Communication and personal selection of pension saver's financial risk

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Merton (2014):

Our approach to saving is all wrong.

- *Monthly income, not net worth.*
- *Do not make employees smarter about investments. We need smarter communication.*
- *Balancing the portfolios.*
  - *Take risk out of the portfolio once the goal is achieved. Avoid achieving goal only to fall below if markets go down.*
  - *Minimum guaranteed income.*

In this first talk of the project, we only consider the simple lump sum case.

Hence, we only consider the last two of Merton's points.

We consider four different people:

- Lisa: The risk taker
- John: The moderate risk taker
- Susan: The moderate risk averse
- James: The risk averse

In a power utility world, Lisa, John, Susan, James would have parameters

$$\rho = -0.25, \quad -1, \quad -4, \quad -10,$$

respectively.

In a non-hedged power utility world without guarantees and other safety measures the investment in stocks would be

	Lisa	John	Susan	James
Percentage in stocks	75%	46%	19%	8%

In this talk we will suggest an approach where a simple question to Lisa, John, Susan and James will tell us what kind of risk they want.

We hedge by optimizing the median return given some guarantee.

All numbers are in 2017 - values, i.e, adjusted for inflation.

In later work to be presented next May 2018, we will argue how such an inflation-hedged lower bound is possible in our pension universe.

- We only consider the simple lump-sum case.
- Lisa, John, Susan and James want to invest $\pounds 10,000$.
- $\Rightarrow 30$ years of investment

We now present the simple instructions.

THE COMMUNICATION

- Your investment has a best-case (BC) and a worst-case (WC).
- You will never drop below your WC.
- Half-the-time you will get the BC and the other half-of-the-time you will get an investment result between WC and BC.
- Use a slider to see which WC suits you best.
  For every WC their is a link to a BC. And the BC increases when the WC decreases.

Which WC will the risk taker Lisa pick?

- $£3,900$ □
- $£6,400$ □
- $£9,100$ □

Which WC will the risk taker Lisa pick?

- $\pounds 3,900$ ☑
- $\pounds 6,400$ ☐
- $\pounds 9,100$ ☐

What is the corresponding BC?

- $£12,320$ $\square$
- $£15,320$ $\square$
- $£16,470$ $\square$

What is the corresponding BC?

- $\pounds 12,320$ ☐
- $\pounds 15,320$ ☐
- $\pounds 16,470$ ☑

Lisa's pick:

Goal: $£16,470$
Forecast: Half of the times you will achieve this goal.

More is not possible.

Guarantee:$£3,900$.

Lisa's median in the un-hedged world, where she holds 75% in stocks would be

$$\textbf{Median} = \pounds 13,496$$

With the new hedging strategy

$$\textbf{Lisa's median} = \pounds 16,470$$

- Lisa has increased her median by $\pounds 2,974$.
- She also has a guarantee of $\pounds 3,900$ (Compare to no guarantee before)
- The price is no upside above $\pounds 16,470$.

In other words:

Lisa has sold her upside above $£16,470$ to secure a guarantee and a higher median.

John's pick:

Goal: $\pounds 15,320$
Forecast: Half of the times you will achieve this goal.

More is not possible.

Guarantee:$\pounds 6,400$.

Susan's pick:

Goal: $\pounds 12,320$
Forecast: Half of the times you will achieve this goal.

More is not possible.

Guarantee:$\pounds 9,100$.

James' pick:

Goal: $£10,940$
Forecast: Half of the times you will achieve this goal.

More is not possible.

Guarantee:$£9,700$.

	Lisa	John	Susan	James
Guarantee (Floor)	$£3,900$	$£6,400$	$£9,100$	$£9,700$
Goal/Max value (Achieved half-of-the-time)	$16,470$	$£15,320$	$£12,320$	$£10,940$

Note that Lisa, John, Susan and James self-selected their risk-profile through a simple exercise.

Do Lisa, John, Susan and James lose anything from this simple communication and hedging strategy?

Not really!

Look at this certainty equivalent table in terms of utility theory.

	Optimal Strategy	Hedged Strategy		
Investor	CE	CE	Guarantee	Goal
Lisa	$£12,756$	$£12,020$	$£3,900$	$£16,470$
John	$£11,643$	$£11,263$	$£6,400$	$£15,320$
Susan	$£10,627$	$£10,415$	$£9,100$	$£12,320$
James	$£10,280$	$£10,169$	$£9,700$	$£10,940$

Table: Comparison of different optimal strategies. Investors are assumed to obey a power utility with parameter $\rho = -0.25, -1, -4, -10$, respectively.

Certainty Equivalents (CE): For which certain amount would you exchange your uncertain terminal lump sum.

Now let us go back to the old world of un-hedged utility optimisation.

What can financial miss-understanding cost?

How much would it cost Lisa if the financial assessment thought she was James?

- Between 5% and 10% ☐
- Between 10% and 15% ☐
- Between 15% and 20% ☐

How much would it cost Lisa if the financial assessment thought she was James?

- Between 5% and 10% ☐
- Between 10% and 15% ☐
- Between 15% and 20% ☑

How much would it cost James if the financial assessment thought she was Lisa?

- Between 10% and 20% ☐
- Between 30% and 40% ☐
- Between 70% and 80% ☐

How much would it cost James if the financial assessment thought she was Lisa?

- Between 10% and 20% ☐
- Between 30% and 40% ☐
- Between 70% and 80% ☑

	Lisa Plan	John Plan	Susan Plan	James Plan
Lisa CE	$£12,756$	$£12,326$	$£11,124$	$£10,536$
John CE	$£11,023$	$£11,643$	$£11,023$	$£10,516$
Susan CE	$£6,156$	$£9,268$	$£10,627$	$£10,437$
James CE	$£2,388$	$£5,958$	$£9,879$	$£10,280$

Table: The impact of miss-communication

The underlying mechanism

- How does an optimal investment strategy look like?
- We need an objective function which we can maximize.
- Maximizing $\mathbb{E}[\text{lump sum}]$ might not be the best idea.

Another perspective.

Q: What number is halfway between 1 and 9?

James: 3

People tend to think on a log-scale (Siegler and Booth, 2004, Dehaene et al., 2008).

The difference between 100 and 110 is the same as the difference between $10,000$ and $11,000$.

It's a 10% increase in both cases.

- So then maximize $\mathbb{E}[\log(\text{lump sum})]$?
  (Which is the same as maximizing your relative return)
- James might still be risk averse: He is more afraid of losses then excited about gains.

Smoothly increase risk aversion + keep scale invariance
$\Rightarrow$ power utility family:

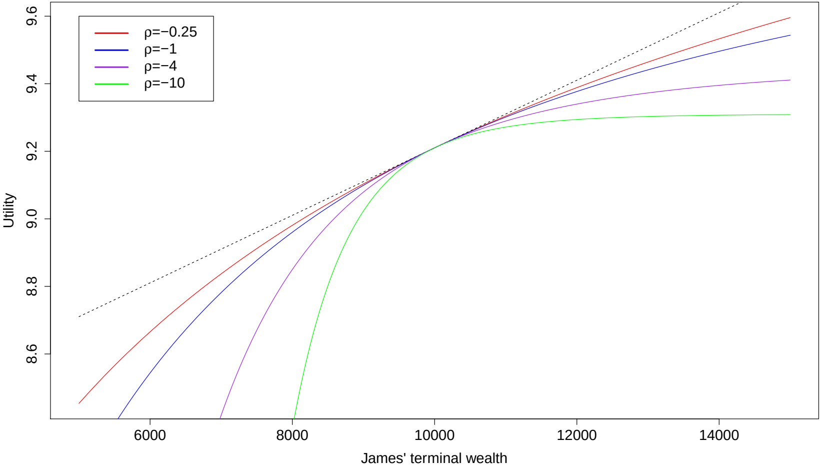
Maximize

$$\mathbb{E}[U(\text{lump sum})]$$

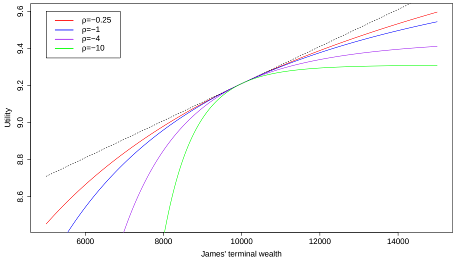
where

$$U(W) = \frac{1}{\rho} W^{\rho}, \quad \rho < 0$$

Different power utility functions for varying ρ



Different power utility functions for varying ρ



ρ	optimal CE	CE $\widehat{\rho} = -0.25$
-0.25	$£13,404$	$£13,404$
-1.00	$£12,009$	$£11,243$
-4.00	$£10,760$	$£5,582$
-10.00	$£10,339$	$£1,922$

CE= Certainty equivalent: For which certain amount would you exchange your uncertain terminal lump sum.

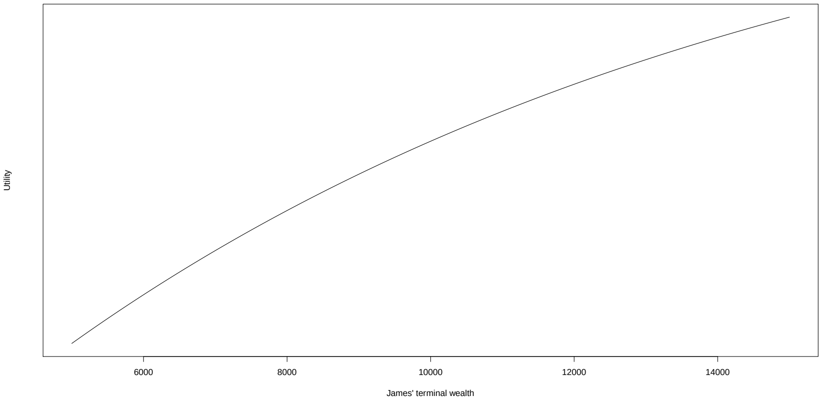
The pension saver might not be in control of his investment strategy.

Risk-appetite misspecification can have a huge effect.

A new idea.

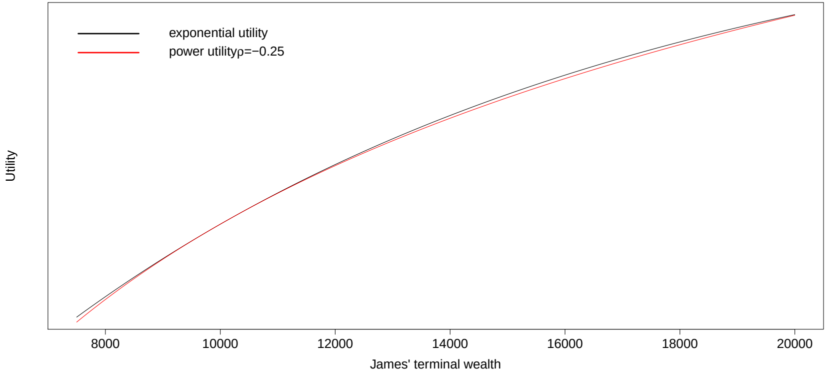
Let's start with a specific exponential utility function.

The exponential utility function



Now we compare it to a power utility function with parameter $\rho = -0.25$.

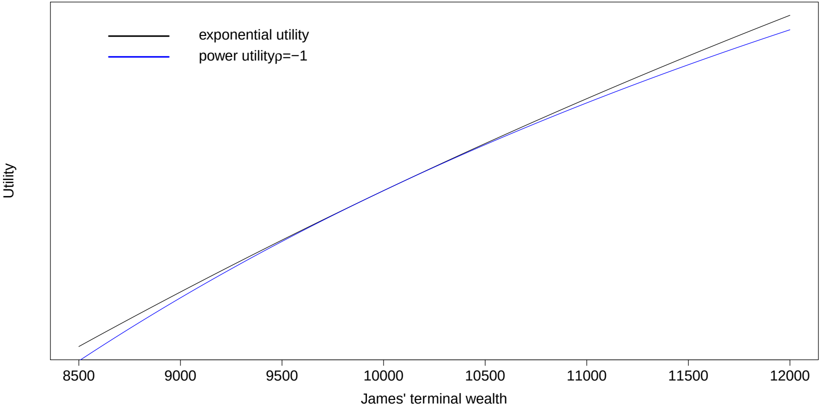
Exponential utility function approximating power utility function



It is a good approximation if the lump sum is in the range from $7,000$ to $20,000$.

Now we compare it to a power utility function with parameter $\rho = -1$.

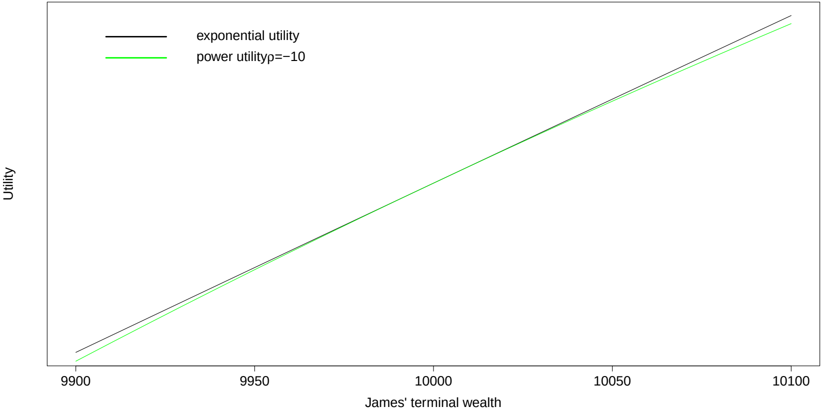
Exponential utility function approximating power utility function



It is a good approximation if the lump sum is in the range from $8,500$ to $12,000$.

...and a power utility function with parameter $\rho = -10$

Exponential utility function approximating power utility function



It is a good approximation if the lump sum is in the range from $9,900$ to $10,100$

- We seem to be able to approximate any power utility function with one exponential utility function if the wealth range is restricted.
- The more risk averse the investor, the more we must restrict the wealth range.
- But a risk averse investor might prefer to exclude those external ranges anyway.

How good is the approximation in our case?

3-dimensional optimisation

Investor	Unconstrained CRRA optimisation		
	ρ	CE_{opt}	$CE_{\rho=-0.25}$
Lisa	–0.25	£12,756	£12,756
John	–1.00	£11,643	£11,023
Susan	–4.00	£10,627	£6,156
James	–10.00	£10,280	£2,388

Constrained CARA optimisation					
CE	G_L	G_U	C	Prob.	% loss
£12,556	£4,050	£60,000	£9,100	0	1.56
£11,576	£5,150	£36,300	£5,100	0	0.57
£10,621	£7,000	£19,800	£1,900	0	0.05
£10,278	£8,350	£14,600	£900	0	0.02

2-dimensional optimisation

Investor	Unconstrained CRRA optimisation		
	ρ	CE_{opt}	$CE_{\rho=-0.25}$
Lisa	–0.25	£12,756	£12,756
John	–1.00	£11,643	£11,023
Susan	–4.00	£10,627	£6,156
James	–10.00	£10,280	£2,388

Constrained CARA optimisation					
CE	G_L	G_U	C	Prob.	$\%$ loss
£12,550	£4,300	£60,000	£10,000	0.00	1.61
£11,324	£6,900	£20,200	£10,000	0.21	2.74
£10,430	£8,850	£11,800	£10,000	0.63	1.85
£10,184	£9,500	£10,700	£10,000	0.69	0.93

1-dimensional optimisation

Investor	Unconstrained CRRA optimisation		
	ρ	CE_{opt}	$CE_{\rho=-0.25}$
Lisa	–0.25	£12,756	£12,756
John	–1.00	£11,643	£11,023
Susan	–4.00	£10,627	£6,156
James	–10.00	£10,280	£2,388

Constrained CARA optimisation					
CE	G_L	G_U	C	Prob.	$\%$ loss
£12,017	£3,650	£16,500	£10,000	0.5	5.79
£11,264	£6,500	£15,200	£10,000	0.5	3.26
£10,416	£9,100	£12,300	£10,000	0.5	1.99
£10,171	£9,650	£11,000	£10,000	0.5	1.06

Why the exponential utility function?

Remember:

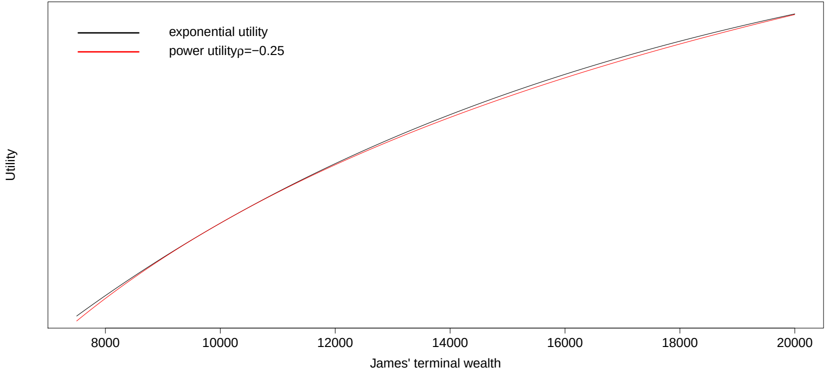


Figure: Exponential utility function approximating power utility function

As a first research output of our project, Donnelly et al., 2016 developed an optimal strategy for the power utility case.

The strategy, however, turned out to be quite complicated. While solvable, the solution spans over several lines and is arguably a black-box.

Theorem. Power utility constraint strategy

Donnelly et al. (2016):

Assume no inflation, if Guarantee $< 10,000 <$ Top, then the optimal strategy π^{**}, i.e., the amount to put into the risky fund, is

$$\pi^{**}(t) = A[1 - \Phi(d_+(t, P(t), \text{Top})) - \Phi(-d_+(t, P(t), \text{Guarantee}))]P(t),$$

where

$$c_p(t, y, G_U) = y\Phi(d_+(t, y, G_U)) - G_U e^{-r(T-t)}\Phi(d_-(t, y, G_U))$$
$$p_p(t, y, G_L) = G_L e^{-r(T-t)}\Phi(-d_-(t, y, G_L)) - y\Phi(-d_+(t, y, G_L))$$
$$d_\pm(t, y, G) = \frac{1}{\sigma A\sqrt{T-t}}\left\{log(\frac{y}{G}) \pm \frac{1}{2}\sigma^2 A^2(T-t)\right\},$$
$$A = \frac{\theta}{\sigma(1-\rho)},$$

where θ is the market price of risk, σ the standard deviation of the risky asset and $P(t)$ is defined as

$$P(t) = P(0)\exp\left\{\left(\theta\sigma A - \frac{1}{2}\sigma^2 A^2\right)t + \sigma A W(t)\right\},$$

and with $P(0)$ defined as solution of

$$10,000 = P(0) - c_p(0, P(0), G_U) + p_p(0, P(0), G_L)$$

The strategy for the constraint exponential utility case is quite simple:

- Every year[1], put your initial amount (here: $\pounds 10,000$) scaled by the probability that you do not hit the boundaries (guaranteed and top amount) into a risky fund.
- Put the rest into a risk-free fund.

[1] Technically, the strategy requires continuous trading.

Theorem. Exponential constraint strategy

Assume no inflation, if Guarantee $< 10,000 <$ Top, then the optimal strategy π^*, i.e., the amount to put into the risky fund, is

$$\pi^*(t) = 10,000 \times Pr(\text{Guarantee} < X(T) < \text{Top}|X(t)).$$

The underlying model

In the period $[0, T], T > 0$, there are two assets one can invest in.

$$\mathrm{d}S_0(t) = rS_0(t), \quad \mathrm{d}S_1(t) = \mu S_1(t)\mathrm{d}t + \sigma S_1(t)\mathrm{d}W_t,$$

where $\mu, \sigma, r > 0$, and $S_0(0) = S_1(0) = 1$ and W is a Brownian motion.

Let

- X_t be the amount of capital invested in the fund at time t.
- π_t be the amount invested in a risky asset, the remainder in risk-free assets.

Hence, we have

$$\begin{aligned} dX(t) &= r\big(X(t) - \pi(t)\big)\,dt + \big(\mu\,dt + \sigma\,dW(t)\big)\pi(t) \\ &= rX(t)\,dt + \big(\theta\,dt + dW(t)\big)\sigma\pi(t), \end{aligned}$$

where $\theta = (\mu - r)/\sigma$ is the market price of risk.

How to find the optimal strategy

The unconstrained case

The unconstrained case

Optimal control theory:

Define the optimal value function,

$$V(t,y) = sup_{\pi}\mathbb{E}\left[U\{Y(T)\}|Y(t)=y, \textit{strategy } \pi \textit{ is used}\right],$$

at time t given that $Y(t)=y$, where $Y(t)=e^{r(T-t)}X(t)$

The dynamics of V can be described via the Hamilton-Jacobi-Bellman equation

$$\sup_{\pi}\left\{V_t + \theta\sigma e^{r(T-t)}\pi(t)V_y + \frac{1}{2}\sigma^2\pi(t)^2 e^{2r(T-t)}V_{yy}\right\} = 0,$$

The unconstrained case

and conclude that the optimal unconstrained strategy is given by

$$\pi^{***}(t,y) = -\frac{\theta}{\sigma}e^{-r(T-t)} \cdot \frac{V_y}{V_{yy}},$$

$$V_t - \frac{\theta^2}{2} \cdot \frac{V_y^2}{V_{yy}} = 0.$$

The unconstrained case

Add the boundary condition

$$V(T, y) = -\frac{1}{\gamma} \exp\left[-\gamma y\right],$$

to find the unique solution

$$V(t, y) = -\frac{1}{\gamma} \exp\left[-\frac{\theta^2}{2}(T-t) - \gamma y\right],$$

leading to the optimal unconstrained strategy

$$\pi^{***}(t, y) = Ce^{-r(T-t)},$$

where $C = \theta/(\gamma\sigma)$.

The constraint case with top G_U and floor G_L

The constraint case with top G_U and floor G_L

Idea: The optimal constraint strategy should be an optimal unconstrained strategy minus a call option plus a put option with strike price G_U and G_L respectively.

The constraint case with top G_U and floor G_L

Define the process

$$P(t) = P(0) + R\big(\theta t + W(t)\big),$$

$R = C\sigma = \theta/\gamma$,
i.e., the optimal unconstrained portfolio at time t but starting with different starting wealth $P(0)$.

The constraint case with top G_U and floor G_L

Step 1

Show that the terminal wealth

$$Y^*(T) = \begin{cases} G_L & \text{if } P(T) < G_L \\ P(T) & \text{if } G_L \leq P(T) \leq G_U \\ G_U & \text{if } P(T) > G_U \end{cases}$$

is feasible and optimal (cf. Grossman and Zhou (1996)).

Note that $Y^*(T)$ equals $P(T)$ minus a terminal call option plus a terminal put option.

The constraint case with top G_U and floor G_L

Step 2

Determine the dynamics ($i.e.\ \int_0^t Y^*(s)\mathrm{d}s$) of the optimal portfolio

$$Y^*(t) = \mathbb{E}_{\mathbb{Q}}(\max\{G_L, \min\{G_U, P_e(0) + R\left(W^{\mathbb{Q}}(t) + \sqrt{T-t}\cdot Z\right)\}\}|\mathcal{F}_t^{\mathbb{Q}}),$$

where $\mathbb{Q}$ is the martingale measure making $W^{\mathbb{Q}}(t) = W(t) + \theta t$ a martingale.

This will lead to the optimal strategy π^*

Research outlook

Research outlook

Accumulation Phase

- Market timing
- A risk-free inflation fund

Decumulation Phase

- Monthly income, not net worth

In both cases

- Risk sharing principal

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