Measuring and Comparing Diversification

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Agenda

- Preliminaries

1. Single Measures of Diversification
   - Examples: Diversification Benefit, Diversification Score
   - Pitfalls
   - Can single measures be used? How can they be used?
2. Factors to consider when comparing Dependencies
3. Examples of alternative comparisons

Preliminaries

Materiality of Correlations

$$\frac{\Delta C}{C} \approx \frac{C_i}{C} \cdot \frac{C_j}{C} \cdot \Delta\rho_{ij} = a_i \cdot a_j \cdot \Delta\rho_{ij}$$

- Assume: Lognormal loss distributions

	A	B	C	D
mean	100	40	200	150
st. dev	30	20	30	40
99.5th %ile	204	121	290	285
Ci=99.5th - mean	104	81	90	135
alpha	0.37	0.29	0.32	0.48
C=99.5th - mean	280			

0.37=104/280

0.48=135/280

LoB	LoB	a_i*a_j
A	D	0.18
C	D	0.16
B	D	0.14
A	C	0.12
A	B	0.11
B	C	0.09

0.18=0.37 * 0.48

- If Cor(A, D) increases by 10% then
  - Approximate method above: capital will increase by ~1.8% (= 0.18 x 0.10)
  - Based on actual distributions and 100K sims capital increases by ~2% ± 0.5%

Simple Measures of Dependencies

Single Measures of Diversification

- A single measure summarising diversification is attractive
  - Management like it
  - Actuaries may like it
- Single measure usually have pitfalls
  - We need to be aware of them

Single measures may still contain useful information, but they may not be appropriate for direct comparisons

Diversification Benefit (D.B.)

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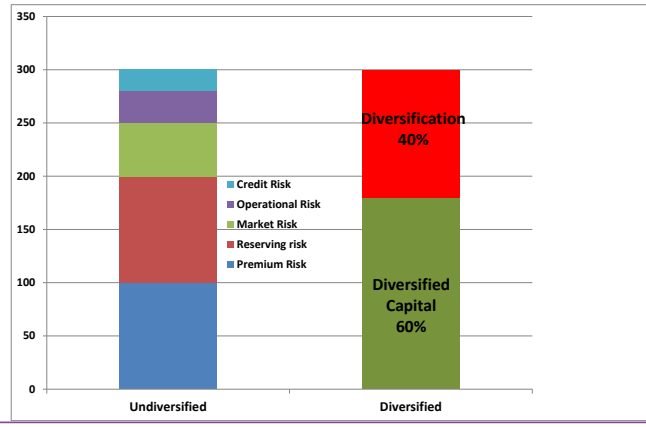
Definition:

(Sum of Undiversified Capital – Diversified Capital)/Sum of Undiversified Capital

Premium Risk	100
Reserving Risk	100
Market Risk	50
Operational Risk	30
Credit Risk	20
Sum of Undiversified	**300**
Diversification Credit	120
Diversified Capital	**180**
Diversification Benefit	**40%**



Diversification Benefit

Intuitive Measure, **but**

1. It only captures diversification at a certain level
2. It may be distorted by double counting of risks
3. It depends on the skewness of the marginal distributions
4. It depends on the number of risks and granularity
5. It may be distorted by expected profit
6. It depends on the relative size of risk charges

None of the above is related to dependencies!

DB: Distortions by Granularity

When comparing Firms the same granularity should be used

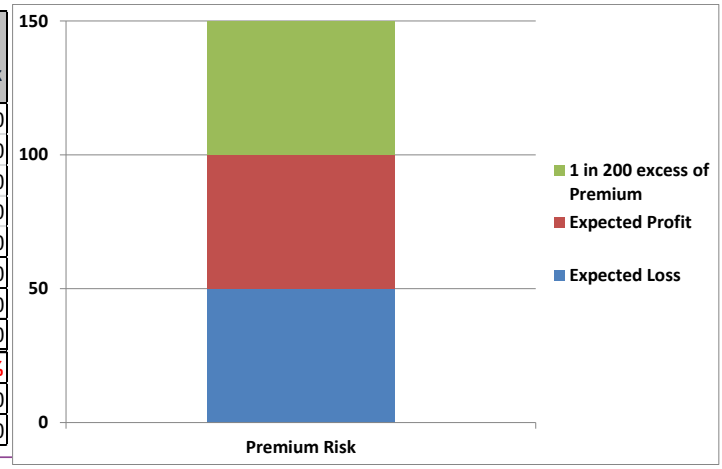
Risk Type	Capital Risk Charge	Risk Type	Capital Risk Charge
Premium Risk	100	Premium Risk non cat	71
		Premium risk cat	71
Reserving Risk	100	Reserving Risk	71
		Reserving Risk PPOs	71
Market Risk	50	Market Risk	50
Operational Risk	30	Operational Risk	30
Credit Risk	20	Credit Risk	20
Sum of Undiversified	**300**	**Sum of Undiversified**	384
Diversification Credit	120	Diversification Credit	204
Diversified Capital	**180**	**Diversified Capital**	180
Dinersification Benefit	40%	Dinersification Benefit	53%

DB: Distortion by expected profit

Suggestion: Expected Profit should be shown separately

- Similarly for other margins

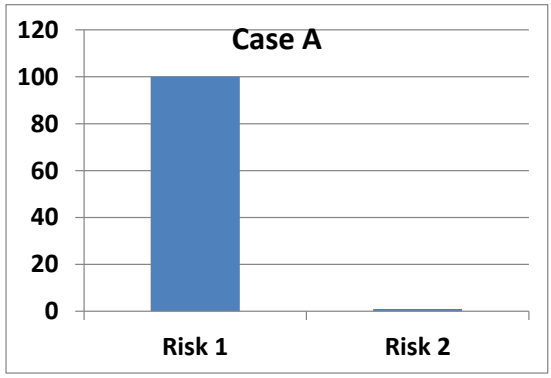
	Profit shown separately	Profit included in premium risk
Premium Risk	100	50
Reserving Risk	100	100
Market Risk	50	50
Operational Risk	30	30
Credit Risk	20	20
Sum of Undiversified	**300**	250
Diversification Credit	120	120
Diversified Capital	**180**	130
Dinersification Benefit	40%	48%
Expected Profit	50	0
Capital Required	130	130

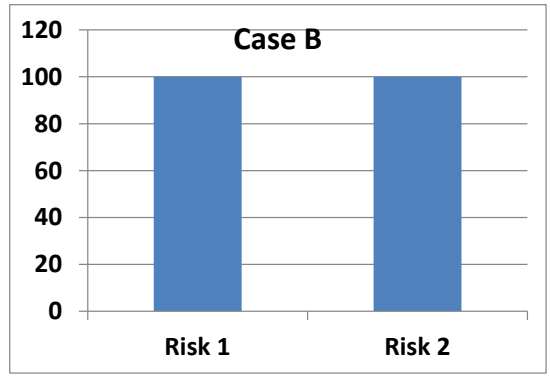


DB: Distortions by Relative Size of Risk Charges

What is the maximum D.B. in each case?

- Case A: No scope for much diversification, insensitive to correlations
- Case B: More scope for diversification, sensitive to correlations





Experiment with simulated risk charges

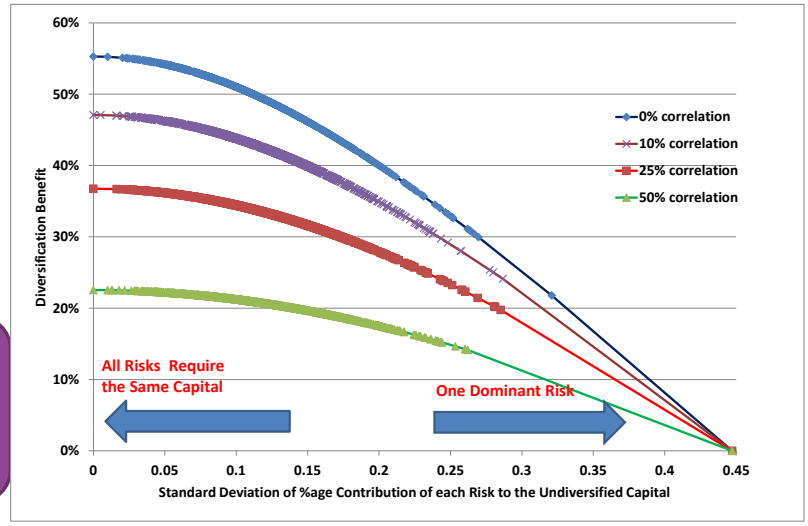
- Simulated randomly 5 capital risk charges for 1,000 Firms
- For each, calculated the s.d. of the risk charges (as %age)
- Low s.d. → similar sizes. High s.d → dominant risk
- Assumed Normality when calculating Diversified Capital

St.Dev. of %age contribution		Risk 1 Premium	Risk 2 Reserving	Risk 3 Market	Risk 4 Operational	Risk 5 Credit	Sum of Charges	Div. Capital	DB
	Firm 1	£ 81	£ 65	£ 97	£ 87	£ 76	£ 406	£ 183	55%
0.03		20%	16%	24%	21%	19%	100%		
	Firm 2	£ 63	£ 11	£ 1	£ 7	£ 10	£ 92	£ 65	29%
0.27		68%	12%	1%	8%	11%	100%		

How DB depends on Relative size of Risks and Cor.

- 5 Risks
- For a given correlation, D.B. is a function of the s.d. of risk charges
- Higher sensitivity of DB to correlations when s.d. is low

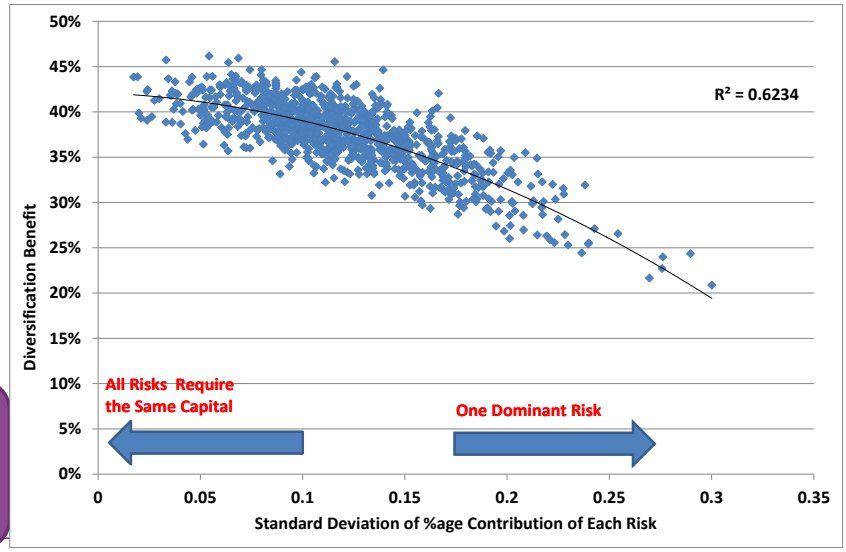
DB varies by correlation, but it can not be used as a measure of dependencies



How DB depends on relative size of risks

- Random correlations between 0% and 35%
- The s.d. of the %age sizes (structure of risk charges) explains 60% of the differences
- **Similar observations for real data!**

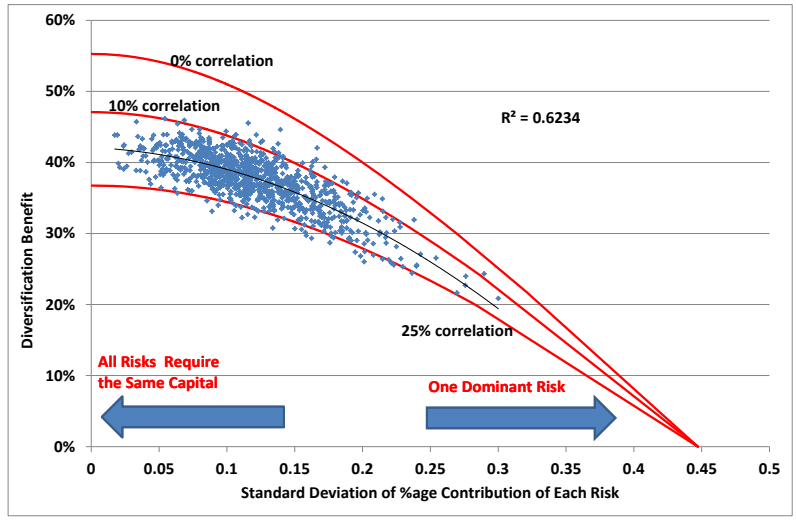
Unadjusted D.B. is not appropriate for comparisons between firms



How DB depends on relative size of risks

- We could overlay the curves for different correlation levels

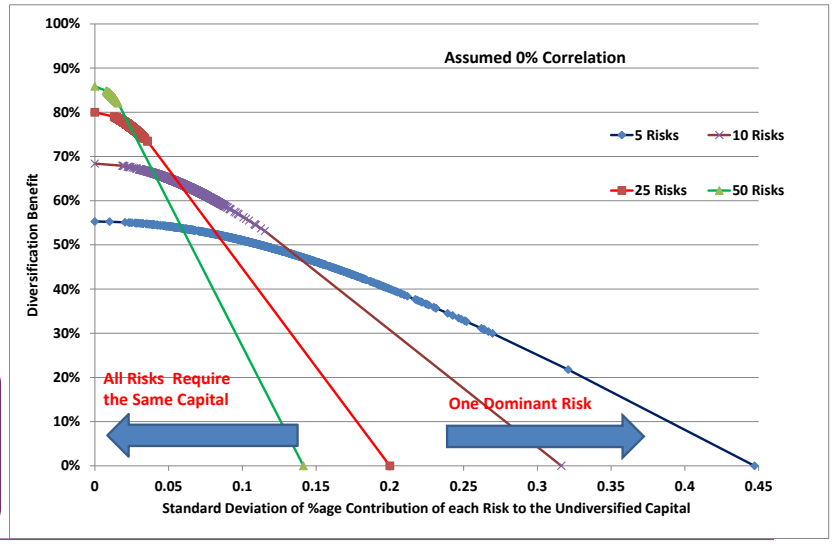
Assessment of diversification could be made either relative to the fitted line or the correlation curves



How DB depends on Relative size of Risks and # of risks

- The larger the number of risks the greater the scope for diversification
- DB becomes even less appropriate for comparisons between firms with different number of risks

When DB is used for comparisons the number of risks and their relative size should be taken into account



Diversification Score (D.S.)

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- $$\frac{Sum\ of\ Undiversified\ Capital\ Charges - Diversified\ Capital}{Sum\ of\ Undiversified\ Capital\ Charges - Capital\ assuming\ independent\ risks}$$

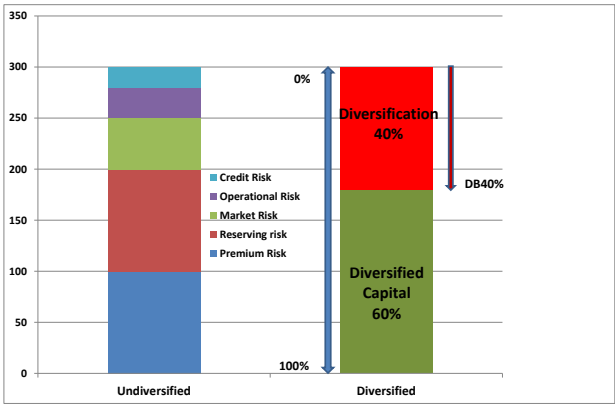
Premium Risk	100
Reserving Risk	100
Market Risk	50
Operational Risk	30
Credit Risk	20
Sum of Undiversified	**300**
Diversification Credit	**120**
Diversified Capital	**180**
Capital for 0% Correl	**150**
Diversification Score	**80%**
Diversification Benefit	**40%**

$$»D.S. = \frac{300-180}{300-150} = 80\%$$

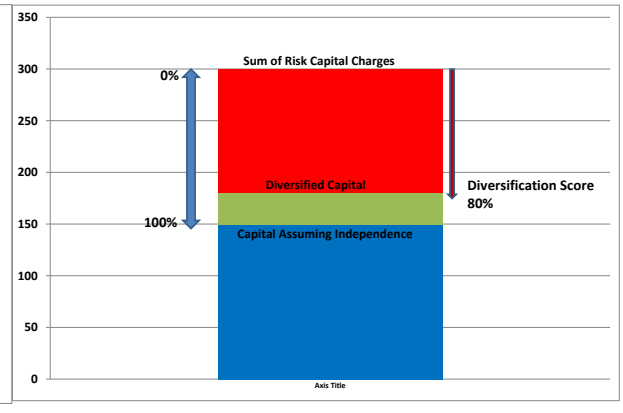
$$»D.B. = \frac{300-180}{300} = 40\%$$

DS: On the scale 0% fully dependent and 100% independent, where does the diversified capital stand?

Diversification Benefit



Diversification Score



Diversification Score (DS)

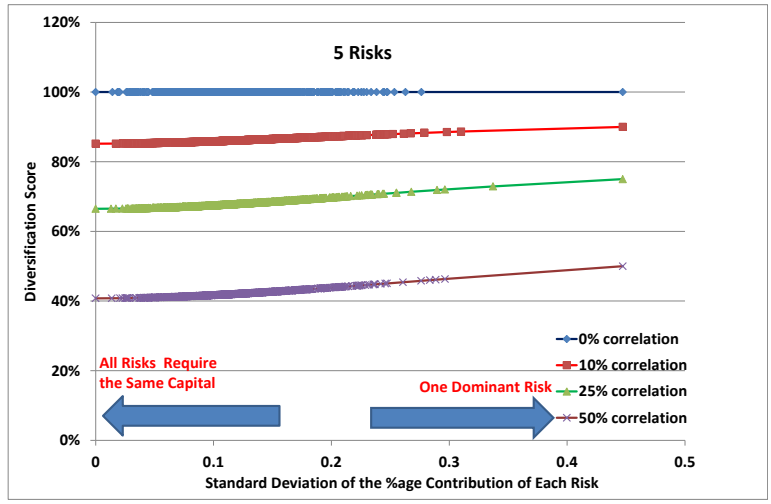
Intuitive Measure, **but**

1. It only captures diversification at a certain level
2. It may be distorted by double counting of risks
3. It is distorted by highly skewed marginal distributions
4. It depends on the number of risks and granularity
5. It may be distorted by expected profit
6. It may be not be possible to calculate capital for 0% cor., but approximately

DS suffers from all the limitations of DB (plus one), BUT it depends less on the relative sizes of risks

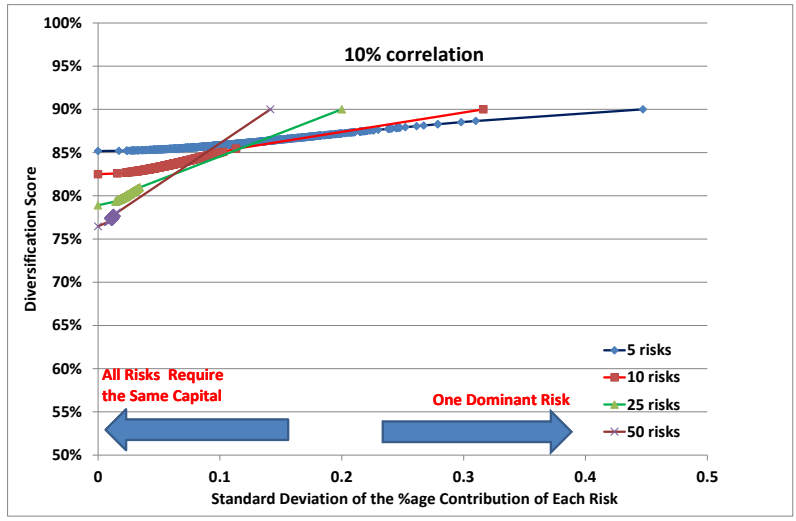
How D.S. Depends on Relative Size of Risks and Cor.

- Diversification Score depends on the relative size of risks, **but less than the D.B.**
- **This makes it more appropriate for comparisons than the D.B.**
- Unlike the D.B. the D.S. increases as few risks become more dominant.



How D.S. Depends on Relative Size and number of Risks

- 10% Cor. assumed
- Diversification Score depends on the number of risks. There is a lot of crossing of the lines
- **D.S. should not be used for comparisons of portfolios without taking into account differences in the number of risks**



Other Ways of Looking at Dependencies

Just Look at the Correlations and Conditional Tail Probabilities

- For a small number of risks this will be easy
- Always have next to the correlation table the materiality of the risk

$$\frac{\Delta C}{C} \approx \frac{C_i}{C} \cdot \frac{C_j}{C} \cdot \Delta\rho_{ij} = a_i \cdot a_j \cdot \Delta\rho_{ij}$$

Correlation Table

		Capital	100	100	50	30	20
Capital			Risk 1	Risk 2	Risk 3	Risk 4	Risk 5
100	Risk 1		100%	20%	10%	10%	10%
100	Risk 2		20%	100%	10%	5%	5%
50	Risk 3		10%	10%	100%	5%	2%
30	Risk 4		10%	5%	5%	100%	5%
20	Risk 5		10%	5%	2%	5%	100%

Materiality Table

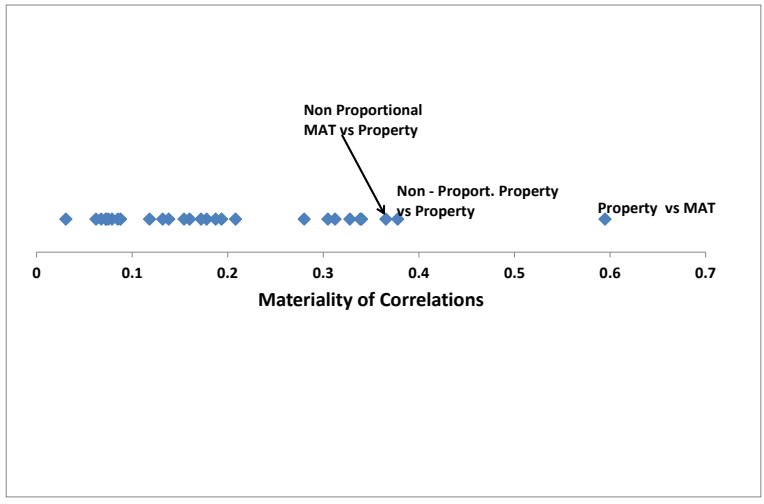
		Risk 1	Risk 2	Risk 3	Risk 4	Risk 5
	Risk 1	0.308642	0.308642	0.154321	0.092593	0.061728
	Risk 2	0.308642	0.308642	0.154321	0.092593	0.061728
	Risk 3	0.154321	0.154321	0.07716	0.046296	0.030864
	Risk 4	0.092593	0.092593	0.046296	0.027778	0.018519
	Risk 5	0.061728	0.061728	0.030864	0.018519	0.012346

Comparisons of Correlations

Comparisons of Correlations between Firms

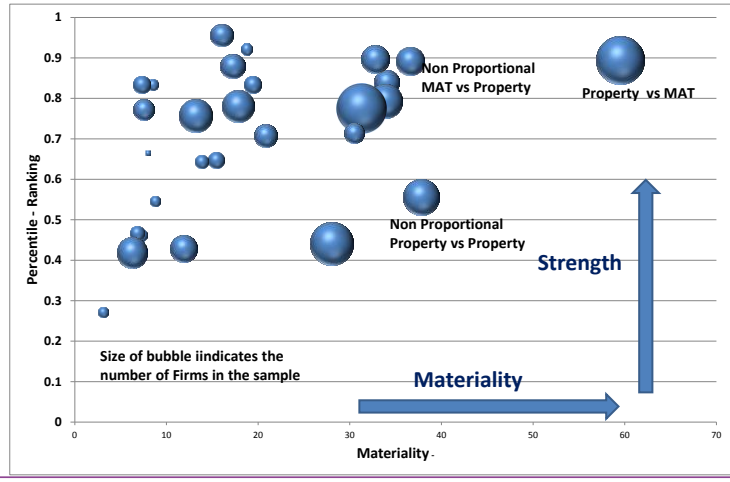
- Comparisons should be made at the same level of granularity
  - Not a common level of granularity among Firms
  - SII classes too broad
- Even at the same level of granularity, different firms have different types of risk

Materiality of Correlations



Presentation of Comparison

- Add another dimension: ranking among a group of Firms
- The y – axis indicates ranking. It does NOT indicate correlation



Limitations of the Comparison

- Correlations vary by Firm for many reasons
  - Differences in the portfolios' risk profiles
  - Size of the portfolio
  - others
- For each correlation the number of Firms in the sample introduces some bias
- The rankings do not provide information on the size of correlations between Firms
- It does not take into account tail dependencies
- Could a weighted average of the rankings, with materiality as the weight, serve as an index?

Conclusions

- Single Measures of Dependencies, such as the diversification benefit, depend on factors unrelated to dependencies and should be used with care
- Dependencies should be examined at different levels of granularity and different aspects of them need to be considered before forming a view
- It will be useful for the profession to develop ways of summarising dependencies

Institute and Faculty of Actuaries

Questions