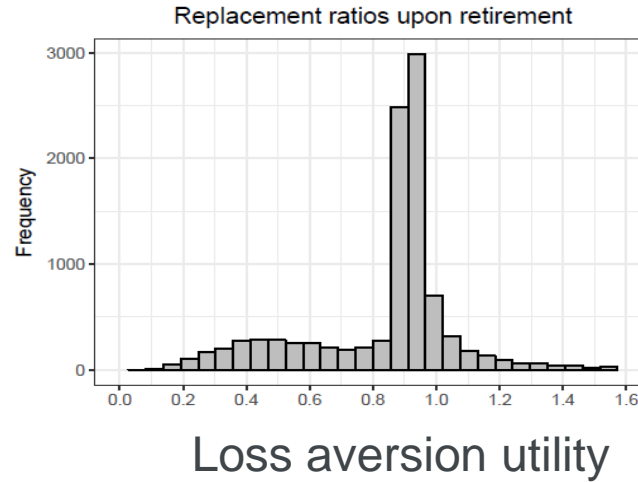
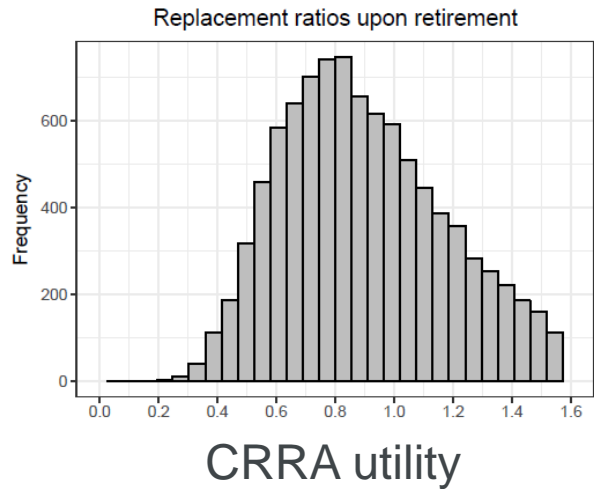
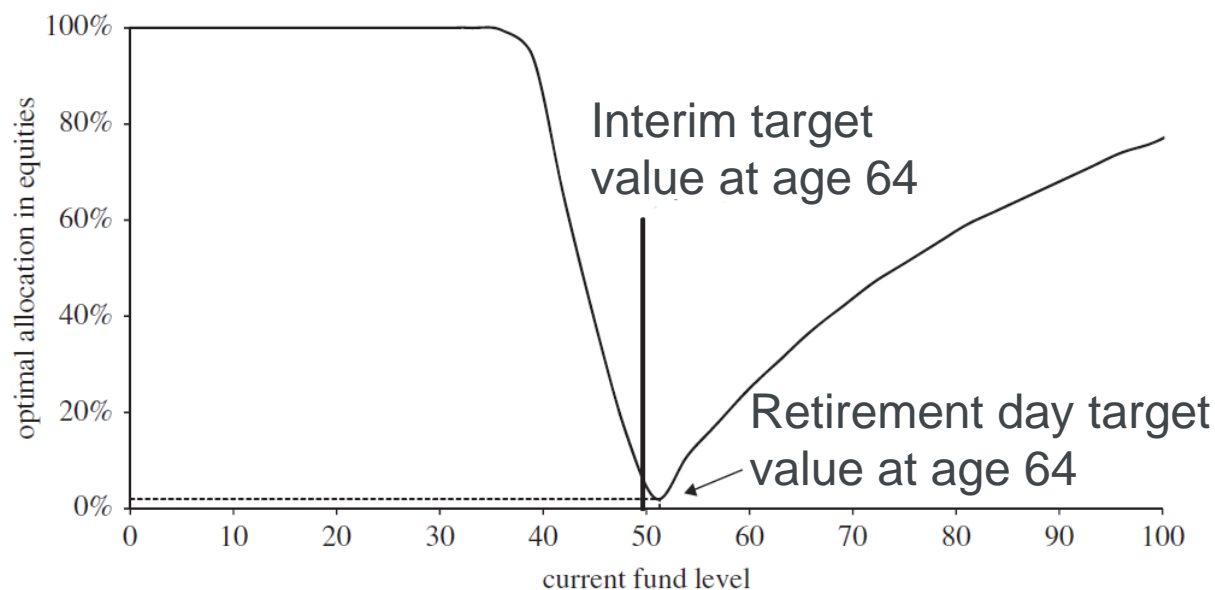


Figure 4 (with labels removed) from Blake et al (2013) – bonds and equities only

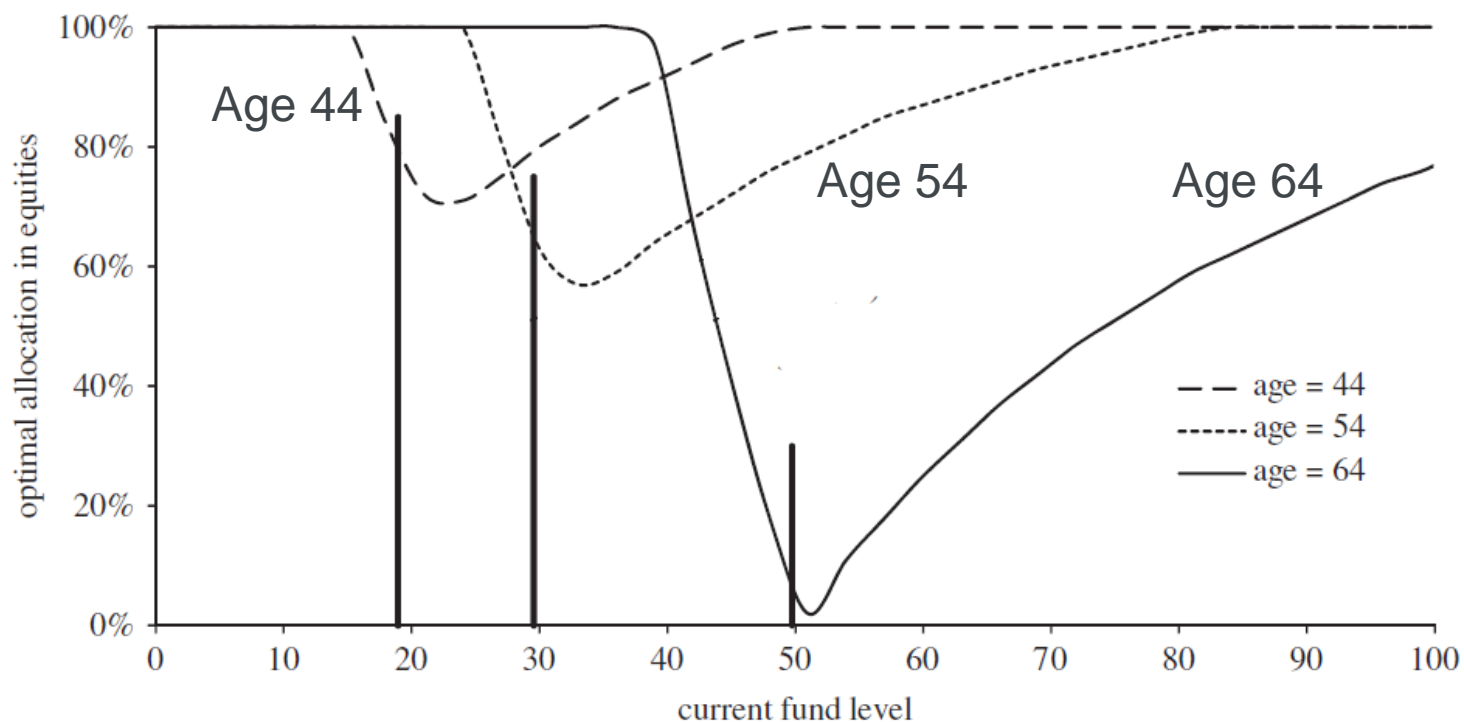
Loss aversion utility investment strategy at age 64, one year before retirement



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Figure 5 (with labels removed) from Blake et al (2013) – bonds and equities only



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Overall comparison (ranked by 50% quantile of replacement ratio)

Replacement Ratios	Quantiles			Mean
	10%	50%	90%	
Loss aversion unconstrained	45.4%	90.9%	103.3%	85.4%
Loss aversion constrained	40.0%	90.1%	99.8%	74.9%
Lifestyle	56.5%	89.2%	144.7%	96.5%
CRRA unconstrained	41.9%	83.8%	184.3%	103.0%
CRRA constrained	48.4%	79.3%	100.0%	77.7%



Overall comparison (ranked by Certainty Equivalent Replacement Ratio)

	Certainty equivalent RR	Prob[RR<40%]	Expected[RR RR<40%]
Loss aversion unconstrained	0.816	7.3%	30.4%
CRRA unconstrained	0.788	1.3%	35.5%
Lifestyle with CERR calculated under loss aversion utility	0.749	8.6%	32.9%
Loss aversion constrained	0.749	0%	N/A
CRRA constrained	0.685	0%	N/A
Lifestyle with CERR calculated under CRRA utility	0.638	8.6%	32.9%

Certainty equivalent RR (CERR) satisfies:

$$U(\text{CERR} \times \text{Salary at age 64} \times \text{annuity payable from age 65}) = EU(\text{Fund value at age 65}).$$



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Summary

- Extended formulations of Blake et al (2013) and Donnelly et al (2018) to include inflation; 3 assets; with and without terminal wealth constraints.
- Distribution is significantly different under loss aversion compared to power utility (also Blake et al 2013).
- Both loss aversion utility and classical lifestyle do well.
- Terminal wealth constraints don't add significant benefit.



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The Actuarial Research Centre (ARC) is the Institute and Faculty of Actuaries' (IFoA) network of actuarial researchers around the world.

The ARC seeks to deliver cutting-edge research programmes that address some of the significant, global challenges in actuarial science, through a partnership of the actuarial profession, the academic community and practitioners.

The **'Minimising Longevity and Investment Risk while Optimising Future Pension Plans'** research programme is being funded by the ARC.

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Motivation slides

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Loss aversion utility

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CRRA utilit

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Questions

Comments

The views expressed in this presentation are those of the presenter.



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