Multi-population mortality models

Fitting, Forecasting, Comparison and Applications

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HERIOT WATT UNIVERSITY

Actuarial Research Centre
Institute and Faculty of Actuaries

Motivation and purpose

- Forecasting joint mortality rates
  - The projections are correlated
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  - Annuities
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  - Reserving
  - Diversifying portfolio risk
  - SCR estimates
- Hedging risk
  - Q-forward contracts
  - S-forward contracts

Data specifics

Six populations considered:

		Exposure to risk at the age of 60 in 2010	
i	Country	Male	Female
1	Austria	47023	49526
2	Belgium	65344	66434
3	Czech Republic	71575	71575
4	Denmark	34420	35132
5	Sweden	59759	59742
6	Switzerland	46527	47078

Data range: 30 ages (from 60 to 89 years old) and 50 calendar years (from 1961 up to 2010)

Importance of data chosen

- Closely related countries
- Similar size and features

Multi-population mortality models

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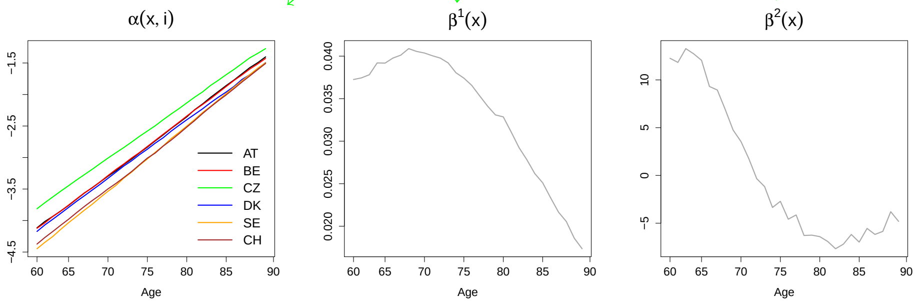
- Model 1 and Model 2 - special cases of Model 0
- Identifiability issue - *Constraints implementation*
- Parameter estimation - *Maximum likelihood estimation*
- Model selection criterion - *BIC*

	BIC value	Rank
Model 0	100043.08	(2)
Model 1	101628.38	(4)
Model 2	101525.12	(3)
Model 3	**99805.38**	**(1)**

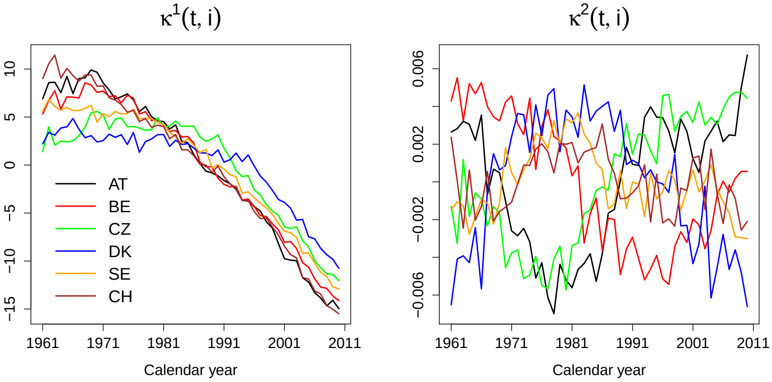
Estimated age dependent parameters in Model 3

$$\log m(x,t,i) = \boxed{\alpha(x,i)} + \boxed{\beta^1(x)}\kappa^1(t,i) + \boxed{\beta^2(x)}\kappa^2(t,i)$$



Estimated time dependent parameters in Model 3

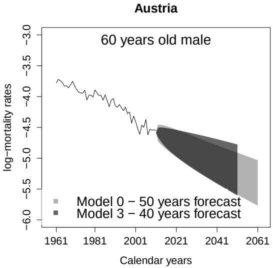
$$\log m(x,t,i) = \alpha(x,i) + \beta^1(x)\boxed{\kappa^1(t,i)} + \beta^2(x)\boxed{\kappa^2(t,i)}$$

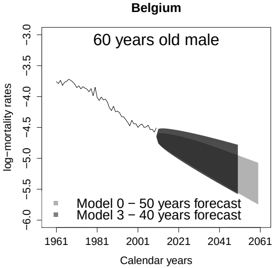


Forecasting joint mortality rates

Points to consider

- Shape of the estimated parameters
- Common time dependent parameters
  - Single variate time series processes
- Country specific time dependent parameters
  - Multivariate time series processes
- Projected joint mortality forecasts





Robustness of the estimated parameters

- Full data set: 1961 - 2010
- Reduced data set: 1961 - 2000
  - One step MLE
  - Two step MLE

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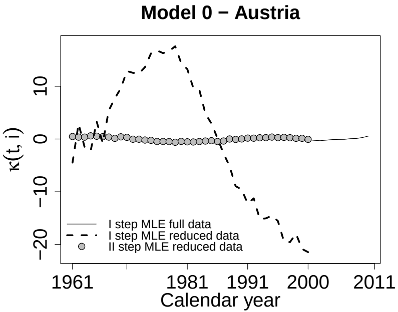
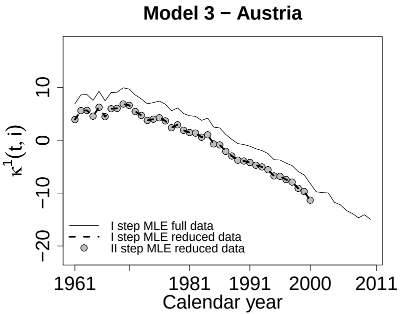
- Models robustness
  - Model 0 suffers from multi maxima problems
  - Model 3 is very robust
- Conclusion
  - Model 3 ranks as the best model

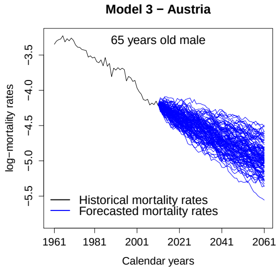


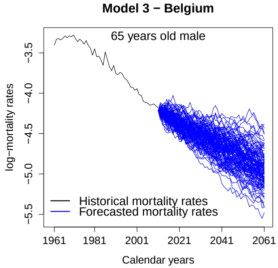


Multi-population mortality models applications

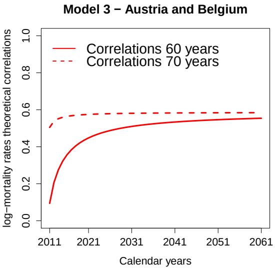
$$\log m(x,t,i) = \alpha(x,i) + \beta^1(x)\kappa^1(t,i) + \beta^2(x)\kappa^2(t,i)$$

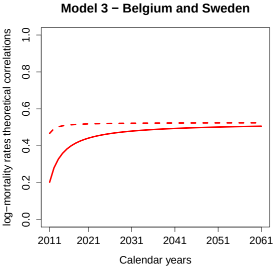
Joint forecasts





Theoretical correlations





Multi-population mortality models applications

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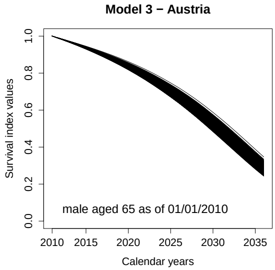
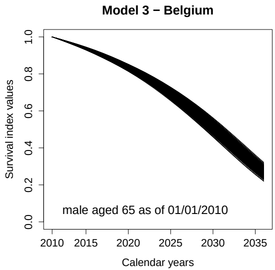
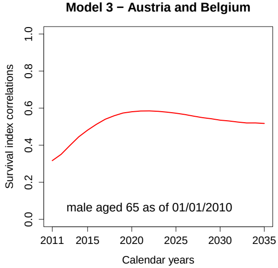
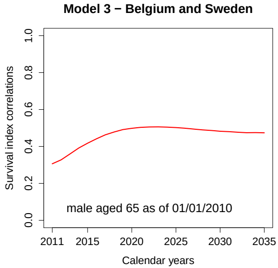
Survival index values





Empirical correlations





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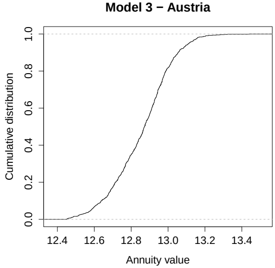
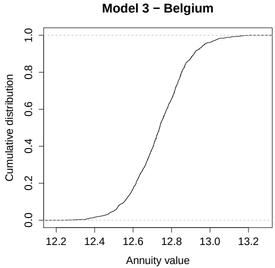
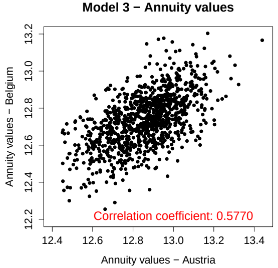
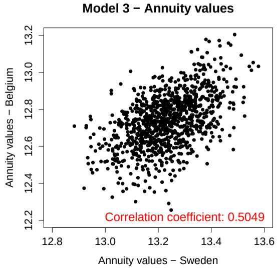
Annuity ECDF





Annuity scatter plots





Multi-population mortality models applications

Other applications of multi-population mortality models include

- Reserving
- Assess benefits of diversification across countries
- SCR values: single and multi country
- Hedging portfolio risk
  - Q-forward contracts
  - S-forward contracts

http://www.macs.hw.ac.uk/ andrewc/papers/Enchev2015.pdf

Paper link