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On the Valuation of Staff Pension Funds. By HENRY WILLIAM MANLY, *Actuary of The Equitable Life Assurance Society, and Ex-President of the Institute of Actuaries. With Tables by* ERNEST CHARLES THOMAS, *of The Gresham Life Assurance Company, Fellow of the Institute of Actuaries, and* JOHN NORMAN LEWIS, *of The London Assurance Corporation, Fellow of the Institute of Actuaries.*

(*Continued from vol.* xxxvi, *page* 276.)

ADDITIONS AND ALTERATIONS.

ON reading over my paper after it was published, I did not feel quite satisfied as to the accuracy of my approximate formula, given on page 235, for the adjustment of N^s_x in respect of the excess of contributions included in the Table as constructed, so I have had the values of ${}^dC_xs_x$, ${}^wC_xs_x$ and ${}^rC_xs_x$, calculated. The constant summation of these columns, namely, $\Sigma{}^dC_xs_x$, $\Sigma{}^wC_xs_x$ and $\Sigma{}^rC_xs_x$, will give the columnar values to be used for making accurate adjustments. These latter values, divided by D^s_x, are the present values of a full year's salary at the end of the year of death, withdrawal, or resignation, equated to a salary of 1 at x; and the columns $\Sigma{}^dC_xs_x$, $\Sigma{}^wC_xs_x$ and $\Sigma{}^rC_xs_x$, will therefore be appropriately represented by the symbols ${}^dM^{ls}_x$, ${}^wM^{ls}_x$ and ${}^rM^{ls}_x$ respectively. These may, on some occasion, be found useful for other purposes; such as when one year's salary is given on death or retirement in lieu of other compensation.

When the correct values are substituted for those used in Section XI of the Valuation Schedule on page 248 (see page 196), it will be seen that a closer approximation would have been

$$\cdot74({}^dM^s_x + {}^wM^s_x + {}^rM^s_x)$$

which does not differ much from my estimate on page 235.

Another alteration which I have seen reason to make is more important, and affects Problems IIIA, XB and XIB. In those problems (pages 221, 238 and 239), I have assumed that the annuity commences at the end of the year of retirement, that the full year's salary in the year of retirement is received, and that the annuity-value is then a'_{x+1}; but if we assume that retirement on the average takes place in the middle of the year, the basis on which the pension is calculated is too large by half the last year's salary. Further, on the assumption that retirement on the average occurs in the middle of the year, the value of the annuity will be $a'_{x+\cdot5}$ instead of a'_{x+1}, and the continuous annuity will be, approximately, $a'_{x+\cdot5}+\cdot5$.

I have recalculated the Tables for valuing the benefits described in the above-mentioned three Problems by substituting $a'_{x+\cdot 5}+\cdot 5$ for a'_{x+1}; and these Tables will be found on pages 207 and 208. The values $\frac{{}^{ra}\mathrm{R}^s_x}{\mathrm{D}^s_x}$ and $\frac{{}^{ra}\mathrm{R}^{ls}_x}{\mathrm{D}^s_x}$ will now be too great by the present value of an annuity of $a'_{x+\cdot 5}+\cdot 5$ based on half the full salary receivable in the year of retirement, that is, by $\frac{1}{2}\Sigma r_x s_x(a'_{x+\cdot 5}+\cdot 5)v^{x+1}\div \mathrm{D}^s_x$ or $\frac{{}^{ra}\mathrm{M}^{ls}}{2\mathrm{D}^s_x}$; and the adjusted values will be too small by half a year's interest.

FURTHER COMMENTS ON ADJUSTMENTS.

It may be useful to give here the results of my more mature judgment respecting adjustments.

I think, as regards the question of discount, a distinction should be made between the accumulation of capital and the reserve for liabilities. As I have already explained (page 235), the periodical contributions and interest on investments are not invested directly they are received, and consequently, on the assumption that the investments are made, on the average, at the end of every quarter, my values of the future contributions (which assume that the investments are made at the end of the year), are over-discounted by $\frac{3}{8}$ths of a year, and have therefore to be multiplied by $(1+\frac{3}{8}i)$. In those Funds where the master contributes half the contributions, the more common rule is for him to pay his share at the end of the year, and my values in that case are consequently over-discounted to the extent of only one-fifth of a year, and have therefore to be multiplied by $(1+\cdot 2i)$. On the other hand, the liabilities have to be met immediately the events provided for happen. Now my values assume that the events happen at the end of the year in which they occur, and that the last year's salary will be received in full—conditions which only apply to the value of the pension on the attainment of the pension age. With that exception, all my values of liabilities, on the assumption that the events will happen in the middle of the year, are too great by the value of half the last year's salary, and are over-discounted by exactly half a year. The correction for half the last year's salary must therefore be first made, and the result multiplied by $(1+\frac{1}{2}i)$.

The proper multiplier for finding the value of the future contributions will therefore be

$$\left\{\mathbb{N}^s_x-\tfrac{1}{2}\left({}^{d}\mathrm{M}^{ls}_x+{}^{w}\mathrm{M}^{ls}_x+{}^{r}\mathrm{M}^{ls}_x\right)\right\}\div \mathrm{D}^s_x$$

multiplied by $(1+\frac{3}{8}i)$ or $(1+\cdot 2i)$ according as the full contributions are received equally over the year and invested quarterly, or half received over the year and half paid in a lump sum at the end of the year.

The multiplier for finding the value of the return of contributions without interest on death will be

$$(1+\tfrac{1}{2}i)\left({}^{d}\mathrm{R}^{s}_{x}-\tfrac{1}{2}{}^{d}\mathrm{M}^{ls}_{x}\right)\div\mathrm{D}^{s}_{x}\,;$$

and, for return with interest, $(1+\frac{1}{2}i)\left({}^{d}\mathbb{N}^{s}_{x}-\frac{1}{2}{}^{d}\mathrm{M}^{ls}_{x}\right)\div\mathrm{D}^{s}_{x}$. Similarly for the return on other modes of exit.

The multipliers for the value of a pension on early retirement will be

$(1+\frac{1}{2}i)\left({}^{ra}\mathrm{R}^{s}_{x}-\frac{1}{2}{}^{ra}\mathrm{M}^{ls}_{x}\right)\div\mathrm{D}^{s}_{x}$ for pension based on average salary,

$(1+\frac{1}{2}i)\left({}^{ra}\mathrm{R}^{ls}_{x}-\frac{1}{2}{}^{ra}\mathrm{M}^{ls}_{x}\right)\div\mathrm{D}^{s}_{x}$ for pension based on last salary.

In Problem IIIA, the formula for the correct value is evidently

$$(1+\tfrac{1}{2}i)\left({}^{ra}\mathrm{R}_{x}-\tfrac{1}{2}{}^{ra}\mathrm{M}_{x}\right)\div\mathrm{D}_{x}.$$

It may be thought that I am now aiming at too much refinement; but on consideration it will be seen that my proposals are all on the side of safety.

Valuation Schedule.

Sections VI, X and XI of the Valuation Schedule, pages 245 and 248, will now be altered as follows:—

Revised Valuation Schedule—(*see page* 245).

		Section VI Annuity of 1 per-cent of Total Salary on Retirement before 65				
Present Age	Number of Members	$\frac{{}^{ra}\mathrm{R}^{s}_{x}}{\mathrm{D}^{s}_{x}}$	$\frac{{}^{ra}\mathrm{M}}{\mathrm{D}_{x}}$	Present Value in respect of Future Contributions (3)×(28)	Present Value in respect of Past Contributions (8)×(29)	Total of last two Cols. (30)+(31)
(1)	(2)	(28)	(29)	(30)	(31)	(32)
20	50	26·782	·271	602·59	16·19	618·78
30	50	28·736	·680	1293·12	254·30	1547·42
40	25	28·299	1·241	919·72	530·28	1450·00
50	15	21·537	1·930	572·88	913·21	1486·09
60	6	7·003	2·185	73·53	576·40	649·93
Totals	146	...	..	3461·84	2290·38	5752·22

REVISED VALUATION SCHEDULE—(*see page* 248).

		Section X Annuity of 1 per-cent of Last Salary and Number of Years of Service, on Retirement before 65				
Present Age	Number of Members	$\frac{{}^{ra}R_x^{ls}}{D_x^s}$	$\frac{{}^{ra}M_x^{ls}}{D_x^s}$	Present Value in respect of Future Service (3)×(55)	Present Value in respect of Past Service (44)×(56)	Present Value in respect of Total Service (57)+(58)
(1)	(2)	(55)	(56)	(57)	(58)	(59)
20	50	42·607	1·136	958·66	88·88	1047·54
30	50	39·426	1·430	1774·17	788·00	2562·17
40	25	34·453	1·863	1119·72	1215·98	2335·70
50	15	23·761	2·315	632·04	1725·60	2357·64
60	6	7·178	2·270	75·37	1014·69	1090·06
Totals	146	...	...	4559·96	4833·15	9393·11

REVISED VALUATION SCHEDULE—(*see page* 248).

		Section XI—Adjustments							
Present Age	Number of Members	$\frac{{}^{w}M_x^{ls}}{2D_x^s}$	(3)×(60)	$\frac{{}^{d}M_x^{ls}}{2D_x^s}$	(3)×(62)	$\frac{{}^{r}M_x^{ls}}{2D_x^s}$	(3)×(64)	$\frac{{}^{ra}M^{ls}}{2D_x^s}$	(3)×(66)
(1)	(2)	(60)	(61)	(62)	(63)	(64)	(65)	(66)	(67)
20	50	·307	6·91	·116	2·61	·064	1·44	·568	12·78
30	50	·118	5·31	·120	5·40	·080	3·60	·715	32·18
40	25	·041	1·33	·127	4·13	·104	3·38	·932	30·29
50	15	·003	·08	·120	3·19	·129	3·43	1·158	30·80
60	6	...	...	·071	·75	·130	1·37	1·135	11·92
Totals	146	...	13·63	...	16·08	...	13·22	...	117·97

As an example of the effect of the alterations I will re-value Fund VIII on page 254.

FUND VIII.			Pension as a Percentage of *Last* Salary for every Year of Service	
			Without	With
			Return of Difference between Total Contributions and Annuity Payments	
BENEFITS :—				
(i) *Pension on attainment of age 65 based on last Salary.*				
(ii) *Pension on retirement before 65 based on last Salary.*				
(iii) *Return of Contributions, with interest, on death before 65.*				
(iv) *Return of Half Contributions, without interest, on withdrawal.*				
Present Value of 1 per-cent of Future Salaries		2250·92		
Deduct :				
Adjustment for ½ years' contributions on withdrawal, death and early retirement, over-estimated = 13·63 + 16·08 + 13·22 .		42·93		
		2207·99		
Add : Adjustment for over-discount (1½ per-cent = $\frac{3}{8}i$)		33·12		
		2241·11		
Deduct :				
1 per-cent of Salaries, with interest, on death before 65 .	1144·36			
½ per-cent of Salaries, without interest, on withdrawal . .	147·32			
	1291·68			
Deduct adjustment for over-estimate of contributions (6·82 + 16·08)	22·90			
	1268·78			
Add adjustment for over-discount (2 per-cent = $\frac{1}{2}i$)	25·37			
		1294·15		
		946·96		
		5		
		4734·80		
Add : Fund		12000·00		
Total Asset to provide Pensions		16734·80		
This has to be divided by—				
Present Value of Pension of 1 per-cent of last Salary on attainment of age 65 .		10850·26		
Present Value of Pension of 1 per-cent of last Salary on retirement before age 65	9393·11			
Deduct adjustment for over-estimate of contribution .	117·97			
	9275·14			
Add adjustment for over-discount (2 per-cent). .	185·50			
		9460·64		
		20310·90		
Which will give a Pension of			·824	·784

A CORRECTION.

Mr. W. A. Robertson, of the Scottish Union and National Office, Edinburgh, points out an error which I am very pleased to acknowledge.

In Problem XIIIA, page 227,

$$^{r}\mathbb{N}_x - {}^{r}\mathbb{N}_{x+15} - 15\,{}^{r}\mathrm{D}_{x+15}$$

is given as the value to be used in the formula to provide for the return of the premiums paid with compound interest on retirement from invalidity before 15 years' service. The correct value is

$$^{r}\mathbb{N}_x - {}^{r}\mathbb{N}_{x+16} - a_{(16)}{}^{r}\mathrm{D}_{x+15}$$

or

$$^{r}\mathbb{N}_x - {}^{r}\mathbb{N}_{x+15} - (1+i)\,a_{(15)}{}^{r}\mathrm{D}_{x+15}$$

This follows from Problem IIA (page 220), where it is shown that the deferred value for the return on death after 5 years is

$$\frac{1}{v^x l_x}\left[\{(1+i)^5+(1+i)^4+\ldots+(1+i)+1\}v\mathrm{D}'_{x+5}+v\mathrm{D}'_{x+6}+\ldots\right]$$

which is the same as

$$\frac{1}{v^x l_x}\left[(1+i)\{(1+i)^4+(1+i)^3+(1+i)^2+(1+i)+1\}v\mathrm{D}'_{x+5} + v\mathrm{D}'_{x+5}+v\mathrm{D}'_{x+6}+\ldots\right]$$

that is,

$$(a_{(6)}{}^{d}\mathrm{D}_{x+5}+{}^{d}\mathbb{N}_{x+6})\div\mathrm{D}_x$$

or

$$\{(1+i)a_{(5)}{}^{d}\mathrm{D}_{x+5}+{}^{d}\mathbb{N}_{x+5}\}\div\mathrm{D}_x$$

and the value of the temporary benefit, for the return on death within 5 years, would therefore be the difference between the immediate benefit $({}^{d}\mathbb{N}_x\div\mathrm{D}_x)$ and the deferred benefit, namely,

$$({}^{d}\mathbb{N}_x - {}^{d}\mathbb{N}_{x+6} - a_{(6)}{}^{d}\mathrm{D}_{x+5})\div\mathrm{D}_x$$

or

$$({}^{d}\mathbb{N}_x - {}^{d}\mathbb{N}_{x+5} - (1+i)a_{(5)}{}^{d}\mathrm{D}_{x+5})\div\mathrm{D}_x$$

By substituting *r* for *d*, the same formula will apply for the return on retirement within 5 years.

A similar error crept into Problem XIIB (page 241), where

$$^{r}\mathbb{N}^s_x - {}^{r}\mathbb{N}^s_{x+15} - (as)_{\overline{15}|}{}^{r}\mathrm{D}_{x+15}$$

should read

$$^{r}\mathbb{N}^s_x - {}^{r}\mathbb{N}^s_{x+16} - as_{(16)}{}^{r}\mathrm{D}_{x+15}$$

or

$$^{r}\mathbb{N}^s_x - {}^{r}\mathbb{N}^s_{x+15} - (1+i)\,(as)_{(15)}{}^{r}\mathrm{D}_{x+15}$$

Mr. Robertson obtained the same results by the direct method of summing the values on pages 219 and 237 for 15 years.

New Tables at 3 and 4 per-cent.

In a postscript which I added to my paper when it was published, I had the satisfaction of stating that, in response to my invitation, a gentleman had kindly volunteered to assist me by undertaking to calculate additional Tables. I have great pleasure in stating now that the gentleman is Mr. John Norman Lewis, at that time in the Scottish Widows' Fund, but now of the London Assurance; and I desire to express the indebtedness, not only of myself, but of the Institute and the profession, to him for his conscientious labours and for the skilful manner in which he has performed the work. Mr. Lewis has calculated the whole of the Tables at 3 per-cent, and Mr. Thomas has completed the Tables at 4 per-cent. It has given me great pleasure to have had these two gentlemen associated with me, and I feel sure that both will, in due time, receive their reward.

In view of the claims on the Editor of the *Journal,* I have reduced the Tables to the smallest number for efficient use. I had to decide between giving simply the Tables of multipliers, or publishing such of the Tables as would be useful for other investigations than those I have made; and after consulting many of my friends engaged in this class of work, who all wished to have the full Tables, I decided to publish those now given.

When we come to construct Tables for Pensions commencing at age 60, it will be found that all the figures in Table 4, after age 59, are altered. As all remaining in the service at age 60 are pensioned, it follows that all the r's after 59 disappear; the d's after 59 are increased by the deaths that occur amongst those who would otherwise retire between 60 and 65; and, of course, the l's are also changed. In these circumstances, I have assumed that Table 4 holds good up to age 60, and that from that age to age 65 the mortality follows my Table 2, after which it merges into the English Life Table No. 3. As a consequence, the N's, M's and R's, in Table 7, no longer apply, and have to be recalculated.

My Table 2 gives a slightly smaller annuity for age 60 than the English Life Table, the reason being that the heavy mortality assumed to prevail amongst those who retire early, in Table 8, col. $q_x^{(r)}$, brings the total mortality, from age 57 to 64, slightly above the English Life rate. If I were to recalculate all the Tables, I should modify $q_x^{(r)}$ after age 50 so as to make the total mortality, after age 56, the same as the English Life rate. As it is, the difference is very small and is not likely to affect a valuation, and certainly would not justify the trouble and labour of recalculating all the Tables.

All the D's, C's and (a')'s, remain the same for the same rate of interest, so these have not been repeated in the Tables for Pension age 60.

Valuation of Funds for Pension Age 60.

One question arises here for consideration, namely, the annuity-value which is to be used for calculating the pensions at age 60. I have used the value according to my Table No. 2, in order that my valuations shall be consistent throughout, but I should not be disposed to recommend it in all cases; for, although the after-life of persons who retire at age 65 may be fairly represented by the English Life Table, I do not think it is a good measure of the vitality of persons who retire at age 60. A man at 60 can adapt himself more readily to new conditions of life than a man at 65 can, so that in most cases I should be inclined to use a Table giving higher annuity-values for valuing the Pensions at 60 and over; but the selection of the Table must be left to individual judgment. On the other hand it must not be forgotten that there will always be a set-off by reason of many of the members remaining in the service after 60 years of age.

Valuation of Funds at 3 per-cent.

On the assumption that the Fund is only making 3 per-cent interest on its investments, and that in those benefits where the contributions are to be returned with compound interest, the same rate is to be allowed, col. (6) in my imaginary particulars will be changed, and the amount of the invested capital will be different.

The following figures must, therefore, be substituted for those in col. (6), page 242:

Total Past Salary, with 3 per-cent Compound Interest														
Present Age														
20			30			40			50			60		
Number of Members	Number of Years' Service	Total Past Salary with 3 per-cent Compound Interest	Number of Members	Number of Years' Service	Total Past Salary with 3 per-cent Compound Interest	Number of Members	Number of Years' Service	Total Past Salary with 3 per-cent Compound Interest	Number of Members	Number of Years' Service	Total Past Salary with 3 per-cent Compound Interest	Number of Members	Number of Years' Service	Total Past Salary with 3 per-cent Compound Interest
10	5	1,722	5	15	3,905	4	25	7,674	5	35	18,630	3	45	18,424
10	4	1,246	5	14	3,756	3	24	5,391	5	33	18,553	2	42	13,342
5	4	676	11	13	9,400	4	23	8,867	3	30	9,048	1	40	13,356
5	4	777	4	13	3,129	6	22	11,900	1	29	3,741	...	...	...
10	3	1,154	15	12	13,230	3	20	6,016	1	20	15,596	...	...	..
5	2	380	4	11	2,963	2	19	3,191	...	...	...	...	...	...
5	1	250	1	11	1,091	1	18	1,848	.	.	.	...	.	...
..	...	...	3	10	2,435	1	17	3,434	..	...	...	...	..	...
	..	...	1	10	1,644	1	10	3,608	...		. .		.	...
	..	..	1	8	719	..	...	...	...	. .	. .		..	..
...	..	6,205	...	...	42,272	...	...	51,929	...	...	65,568	..	...	44,122

And I shall assume that the Fund has an invested capital of £10,500.

Strictly speaking, the capital would not be the same in each of the Funds if the respective benefits had been in operation in past years; because the payment out in each case would have been different, so that to be quite accurate it should be understood that my valuations apply to a single existing Fund now possessing a capital of £12,000 or £10,500, according as the interest assumed is 4 per-cent or 3 per-cent; and the results produced show the changes which would have to be made in the scale of pension caused by a change in the benefits.

In order to avoid repetition, I propose in future to refer to the benefits by numbers.

Table of Benefits.

I. Pension on attainment of fixed age 60 or 65.

II. Ditto with return of excess of contributions, without interest, over annuity payments.

III. Return of half the contributions, without interest, on withdrawal.

IV. Return of total contributions, without interest, on withdrawal.

V. Return of total contributions, without interest, on death before pension age.

VI. Return of total contributions, with interest, on death before pension age.

VII. Return of total contributions, without interest, on retirement before pension age.

VIII. Return of total contributions, with interest, on retirement before pension age.

IX. Pension according to scale on retirement before pension age.

TABLE G.

Showing the alterations in the Pension Benefit as the result of changing existing or introducing other Benefits in an established Fund: Contributions being 5 *per-cent of Salary. The Funds referred to in the Table are those on pages* 251–4.

Pension Age		65				60			
Rate of guaranteed Interest		4 per-cent		3 per-cent		4 per-cent		3 per-cent	
Amount of Fund		12,000		10,500		12,000		10,500	
FUND	BENEFITS other than Pension on attainment of Pension Age	SCALE OF PENSION, being the percentage of Average Salary or of Last Salary for every year of Service							
		I	II	I	II	I	II	I	II
Pension based on Average Salary									
I	—	3·568	3·530	2·665	2·624	1·630	1·583	1·247	1·208
II	V	3·057	3·017	2·213	2·166	1·465	1·426	1·099	1·055
III	V, VII	2·552	2·509	1·767	1·713	1·377	1·337	1·018	·971
IV	IV, V, VII	2·347	2·301	1·589	1·529	1·271	1·227	·929	·877
V	IV, V, IX	1·506	1·442	1·094	1·012	1·165	1·119	·868	·813
VI	III, VI, IX	1·366	1·297	1·009	·920	1·112	1·064	·846	·790
Pension based on Last Salary									
VII	III, V, VIII	1·188	1·157	·833	·794	·940	·919	·563	·533
VIII	III, VI, IX	·824	·784	·603	·551	·810	·786	·512	·478

STANDARD SCALE OF BENEFITS WHICH CAN BE GIVEN FOR 5 PER-CENT OF SALARY.

The above Funds, however, do not give us all the information we require. What we frequently want to know is:—Assuming certain benefits to be agreed upon, what is the pension which ought to be given for a certain fixed contribution of salary? For this purpose I think we might take 20 as a fair average age at entry on which to base a standard pension, and the following Table will give the desired information:—

TABLE H.

Showing the Standard Pension Benefit for 5 per-cent of Salary after allowing for various other Benefits.

Pension Age .	65		60		65		60	
Rate of Interest	4%	3%	4%	3%	4%	3%	4%	3%
BENEFITS other than Pension on attainment of Pension Age	SCALE OF PENSION; being the percentage of *Average* Salary for every year of Service				SCALE OF PENSION; being the percentage of *Last* Salary for every year of Service			
	I	I	I	I	I	I	I	I
0	4·588	3·301	2·115	1·564	2·795	2·011	1·310	·969
V	3·873	2·711	1·852	1·346	2·359	1·652	1·148	·834
V, VII	3·316	2·220	1·733	1·242	2·020	1·352	1·074	·770
III, V, VII	2·971	1·982	1·568	1·124	1·810	1·207	·971	·696
IV, V, VII	2·626	1·744	1·403	1·006	1·599	1·062	·869	·623
III, VI, VII	2·422	1·670	1·395	1·024	1·475	1·017	·864	·634
IV, VI, VII	2·078	1·498	1·230	·906	1·265	·912	·762	·561
III, VI, VIII	1·877	1·346	1·299	·967	1·143	·820	·804	·599
IV, VI, VIII	1·532	1·108	1·133	·849	·933	·675	·702	·526
IX	2·287	1·693	1·727	1·293	1·410	1·039	1·077	·804
V, IX	1·931	1·390	1·512	1·113	1·190	·853	·943	·692
III, V, IX	1·759	1·268	1·378	1·015	1·084	·779	·859	·631
IV, V, IX	1·587	1·146	1·243	·917	·978	·704	·775	·570
III, VI, IX	1·485	1·108	1·236	·932	·915	·680	·771	·580
IV, VI, IX	1·313	·986	1·102	·835	·809	·605	·687	·519

To ascertain the Pension when the excess of the accumulated contributions without interest over the annuity payments is returned, the formula in Problem VIIB must be applied to the above figures as explained on page 255.

I will conclude with the investigation of three problems about which inquiry is sometimes made.

Problem XIIIB.—What proportion of salary, commencing at age 20, is required to provide a pension of two-thirds of the last salary, to those only who reach the age of 65?

Equating contributions to liability, we have

$$P\left\{\mathbb{N}^s_{20}-\tfrac{1}{2}\left({}^d\mathrm{M}^{ls}_{20}+{}^w\mathrm{M}^{ls}_{20}+{}^r\mathrm{M}^{ls}_{20}\right)\right\}(1+\tfrac{3}{8}i)=\tfrac{2}{3}s_{64}(\mathrm{N}_{65}+\tfrac{1}{2}\mathrm{D}_{65})$$

$$P=\frac{\tfrac{2}{3}s_{64}(\mathrm{N}_{65}+\tfrac{1}{2}\mathrm{D}_{65})}{(1+\tfrac{3}{8}i)\left\{\mathbb{N}^s_{20}-\tfrac{1}{2}\left({}^d\mathrm{M}^{ls}_{20}+{}^w\mathrm{M}^{ls}_{20}+{}^r\mathrm{M}^{ls}_{20}\right)\right\}}$$

If interest is guaranteed at 4 per-cent, free of income tax, and there are no expenses, then $P=\cdot026505$: say 2·65 per-cent.

If interest is guaranteed at 3 per-cent, with the same conditions, then $P=\cdot03683$: say 3·683 per-cent.

Problem XIVB.—What proportion of salary, commencing at age 20, is required to provide a pension of $\frac{1}{60}$th of last salary for every year of service not exceeding 40, on retirement from ill-health before the age of 65, and on compulsory retirement at 65? This is practically the Government scale.

This is the same as Problem XIIIB with an addition to the numerator of a provision for early retirement.

As no more than 40 years' service is to be reckoned, we shall have to divide the provision for early retirement into two parts, namely, before and after 40 years' service.

The provision for early retirement during the first 40 years is $\frac{1}{60}\left({}^{ra}\mathrm{R}^{ls}_{20}-{}^{ra}\mathrm{R}^{ls}_{60}-40\,{}^{ra}\mathrm{M}^{ls}_{60}\right)$.

For the last 5 years it is the insurance of an annuity of $\frac{40}{60}$ths of the last salary $=\frac{40}{60}{}^{ra}\mathrm{M}^{ls}_{60}$.

And these, being added together, will give $\frac{1}{60}\left({}^{ra}\mathrm{R}^{ls}_{20}-{}^{ra}\mathrm{R}^{ls}_{60}\right)$.

The correction, beyond the allowance for over-discount, will be $-\frac{1}{2}{}^{ra}\mathrm{M}^{ls}_{20}$, because the above formula provides, whenever retirement takes place before 65, whether before or after 40 years' service, an excess of an annuity of half the last year's salary.

The complete formula, therefore, is

$$P=\frac{\tfrac{2}{3}s_{64}(\mathrm{N}_{65}+\tfrac{1}{2}\mathrm{D}_{65})+\tfrac{1}{60}\left({}^{ra}\mathrm{R}^{ls}_{20}-{}^{ra}\mathrm{R}^{ls}_{60}-\tfrac{1}{2}{}^{ra}\mathrm{M}^{ls}_{20}\right)(1+\tfrac{1}{2}i)}{(1+\tfrac{3}{8}i)\left\{\mathbb{N}^s_{20}-\tfrac{1}{2}\left({}^d\mathrm{M}^{ls}_{20}+{}^w\mathrm{M}^{ls}_{20}+{}^r\mathrm{M}^{ls}_{20}\right)\right\}}$$

If 4 per-cent interest, free of income tax, is guaranteed, and there are no expenses, then P = ·054616: say 5·462 per-cent.

If 3 per-cent is guaranteed, then P = ·073944: say 7·394 per-cent.

Problem XVB.—The same as Problem XIVB, only substituting "average salary for 40 years or for term of service if less than 40 years" for "last salary."

If the number of years' service exceeds 40, the employee will certainly select the last 40 years on which to base his average, and consequently the portion of the formula on the liability side representing the value of retirement at age 65 will be

$$\tfrac{1}{60}\Sigma s_{25}(N_{65}+\tfrac{1}{2}D_{65}).$$

When we come to consider the terms in the formula for retirement before 65, we have to divide it, as before, into two parts, and consider retirement before and after 40 years separately.

For the first 40 years' service we have—

$$\tfrac{1}{60}\{{}^{ra}R^{s}_{20}-{}^{ra}R^{s}_{60}-(s_{20}+s_{21}+\ldots s_{59})^{ra}M_{60}\}$$

and for service after 40 years,

$$\tfrac{1}{60}\{(s_{21}+\ldots s_{60})^{ra}C_{60}+(s_{22}+\ldots s_{61})^{ra}C_{61}+(s_{23}+\ldots s_{62})^{ra}C_{62}$$
$$+(s_{24}+\ldots s_{63})^{ra}C_{63}+(s_{25}+\ldots s_{64})^{ra}C_{64}\}$$

Now we have—

$$(s_{21}+\ldots+s_{60})^{ra}C_{60}=(s_{20}+\ldots+s_{59})^{ra}C_{60}$$
$$+(s_{60}-s_{20})^{ra}C_{60}$$

$$(s_{22}+\ldots+s_{61})^{ra}C_{61}=(s_{20}+\ldots+s_{59})^{ra}C_{61}$$
$$+(s_{60}+s_{61}-s_{20}-s_{21})^{ra}C_{61}$$

$$(s_{23}+\ldots+s_{62})^{ra}C_{62}=(s_{20}+\ldots+s_{59})^{ra}C_{62}$$
$$+(s_{60}+s_{61}+s_{62}-s_{20}-s_{21}-s_{22})^{ra}C_{62}$$

$$(s_{24}+\ldots+s_{63})^{ra}C_{63}=(s_{20}+\ldots+s_{59})^{ra}C_{63}$$
$$+(s_{60}+s_{61}+s_{62}+s_{63}-s_{20}-s_{21}-s_{22}-s_{23})^{ra}C_{63}$$

$$(s_{25}+\ldots+s_{64})^{ra}C_{64}=(s_{20}+\ldots+s_{59})^{ra}C_{64}$$
$$+(s_{60}+s_{61}+s_{62}+s_{63}+s_{64}-s_{20}-s_{21}-s_{22}-s_{23}-s_{24})^{ra}C_{64}$$

Summing the values on the right-hand side, we have—

$$(s_{20}+\ldots+s_{59})^{ra}M_{60}+{}^{ra}R^{s}_{60}-({}^{ra}M_{60}s_{20}+{}^{ra}M_{61}s_{21}+\ldots+{}^{ra}M_{64}s_{24})$$

so that those terms of the formula which represent the pension on early retirement will be—

$$\tfrac{1}{60}\{{}^{ra}R^{s}_{20}-({}^{ra}M_{60}s_{20}+{}^{ra}M_{61}s_{21}+\ldots+{}^{ra}M_{64}s_{64})\}$$

which, on consideration, will be found to be self-evident; for it is $\frac{1}{60}$th of—

The present value of an annuity based on the total salary from age 20 to 65;

Less the present value of an annuity of the salary at age 20 to all those whose service will exceed 40 years; the present value of an annuity of the salary at age 21 to all those whose service will exceed 41 years; and so on:

thus cutting off the first year's salary for those who retire in the 41st year of service, the first two years' salary for those who retire in the 42nd year of service, &c.; so that each of those whose service exceeds 40 years will receive a pension based on $\frac{1}{60}$th of his total salary dating back for 40 years from the end of the year of his retirement.

The complete formula, therefore, is—

$$P=\frac{\tfrac{1}{60}\Sigma_{25}(N_{65}+\tfrac{1}{2}D_{65})+\tfrac{1}{60}\{{}^{ra}R^{s}_{20}-({}^{ra}M_{60}s_{20}+{}^{ra}M_{61}s_{21}+\ldots+{}^{ra}M_{64}s_{24})-\tfrac{1}{2}{}^{ra}M^{ls}_{20}\}(1+\tfrac{1}{2}i)}{(1+\tfrac{3}{8}i)\{N^{s}_{20}-\tfrac{1}{2}({}^{d}M^{ls}_{20}+{}^{w}M^{ls}_{20}+{}^{r}M^{ls}_{20})\}}$$

If interest is guaranteed at 4 per-cent, free of income tax, and there are no expenses, then $P=\cdot035347$, say 3·535 per-cent. If interest is guaranteed at 3 per-cent with the same conditions, then $P=\cdot04773$, say 4·773 per-cent.

I should like to direct attention to the "Actuarial Note" by Mr. H. T. Adlard, in vol. xxxvi, page 389, as affording the means of solving the very difficult problem of a return of contributions on death, withdrawal, or retirement, with compound interest at a different rate to that used in the valuation.

Hypothetical Experience of Staff Pension Fund.

TABLE 18.

Commutation Columns for finding the Value of one Year's Salary on Death, Withdrawal, and Retirement; the Value of a Pension of 1 for each Year of Service on Early Retirement; and the Value of a Pension on Early Retirement based on Average Salary or Last Salary and Number of Years' Service.

PENSION AGE 65. INTEREST 4 PER-CENT.

Age (x)	${}^{d}M_x^{ls} = \Sigma\,{}^{d}C_x s_x$	${}^{w}M_x^{ls} = \Sigma\,{}^{w}C_x s_x$	${}^{r}M_x^{ls} = \Sigma\,{}^{r}C_x s_x$	${}^{ra}M_x$*	${}^{ra}R_x = \Sigma\,{}^{ra}M_x$	${}^{ra}M_x^{s} = {}^{ra}M_x \times s_x$	${}^{ra}R_x^{s} = \Sigma\,{}^{ra}M_x^{s}$	${}^{ra}M_x^{ls}$†	${}^{ra}R_x^{ls} = \Sigma\,{}^{ra}M_x^{ls}$	Age (x)
15	64,737	259,869	33,017	1,572	64,373	31,436	7,218,757	296,255	12,590,444	15
16	63,969	243,849	33,017	1,572	62,801	39,295	7,187,321	296,255	12,294,189	16
17	63,096	222,749	33,017	1,572	61,229	47,154	7,148,026	296,255	11,997,934	17
18	62,148	200,219	33,017	1,572	59,657	55,013	7,100,872	296,255	11,701,679	18
19	61,168	179,149	33,017	1,572	58,085	62,872	7,045,859	296,255	11,405,424	19
20	60,164	159,909	33,017	1,572	56,513	70,731	6,982,987	296,255	11,109,169	20
21	59,138	142,404	33,017	1,572	54,941	78,591	6,912,256	296,255	10,812,914	21
22	58,103	126,754	33,017	1,572	53,369	86,450	6,833,665	296,255	10,516,659	22
23	57,052	112,784	33,017	1,572	51,797	94,309	6,747,215	296,255	10,220,404	23
24	55,996	100,304	33,017	1,572	50,225	102,168	6,652,906	296,255	9,924,149	24
25	54,949	89,124	33,017	1,572	48,653	110,027	6,550,738	296,255	9,627,894	25
26	53,892	79,114	33,017	1,572	47,081	116,314	6,440,711	296,255	9,331,639	26
27	52,812	70,234	33,017	1,572	45,509	122,601	6,324,397	296,255	9,035,384	27
28	51,743	62,278	33,017	1,572	43,937	128,888	6,201,796	296,255	8,739,129	28
29	50,693	55,226	32,965	1,567	42,365	134,765	6,072,908	295,864	8,442,874	29
30	49,635	48,862	32,911	1,562	40,798	140,613	5,938,143	295,462	8,147,000	30
31	48,564	43,192	32,831	1,556	39,236	146,225	5,797,530	294,851	7,851,548	31
32	47,492	38,116	32,724	1,547	37,680	151,583	5,651,305	294,022	7,556,697	32
33	46,414	33,608	32,621	1,538	36,133	156,892	5,499,722	293,213	7,262,675	33
34	45,343	29,528	32,491	1,528	34,595	161,937	5,342,830	292,188	6,969,462	34
35	44,272	25,818	32,357	1,518	33,067	166,927	5,180,893	291,108	6,677,274	35
36	43,172	22,518	32,196	1,506	31,549	171,645	5,013,966	289,804	6,386,166	36
37	42,078	19,554	32,037	1,494	30,043	176,310	4,842,321	288,492	6,096,362	37
38	40,957	16,840	31,850	1,481	28,549	180,681	4,666,011	286,940	5,807,870	38
39	39,822	14,400	31,639	1,466	27,068	184,771	4,485,330	285,160	5,520,930	39
40	38,663	12,258	31,402	1,450	25,602	188,555	4,300,559	283,142	5,235,770	40
41	37,493	10,338	31,142	1,433	24,152	192,046	4,112,004	280,901	4,952,628	41
42	36,300	8,566	30,858	1,415	22,719	195,227	3,919,958	278,423	4,671,727	42
43	35,072	7,048	30,552	1,395	21,304	198,106	3,724,731	275,721	4,393,304	43
44	33,837	5,770	30,224	1,375	19,909	200,677	3,526,625	272,795	4,117,583	44
45	32,567	4,602	29,873	1,353	18,534	202,932	3,325,948	269,638	3,844,788	45
46	31,232	3,552	29,479	1,329	17,181	204,661	3,123,016	266,052	3,575,150	46
47	29,861	2,628	29,040	1,303	15,852	205,868	2,918,355	262,032	3,309,098	47
48	28,471	1,838	28,560	1,275	14,549	206,530	2,712,487	257,594	3,047,066	48
49	27,029	1,190	28,038	1,245	13,274	206,657	2,505,957	252,743	2,789,472	49
50	25,535	692	27,454	1,212	12,029	206,035	2,299,300	247,274	2,536,729	50
51	24,022	352	26,809	1,176	10,817	204,671	2,093,265	241,204	2,289,455	51
52	22,491	178	26,036	1,137	9,641	202,379	1,888,594	234,364	2,048,251	52
53	20,907	...	25,285	1,094	8,504	199,170	1,686,215	226,779	1,813,887	53
54	19,287	...	24,387	1,048	7,410	194,844	1,487,045	218,264	1,587,108	54
55	17,632	...	23,398	997	6,362	189,432	1,292,201	208,863	1,368,844	55
56	15,922	...	22,300	942	5,365	182,767	1,102,769	198,430	1,159,981	56
57	14,157	...	21,056	881	4,423	174,517	920,002	186,654	961,551	57
58	12,375	...	19,652	805	3,542	162,648	745,485	173,431	774,897	58
59	10,557	...	18,034	730	2,737	150,458	582,837	158,319	601,466	59
60	8,724	...	16,075	642	2,007	134,887	432,379	140,177	443,147	60
61	6,876	...	13,715	539	1,365	115,301	297,492	118,603	302,976	61
62	5,057	...	10,781	416	826	90,647	182,191	92,286	184,367	62
63	3,269	...	7,299	276	410	61,181	91,544	61,718	92,081	63
64	1,582	...	3,636	134	134	30,363	30,363	30,363	30,363	64

* ${}^{ra}M_x = \Sigma\{{}^{r}C_x \times (a'_{x+\cdot5} + \cdot5)\}$ † ${}^{ra}M_x^{ls} = \Sigma\{{}^{r}C_x s_x (a'_{x+\cdot5} + \cdot5)\}$

Hypothetical Experience of Staff Pension Fund.

TABLE 19—(*continuation of Table* 17).

Multipliers for use in a Valuation.

PENSION AGE 65. INTEREST 4 PER-CENT.

Age (x)	${}^{d}M^{ls}_{x} \div D^{s}_{x}$	${}^{w}M^{ls}_{x} \div D^{s}_{x}$	${}^{r}M^{ls}_{x} \div D^{s}_{x}$	${}^{ra}M_{x} \div D_{x}$	${}^{ra}R_{x} \div D_{x}$	${}^{ra}R^{s}_{x} \div D^{s}_{x}$	${}^{ra}M^{ls}_{x} \div D^{s}_{x}$	${}^{ra}R^{ls}_{x} \div D^{s}_{x}$	Age (x)
15	·291	1·170	·149	·142	5·796	32·499	1·334	56·683	15
16	·260	0·991	·134	·160	6·382	29·214	1·204	49·971	16
17	·245	·865	·128	·183	7·135	27·764	1·151	46·598	17
18	·238	·765	·126	·210	7·987	27·163	1·133	44·764	18
19	·233	·684	·126	·240	8·867	26·889	1·131	43·524	19
20	·231	·613	·127	·271	9·754	26·782	1·136	42·607	20
21	·229	·552	·128	·305	10·650	26·797	1·149	41·919	21
22	·228	·498	·130	·340	11·532	26·847	1·164	41·318	22
23	·228	·450	·132	·376	12·406	26·935	1·183	40·800	23
24	·227	·407	·134	·415	13·252	27·006	1·203	40·285	24
25	·227	·369	·137	·455	14·082	27·086	1·225	39·809	25
26	·230	·338	·141	·497	14·880	27·580	1·265	39·855	26
27	·233	·310	·146	·541	15·650	27·882	1·306	39·834	27
28	·235	·283	·150	·586	16·388	28·210	1·348	39·752	28
29	·238	·259	·155	·632	17·090	28·485	1·388	39·602	29
30	·240	·236	·159	·680	17·769	28·736	1·430	39·426	30
31	·242	·225	·164	·730	18·403	28·928	1·471	39·179	31
32	·244	·196	·168	·780	18·992	29·066	1·512	38·866	32
33	·246	·178	·173	·832	19·542	29·161	1·555	38·509	33
34	·248	·161	·178	·886	20·055	29·220	1·598	38·116	34
35	·250	·146	·182	·941	20·500	29·200	1·641	37·633	35
36	·251	·131	·188	·997	20·893	29·127	1·684	37·099	36
37	·252	·117	·192	1·057	21·247	29·022	1·729	36·537	37
38	·253	·104	·197	1·117	21·530	28·843	1·774	35·902	38
39	·254	·092	·202	1·178	21·742	28·592	1·818	35·195	39
40	·254	·081	·207	1·241	21·900	28·299	1·863	34·453	40
41	·255	·070	·212	1·305	21·996	27·948	1·909	33·661	41
42	·255	·060	·217	1·371	22·015	27·525	1·955	32·803	42
43	·255	·051	·222	1·438	21·963	27·042	2·002	31·896	43
44	·254	·043	·227	1·507	21·830	26·486	2·049	30·924	44
45	·253	·036	·232	1·577	21·601	25·843	2·095	29·874	45
46	·251	·029	·237	1·647	21·290	25·129	2·141	28·768	46
47	·249	·022	·243	1·719	20·913	24·368	2·188	27·630	47
48	·246	·015	·247	1·788	20·405	23·484	2·230	26·380	48
49	·243	·011	·253	1·861	19·841	22·566	2·276	25·118	49
50	·239	·006	·257	1·930	19·154	21·537	2·315	23·761	50
51	·234	·003	·261	1·994	18·334	20·391	2·350	22·302	51
52	·228	·002	·265	2·056	17·434	19·186	2·381	20·809	52
53	·222	...	·269	2·117	16·449	17·920	2·410	19·278	53
54	·215	...	·272	2·169	15·342	16·552	2·429	17·666	54
55	·206	...	·273	2·211	14·106	15·080	2·437	15·974	55
56	·196	...	·275	2·254	12·835	13·599	2·447	14·305	56
57	·185	...	·275	2·278	11·429	12·064	2·436	12·549	57
58	·172	...	·273	2·262	9·950	10·367	2·412	10·776	58
59	·158	...	·269	2·247	8·422	8·706	2·365	8·984	59
60	·141	...	·260	2·185	6·827	7·003	2·270	7·178	60
61	·122	...	·244	2·049	5·190	5·286	2·107	5·383	61
62	·101	...	·215	1·808	3·591	3·634	1·841	3·677	62
63	·075	...	·167	1·399	2·081	2·093	1·411	2·105	63
64	·042	...	·097	·809	·809	·809	·809	·809	64

Hypothetical Experience of Staff Pension Fund.

TABLE 20.

Simple Commutation Columns (according to Table 4), and Commutation Columns for valuing Benefits on Death and Early Retirement.

PENSION AGE 60. INTEREST 4 PER-CENT.

Age (x)	$N_x^{(4)}$	$M_x^{(4)}$	$R_x^{(4)}$	${}^dM_x = \Sigma_x^{59} C_x$	${}^dR_x = \Sigma {}^dM_x$	${}^rM_x = \Sigma_x^{59} {}^rC_x$	${}^rR_x = \Sigma {}^rM_x$	Age (x)
15	104,449	785·6	20777·6	603·4	10481·6	100·84	3582·49	15
16	94,608	747·2	19992·0	565·0	9878·2	100·84	3481·65	16
17	86,026	712·3	19244·8	530·1	9313·2	100·84	3380·81	17
18	78,557	680·7	18532·5	498·5	8783·1	100·84	3279·97	18
19	72,006	652·7	17851·8	470·5	8284·6	100·84	3179·13	19
20	66,212	627·6	17199·1	445·4	7814·1	100·84	3078·29	20
21	61,053	604·8	16571·5	422·6	7368·7	100·84	2977·45	21
22	56,425	584·1	15966·7	401·9	6946·1	100·84	2876·61	22
23	52,250	565·0	15382·6	382·8	6544·2	100·84	2775·77	23
24	48,460	547·4	14817·6	365·2	6161·4	100·84	2674·93	24
25	45,005	531·3	14270·2	349·1	5796·2	100·84	2574·09	25
26	41,841	516·2	13738·9	334·0	5447·1	100·84	2473·25	26
27	38,933	501·6	13222·7	319·4	5113·1	100·84	2372·41	27
28	36,252	487·9	12721·1	305·7	4793·7	100·84	2271·57	28
29	33,773	475·1	12233·2	292·9	4488·0	100·20	2170·73	29
30	31,477	462·8	11758·1	280·6	4195·1	99·58	2070·53	30
31	29,345	450·9	11295·3	268·7	3914·5	98·69	1970·95	31
32	27,361	439·5	10844·4	257·3	3645·8	97·55	1872·26	32
33	25,512	428·5	10404·9	246·3	3388·5	96·45	1774·71	33
34	23,787	418·0	9976·4	235·8	3142·2	95·13	1678·26	34
35	22,174	407·9	9558·4	225·7	2906·4	93·86	1583·13	35
36	20,664	397·9	9150·5	215·7	2680·7	92·40	1489·27	36
37	19,250	388·3	8752·6	206·1	2465·0	91·00	1396·87	37
38	17,924	378·8	8364·3	196·6	2258·9	89·42	1305·87	38
39	16,679	369·5	7985·5	187·3	2062·3	87·69	1216·45	39
40	15,510	360·3	7616·0	178·1	1875·0	85·81	1128·76	40
41	14,412	351·3	7255·7	169·1	1696·9	83·81	1042·95	41
42	13,380	342·4	6904·4	160·2	1527·8	81·69	959·14	42
43	12,410	333·5	6562·0	151·3	1367·6	79·47	877·45	43
44	11,498	324·8	6228·5	142·6	1216·3	77·16	797·98	44
45	10,640	316·1	5903·7	133·9	1073·7	74·76	720·82	45
46	9,833	307·2	5587·6	125·0	939·8	72·13	646·06	46
47	9,075	298·3	5280·4	116·1	814·8	69·28	573·93	47
48	8,362	289·5	4982·1	107·3	698·7	66·24	504·65	48
49	7,693	280·6	4692·6	98·4	591·4	63·02	438·41	49
50	7,065	271·6	4412·0	89·4	493·0	59·50	375·39	50
51	6,475	262·7	4140·4	80·5	403·6	55·71	315·89	51
52	5,922	253·9	3877·7	71·7	323·1	51·55	260·18	52
53	5,405	245·0	3623·8	62·8	251·4	47·05	208·63	53
54	4,922	236·1	3378·8	53·9	188·6	42·12	161·58	54
55	4,471	227·2	3142·7	45·0	134·7	36·80	119·46	55
56	4,053	218·2	2915·5	36·0	89·7	31·02	82·66	56
57	3,666	209·1	2697·3	26·9	53·7	24·61	51·64	57
58	3,310	200·1	2488·2	17·9	26·8	17·52	27·03	58
59	2,985	191·1	2288·1	8·9	8·9	9·51	9·51	59
60	2,691	182·2	2097·0	...	...	...	...	60

NOTE.—$D_x^{(4)}$ and $C_x^{(4)}$ are the same as in Table 7. ${}^dC_x$, ${}^wC_x$, ${}^wM_x$, ${}^wR_x$ and ${}^rC_x$ are the same as in Table 9.

Hypothetical Experience of Staff Pension Fund.

TABLE 21.

Commutation Tables for finding the Values of the Return of Contributions of 1 per annum, with Compound Interest at 4 per-cent, on Death or Early Retirement, and for finding the Value of a Pension of 1 for each Year of Service on Early Retirement.

PENSION AGE 60. INTEREST 4 PER-CENT.

Age (x)	${}^{d}D_x = l'_x \times v^{x+1}$	${}^{d}N_x = \Sigma {}^{d}D_x$	${}^{r}D_x = \Sigma r_x \times v^{x+1}$	${}^{r}N_x = \Sigma {}^{r}D_x$	${}^{ra}M_x = \Sigma\{{}^{r}C_x \times (a'_{x+\cdot 5}+\cdot 5)\}$	${}^{ra}R_x = \Sigma {}^{ra}M_x$	Age (x)
15	1,333	19,575	410	8,128	930	33,469	15
16	1,245	18,242	394	7,718	930	32,539	16
17	1,163	16,997	379	7,324	930	31,609	17
18	1,088	15,834	364	6,945	930	30,679	18
19	1,020	14,746	350	6,581	930	29,749	19
20	956	13,726	337	6,231	930	28,819	20
21	898	12,770	324	5,894	930	27,889	21
22	843	11,872	311	5,570	930	26,959	22
23	792	11,029	299	5,259	930	26,029	23
24	745	10,237	288	4,960	930	25,099	24
25	701	9,492	277	4,672	930	24,169	25
26	659	8,791	266	4,395	930	23,239	26
27	620	8,132	256	4,129	930	22,309	27
28	583	7,512	246	3,873	930	21,379	28
29	548	6,929	236	3,627	925	20,449	29
30	515	6,381	226	3,391	920	19,524	30
31	484	5,866	217	3,165	913	18,604	31
32	454	5,382	207	2,948	905	17,691	32
33	426	4,928	198	2,741	896	16,786	33
34	400	4,502	189	2,543	885	15,890	34
35	375	4,102	181	2,354	875	15,005	35
36	351	3,727	172	2,173	863	14,130	36
37	328	3,376	164	2,001	852	13,267	37
38	306	3,048	157	1,837	839	12,415	38
39	286	2,742	149	1,680	824	11,576	39
40	266	2,456	141	1,531	808	10,752	40
41	247	2,190	134	1,390	791	9,944	41
42	229	1,943	127	1,256	772	9,153	42
43	212	1,714	120	1,129	753	8,381	43
44	195	1,502	113	1,009	732	7,628	44
45	179	1,307	106	896	711	6,896	45
46	164	1,128	100	790	687	6,185	46
47	149	964	93	690	661	5,498	47
48	135	815	87	597	633	4,837	48
49	121	680	80	510	603	4,204	49
50	108	559	74	430	570	3,601	50
51	95	451	67	356	534	3,031	51
52	83	356	61	289	495	2,497	52
53	71	273	54	228	452	2,002	53
54	60	202	47	174	405	1,550	54
55	49	142	40	127	355	1,145	55
56	38	93	33	87	300	790	56
57	28	55	26	54	239	490	57
58	18	27	18	28	163	251	58
59	9	9	10	10	88	88	59

Hypothetical Experience of Staff Pension Fund.

TABLE 22.

Table of Average Salaries and various combinations thereof.

PENSION AGE 60. INTEREST 4 PER-CENT.

Age (x)	s_x	Σs_x	$\Sigma s_x \div s_x$	$s_{59} \div s_x$	$s_x v^x$	$\Sigma s_x v^x$	$(1+i)^{x-1}$	Age (x)
15	20	5,255	262·75	10·300	11·11	1083·39	1·732	15
16	25	5,235	209·40	8·240	13·35	1072·28	1·801	16
17	30	5,210	173·67	6·867	15·40	1058·93	1·873	17
18	35	5,180	148·00	5·886	17·28	1043·53	1·948	18
19	40	5,145	128·63	5·150	18·98	1026·25	2·026	19
20	45	5,105	113·44	4·578	20·54	1007·27	2·107	20
21	50	5,060	101·20	4·120	21·94	986·73	2·191	21
22	55	5,010	91·09	3·746	23·21	964·79	2·279	22
23	60	4,955	82·58	3·433	24·34	941·58	2·370	23
24	65	4,895	75·31	3·169	25·36	917·24	2·465	24
25	70	4,830	69·00	2·943	26·26	891·88	2·563	25
26	74	4,760	64·33	2·784	26·69	865·62	2·666	26
27	78	4,686	60·08	2·641	27·05	838·93	2·773	27
28	82	4,608	56·20	2·512	27·35	811·88	2·883	28
29	86	4,526	52·63	2·395	27·58	784·53	2·999	29
30	90	4,440	49·33	2·289	27·75	756·95	3·119	30
31	94	4,350	46·28	2·192	27·87	729·20	3·243	31
32	98	4,256	43·43	2·102	27·94	701·33	3·373	32
33	102	4,158	40·76	2·020	27·96	673·39	3·508	33
34	106	4,056	38·26	1·943	27·94	645·43	3·648	34
35	110	3,950	35·91	1·873	27·87	617·49	3·794	35
36	114	3,840	33·68	1·807	27·78	589·62	3·946	36
37	118	3,726	31·58	1·746	27·65	561·84	4·104	37
38	122	3,608	29·57	1·689	27·49	534·19	4·268	38
39	126	3,486	27·67	1·635	27·29	506·70	4·439	39
40	130	3,360	25·85	1·585	27·08	479·41	4·616	40
41	134	3,230	24·11	1·537	26·84	452·33	4·801	41
42	138	3,096	22·44	1·493	26·58	425·49	4·993	42
43	142	2,958	20·83	1·451	26·30	398·91	5·193	43
44	146	2,816	19·29	1·411	25·99	372·61	5·401	44
45	150	2,670	17·80	1·373	25·68	346·62	5·617	45
46	154	2,520	16·36	1·338	25·35	320·94	5·841	46
47	158	2,366	14·97	1·304	25·01	295·59	6·075	47
48	162	2,208	13·63	1·272	24·66	270·58	6·318	48
49	166	2,046	12·33	1·241	24·29	245·92	6·571	49
50	170	1,880	11·06	1·212	23·92	221·63	6·833	50
51	174	1,710	9·83	1·184	23·54	197·71	7·107	51
52	178	1,536	8·63	1·157	23·16	174·17	7·391	52
53	182	1,358	7·46	1·132	22·77	151·01	7·687	53
54	186	1,176	6·32	1·108	22·38	128·24	7·994	54
55	190	990	5·21	1·084	21·98	105·86	8·314	55
56	194	800	4·12	1·062	21·57	83·88	8·646	56
57	198	606	3·06	1·040	21·17	62·31	8·992	57
58	202	408	2·02	1·020	20·77	41·14	9·352	58
59	206	206	1·00	1·000	20·37	20·37	9·726	59

Hypothetical Experience of Staff Pension Fund.

TABLE 23.

Commutation Columns for finding the Present Values of Future Salary, and Return of Contributions on Death and Early Retirement, without Interest.

PENSION AGE 60. INTEREST 4 PER-CENT.

Age (x)	$N^s_x = \Sigma^{59}_x(vD^s_x)$	${}^dM^s_x = {}^dM_x \times s_x$	${}^dR^s_x = \Sigma\,{}^dM^s_x$	${}^rM^s_x = {}^rM_x \times s_x$	${}^rR^s_x = \Sigma\,{}^rM^s_x$	Age (x)
15	7,354,555	12,068	851,190	2,017	362,839	15
16	7,140,978	14,125	839,122	2,521	360,822	16
17	6,904,416	15,903	824,997	3,025	358,301	17
18	6,656,858	17,448	809,094	3,529	355,276	18
19	6,405,497	18,820	791,646	4,034	351,747	19
20	6,153,535	20,043	772,826	4,538	347,713	20
21	5,902,833	21,130	752,783	5,042	343,175	21
22	5,654,804	22,105	731,653	5,546	338,133	22
23	5,410,054	22,968	709,548	6,050	332,587	23
24	5,169,189	23,738	686,580	6,555	326,537	24
25	4,932,314	24,437	662,842	7,059	319,982	25
26	4,699,766	24,716	638,405	7,463	312,923	26
27	4,474,635	24,913	613,689	7,866	305,460	27
28	4,256,535	25,067	588,776	8,269	297,594	28
29	4,045,148	25,189	563,709	8,617	289,325	29
30	3,840,154	25,254	538,520	8,962	280,708	30
31	3,641,462	25,258	513,266	9,277	271,746	31
32	3,448,762	25,215	488,008	9,560	262,469	32
33	3,261,808	25,123	462,793	9,838	252,909	33
34	3,080,464	24,995	437,670	10,084	243,071	34
35	2,904,647	24,827	412,675	10,325	232,987	35
36	2,734,041	24,590	387,848	10,534	222,662	36
37	2,568,522	24,320	363,258	10,738	212,128	37
38	2,408,087	23,985	338,938	10,909	201,390	38
39	2,252,537	23,600	314,953	11,049	190,481	39
40	2,101,700	23,153	291,353	11,155	179,432	40
41	1,955,575	22,659	268,200	11,231	168,277	41
42	1,814,102	22,108	245,541	11,273	157,046	42
43	1,677,164	21,485	223,433	11,285	145,773	43
44	1,544,722	20,820	201,948	11,265	134,488	44
45	1,416,691	20,085	181,128	11,214	123,223	45
46	1,292,941	19,250	161,043	11,108	112,009	46
47	1,173,443	18,344	141,793	10,946	100,901	47
48	1,058,285	17,383	123,449	10,731	89,955	48
49	947,222	16,334	106,066	10,461	79,224	49
50	840,439	15,198	89,732	10,115	68,763	50
51	737,785	14,007	74,534	9,694	58,648	51
52	639,073	12,763	60,527	9,176	48,954	52
53	544,425	11,430	47,764	8,563	39,778	53
54	453,950	10,025	36,334	7,834	31,215	54
55	367,567	8,550	26,309	6,992	23,381	55
56	285,173	6,984	17,759	6,018	16,389	56
57	207,200	5,326	10,775	4,873	10,371	57
58	133,521	3,616	5,449	3,539	5,498	58
59	64,375	1,833	1,833	1,959	1,959	59

NOTE.—The values of D^s_x will be found in Table 13, and the values of ${}^wM^s_x$ and ${}^wR^s_x$ are the same as in Table 14.

Hypothetical Experience of Staff Pension Fund.

TABLE 24.

Commutation Columns for finding the Values of Return of Contributions, with Compound Interest, on Death or Early Retirement.

PENSION AGE 60. INTEREST 4 PER-CENT.

Age (x)	${}^{d}\mathrm{D}^{s}_{x} = {}^{d}\mathrm{D}_{x} \times s_{x}$	${}^{d}\mathrm{N}^{s}_{x} = \Sigma\,{}^{d}\mathrm{D}^{s}_{x}$	${}^{r}\mathrm{D}^{s}_{x} = {}^{r}\mathrm{D}_{x} \times s_{x}$	${}^{r}\mathrm{N}^{s}_{x} = \Sigma\,{}^{r}\mathrm{D}^{s}_{x}$	Age (x)
15	26,660	1,435,361	8,200	678,082	15
16	31,125	1,408,701	9,850	669,882	16
17	34,890	1,377,576	11,370	660,032	17
18	38,080	1,342,686	12,740	648,662	18
19	40,800	1,304,606	14,000	635,922	19
20	43,020	1,263,806	15,165	621,922	20
21	44,900	1,220,786	16,200	606,757	21
22	46,365	1,175,886	17,105	590,557	22
23	47,520	1,129,521	17,940	573,452	23
24	48,425	1,082,001	18,720	555,512	24
25	49,070	1,033,576	19,390	536,792	25
26	48,766	984,506	19,684	517,402	26
27	48,360	935,740	19,968	497,718	27
28	47,806	887,380	20,172	477,750	28
29	47,128	839,574	20,296	457,578	29
30	46,350	792,446	20,340	437,282	30
31	45,496	746,096	20,398	416,942	31
32	44,492	700,600	20,286	396,544	32
33	43,452	656,108	20,196	376,258	33
34	42,400	612,656	20,034	356,062	34
35	41,250	570,256	19,910	336,028	35
36	40,014	529,006	19,608	316,118	36
37	38,704	488,992	19,352	296,510	37
38	37,332	450,288	19,154	277,158	38
39	36,036	412,956	18,774	258,004	39
40	34,580	376,920	18,330	239,230	40
41	33,098	342,340	17,956	220,900	41
42	31,602	309,242	17,526	202,944	42
43	30,104	277,640	17,040	185,418	43
44	28,470	247,536	16,498	168,378	44
45	26,850	219,066	15,900	151,880	45
46	25,256	192,216	15,400	135,980	46
47	23,542	166,960	14,694	120,580	47
48	21,870	143,418	14,094	105,886	48
49	20,086	121,548	13,280	91,792	49
50	18,360	101,462	12,580	78,512	50
51	16,530	83,102	11,658	65,932	51
52	14,774	66,572	10,858	54,274	52
53	12,922	51,798	9,828	43,416	53
54	11,160	38,876	8,742	33,588	54
55	9,310	27,716	7,600	24,846	55
56	7,372	18,406	6,402	17,246	56
57	5,544	11,034	5,148	10,844	57
58	3,636	5,490	3,636	5,696	58
59	1,854	1,854	2,060	2,060	59

Hypothetical Experience of Staff Pension Fund.

TABLE 25.

Commutation Columns for finding the Present Value of the Last Year's Salary on Death or Retirement; and the Present Value of a Pension based on Average Salary and Last Salary.

PENSION AGE 60. INTEREST 4 PER-CENT.

Age (x)	${}^{d}M_x^{ls} = \Sigma({}^{d}C_x s_x)$	${}^{r}M_x^{ls} = \Sigma({}^{r}C_x s_x)$	${}^{ra}M_x^{s} = {}^{ra}M_x \times s_x$	${}^{ra}R_x^{s} = \Sigma^{ra}M_x^{s}$	${}^{ra}M_x^{ls} = \Sigma\{{}^{r}C_x s_x \times (a'_{x+\cdot5} + \cdot5)\}$	${}^{ra}R_x^{ls} = \Sigma^{ra}M_x^{ls}$	Age (x)
15	56,013	16,942	18,600	3,411,828	156,078	5,839,331	15
16	55,245	16,942	23,250	3,393,228	156,078	5,683,253	16
17	54,372	16,942	27,900	3,369,978	156,078	5,527,175	17
18	53,424	16,942	32,550	3,342,078	156,078	5,371,097	18
19	52,444	16,942	37,200	3,309,528	156,078	5,215,019	19
20	51,440	16,942	41,850	3,272,328	156,078	5,058,941	20
21	50,414	16,942	46,500	3,230,478	156,078	4,902,863	21
22	49,379	16,942	51,150	3,183,978	156,078	4,746,789	22
23	48,328	16,942	55,800	3,132,828	156,078	4,590,707	23
24	47,272	16,942	60,450	3,077,028	156,078	4,434,629	24
25	46,225	16,942	65,100	3,016,578	156,078	4,278,551	25
26	45,168	16,942	68,820	2,951,478	156,078	4,122,473	26
27	44,088	16,942	72,540	2,882,658	156,078	3,966,395	27
28	43,019	16,942	76,260	2,810,118	156,078	3,810,317	28
29	41,969	16,890	79,550	2,733,858	155,687	3,654,239	29
30	40,911	16,836	82,800	2,654,308	155,285	3,498,552	30
31	39,840	16,756	85,822	2,571,508	154,674	3,343,267	31
32	38,768	16,649	88,690	2,485,686	153,845	3,188,593	32
33	37,690	16,546	91,392	2,396,996	153,036	3,034,748	33
34	36,619	16,416	93,810	2,305,604	152,011	2,881,712	34
35	35,548	16,282	96,250	2,211,794	150,931	2,729,701	35
36	34,448	16,121	98,382	2,115,544	149,627	2,578,770	36
37	33,354	15,961	100,536	2,017,162	148,315	2,429,143	37
38	32,233	15,775	102,358	1,916,626	146,770	2,280,828	38
39	31,098	15,564	103,824	1,814,268	144,983	2,134,058	39
40	29,939	15,327	105,040	1,710,444	142,965	1,989,075	40
41	28,769	15,067	105,994	1,605,404	140,723	1,846,110	41
42	27,576	14,783	106,536	1,499,410	138,246	1,705,387	42
43	26,348	14,477	106,926	1,392,874	135,544	1,567,141	43
44	25,113	14,149	106,872	1,285,948	132,618	1,431,597	44
45	23,843	13,798	106,650	1,179,076	129,461	1,298,979	45
46	22,508	13,404	105,798	1,072,426	125,875	1,169,518	46
47	21,137	12,965	104,438	966,628	121,854	1,043,643	47
48	19,747	12,484	102,546	862,190	117,416	921,789	48
49	18,305	11,963	100,098	759,644	112,565	804,373	49
50	16,811	11,379	96,900	659,546	107,096	691,808	50
51	15,298	10,734	92,916	562,646	101,027	584,712	51
52	13,767	10,010	88,110	469,730	94,187	483,685	52
53	12,183	9,209	82,264	381,620	86,602	389,498	53
54	10,563	8,312	75,330	299,356	78,086	302,896	54
55	8,908	7,323	67,450	224,026	68,686	224,810	55
56	7,198	6,224	58,200	156,576	58,253	156,124	56
57	5,433	4,981	47,322	98,376	46,477	97,871	57
58	3,651	3,577	32,926	51,054	33,253	51,394	58
59	1,833	1,959	18,128	18,128	18,141	18,141	59

NOTE.—The values of ${}^{w}M_x^{ls}$ will be found in Table 18.

Hypothetical Experience of Staff Pension Fund.

TABLE 26.

Multipliers for use in a Valuation.

PENSION AGE 60. INTEREST 4 PER-CENT.

Age (x)	$(N_{60}+\frac{1}{2}D_{60}) \div D_x$	$N^s_x \div D^s_x$	${}^dM_x \div D_x$	${}^dR^s_x \div D^s_x$	${}^rM_x \div D_x$	${}^rR^s_x \div D^s_x$	${}^dD_x \div D_x$	${}^dN^s_x \div D^s_x$	Age (x)
15	·255	33·111	·054	3·832	·009	1·633	·120	6·462	15
16	·288	29·026	·057	3·411	·010	1·467	·127	5·726	16
17	·331	26·818	·062	3·204	·012	1·392	·136	5·351	17
18	·380	25·465	·067	3·095	·014	1·359	·146	5·136	18
19	·433	24·444	·072	3·021	·015	1·342	·156	4·979	19
20	·490	23·601	·077	2·964	·017	1·334	·165	4·847	20
21	·550	22·883	·082	2·918	·020	1·330	·174	4·733	21
22	·613	22·216	·087	2·874	·022	1·328	·182	4·620	22
23	·680	21·597	·092	2·833	·024	1·328	·190	4·509	23
24	·749	20·983	·096	2·787	·027	1·326	·197	4·392	24
25	·821	20·394	·101	2·741	·029	1·323	·203	4·274	25
26	·897	20·073	·106	2·727	·032	1·337	·208	4·205	26
27	·976	19·728	·110	2·706	·035	1·347	·213	4·126	27
28	1·058	19·362	·114	2·678	·038	1·354	·217	4·037	28
29	1·144	18·974	·118	2·644	·040	1·357	·221	3,938	29
30	1·236	18·584	·122	2·606	·043	1·359	·224	3·835	30
31	1·331	18·170	·126	2·561	·046	1·356	·227	3·723	31
32	1·430	17·738	·130	2·510	·049	1·350	·229	3·603	32
33	1·534	17·295	·133	2·454	·052	1·341	·230	3·479	33
34	1·645	16·847	·137	2·394	·055	1·329	·232	3·351	34
35	1·759	16·370	·140	2·326	·058	1·313	·232	3·214	35
36	1·879	15·883	·143	2·253	·061	1·294	·232	3·073	36
37	2·006	15·394	·146	2·177	·064	1·271	·232	2·931	37
38	2·140	14·886	·148	2·095	·067	1·245	·231	2·784	38
39	2·279	14·359	·150	2·008	·070	1·214	·230	2·633	39
40	2·427	13·830	·152	1·917	·073	1·181	·228	2·480	40
41	2·584	13·291	·154	1·823	·076	1·144	·225	2·327	41
42	2·749	12·738	·155	1·724	·079	1·103	·222	2·171	42
43	2·925	12·176	·156	1·622	·082	1·058	·219	2·016	43
44	3·111	11·601	·156	1·517	·085	1·010	·214	1·859	44
45	3·307	11·008	·156	1·407	·087	·957	·209	1·702	45
46	3·516	10·404	·155	1·296	·089	·901	·203	1·547	46
47	3·743	9·798	·153	1·184	·091	·842	·197	1·394	47
48	3·979	9·162	·150	1·069	·093	·779	·189	1·242	48
49	4·241	8·529	·147	·955	·094	·713	·181	1·095	49
50	4·518	7·872	·142	·841	·095	·644	·172	·950	50
51	4·809	7·187	·136	·726	·094	·571	·161	·809	51
52	5·130	6·492	·130	·615	·093	·497	·150	·676	52
53	5·487	5·786	·121	·508	·091	·423	·137	·550	53
54	5·874	5·053	·112	·404	·087	·347	·124	·433	54
55	6·290	4·290	·100	·307	·082	·273	·109	·323	55
56	6·787	3·517	·086	·219	·074	·202	·091	·227	56
57	7·331	2·704	·070	·141	·064	·135	·072	·144	57
58	7·969	1·857	·050	·076	·049	·076	·051	·076	58
59	8·729	·962	·027	·027	·029	·029	·027	·027	59

NOTE.—$N_{60}=D_{60}\times a_{60}$ (No. 2 Table). ${}^wM_x \div D_x$ and ${}^wR^s \div D^s_x$ are the same as in Table 17.

Hypothetical Experience of Staff Pension Fund.

TABLE 26—(*continued*).

Multipliers for use in a Valuation.

PENSION AGE 60. INTEREST 4 PER-CENT.

Age (x)	${}^{r}D_x \div D_x$	${}^{r}N^s_x \div D^s_x$	${}^{ra}M_x \div D_x$	${}^{ra}R^s_x \div D^s_x$	${}^{ra}M^{ls}_x \div D^s_x$	${}^{ra}R^{ls}_x \div D^s_x$	${}^{d}M^{ls}_x \div D^s_x$	${}^{r}M^{ls}_x \div D^s_x$	Age (x)
15	·037	3·053	·084	15·360	·703	26·289	·252	·076	15
16	·040	2·723	·095	13·792	·634	23·100	·225	·069	16
17	·044	2·564	·108	13·089	·606	21·468	·211	·066	17
18	·049	2·481	·125	12·784	·597	20·546	·204	·065	18
19	·053	2·427	·142	12·630	·596	19·902	·200	·065	19
20	·058	2·385	·161	12·550	·598	19·403	·197	·065	20
21	·063	2·352	·180	12·524	·605	19·007	·195	·066	21
22	·067	2·320	·201	12·509	·613	18·649	·194	·067	22
23	·072	2·289	·223	12·506	·623	18·326	·193	·068	23
24	·076	2·255	·245	12·491	·634	18·001	·192	·069	24
25	·080	2·220	·269	12·473	·645	17·691	·191	·070	25
26	·084	2·210	·294	12·606	·667	17·607	·193	·072	26
27	·088	2·194	·320	12·709	·688	17·487	·194	·075	27
28	·092	2·173	·347	12·782	·710	17·332	·196	·077	28
29	·095	2·146	·373	12·824	·730	17·140	·197	·079	29
30	·098	2·116	·401	12·845	·752	16·931	·198	·082	30
31	·102	2·080	·428	12·831	·772	16·682	·199	·084	31
32	·104	2·040	·456	12·785	·791	16·400	·199	·086	32
33	·107	1·995	·485	12·710	·811	16·091	·200	·088	33
34	·110	1·947	·513	12·609	·831	15·760	·200	·090	34
35	·112	1·894	·543	12·466	·851	15·385	·200	·092	35
36	·114	1·836	·572	12·289	·869	14·981	·200	·095	36
37	·116	1·777	·603	12·090	·889	14·558	·200	·096	37
38	·118	1·713	·633	11·848	·907	14·099	·199	·098	38
39	·120	1·645	·662	11·566	·924	13·604	·198	·099	39
40	·121	1·574	·691	11·255	·941	13·089	·197	·101	40
41	·122	1·501	·720	10·911	·956	12·547	·196	·102	41
42	·123	1·425	·748	10·528	·971	11·975	·194	·104	42
43	·124	1·346	·776	10·113	·984	11·377	·191	·105	43
44	·124	1·265	·803	9·658	·996	10·752	·189	·106	44
45	·124	1·180	·829	9·161	1·006	10·093	·185	·107	45
46	·124	1·094	·851	8·629	1·013	9·410	·181	·108	46
47	·123	1·007	·872	8·071	1·017	8·714	·176	·108	47
48	·122	·917	·888	7·464	1·017	7·980	·171	·108	48
49	·120	·827	·901	6·841	1·014	7·243	·165	·108	49
50	·118	·735	·908	6·178	1·003	6·480	·157	·107	50
51	·114	·642	·905	5·481	·984	5·696	·149	·105	51
52	·110	·551	·895	4·772	·957	4·914	·140	·102	52
53	·104	·461	·874	4·056	·920	4·139	·129	·098	53
54	·097	·374	·839	3·332	·869	3·372	·118	·093	54
55	·089	·290	·787	2·614	·802	2·624	·104	·086	55
56	·079	·213	·718	1·931	·718	1·925	·089	·077	56
57	·065	·142	·618	1·284	·607	1·277	·071	·065	57
58	·051	·079	·458	·710	·462	·715	·051	·050	58
59	·029	·029	·271	·271	·271	·271	·027	·029	59

NOTE.—${}^{w}M^{ls}_x \div D^s_x$ will be found in Table 19.

Hypothetical Experience of Staff Pension Fund.

TABLE 27.

Simple Commutation Columns (according to Table 2).

INTEREST 3 PER-CENT.

Age (x)	$D_x^{(2)}$	$N_x^{(2)}$	$C_x^{(2)}$	$M_x^{(2)}$	$R_x^{(2)}$	Age (x)
15	12,837	303,898	44·9	3612·3	135941·9	15
16	12,418	291,480	44·8	3567·4	132329·6	16
17	12,012	279,468	44·1	3522·6	128762·2	17
18	11,618	267,850	43·9	3478·5	125239·6	18
19	11,236	256,614	43·7	3434·6	121761·1	19
20	10,865	245,749	43·0	3390·9	118326·5	20
21	10,505	235,244	42·8	3347·9	114935·6	21
22	10,157	225,087	42·6	3305·1	111587·7	22
23	9,818	215,269	41·8	3262·5	108282·6	23
24	9,491	205,778	41·6	3220·7	105020·1	24
25	9,172	196,606	40·8	3179·1	101799·4	25
26	8,864	187,742	41·4	3138·3	98620·3	26
27	8,565	179,177	40·6	3096·9	95482·0	27
28	8,275	170,902	40·3	3056·3	92385·1	28
29	7,993	162,909	40·4	3016·0	89328·8	29
30	7,720	155,189	40·4	2975·6	86312·8	30
31	7,455	147,734	41·2	2935·2	83337·2	31
32	7,197	140,537	41·1	2894·0	80402·0	32
33	6,946	133,591	41·0	2852·9	77508·0	33
34	6,703	126,888	41·6	2811·9	74655·1	34
35	6,466	120,422	42·1	2770·3	71843·2	35
36	6,235	114,187	41·9	2728·2	69072·9	36
37	6,012	108,175	43·3	2686·3	66344·7	37
38	5,794	102,381	43·9	2643·0	63658·4	38
39	5,581	96,800	43·8	2599·1	61015·4	39
40	5,375	91,425	45·5	2555·3	58416·3	40
41	5,172	86,253	45·1	2509·8	55861·0	41
42	4,977	81,276	47·4	2464·7	53351·2	42
43	4,784	76,492	47·4	2417·3	50886·5	43
44	4,598	71,894	49·2	2369·9	48469·2	44
45	4,414	67,480	50·6	2320·7	46099·3	45
46	4,235	63,245	51·8	2270·1	43778·6	46
47	4,060	59,185	52·5	2218·3	41508·5	47
48	3,889	55,296	54·3	2165·8	39290·2	48
49	3,722	51,574	56·3	2111·5	37124·4	49
50	3,557	48,017	56·9	2055·2	35012·9	50
51	3,397	44,620	57·6	1998·3	32957·7	51
52	3,240	41,380	58·9	1940·7	30959·4	52
53	3,087	38,293	60·0	1881·8	29018·7	53
54	2,937	35,356	61·6	1821·8	27136·9	54
55	2,790	32,566	63·0	1760·2	25315·1	55
56	2,646	29,920	65·3	1697·2	23554·9	56
57	2,503	27,417	66·4	1631·9	21857·7	57
58	2,364	25,053	68·0	1565·5	20225·8	58
59	2,227	22,826	68·7	1497·5	18660·3	59
60	2,093	20,733	70·4	1428·8	17162·8	60
61	1,962	18,771	70·9	1358·4	15734·0	61
62	1,834	16,937	71·5	1287·5	14375·6	62
63	1,709	15,228	71·5	1216·0	13088·1	63
64	1,588	13,640	69·8	1144·5	11872·1	64
65	{ 1,472 (×8·266) }	12,168	67·2	1074·7	10727·6	65

Hypothetical Experience of Staff Pension Fund.

TABLE 28.

Simple Commutation Columns (according to Table 3).

INTEREST 3 PER-CENT.

Age (x)	$D_x^{(3)}$	$N_x^{(3)}$	$C_x^{(3)}$	$M_x^{(3)}$	$R_x^{(3)}$	Age (x)
15	12,837	139,365	44·9	1304·1	41772·2	15
16	11,484	127,881	41·1	1259·2	40468·1	16
17	10,113	117,768	37·6	1218·1	39208·9	17
18	8,888	108,880	33·6	1180·5	37990·8	18
19	7,872	101,008	30·5	1146·9	36810·3	19
20	7,029	93,979	28·0	1116·4	35663·4	20
21	6,320	87,659	25·6	1088·4	34547·0	21
22	5,723	81,936	23·8	1062·8	33458·6	22
23	5,215	76,721	22·1	1039·0	32395·8	23
24	4,779	71,942	20·5	1016·9	31356·8	24
25	4,400	67,542	19·5	996·4	30339·9	25
26	4,069	63,473	18·9	976·9	29343·5	26
27	3,775	59,698	17·9	958·0	28366·6	27
28	3,514	56,184	17·0	940·1	27408·6	28
29	3,280	52,904	16·5	923·1	26468·5	29
30	3,070	49,834	16·0	906·6	25545·4	30
31	2,880	46,954	15·9	890·6	24638·8	31
32	2,706	44,248	15·5	874·7	23748·2	32
33	2,548	41,700	15·0	859·2	22873·5	33
34	2,403	39,297	14·9	844·2	22014·3	34
35	2,269	37,028	14·8	829·3	21170·1	35
36	2,145	34,883	14·4	814·5	20340·8	36
37	2,031	32,852	14·6	800·1	19526·3	37
38	1,925	30,927	14·5	785·5	18726·2	38
39	1,826	29,101	14·4	771·0	17940·7	39
40	1,733	27,368	14·6	756·6	17169·7	40
41	1,646	25,722	14·4	742·0	16413·1	41
42	1,564	24,158	14·9	727·6	15671·1	42
43	1,487	22,671	14·7	712·7	14943·5	43
44	1,415	21,256	15·1	698·0	14230·8	44
45	1,346	19,910	15·4	682·9	13532·8	45
46	1,281	18,629	15·7	667·5	12849·9	46
47	1,219	17,410	15·7	651·8	12182·4	47
48	1,161	16,249	16·2	636·1	11530·6	48
49	1,105	15,144	16·7	619·9	10894·5	49
50	1,051	14,093	16·8	603·2	10274·6	50
51	1,001	13,092	17·0	586·4	9671·4	51
52	952	12,140	17·3	569·4	9085·0	52
53	906	11,234	17·6	552·1	8515·6	53
54	862	10,372	18·1	534·5	7963·5	54
55	818	9,554	18·5	516·4	7429·0	55
56	776	8,778	19·1	497·9	6912·6	56
57	734	8,044	19·4	478·8	6414·7	57
58	693	7,351	19·9	459·4	5935·9	58
59	653	6,698	20·2	439·5	5476·5	59
60	614	6,084	20·6	419·3	5037·0	60
61	576	5,508	20·8	398·7	4617·7	61
62	538	4,970	21·0	377·9	4219·0	62
63	501	4,469	21·0	356·9	3841·1	63
64	466	4,003	20·5	335·9	3484·2	64
65	{ 432 ×8·266 }	3,571	19·8	315·4	3148·3	65

Hypothetical Experience of Staff Pension Fund.

TABLE 29.

Simple Commutation Columns (according to Table 4).

INTEREST 3 PER-CENT.

Age (x)	$D_x^{(4)}$	$N_x^{(4)}$	$C_x^{(4)}$	$M_x^{(4)}$	$R_x^{(4)}$	Age (x)
15	12,837	136,322	44·9	1092·3	30684·9	15
16	11,484	124,838	41·1	1047·4	29592·6	16
17	10,113	114,725	37·6	1006·3	28545·2	17
18	8,888	105,837	33·6	968·7	27538·9	18
19	7,872	97,965	30·5	935·1	26570·2	19
20	7,029	90,936	28·0	904·6	25635·1	20
21	6,320	84,616	25·6	876·6	24730·5	21
22	5,723	78,893	23·8	851·0	23853·9	22
23	5,215	73,678	22·1	827·2	23002·9	23
24	4,779	68,899	20·5	805·1	22175·7	24
25	4,400	64,499	19·5	784·6	21370·6	25
26	4,069	60,430	18·9	765·1	20586·0	26
27	3,775	56,655	17·9	746·2	19820·9	27
28	3,514	53,141	17·0	728·3	19074·7	28
29	3,279	49,862	16·5	711·3	18346·4	29
30	3,068	46,794	16·0	694·8	17635·1	30
31	2,877	43,917	15·5	678·8	16940·3	31
32	2,702	41,215	15·1	663·3	16261·5	32
33	2,543	38,672	14·6	648·2	15598·2	33
34	2,396	36,276	14·2	633·6	14950·0	34
35	2,262	34,014	14·1	619·4	14316·4	35
36	2,137	31,877	13·7	605·3	13697·0	36
37	2,022	29,855	13·7	591·6	13091·7	37
38	1,914	27,941	13·6	577·9	12500·1	38
39	1,814	26,127	13·5	564·3	11922·2	39
40	1,720	24,407	13·4	550·8	11357·9	40
41	1,631	22,776	13·3	537·4	10807·1	41
42	1,548	21,228	13·5	524·1	10269·7	42
43	1,470	19,758	13·3	510·6	9745·6	43
44	1,395	18,363	13·5	497·3	9235·0	44
45	1,325	17,038	13·9	483·8	8737·7	45
46	1,258	15,780	14·0	469·9	8253·9	46
47	1,194	14,586	14·0	455·9	7784·0	47
48	1,133	13,453	14·3	441·9	7328·1	48
49	1,075	12,378	14·6	427·6	6886·2	49
50	1,019	11,359	14·6	413·0	6458·6	50
51	965	10,394	14·6	398·4	6045·6	51
52	913	9,481	14·8	383·8	5647·2	52
53	863	8,618	15·0	369·0	5263·4	53
54	814	7,804	15·2	354·0	4894·4	54
55	766	7,038	15·5	338·8	4540·4	55
56	718	6,320	15·8	323·3	4201·6	56
57	671	5,649	15·8	307·5	3878·3	57
58	623	5,026	15·9	291·7	3570·8	58
59	575	4,451	16·0	275·8	3279·1	59
60	525	3,926	15·8	259·8	3003·3	60
61	474	3,452	15·5	244·0	2743·5	61
62	419	3,033	15·1	228·5	2499·5	62
63	363	2,670	14·2	213·4	2271·0	63
64	307	2,363	13·0	199·2	2057·6	64
65	255	2,108	11·7	186·2	1858·4	65

Hypothetical Experience of Staff Pension Fund.

TABLE 30.

Commutation Columns for Valuing the Return of Contributions of 1 per annum on Death, Withdrawal, or Early Retirement.

PENSION AGE 65. INTEREST 3 PER-CENT.

Age (x)	${}^dC_x = C_x^{(4)}$	${}^dM_x = \Sigma{}^dC_x$	${}^dR_x = \Sigma{}^dM_x$	${}^wC_x$*	${}^wM_x = \Sigma{}^wC_x$	${}^wR_x = \Sigma{}^wM_x$	${}^rC_x$†	${}^rM_x = \Sigma{}^rC_x$	${}^rR_x = \Sigma{}^rM_x$	Age (x)
15	44·9	906·1	19,517	935	7,102	46,205	...	300·2	12,482	15
16	41·1	861·2	18,610	995	6,167	39,103	...	300·2	12,182	16
17	37·6	820·1	17,749	893	5,172	32,936	...	300·2	11,882	17
18	33·6	782·5	16,929	723	4,279	27,764	...	300·2	11,582	18
19	30·5	748·9	16,147	584	3,556	23,485	...	300·2	11,281	19
20	28·0	718·4	15,398	477	2,972	19,929	...	300·2	10,981	20
21	25·6	690·4	14,679	387	2,495	16,957	...	300·2	10,681	21
22	23·8	664·8	13,989	317	2,108	14,462	..	300·2	10,381	22
23	22·1	641·0	13,324	263	1,791	12,354	...	300·2	10,081	23
24	20·5	618·9	12,683	219	1,528	10,563	...	300·2	9,780	24
25	19·5	598·4	12,064	184	1,309	9,035	...	300·2	9,480	25
26	18·9	578·9	11,466	156	1,125	7,726	...	300·2	9,180	26
27	17·9	560·0	10,887	133	969	6,601	...	300·2	8,880	27
28	17·0	542·1	10,327	114	836	5,632	·85	300·2	8,580	28
29	16·5	525·1	9,785	98	722	4,796	·82	299·4	8,279	29
30	16·0	508·6	9,260	85	624	4,074	1·20	298·5	7,980	30
31	15·5	492·6	8,751	74	539	3,450	1·55	297·3	7,682	31
32	15·1	477·1	8,259	64	465	2,911	1·51	295·8	7,384	32
33	14·6	462·0	7,781	56	401	2,446	1·83	294·3	7,088	33
34	14·2	447·4	7,319	49	345	2,045	1·78	292·4	6,794	34
35	14·1	433·2	6,872	43	296	1,700	2·07	290·7	6,502	35
36	13·7	419·1	6,439	38	253	1,404	2·01	288·6	6,211	36
37	13·7	405·4	6,020	33	215	1,151	2·28	286·6	5,923	37
38	13·6	391·7	5,614	28	182	936	2·53	284·3	5,636	38
39	13·5	378·1	5,223	25	154	754	2·76	281·8	5,352	39
40	13·4	364·6	4,845	22	129	600	2·98	279·0	5,070	40
41	13·3	351·2	4,480	19	107	471	3·18	276·0	4,791	41
42	13·5	337·9	4,129	17	88	364	3·37	272·9	4,515	42
43	13·3	324·4	3,791	14	71	276	3·54	269·5	4,242	43
44	13·5	311·1	3,466	12	57	205	3·70	265·9	3,972	44
45	13·9	297·6	3,155	11	45	148	4·11	262·2	3,707	45
46	14·0	283·7	2,858	9	34	103	4·49	258·1	3,444	46
47	14·0	269·7	2,574	7	25	69	4·84	253·6	3,186	47
48	14·3	255·7	2,304	6	18	44	5·17	248·8	2,933	48
49	14·6	241·4	2,049	5	12	26	5·70	243·6	2,684	49
50	14·6	226·8	1,807	3	7	14	6·20	237·9	2,440	50
51	14·6	212·2	1,580	2	4	7	6·88	231·7	2,202	51
52	14·8	197·6	1,368	1	2	3	7·52	224·9	1,970	52
53	15·0	182·8	1,171	1	1	1	8·31	217·3	1,746	53
54	15·2	167·8	988	...	...	...	9·05	209·0	1,528	54
55	15·5	152·6	820	...	...	...	9·93	200·0	1,319	55
56	15·8	137·1	667	...	...	...	11·13	190·0	1,119	56
57	15·8	121·3	530	...	...	...	12·42	178·9	929	57
58	15·9	105·5	409	...	...	...	14·16	166·5	750	58
59	16·0	89·6	304	...	...	...	16·97	152·3	584	59
60	15·8	73·6	214	...	...	...	20·27	135·4	432	60
61	15·5	57·8	140	...	...	...	24·96	115·1	296	61
62	15·1	42·3	83	...	...	...	29·36	90·1	181	62
63	14·2	27·2	40	...	...	...	30·61	60·8	91	63
64	13·0	13·0	13	...	...	...	30·16	30·2	30	64

* ${}^wC_x = w_x \times v^{x+1}$ † ${}^rC_x = r_x \times v^{x+1}$

Hypothetical Experience of Staff Pension Fund.

TABLE 31.

Table for Valuing Return of Contributions of 1 *per annum with Compound Interest at* 3 *per-cent on Death or Early Retirement; the Values of Annuities of* 1 *per annum on Early Retirement from Ill-health; and for finding the Value of a Pension of* 1 *for each Year of Service on Early Retirement.*

PENSION AGE 65. INTEREST 3 PER-CENT.

Age (x)	${}^{d}D_x$*	${}^{d}N_x = \Sigma {}^{d}D_x$	${}^{r}D_x$†	${}^{r}N_x = \Sigma {}^{r}D_x$	$D_x^{(r)}$	$N_x^{(r)}$	a'_x	${}^{ra}C_x$‡	${}^{ra}M_x = \Sigma {}^{ra}C_x$	${}^{ra}R_x = \Sigma {}^{ra}M_x$	Age (x)
15	1,851	33,337	1,025	25,171	...	...	...	...	2,908	120,856	15
16	1,753	31,486	995	24,146	...	...	...	...	2,908	117,948	16
17	1,662	29,733	966	23,151	...	...	...	...	2,908	115,040	17
18	1,577	28,071	938	22,185	...	...	...	...	2,908	112,132	18
19	1,499	26,494	910	21,247	...	...	...	...	2,908	109,225	19
20	1,426	24,995	884	20,337	...	...	...	...	2,908	106,317	20
21	1,357	23,569	858	19,453	...	...	...	...	2,908	103,409	21
22	1,293	22,212	833	18,595	...	...	...	...	2,908	100,501	22
23	1,232	20,919	809	17,762	...	...	...	...	2,908	97,594	23
24	1,174	19,687	785	16,953	...	...	...	...	2,908	94,686	24
25	1,120	18,513	762	16,168	...	...	...	...	2,908	91,778	25
26	1,069	17,393	740	15,406	...	...	...	...	2,908	88,870	26
27	1,019	16,324	719	14,666	...	...	...	...	2,908	85,963	27
28	972	15,305	698	13,947	...	...	...	6·9	2,908	83,055	28
29	972	14,333	676	13,249	8,487	64,788	7·63	6·7	2,901	80,147	29
30	884	13,406	656	12,573	7,416	57,372	7·74	10·0	2,894	77,246	30
31	843	12,522	636	11,917	6,487	50,885	7·84	13·0	2,884	74,352	31
32	803	11,679	616	11,281	5,681	45,204	7·96	12·9	2,871	71,468	32
33	765	10,876	596	10,665	4,983	40,221	8·07	15·8	2,858	68,596	33
34	729	10,111	577	10,069	4,378	35,843	8·19	15·6	2,843	65,738	34
35	694	9,382	559	9,492	3,853	31,990	8·30	18·3	2,827	62,896	35
36	660	8,688	540	8,933	3,397	28,593	8·42	18·1	2,809	60,069	36
37	627	8,028	523	8,393	2,999	25,594	8·53	20·7	2,791	57,260	37
38	596	7,401	505	7,870	2,653	22,941	8·65	23·3	2,770	54,469	38
39	565	6,805	488	7,365	2,350	20,591	8·76	25·8	2,747	51,700	39
40	536	6,240	471	6,877	2,086	18,505	8·87	28·1	2,721	48,953	40
41	507	5,704	455	6,406	1,854	16,651	8·98	30·3	2,693	46,232	41
42	479	5,197	438	5,951	1,650	15,001	9·09	32·5	2,662	43,540	42
43	452	4,718	422	5,513	1,472	13,529	9·19	34·5	2,630	40,877	43
44	426	4,266	406	5,091	1,314	12,215	9·29	36·4	2,595	38,247	44
45	401	3,840	391	4,685	1,176	11,039	9·39	40·8	2,559	35,652	45
46	376	3,439	376	4,294	1,054	9,985	9·47	45·0	2,518	33,093	46
47	351	3,063	360	3,918	946	9,039	9·56	48·9	2,473	30,575	47
48	327	2,712	345	3,558	850	8,189	9·63	52·5	2,424	28,102	48
49	304	2,385	330	3,213	766	7,423	9·69	58·3	2,372	25,677	49
50	281	2,081	315	2,883	691	6,732	9·75	63·7	2,314	23,306	50
51	258	1,800	300	2,568	624	6,108	9·79	70·9	2,250	20,992	51
52	237	1,542	284	2,268	565	5,543	9·82	77·6	2,179	18,742	52
53	215	1,305	269	1,984	512	5,031	9·83	85·8	2,101	16,563	53
54	195	1,090	253	1,715	465	4,566	9·83	93·4	2,015	14,462	54
55	174	895	237	1,462	422	4,144	9·81	102·2	1,922	12,447	55
56	154	721	220	1,225	385	3,759	9·77	114·0	1,820	10,525	56
57	134	567	203	1,005	351	3,408	9·72	126·4	1,706	8,705	57
58	115	433	185	802	320	3,088	9·64	142·9	1,579	6,999	58
59	96	318	166	617	293	2,795	9·53	169·2	1,437	5,419	59
60	78	222	145	451	269	2,526	9·40	199·1	1,267	3,983	60
61	60	144	121	306	247	2,279	9·24	240·9	1,068	2,715	61
62	43	84	93	185	227	2,052	9·05	277·2	828	1,647	62
63	28	41	62	92	209	1,843	8·82	281·3	550	819	63
64	13	13	30	30	193	1,650	8·56	269·0	269	269	64
65	...	...	...	...	178	1,472	8·27	...	...	...	65

* ${}^{d}D_x = l'_x \times v^{x+1}$ † ${}^{r}D_x = \Sigma r_x \times v^{x+1}$ ‡ ${}^{ra}C_x = {}^{r}C_x \times (a'_{x+\cdot 5} + \cdot 5)$

Hypothetical Experience of Staff Pension Fund.

TABLE 32.

Table for finding the Accumulation of Average Salary at 3 per-cent Compound Interest, and for finding the Value of the last year's Full Salary on Death, Withdrawal, or Early Retirement.

PENSION AGE 65. INTEREST 3 PER-CENT.

Age (x)	s_xv^x	Σs_xv^x	$(1+i)^{x+1}$	${}^dC_x^s = {}^dC_x \times s_x$	${}^dM_x^{ls} = \Sigma {}^dC_x^s$	${}^wC_x^s = {}^wC_x \times s_x$	${}^wM_x^{ls} = \Sigma {}^wC_x^s$	${}^rC_x^s = {}^rC_x \times s_x$	${}^rM_x^{ls} = \Sigma {}^rC_x^s$	Age (x)
15	12·84	1740·60	1·5126	898	99,718	18,700	329,829	...	57,736	15
16	15·58	1727·76	1·5580	1,028	98,820	24,875	311,129	...	57,736	16
17	18·15	1712·18	1·6047	1,128	97,792	26,790	286,254	...	57,736	17
18	20·56	1694·03	1·6528	1,176	96,664	25,305	259,464	...	57,736	18
19	22·81	1673·47	1·7024	1,220	95,488	23,360	234,159	...	57,736	19
20	24·92	1650·66	1·7535	1,260	94,268	21,465	210,799	...	57,736	20
21	26·88	1625·74	1·8061	1,280	93,008	19,350	189,334	...	57,736	21
22	28·70	1598·86	1·8603	1,309	91,728	17,435	169,984	...	57,736	22
23	30·40	1570·16	1·9161	1,326	90,419	15,780	152,549	...	57,736	23
24	31·98	1539·76	1·9736	1,333	89,093	14,235	136,769	...	57,736	24
25	33·43	1507·78	2·0328	1,365	87,760	12,880	122,534	...	57,736	25
26	34·31	1474·35	2·0938	1,399	86,395	11,544	109,654	...	57,736	26
27	35·11	1440·04	2·1566	1,396	84,996	10,374	98,110	...	57,736	27
28	35·84	1404·93	2·2213	1,394	83,600	9,348	87,736	70	57,736	28
29	36·49	1369·09	2·2879	1,419	82,206	8,428	78,388	71	57,667	29
30	37·08	1332·60	2·3566	1,440	80,787	7,650	69,960	108	57,596	30
31	37·60	1295·52	2·4273	1,457	79,347	6,956	62,310	146	57,488	31
32	38·06	1257·92	2·5001	1,480	77,890	6,272	55,354	148	57,343	32
33	38·46	1219·86	2·5751	1,489	76,410	5,712	49,082	187	57,195	33
34	38·80	1181·40	2·6523	1,505	74,921	5,194	43,370	189	57,008	34
35	39·09	1142·60	2·7319	1,551	73,416	4,730	38,176	230	56,819	35
36	39·33	1103·51	2·8139	1,562	71,865	4,332	33,446	229	56,589	36
37	39·53	1064·18	2·8983	1,617	70,303	3,894	29,114	269	56,360	37
38	39·68	1024·65	2·9852	1,659	68,686	3,416	25,220	309	56,091	38
39	39·79	984·97	3·0748	1,710	67,027	3,150	21,804	348	55,783	39
40	39·85	945·18	3·1670	1,742	65,317	2,860	18,654	387	55,435	40
41	39·88	905·33	3·2620	1,782	63,575	2,546	15,794	426	55,047	41
42	39·88	865·45	3·3599	1,863	61,793	2,346	13,248	465	54,621	42
43	39·84	825·57	3·4607	1,889	59,930	1,988	10,902	503	54,156	43
44	39·77	785·73	3·5645	1,971	58,041	1,752	8,914	540	53,653	44
45	39·67	745·96	3·6715	2,085	56,070	1,650	7,162	617	53,113	45
46	39·54	706·29	3·7816	2,156	53,985	1,386	5,512	692	52,497	46
47	39·38	666·75	3·8950	2,212	51,829	1,106	4,126	765	51,805	47
48	39·20	627·37	4·0119	2,317	49,617	972	3,020	838	51,041	48
49	39·00	588·17	4·1323	2,424	47,300	830	2,048	946	50,203	49
50	38·78	549·17	4·2562	2,482	44,876	510	1,218	1,054	49,257	50
51	38·53	510·39	4·3839	2,540	42,394	348	708	1,197	48,203	51
52	38·27	471·86	4·5154	2,634	39,854	178	360	1,339	47,006	52
53	37·99	433·59	4·6509	2,730	37,220	182	182	1,512	45,667	53
54	37·70	395·60	4·7904	2,827	34,490	...	...	1,683	44,155	54
55	37·39	357·90	4·9341	2,945	31,663		...	1,887	42,471	55
56	37·06	320·51	5·0821	3,065	28,718	...	...	2,159	40,585	56
57	36·72	283·45	5·2346	3,128	25,653	...	...	2,459	38,426	57
58	36·37	246·73	5·3917	3,212	22,525	.	...	2,860	35,966	58
59	36·01	210·36	5·5534	3,296	19,313	..	..	3,496	33,106	59
60	35·64	174·35	5·7200	3,318	16,017	...	..	4,257	29,610	60
61	35·27	138·71	5·8916	3,317	12,699	...	...	5,341	25,354	61
62	34·88	103·44	6·0684	3,292	9,382	...	...	6,401	20,012	62
63	34·48	68·56	6·2504	3,152	6,090	...	...	6,795	13,612	63
64	34·08	34·08	6·4379	2,938	2,938	...	...	6,816	6,816	64

Hypothetical Experience of Staff Pension Fund.

TABLE 33.

Commutation Columns for finding the Present Value of Future Salary, and Return of Contributions at Death, without Interest.

PENSION AGE 65. INTEREST 3 PER-CENT.

Age (x)	$D^s_x = D^{(4)}_x \times s_x$	$vD^s_x = D^s_x \div 1{\cdot}03$	$N^s_x = \Sigma(v.D^s_x)$	${}^dM^s_x = {}^dM_x \times s_x$	${}^dR^s_x = \Sigma {}^dM^s_x$	Age (x)
15	256,740	249,262	10,654,124	18,122	1,779,340	15
16	287,100	278,738	10,404,862	21,530	1,761,218	16
17	303,390	294,553	10,126,124	24,603	1,739,688	17
18	311,080	302,020	9,831,571	27,388	1,715,085	18
19	314,880	305,709	9,529,551	29,956	1,687,697	19
20	316,305	307,093	9,223,842	32,328	1,657,741	20
21	316,000	306,796	8,916,749	34,520	1,625,413	21
22	314,765	305,598	8,609,953	36,564	1,590,893	22
23	312,900	303,786	8,304,355	38,460	1,554,329	23
24	310,635	301,587	8,000,569	40,229	1,515,869	24
25	308,000	299,030	7,698,982	41,888	1,475,640	25
26	301,106	292,337	7,399,952	42,839	1,433,752	26
27	294,450	285,874	7,107,615	43,680	1,390,913	27
28	288,148	279,756	6,821,741	44,452	1,347,233	28
29	281,994	273,781	6,541,985	45,159	1,302,781	29
30	276,120	268,078	6,268,204	45,774	1,257,622	30
31	270,438	262,561	6,000,126	46,304	1,211,848	31
32	264,796	257,083	5,737,565	46,756	1,165,544	32
33	259,386	251,831	5,480,482	47,124	1,118,788	33
34	253,976	246,579	5,228,651	47,424	1,071,664	34
35	248,820	241,573	4,982,072	47,652	1,024,240	35
36	243,618	236,523	4,740,499	47,777	976,588	36
37	238,596	231,647	4,503,976	47,837	928,811	37
38	233,508	226,707	4,272,329	47,787	880,974	38
39	228,564	221,907	4,045,622	47,641	833,187	39
40	223,600	217,087	3,823,715	47,398	785,546	40
41	218,554	212,189	3,606,628	47,061	738,148	41
42	213,624	207,402	3,394,439	46,630	691,087	42
43	208,740	202,660	3,187,037	46,065	644,457	43
44	203,670	197,738	2,984,377	45,421	598,392	44
45	198,750	192,961	2,786,639	44,640	552,971	45
46	193,732	188,090	2,593,678	43,691	508,331	46
47	188,652	183,157	2,405,588	42,613	464,640	47
48	183,546	178,200	2,222,431	41,423	422,027	48
49	178,450	173,253	2,044,231	40,072	380,604	49
50	173,230	168,185	1,870,978	38,556	340,532	50
51	167,910	163,019	1,702,793	36,923	301,976	51
52	162,514	157,781	1,539,774	35,173	265,053	52
53	157,066	152,491	1,381,993	33,270	229,880	53
54	151,404	146,994	1,229,502	31,211	196,610	54
55	145,540	141,301	1,082,508	28,994	165,399	55
56	139,292	135,235	941,207	26,597	136,405	56
57	132,858	128,988	805,972	24,017	109,808	57
58	125,846	122,181	676,984	21,311	85,791	58
59	118,450	115,000	554,803	18,458	64,480	59
60	110,250	107,039	439,803	15,456	46,022	60
61	101,436	98,482	332,764	12,369	30,566	61
62	91,342	88,682	234,282	9,221	18,197	62
63	80,586	78,239	145,600	6,038	8,976	63
64	69,382	67,361	67,361	2,938	2,938	64

Hypothetical Experience of Staff Pension Fund.

TABLE 34.

Commutation Columns for finding the Present Values of Return of Contributions on Withdrawal and Early Retirement.

PENSION AGE 65. INTEREST 3 PER-CENT.

Age (x)	${}^{w}M^{s}_{x}$ $={}^{w}M_{x}\times s_{x}$	${}^{w}R^{s}_{x}$ $=\Sigma{}^{w}M^{s}_{x}$	${}^{r}M^{s}_{x}$ $={}^{r}M_{x}\times s_{x}$	${}^{r}R^{s}_{x}$ $=\Sigma{}^{r}M^{s}_{x}$	Age (x)
15	142,040	2,148,868	6004·0	1413732·1	15
16	154,175	2,006,828	7505·0	1407728·1	16
17	155,160	1,852,653	9006·0	1400223·1	17
18	149,765	1,697,493	10507·0	1391217·1	18
19	142,240	1,547,728	12008·0	1380710·1	19
20	133,740	1,405,488	13509·0	1368702·1	20
21	124,750	1,271,748	15010·0	1355193·1	21
22	115,940	1,146,998	16511·0	1340183·1	22
23	107,460	1,031,058	18012·0	1323672·1	23
24	99,320	923,598	19513·0	1305660·1	24
25	91,630	824,278	21014·0	1286147·1	25
26	83,250	732,648	22214·8	1265133·1	26
27	75,582	649,398	23415·6	1242918·3	27
28	68,552	573,816	24616·4	1219502·7	28
29	62,092	505,264	25744·1	1194886·3	29
30	56,160	443,172	26867·7	1169142·2	30
31	50,666	387,012	27949·0	1142274·5	31
32	45,570	336,346	28986·4	1114325·5	32
33	40,902	290,776	30015·5	1085339·1	33
34	36,570	249,874	30998·6	1055323·6	34
35	32,560	213,304	31972·6	1024325·0	35
36	28,842	180,744	32899·3	992352·4	36
37	25,370	151,902	33816·4	959453·1	37
38	22,204	126,532	34684·6	925636·7	38
39	19,404	104,328	35503·0	890952·1	39
40	16,770	84,924	36271·3	855449·1	40
41	14,338	68,154	36988·0	819177·8	41
42	12,144	53,816	37653·3	782189·8	42
43	10,082	41,672	38266·2	744536·5	43
44	8,322	31,590	38827·2	706270·3	44
45	6,750	23,268	39336·0	667443·1	45
46	5,236	16,518	39752·0	628107·1	46
47	3,950	11,282	40075·1	588355·1	47
48	2,916	7,332	40305·6	548280·0	48
49	1,992	4,416	40442·6	507974·4	49
50	1,190	2,424	40448·1	467531·8	50
51	696	1,234	40321·0	427083·7	51
52	356	538	40023·3	386762·7	52
53	182	182	39554·1	346739·4	53
54	...	...	38877·7	307185·3	54
55	...	...	37994·3	268307·6	55
56	...	...	36867·8	230313·3	56
57	...	...	35424·2	193445·5	57
58	...	...	33631·0	158021·3	58
59	...	...	31380·0	124390·3	59
60	...	...	28425·6	93010·3	60
61	...	...	24629·3	64584·7	61
62	...	...	19648·3	39955·4	62
63	...	...	13490·9	20307·1	63
64	...	...	6816·2	6816·2	64

Hypothetical Experience of Staff Pension Fund.

TABLE 35.

Commutation Columns for finding the Values of Return of Contributions, with Compound Interest at 3 per-cent on Death or Early Retirement.

PENSION AGE 65. INTEREST 3 PER-CENT.

Age (x)	${}^dD^s_x = {}^dD_x \times s_x$	${}^dN^s_x = \Sigma {}^dD^s_x$	${}^rD^s_x = {}^rD_x \times s_x$	${}^rN^s_x = \Sigma {}^rD^s_x$	Age (x)
15	37,020	2,755,427	20,500	2,395,105	15
16	43,825	2,718,407	24,875	2,374,605	16
17	49,860	2,674,582	28,980	2,349,730	17
18	55,195	2,624,722	32,830	2,320,750	18
19	59,960	2,569,527	36,400	2,287,920	19
20	64,170	2,509,567	39,780	2,251,520	20
21	67,850	2,445,397	42,900	2,211,740	21
22	71,115	2,377,547	45,815	2,168,840	22
23	73,920	5,306,432	48,540	2,123,025	23
24	76,310	2,232,512	51,025	2,074,485	24
25	78,400	2,156,202	53,340	2,023,460	25
26	79,106	2,077,802	54,760	1,970,120	26
27	79,482	1,998,696	56,082	1,915,360	27
28	79,704	1,919,214	57,236	1,859,278	28
29	79,722	1,839,510	58,136	1,802,042	29
30	79,560	1,759,788	59,040	1,743,906	30
31	79,242	1,680,228	59,784	1,684,866	31
32	78,694	1,600,986	60,368	1,625,082	32
33	78,030	1,522,292	60,792	1,564,714	33
34	77,274	1,444,262	61,162	1,503,922	34
35	76,340	1,366,988	61,490	1,442,760	35
36	75,240	1,290,648	61,560	1,381,270	36
37	73,986	1,215,408	61,714	1,319,710	37
38	72,712	1,141,422	61,610	1,257,996	38
39	71,190	1,068,710	61,488	1,196,386	39
40	69,680	997,520	61,230	1,134,898	40
41	67,938	927,840	60,970	1,073,668	41
42	66,102	859,902	60,444	1,012,698	42
43	64,184	793,800	59,924	952,254	43
44	62,196	729,616	59,276	892,330	44
45	60,150	667,420	58,650	833,054	45
46	57,904	607,270	57,904	774,404	46
47	55,458	549,366	56,880	716,500	47
48	52,974	493,908	55,890	659,620	48
49	50,464	440,934	54,780	603,730	49
50	47,770	390,470	53,550	548,950	50
51	44,892	342,700	52,200	495,400	51
52	42,186	297,808	50,552	443,200	52
53	39,130	255,622	48,958	392,648	53
54	36,270	216,492	47,058	343,690	54
55	33,060	180,222	45,030	296,632	55
56	29,876	147,162	42,680	251,602	56
57	26,532	117,286	40,194	208,922	57
58	23,230	90,754	37,370	168,728	58
59	19,776	67,524	34,196	131,358	59
60	16,380	47,748	30,450	97,162	60
61	12,840	31,368	25,894	66,712	61
62	9,374	18,528	20,274	40,818	62
63	6,216	9,154	13,764	20,544	63
64	2,938	2,938	6,780	6,780	64

Hypothetical Experience of Staff Pension Fund.

TABLE 36.

Commutation Columns for finding the Values of Pensions on Early Retirement, based on number of Years' Service and (a) Average Salary. (b) Last Salary.

PENSION AGE 65. INTEREST 3 PER-CENT.

Age (x)	AVERAGE SALARY		LAST SALARY			Age (x)
	${}^{ra}M^{s}_{x}$ $={}^{ra}M_{x}\times s_{x}$	${}^{ra}R^{s}_{x}$ $=\Sigma^{ra}M^{s}_{x}$	${}^{ra}C^{ls}_{x}$ $={}^{r}C_{x}.s_{x}(a'_{x+\cdot 5}+\cdot 5)$	${}^{ra}M^{ls}_{x}$ $=\Sigma^{ra}C^{ls}_{x}$	${}^{ra}R^{ls}_{x}$ $=\Sigma^{ra}M^{ls}_{x}$	
15	58,155	13,683,338	...	559,028	24,017,405	15
16	72,694	13,625,183	...	559,028	23,458,377	16
17	87,233	13,552,489	...	559,028	22,899,349	17
18	101,772	13,465,256	...	559,028	22,340,321	18
19	116,311	13,363,484	...	559,028	21,781,293	19
20	130,850	13,247,173	...	559,028	21,222,265	20
21	145,389	13,116,323	...	559,028	20,663,237	21
22	159,927	12,970,934	...	559,028	20,104,209	22
23	174,466	12,811,007	...	559,028	19,545,181	23
24	189,005	12,636,541	...	559,028	18,986,153	24
25	203,544	12,447,536	...	559,028	18,427,125	25
26	215,175	12,243,992	...	559,028	17,868,097	26
27	226,806	12,028,817	...	559,028	17,309,069	27
28	238,437	11,802,011	563·2	559,028	16,750,041	28
29	249,477	11,563,574	577·4	558,465	16,191,013	29
30	260,476	11,314,097	895·3	557,887	15,632,548	30
31	271,118	11,053,621	1223·9	556,992	15,074,661	31
32	281,379	10,782,503	1261·0	555,768	14,517,669	32
33	291,551	10,501,124	1611·2	554,507	13,961,901	33
34	301,310	10,209,573	1651·1	552,896	13,407,394	34
35	310,967	9,908,263	2036·0	551,245	12,854,498	35
36	320,184	9,597,296	2057·3	549,209	12,303,253	36
37	329,288	9,277,112	2445·2	547,151	11,754,044	37
38	337,922	8,947,824	2843·1	544,706	11,206,893	38
39	346,064	8,609,902	3241·5	541,863	10,662,187	39
40	353,703	8,263,838	3653·2	538,622	10,120,324	40
41	360,820	7,910,135	4065·0	534,968	9,581,702	41
42	367,404	7,549,315	4483·6	530,903	9,046,734	42
43	373,440	7,181,911	4896·3	526,420	8,515,831	43
44	378,925	6,808,471	5315·6	521,524	7,989,411	44
45	383,846	6,429,546	6121·8	516,208	7,467,887	45
46	387,797	6,045,700	6928·8	510,086	6,951,679	46
47	390,761	5,657,903	7723·5	503,157	6,441,593	47
48	392,735	5,267,142	8509·0	495,434	5,938,436	48
49	410,312	4,874,407	9670·2	486,925	5,443,002	49
50	398,297	4,464,095	10824·6	477,255	4,956,077	50
51	391,472	4,065,798	12342·1	466,430	4,478,822	51
52	387,846	3,674,326	13814·4	454,088	4,012,392	52
53	382,437	3,286,480	15623·1	440,274	3,558,304	53
54	374,876	2,904,043	17371·7	424,650	3,118,030	54
55	365,191	2,529,167	19414·1	407,279	2,693,380	55
56	353,057	2,163,976	22110·2	387,865	2,286,101	56
57	337,770	1,810,919	25034·7	365,754	1,898,236	57
58	319,053	1,473,149	28860·4	340,720	1,532,482	58
59	295,940	1,154,096	34853·1	311,859	1,191,762	59
60	266,156	858,156	41800·8	277,006	879,903	60
61	228,629	592,000	51544·5	235,205	602,897	61
62	180,395	363,371	60420·7	183,661	367,692	62
63	122,175	182,976	62449·7	123,240	184,031	63
64	60,801	60,801	60790·5	60,791	60,791	64

Hypothetical Experience of Staff Pension Fund.

TABLE 37.

Multipliers for use in a Valuation.

PENSION AGE 65. INTEREST 3 PER-CENT.

Age (x)	$\frac{N_{65}+\frac{1}{2}D_{65}}{D_x}$	$\frac{N^s_x}{D^s_x}$	$\frac{{}^dM_x}{D_x}$	$\frac{{}^dR^s_x}{D^s_x}$	$\frac{{}^wM_x}{D_x}$	$\frac{{}^wR^s_x}{D^s_x}$	$\frac{{}^rM_x}{D_x}$	$\frac{{}^rR^s_x}{D^s_x}$	$\frac{{}^dD_x}{D_x}$	$\frac{{}^dN^s_x}{D^s_x}$	Age (x)
15	·174	41·498	·071	6·931	·553	8·370	·023	5·506	·144	10·732	15
16	·195	36·241	·075	6·135	·537	6·990	·026	4·903	·153	9·468	16
17	·221	33·377	·081	5·734	·511	6·107	·030	4·615	·164	8·816	17
18	·252	31·605	·088	5·513	·481	5·457	·034	4·472	·177	8·437	18
19	·284	30·264	·095	5·360	·452	4·915	·038	4·385	·190	8·160	19
20	·318	29·161	·102	5·241	·423	4·443	·043	4·327	·203	7·934	20
21	·354	28·218	·109	5·144	·395	4·025	·048	4·289	·215	7·739	21
22	·391	27·354	·116	5·054	·368	3·644	·052	4·258	·226	7·553	22
23	·429	26·540	·123	4·968	·343	3·295	·058	4·230	·236	7·371	23
24	·468	25·756	·130	4·880	·320	2·973	·063	4·203	·246	7·187	24
25	·508	24·997	·136	4·791	·298	2·676	·068	4·176	·255	7·001	25
26	·549	24·575	·142	4·762	·276	2·433	·074	4·202	·263	6·901	26
27	·592	24·139	·148	4·724	·257	2·205	·080	4·221	·270	6·788	27
28	·636	23·674	·154	4·675	·238	1·991	·085	4·232	·277	6·661	28
29	·682	23·199	·160	4·620	·220	1·792	·091	4·237	·283	6·523	29
30	·729	22·701	·166	4·555	·203	1·605	·097	4·234	·288	6·373	30
31	·777	22·187	·171	4·481	·187	1·431	·103	4·224	·293	6·213	31
32	·827	21·668	·177	4·402	·172	1·270	·109	4·208	·297	6·046	32
33	·879	21·129	·182	4·313	·158	1·121	·116	4·184	·301	5·869	33
34	·933	20·587	·187	4·220	·144	·984	·122	4·155	·304	5·687	34
35	·988	20·023	·192	4·116	·131	·857	·129	4·117	·307	5·494	35
36	1·046	19·459	·196	4·009	·118	·742	·135	4·073	·309	5·298	36
37	1·106	18·877	·201	3·893	·106	·637	·142	4·021	·310	5·094	37
38	1·168	18·296	·205	3·773	·095	·542	·149	3·964	·311	4·888	38
39	1·232	17·700	·208	3·645	·085	·456	·155	3·898	·311	4·676	39
40	1·300	17·101	·212	3·513	·075	·380	·162	3·826	·312	4·461	40
41	1·370	16·502	·215	3·377	·066	·312	·169	3·748	·311	4·245	41
42	1·444	15·890	·218	3·235	·057	·252	·176	3·662	·309	4·025	42
43	1·521	15·268	·221	3·087	·048	·200	·183	3·567	·308	3·803	43
44	1·602	14·653	·223	2·938	·041	·155	·191	3·468	·305	3·582	44
45	1·687	14·021	·225	2·782	·034	·117	·198	3·358	·303	3·358	45
46	1·777	13·388	·226	2·624	·027	·085	·205	3·242	·299	3·135	46
47	1·872	12·751	·226	2·463	·021	·060	·212	3·119	·294	2·912	47
48	1·973	12·108	·226	2·299	·016	·040	·220	2·987	·289	2·691	48
49	2·080	11·455	·225	2·133	·011	·025	·227	2·847	·283	2·471	49
50	2·194	10·801	·223	1·966	·007	·014	·234	2·699	·276	2·254	50
51	2·317	10·141	·220	1·798	·004	·007	·240	2·543	·267	2·041	51
52	2·448	9·475	·216	1·631	·002	·003	·246	2·380	·260	1·833	52
53	2·590	8·799	·212	1·464	·001	·001	·252	2·208	·249	1·628	53
54	2·746	8·121	·206	1·299	...	...	·257	2·029	·240	1·430	54
55	2·918	7·438	·199	1·136	...	...	·261	1·844	·227	1·238	55
56	3·114	6·757	·191	·979	...	...	·265	1·653	·214	1·057	56
57	3·332	6·066	·181	·827	...	...	·267	1·456	·200	·883	57
58	3·588	5·379	·169	·682	..	...	·267	1·256	·185	·721	58
59	3·888	4·684	·156	·544	...	...	·265	1·050	·167	·570	59
60	4·258	3·989	·140	·417	...	...	·258	·844	·149	·433	60
61	4·716	3·281	·122	·301	...	..	·243	·637	·127	·309	61
62	5·335	2·565	·101	·199	...	...	·215	·437	·104	·203	62
63	6·158	1·807	·075	·111	...	...	·167	·252	·077	·114	63
64	7·282	·971	·042	·042	...	...	·098	·098	·042	·042	64

Hypothetical Experience of Staff Pension Fund.

Table 37—*continued.*

Multipliers for use in a Valuation.

Pension Age 65. Interest 3 per-cent.

Age (x)	$\frac{{}^{r}D_x}{D_x}$	$\frac{{}^{r}N^s_x}{D^s_x}$	$\frac{{}^{ra}M^s_x}{D^s_x}$	$\frac{{}^{ra}R^s_x}{D^s_x}$	$\frac{{}^{ra}M^{ls}_x}{D^s_x}$	$\frac{{}^{ra}R^{ls}_x}{D^s_x}$	$\frac{{}^{d}M^{ls}_x}{D^s_x}$	$\frac{{}^{w}M^{ls}_x}{D^s_x}$	$\frac{{}^{r}M^{ls}_x}{D^s_x}$	Age (x)
15	·080	9·329	·227	53·296	2·177	93·548	·388	1·285	·225	15
16	·087	8·271	·253	47·458	1·947	81·708	·344	1·084	·201	16
17	·096	7·745	·288	44·670	1·843	75·478	·322	·944	·190	17
18	·106	7·460	·327	43·286	1·797	71·815	·311	·834	·186	18
19	·116	7·266	·369	42·440	1·775	69·173	·303	·744	·183	19
20	·126	7·118	·414	41·881	1·767	67·094	·298	·666	·183	20
21	·136	6·999	·460	41·507	1·769	65·390	·294	·599	·183	21
22	·146	6·890	·508	41·169	1·774	63·811	·291	·540	·183	22
23	·155	6·785	·558	40·943	1·787	62·465	·289	·488	·184	23
24	·164	6·678	·608	40·680	1·800	61·121	·287	·440	·186	24
25	·173	6·570	·661	40·414	1·815	59·828	·285	·398	·187	25
26	·182	6·543	·715	40·663	1·856	59·342	·287	·364	·192	26
27	·190	6·505	·770	40·852	1·902	58·785	·289	·333	·196	27
28	·199	6·453	·827	40·958	1·940	58·130	·290	·304	·200	28
29	·206	6·390	·885	41·007	1·981	57·416	·292	·278	·205	29
30	·214	6·316	·943	40·975	2·020	56·615	·293	·253	·209	30
31	·221	6·230	1·003	40·873	2·060	55·742	·293	·230	·213	31
32	·228	6·137	1·063	40·720	2·098	54·826	·294	·209	·217	32
33	·234	6·032	1·124	40·485	2·138	53·827	·295	·189	·221	33
34	·241	5·908	1·186	40·199	2·176	52·790	·295	·171	·224	34
35	·247	5·785	1·250	39·821	2·215	51·662	·295	·153	·228	35
36	·253	5·670	1·314	39·395	2·254	50·502	·295	·137	·232	36
37	·259	5·531	1·380	38·881	2·294	49·262	·295	·122	·236	37
38	·264	5·387	1·447	38·319	2·333	47·994	·294	·108	·240	38
39	·269	5·234	1·514	37·670	2·371	46·649	·293	·095	·244	39
40	·274	5·076	1·582	36·958	2·409	45·261	·292	·083	·248	40
41	·279	4·913	1·651	36·193	2·448	43·841	·291	·072	·252	41
42	·283	4·741	1·715	35·339	2·485	42·349	·289	·062	·256	42
43	·287	4·562	1·789	34·406	2·522	40·796	·287	·052	·259	43
44	·291	4·381	1·860	33·429	2·561	39·227	·285	·044	·263	44
45	·295	4·191	1·931	32·350	2·597	37·574	·282	·036	·267	45
46	·299	3·997	2·002	31·207	2·633	35·883	·279	·028	·271	46
47	·302	3·798	2·071	29·991	2·667	34·145	·275	·022	·275	47
48	·305	3·594	2·140	28·697	2·699	32·354	·270	·016	·278	48
49	·307	3·383	2·299	27·315	2·729	30·502	·265	·011	·281	49
50	·309	3·169	2·299	25·770	2·755	28·610	·259	·007	·284	50
51	·311	2·948	2·331	24·214	2·778	26·674	·252	·004	·287	51
52	·311	2·727	2·387	22·609	2·794	24·690	·245	·002	·289	52
53	·311	2·500	2·435	20·924	2·803	22·655	·237	·001	·291	53
54	·311	2·270	2·476	19·181	2·805	20·594	·228	...	·292	54
55	·309	2·038	2·509	17·378	2·799	18·506	·218	...	·292	55
56	·306	1·806	2·535	15·535	2·785	16·412	·206	...	·291	56
57	·303	1·573	2·542	13·630	2·753	14·288	·193	...	·289	57
58	·297	1·341	2·535	11·706	2·707	12·177	·179	...	·286	58
59	·289	1·109	2·498	9·743	2·633	10·061	·163	...	·279	59
60	·276	·881	2·414	7·784	2·513	7·981	·145	...	·269	60
61	·255	·658	2·254	5·836	2·319	5·944	·125	...	·250	61
62	·222	·447	1·975	3·978	2·011	4·025	·103	...	·219	62
63	·171	·255	1·516	2·271	1·529	2·284	·076	...	·169	63
64	·098	·098	·876	·876	·876	·876	·042	...	·098	64

Hypothetical Experience of Staff Pension Fund.

TABLE 38.

Simple Commutation Columns (according to Table 4), and Commutation Columns for valuing Benefits on Death and Early Retirement.

PENSION AGE 60. INTEREST 3 PER-CENT.

Age (x)	$N_x^{(4)}$	$M_x^{(4)}$	$R_x^{(4)}$	${}^dM_x = \Sigma_x^{59}C_x$	${}^dR_x = \Sigma{}^dM_x$	${}^rM_x = \Sigma_x^{59}{}^rC_x$	${}^rR_x = \Sigma{}^rM_x$	Age (x)
15	137,597	1190·9	36423·6	832·5	15990·6	164·84	5959·55	15
16	126,113	1146·0	35232·7	787·6	15158·1	164·84	5794·71	16
17	116,000	1104·9	34086·7	746·5	14370·5	164·84	5629·87	17
18	107,112	1067·3	32981·8	708·9	13624·0	164·84	5465·03	18
19	99,240	1033·7	31914·5	675·3	12915·1	164·84	5300·19	19
20	92,211	1003·2	30880·8	644·8	12239·8	164·84	5135·35	20
21	85,891	975·2	29877·6	616·8	11595·0	164·84	4970·51	21
22	80,168	949·6	28902·4	591·2	10978·2	164·84	4805·67	22
23	74,953	925·8	27952·8	567·4	10387·0	164·84	4640·83	23
24	70,174	903·7	27027·0	545·3	9819·6	164·84	4475·99	24
25	65,774	883·2	26123·3	524·8	9274·3	164·84	4311·15	25
26	61,705	863·7	25240·1	505·3	8749·5	164·84	4146·31	26
27	57,930	844·8	24376·4	486·4	8244·2	164·84	3981·47	27
28	54,416	826·9	23531·6	468·5	7757·8	164·84	3816·63	28
29	51,137	809·9	22704·7	451·5	7289·3	163·99	3651·79	29
30	48,069	793·4	21894·8	435·0	6837·8	163·17	3487·80	30
31	45,192	777·4	21101·4	419·0	6402·8	161·97	3324·63	31
32	42,490	761·9	20324·0	403·5	5983·8	160·42	3162·66	32
33	39,947	746·8	19562·1	388·4	5580·3	158·91	3002·24	33
34	37,551	732·2	18815·3	373·8	5191·9	157·08	2843·33	34
35	35,289	718·0	18083·1	359·6	4818·1	155·30	2686·25	35
36	33,152	703·9	17365·1	345·5	4458·5	153·23	2530·95	36
37	31,130	690·2	16661·2	331·8	4113·0	151·22	2377·72	37
38	29,216	676·5	15971·0	318·1	3781·2	148·94	2226·50	38
39	27,402	662·9	15294·5	304·5	3463·1	146·41	2077·56	39
40	25,682	649·4	14631·6	291·0	3158·6	143·65	1931·15	40
41	24,051	636·0	13982·2	277·6	2867·6	140·67	1787·50	41
42	22,503	622·7	13346·2	264·3	2590·0	137·49	1646·83	42
43	21,033	609·2	12723·5	250·8	2325·7	134·12	1509·34	43
44	19,638	595·9	12114·3	237·5	2074·9	130·58	1375·22	44
45	18,313	582·4	11518·4	224·0	1837·4	126·88	1244·64	45
46	17,055	568·5	10936·0	210·1	1613·4	122·77	1117·76	46
47	15,861	554·5	10367·5	196·1	1403·3	118·28	994·99	47
48	14,728	540·5	9813·0	182·1	1207·2	113·44	876·71	48
49	13,653	526·2	9272·5	167·8	1025·1	108·27	763·27	49
50	12,634	511·6	8746·3	153·2	857·3	102·57	655·00	50
51	11,669	497·0	8234·7	138·6	704·1	96·37	552·43	51
52	10,756	482·4	7737·7	124·0	565·5	89·49	456·06	52
53	9,893	467·6	7255·3	109·2	441·5	81·97	366·57	53
54	9,079	452·6	6787·7	94·2	332·3	73·66	284·60	54
55	8,313	437·4	6335·1	79·0	238·1	64·61	210·94	55
56	7,595	421·9	5897·7	63·5	159·1	54·68	146·33	56
57	6,924	406·1	5475·8	47·7	95·6	43·55	91·65	57
58	6,301	390·3	5069·7	31·9	47·9	31·13	48·10	58
59	5,726	374·4	4679·4	16·0	16·0	16·97	16·97	59
60	5,201	358·4	4305·0	...	...	...	...	

NOTE.—$D_x^{(4)}$ and $C_x^{(4)}$ are the same as in Table 29. ${}^dC_x$, ${}^wC_x$, ${}^rC_x$, ${}^wM_x$, and ${}^wR_x$ are the same as in Table 30.

Hypothetical Experience of Staff Pension Fund.

TABLE 39.

Commutation Tables for finding the Values of the Return of Contributions of 1 per annum, with Compound Interest at 3 per-cent, on Death or Early Retirement, and for finding the Value of a Pension of 1 for each year of service on Early Retirement.

PENSION AGE 60. INTEREST 3 PER-CENT.

Age (x)	${}^{d}D_x = l'_x \times v^{x+1}$	${}^{d}N_x = \Sigma {}^{d}D_x$	${}^{r}D_x = \Sigma r_x \times v^{x+1}$	${}^{r}N_x = \Sigma {}^{r}D_x$	${}^{ra}M_x = \Sigma\{{}^{r}C_x \times (a'_{x+\cdot5} + \cdot5)\}$	${}^{ra}R_x = \Sigma {}^{ra}M_x$	Age (x)
15	1,556	25,676	478	10,912	1,641	59,843	15
16	1,467	24,120	464	10,434	1,641	58,202	16
17	1,385	22,653	451	9,970	1,641	56,562	17
18	1,308	21,268	437	9,519	1,641	54,921	18
19	1,237	19,960	425	9,082	1,641	53,281	19
20	1,171	18,723	412	8,657	1,641	51,640	20
21	1,110	17,552	400	8,245	1,641	50,000	21
22	1,053	16,442	389	7,845	1,641	48,359	22
23	999	15,389	377	7,456	1,641	46,719	23
24	949	14,390	366	7,079	1,641	45,078	24
25	901	13,441	356	6,713	1,641	43,438	25
26	856	12,540	345	6,357	1,641	41,797	26
27	813	11,684	335	6,012	1,641	40,157	27
28	771	10,871	325	5,677	1,641	38,516	28
29	733	10,100	315	5,352	1,634	36,876	29
30	695	9,367	305	5,037	1,627	35,242	30
31	659	8,672	295	4,732	1,617	33,615	31
32	625	8,013	285	4,437	1,604	31,999	32
33	592	7,388	275	4,152	1,591	30,395	33
34	561	6,796	265	3,877	1,575	28,804	34
35	531	6,235	256	3,612	1,560	27,229	35
36	501	5,704	247	3,356	1,541	25,669	36
37	474	5,203	237	3,109	1,523	24,128	37
38	446	4,729	228	2,872	1,503	22,604	38
39	420	4,283	219	2,644	1,479	21,102	39
40	395	3,863	210	2,425	1,453	19,623	40
41	370	3,468	201	2,215	1,425	18,169	41
42	347	3,098	192	2,014	1,395	16,744	42
43	324	2,751	183	1,822	1,363	15,349	43
44	301	2,427	175	1,639	1,328	13,987	44
45	279	2,126	166	1,464	1,292	12,659	45
46	258	1,847	157	1,298	1,251	11,367	46
47	237	1,589	148	1,141	1,206	10,116	47
48	216	1,352	139	993	1,157	8,910	48
49	196	1,136	130	854	1,104	7,753	49
50	176	940	121	724	1,046	6,649	50
51	157	764	111	603	982	5,603	51
52	138	607	101	492	912	4,621	52
53	120	469	91	391	834	3,709	53
54	102	349	80	300	748	2,875	54
55	84	247	69	220	655	2,127	55
56	66	163	57	151	553	1,472	56
57	49	97	45	94	439	920	57
58	32	48	32	49	312	481	58
59	16	16	17	17	169	169	59

Hypothetical Experience of Staff Pension Fund.

Table 40.

Table for finding Accumulation of Salary at Compound Interest; and Commutation Columns for finding the Present Value of the Return of Contributions on Death and Early Retirement Without Interest.

Pension Age 60. Interest 3 per-cent.

Age (x)	$s_x v^x$	$\Sigma s_x v^x$	$(1+i)^{x-1}$	${}^dM^s_x = {}^dM_x \times s_x$	${}^dR^s_x = \Sigma {}^dM^s_x$	${}^rM^s_x = {}^rM_x \times s_x$	${}^rR^s_x = \Sigma {}^rM^s_x$	Age (x)
15	12·84	1566·25	1·5126	16,650	1,346,547	3,297	609,405	15
16	15·58	1553·41	1·5580	19,690	1,329,897	4,121	606,108	16
17	18·15	1537·83	1·6047	22,395	1,310,207	4,945	601,987	17
18	20·56	1519·68	1·6528	24,812	1,287,812	5,769	597,042	18
19	22·81	1499·12	1·7024	27,012	1,263,000	6,594	591,273	19
20	24·92	1476·31	1·7535	29,016	1,235,988	7,418	584,679	20
21	26·88	1451·39	1·8061	30,840	1,206,972	8,242	577,261	21
22	28·70	1424·51	1·8603	32,516	1,176,132	9,066	569,019	22
23	30·40	1395·81	1·9161	34,044	1,143,616	9,890	559,953	23
24	31·98	1365·41	1·9736	35,445	1,109,572	10,715	550,063	24
25	33·43	1333·43	2·0328	36,736	1,074,127	11,539	539,348	25
26	34·31	1300·00	2·0938	37,392	1,037,391	12,198	527,809	26
27	35·11	1265·69	2·1566	37,939	999,999	12,858	515,611	27
28	35·84	1230·58	2·2213	38,417	962,060	13,517	502,754	28
29	36·49	1194·74	2·2879	38,829	923,643	14,103	489,237	29
30	37·08	1158·25	2·3566	39,150	884,814	14,685	475,134	30
31	37·60	1121·17	2·4273	39,386	845,664	15,225	460,448	31
32	38·06	1083·57	2·5001	39,543	806,278	15,721	445,223	32
33	38·46	1045·51	2·5751	39,617	766,735	16,209	429,502	33
34	38·80	1007·05	2·6523	39,623	727,118	16,651	413,293	34
35	39·09	968·25	2·7319	39,556	687,495	17,083	396,643	35
36	39·33	929·16	2·8139	39,387	647,939	17,468	379,560	36
37	39·53	889·83	2·8983	39,152	608,552	17,844	362,092	37
38	39·68	850·30	2·9852	38,808	569,400	18,171	344,248	38
39	39·79	810·62	3·0748	38,367	530,592	18,448	326,077	39
40	39·85	770·83	3·1670	37,830	492,225	18,675	307,629	40
41	39·88	730·98	3·2620	37,198	454,395	18,850	288,955	41
42	39·88	691·10	3·3599	36,473	417,197	18,974	270,105	42
43	39·84	651·22	3·4607	35,614	380,724	19,045	251,131	43
44	39·77	611·38	3·5645	34,675	345,110	19,065	232,086	44
45	39·67	571·61	3·6715	33,600	310,435	19,032	213,022	45
46	39·54	531·94	3·7816	32,355	276,835	18,907	193,990	46
47	39·38	492·40	3·8950	30,984	244,480	18,688	175,083	47
48	39·20	453·02	4·0119	29,500	213,496	18,377	156,395	48
49	39·00	413·82	4·1323	27,855	183,996	17,973	138,017	49
50	38·78	374·82	4·2562	26,044	156,141	17,437	120,045	50
51	38·53	336·04	4·3839	24,116	130,097	16,768	102,608	51
52	38·27	297·51	4·5154	22,072	105,981	15,929	85,839	52
53	37·99	259·24	4·6509	19,874	83,909	14,919	69,910	53
54	37·70	221·25	4·7904	17,521	64,035	13,701	54,992	54
55	37·39	183·55	4·9341	15,010	46,514	12,276	41,291	55
56	37·06	146·16	5·0821	12,319	31,504	10,608	29,015	56
57	36·72	109·10	5·2346	9,445	19,185	8,623	18,407	57
58	36·37	72·38	5·3917	6,444	9,740	6,288	9,784	58
59	36·01	36·01	5·5534	3,296	3,296	3,496	3,496	59

Hypothetical Experience of Staff Pension Fund.

TABLE 41.

Commutation Columns for finding the Present Values of Future Salary, and of Return of Contributions, with Compound Interest, on Death or Early Retirement.

PENSION AGE 60. INTEREST 3 PER-CENT.

Age (x)	$N^s_x = \Sigma^{59}_x (vD^s_x)$	${}^dD^s_x = {}^dD_x \times s_x$	${}^dN^s_x = \Sigma {}^dD^s_x$	${}^rD^s_x = {}^rD_x \times s_x$	${}^rN^s_x = \Sigma {}^rD^s_x$	Age (x)
15	10,214,321	31,120	1,988,978	9,560	963,768	15
16	9,965,059	36,675	1,957,858	11,600	954,208	16
17	9,686,321	41,550	1,921,183	13,530	942,608	17
18	9,391,768	45,780	1,879,633	15,295	929,078	18
19	9,089,748	49,480	1,833,853	17,000	913,783	19
20	8,784,039	52,695	1,784,373	18,540	896,783	20
21	8,476,946	55,500	1,731,678	20,000	878,243	21
22	8,170,150	57,915	1,676,178	21,395	858,243	22
23	7,864,552	59,940	1,618,263	22,620	836,848	23
24	7,560,766	61,685	1,558,323	23,790	814,228	24
25	7,259,179	63,070	1,496,638	24,920	790,438	25
26	6,960,149	63,344	1,433,568	25,530	765,518	26
27	6,667,812	63,414	1,370,224	26,130	739,988	27
28	6,381,938	63,222	1,306,810	26,650	713,858	28
29	6,102,182	63,038	1,243,588	27,090	687,208	29
30	5,828,401	62,550	1,180,550	27,450	660,118	30
31	5,560,323	61,946	1,118,000	27,730	632,668	31
32	5,297,762	61,250	1,056,054	27,930	604,938	32
33	5,040,679	60,384	994,804	28,050	577,008	33
34	4,788,848	59,466	934,420	28,090	548,958	34
35	4,542,269	58,410	874,954	28,160	520,868	35
36	4,300,696	57,114	816,544	28,158	492,708	36
37	4,064,173	55,932	759,430	27,966	464,550	37
38	3,832,526	54,412	703,498	27,816	436,584	38
39	3,605,819	52,920	649,086	27,594	408,768	39
40	3,383,912	51,350	596,166	27,300	381,174	40
41	3,166,825	49,580	544,816	26,934	353,874	41
42	2,954,636	47,886	495,236	26,496	326,940	42
43	2,747,234	46,008	447,350	25,986	300,444	43
44	2,544,574	43,946	401,342	25,550	274,458	44
45	2,346,836	41,850	357,396	24,900	248,908	45
46	2,153,875	39,732	315,546	24,178	224,008	46
47	1,965,785	37,446	275,814	23,384	199,830	47
48	1,782,628	34,992	238,368	22,518	176,446	48
49	1,604,428	32,536	203,376	21,580	153,928	49
50	1,431,175	29,920	170,840	20,570	132,348	50
51	1,262,990	27,318	140,920	19,314	111,778	51
52	1,099,971	24,564	113,602	17,978	92,464	52
53	942,190	21,840	89,038	16,562	74,486	53
54	789,699	18,972	67,198	14,880	57,924	54
55	642,705	15,960	48,226	13,110	43,044	55
56	501,404	12,804	32,266	11,058	29,934	56
57	366,169	9,702	19,462	8,910	18,876	57
58	237,181	6,464	9,760	6,464	9,966	58
59	115,000	3,296	3,296	3,502	3,502	59

Hypothetical Experience of Staff Pension Fund.

TABLE 42.

Commutation Columns for finding the Present Value of the Last Year's Salary on Death or Early Retirement; and the Present Value of a Pension based on Average Salary and Last Salary.

PENSION AGE 60. INTEREST 3 PER-CENT.

Age (x)	${}^{d}M_x^{ls} = \Sigma({}^{d}C_x s_x)$	${}^{r}M_x^{ls} = \Sigma({}^{r}C_x s_x)$	${}^{ra}M_x^{s} = {}^{ra}M_x \times s_x$	${}^{ra}R_x^{s} = \Sigma {}^{ra}M_x^{s}$	${}^{ra}M_x^{ls} = \Sigma\{{}^{r}C_x s_x \times (a'_{x+\cdot5} + \cdot5)\}$	${}^{ra}R_x^{ls} = \Sigma {}^{ra}M_x^{ls}$	Age (x)
15	83,701	28,126	32,810	6,143,619	282,022	10,672,226	15
16	82,803	28,126	41,013	6,110,809	282,022	10,390,204	16
17	81,775	28,126	49,215	6,069,796	282,022	10,108,182	17
18	80,647	28,126	57,418	6,020,581	282,022	9,826,160	18
19	79,471	28,126	65,620	5,963,163	282,022	9,544,138	19
20	78,251	28,126	73,823	5,897,543	282,022	9,262,116	20
21	76,991	28,126	82,025	5,823,720	282,022	8,980,094	21
22	75,711	28,126	90,228	5,741,695	282,022	8,698,072	22
23	74,402	28,126	98,430	5,651,467	282,022	8,416,050	23
24	73,076	28,126	106,633	5,553,037	282,022	8,134,028	24
25	71,743	28,126	114,835	5,446,404	282,022	7,852,006	25
26	70,378	28,126	121,397	5,331,569	282,022	7,569,984	26
27	68,979	28,126	127,959	5,210,172	282,022	7,287,962	27
28	67,583	28,126	134,521	5,082,213	282,022	7,005,940	28
29	66,189	28,057	140,490	4,947,692	281,458	6,723,918	29
30	64,770	27,986	146,421	4,807,202	280,881	6,442,460	30
31	63,330	27,878	151,989	4,660,781	279,986	6,161,579	31
32	61,873	27,732	157,182	4,508,792	278,762	5,881,593	32
33	60,393	27,584	162,282	4,351,610	277,501	5,602,831	33
34	58,904	27,398	166,971	4,189,328	275,890	5,325,330	34
35	57,399	27,209	171,556	4,022,357	274,239	5,049,440	35
36	55,848	26,979	175,708	3,850,801	272,203	4,775,201	36
37	54,286	26,750	179,738	3,675,093	270,145	4,502,998	37
38	52,669	26,481	183,305	3,495,355	267,700	4,232,853	38
39	51,010	26,172	186,379	3,312,050	264,857	3,965,153	39
40	49,300	25,825	188,942	3,125,671	261,615	3,700,296	40
41	47,558	25,437	190,990	2,936,729	257,962	3,438,681	41
42	45,776	25,011	192,510	2,745,739	253,897	3,180,719	42
43	43,913	24,546	193,475	2,553,229	249,414	2,926,822	43
44	42,024	24,043	193,888	2,359,754	244,517	2,677,408	44
45	40,053	23,503	193,740	2,165,866	239,202	2,432,891	45
46	37,968	22,887	192,623	1,972,126	233,080	2,193,689	46
47	35,812	22,195	190,516	1,779,503	226,151	1,960,609	47
48	33,600	21,430	187,418	1,588,987	218,428	1,734,458	48
49	31,283	20,593	183,330	1,401,569	209,919	1,516,030	49
50	28,859	19,647	177,837	1,218,239	200,248	1,306,111	50
51	26,377	18,593	170,938	1,040,402	189,424	1,105,863	51
52	23,837	17,396	162,247	869,464	177,082	916,439	52
53	21,203	16,057	151,770	707,217	163,267	739,357	53
54	18,473	14,545	139,147	555,447	147,644	576,090	54
55	15,646	12,861	124,393	416,300	130,273	428,446	55
56	12,701	10,975	107,185	291,907	110,858	298,173	56
57	9,636	8,815	86,823	184,722	88,748	187,315	57
58	6,508	6,356	63,044	97,899	63,714	98,567	58
59	3,296	3,496	34,855	34,855	34,853	34,853	59

NOTE.—${}^{w}M_x^{ls}$ will be found in Table 32.

Hypothetical Experience of Staff Pension Fund.

TABLE 43.

Multipliers for use in a Valuation.

PENSION AGE 60. INTEREST 3 PER-CENT.

Age (x)	$\frac{N_{60}+\frac{1}{2}D_{60}}{D_x}$	$\frac{\bar{N}^s_x}{D^s_x}$	$\frac{{}^dM_x}{D_x}$	$\frac{{}^d\bar{R}^s_x}{D^s_x}$	$\frac{{}^rM_x}{D_x}$	$\frac{{}^r\bar{R}^s_x}{D^s_x}$	$\frac{{}^dD_x}{D_x}$	$\frac{{}^d\bar{N}^s_x}{D^s_x}$	Age (x)
15	·426	39·785	·065	5·245	·013	2·374	·121	7·747	15
16	·476	34·709	·069	4·632	·014	2·111	·128	6·820	16
17	·540	31·927	·074	4·319	·016	1·984	·137	6·332	17
18	·615	30·191	·080	4·140	·019	1·919	·147	6·042	18
19	·694	28·867	·086	4·011	·021	1·878	·157	5·824	19
20	·777	27·771	·092	3·908	·023	1·848	·167	5·641	20
21	·864	26·826	·098	3·820	·026	1·827	·176	5·480	21
22	·955	25·956	·103	3·737	·029	1·808	·184	5·325	22
23	1·048	25·134	·109	3·655	·032	1·790	·192	5·172	23
24	1·143	24·340	·114	3·572	·034	1·771	·199	5·017	24
25	1·242	23·569	·119	3·487	·037	1·751	·205	4·859	25
26	1·343	23·115	·124	3·445	·041	1·753	·210	4·761	26
27	1·447	22·645	·129	3·396	·044	1·751	·215	4·653	27
28	1·555	22·148	·133	3·339	·047	1·745	·219	4·535	28
29	1·666	21·639	·138	3·275	·050	1·735	·224	4·410	29
30	1·781	21·108	·142	3·204	·053	1·721	·227	4·276	30
31	1·899	20·560	·146	3·127	·056	1·703	·229	4·134	31
32	2·022	20·007	·149	3·045	·059	1·681	·231	3·988	32
33	2·148	19·433	·153	2·956	·062	1·656	·233	3·835	33
34	2·280	18·856	·156	2·863	·066	1·627	·234	3·679	34
35	2·415	18·255	·159	2·763	·069	1·594	·235	3·516	35
36	2·556	17·654	·162	2·660	·072	1·558	·234	3·352	36
37	2·702	17·034	·164	2·551	·075	1·518	·234	3·183	37
38	2·854	16·413	·166	2·438	·078	1·474	·233	3·013	38
39	3·012	15·776	·168	2·321	·081	1·427	·232	2·840	39
40	3·176	15·134	·169	2·201	·084	1·376	·230	2·666	40
41	3·350	14·490	·170	2·079	·086	1·322	·227	2·493	41
42	3·529	13·831	·171	1·953	·088	1·264	·224	2·318	42
43	3·716	13·161	·171	1·824	·091	1·203	·220	2·143	43
44	3·916	12·494	·170	1·694	·094	1·140	·216	1·971	44
45	4·123	11·808	·169	1·562	·096	1·072	·211	1·798	45
46	4·343	11·118	·167	1·429	·098	1·001	·205	1·629	46
47	4·575	10·420	·164	1·296	·099	·928	·199	1·462	47
48	4·822	9·712	·161	1·163	·100	·852	·191	1·299	48
49	5·082	8·991	·156	1·031	·101	·773	·182	1·140	49
50	5·361	8·262	·150	·901	·101	·693	·173	·986	50
51	5·661	7·522	·144	·775	·100	·611	·163	·839	51
52	5·984	6·768	·136	·652	·098	·528	·151	·699	52
53	6·330	5·999	·127	·534	·095	·445	·139	·567	53
54	6·711	5·216	·116	·423	·090	·363	·125	·444	54
55	7·132	4·416	·103	·320	.084	·284	·110	·331	55
56	7·609	3·600	·088	·226	·076	·208	·092	·232	56
57	8·142	2·756	·071	·144	·065	·139	·073	·146	57
58	8·769	1·885	·051	·077	·050	·078	·051	·078	58
59	9·501	·971	·028	·028	·030	·030	·028	·028	59

Hypothetical Experience of Staff Pension Fund.

TABLE 43—(*continued*).

Multipliers for use in a Valuation.

PENSION AGE 60. INTEREST 3 PER-CENT.

Age (x)	$\frac{{}^{r}D_x}{D_x}$	$\frac{{}^{r}N^s_x}{D^s_x}$	$\frac{{}^{ra}M_x}{D_x}$	$\frac{{}^{ra}R^s_x}{D^s_x}$	$\frac{{}^{ra}M^{ls}_x}{D^s_x}$	$\frac{{}^{ra}R^{ls}_x}{D^s_x}$	$\frac{{}^{d}M^{ls}_x}{D^s_x}$	$\frac{{}^{r}M^{ls}_x}{D^s_x}$	Age (x)
15	·037	3·754	·128	23·929	1·098	41·568	·326	·110	15
16	·040	3·324	·143	21·285	·983	36·190	·288	·098	16
17	·045	3·107	·162	20·007	·929	33·318	·270	·093	17
18	·049	2·987	·185	19·354	·907	31·587	·259	·090	18
19	·054	2·902	·208	18·938	·896	30·310	·252	·089	19
20	·059	2·835	·233	18·645	·892	29·282	·247	·089	20
21	·063	2·779	·260	18·430	·892	28·418	·244	·089	21
22	·068	2·727	·287	18·241	·895	27·608	·240	·089	22
23	·072	2·675	·315	18·062	·901	26·897	·238	·090	23
24	·077	2·621	·343	17·876	·908	26·185	·235	·091	24
25	·081	2·566	·373	17·683	·916	25·494	·233	·091	25
26	·085	2·542	·403	17·707	·937	25·141	·234	·093	26
27	·089	2·513	·435	17·695	·958	24·751	·234	·096	27
28	·092	2·477	·467	17·638	·979	24·314	·235	·098	28
29	·096	2·437	·498	17·545	·998	23·844	·235	·100	29
30	·099	2·391	·530	17·410	1·017	23·332	·235	·101	30
31	·103	2·339	·562	17·234	1·035	22·784	·234	·103	31
32	·105	2·285	·594	17·027	1·053	22·212	·234	·105	32
33	·108	2·225	·626	16·777	1·070	21·600	·233	·106	33
34	·111	2·162	·657	16·495	1·086	20·968	·232	·108	34
35	·113	2·093	·690	16·166	1·102	20·294	·231	·109	35
36	·116	2·023	·721	15·807	1·117	19·601	·229	·111	36
37	·117	1·947	·753	15·403	1·132	18·872	·228	·112	37
38	·119	1·870	·785	14·969	1·146	18·127	·226	·113	38
39	·121	1·788	·815	14·491	1·159	17·348	·223	·114	39
40	·122	1·705	·845	13·979	1·170	16·549	·220	·115	40
41	·123	1·619	·874	13·437	1·180	15·734	·218	·116	41
42	·124	1·530	·901	12·853	1·189	14·889	·214	·117	42
43	·124	1·439	·927	12·232	1·195	14·021	·210	·118	43
44	·125	1·348	·952	11·586	1·200	13·146	·206	·118	44
45	·125	1·252	·975	10·897	1·204	12·241	·202	·118	45
46	·125	1·156	·994	10·180	1·203	11·323	·196	·118	46
47	·124	1·059	1·010	9·433	1·199	10·393	·190	·118	47
48	·123	·961	1·021	8·657	1·190	9·450	·183	·117	48
49	·121	·863	1·027	7·854	1·176	8·496	·175	·115	49
50	·119	·764	1·027	7·033	1·156	7·540	·167	·113	50
51	·115	·666	1·018	6·196	1·128	6·586	·157	·111	51
52	·111	·569	·999	5·350	1·090	5·639	·147	·107	52
53	·105	·474	·966	4·503	1·039	4·707	·135	·102	53
54	·098	·383	·919	3·669	·975	3·805	·122	·096	54
55	·090	·296	·855	2·860	·895	2·944	·108	·088	55
56	·079	·215	·770	2·096	·796	2·141	·091	·079	56
57	·067	·142	·654	1·390	·668	1·410	·073	·066	57
58	·051	·079	·501	·778	·506	·783	·052	·051	58
59	·030	·030	·294	·294	·294	·294	·028	·030	59

NOTE.—The values of ${}^{w}M^{ls}_x \div D^s_x$ will be found in Table 37.

APPENDIX I.

EXPLANATION OF THE NOTATION USED IN THE PAPER.

In all the simple investigations throughout the paper I have used the Institute notation with one exception, namely, the symbol to represent the amount of an annuity-certain of 1 per annum for n years. That symbol in the Institute notation is $s_{\overline{n}|}$; but as I wished to use s to represent salary, and the similarity might lead to confusion, I decided to substitute $a_{(n)}$ for $s_{\overline{n}|}$.

The new elementary symbols are :—

w_x = the number of persons withdrawing (by resignation or dismissal) between the ages of x and $x+1$.

r_x = the number of persons retiring before the pension age (on pension or with compensation) from ill-health between the ages of x and $x+1$.

s_x = the salary receivable between the ages x and $x+1$.

ls = last salary; that is, the full year's salary receivable in the year of death, withdrawal, or retirement. This is only used in combination with commutation symbols.

$(as)_{(n)}$ = the amount of annual salary for n years accumulated at compound interest.

a'_x = the present value of an annuity of 1 on the life of an invalid of the age x who has retired from ill-health.

ra = annuity on early retirement (before the pension age is reached). This is only used in combination with commutation symbols.

An index consisting of a figure or letter in a bracket indicates the Experience Table from which the values are taken, or on which they are based. These tables are called 2, 3, 4, and (r). Nos. 2, 3, and 4 are different arrangements of the same experience, and (r) is the experience representing the mortality prevailing amongst those who retire from ill-health before the pension age, and is to be found in Table No. 8. Thus—

$$\left.\begin{matrix} l_x^{(2)} \\ d_x^{(2)} \\ D_x^{(2)} \\ N_x^{(2)} \\ \&c. \end{matrix}\right\} \text{ are the values of } \left\{\begin{matrix} l_x \\ d_x \\ D_x \\ N_x \\ \&c. \end{matrix}\right\} \text{ by Experience Table No. 2.}$$

$$\left.\begin{array}{l} l_x^{(r)} \\ q_x^{(r)} \\ d_x^{(r)} \\ \mathrm{D}_x^{(r)} \\ \mathrm{N}_x^{(r)} \\ \&c. \end{array}\right\} \text{ are the values of } \left.\begin{array}{l} l_x \\ q_x \\ d_x \\ \mathrm{D}_x \\ \mathrm{N}_x \\ \&c. \end{array}\right\} \text{ by the Invalid Mortality Table No. 8.}$$

Where no index is used the values are based on Experience Table No. 4, which is the basis of all the working formulas.

The only successful way of producing tables for solving the problems which are dealt with in this paper is by the use of the columnar method, and I therefore looked to the columnar symbols to represent the several values to be tabulated.

To the student the columnar symbols should be essentially pictorial, and this will be so if the origin of the symbols is kept in mind. I think there is little doubt that when Griffith Davies adopted D and N as the titles of his columns he selected the initial letters of the words denominator and numerator, so that $\frac{\mathrm{N}}{\mathrm{D}} = \frac{\text{Numerator}}{\text{Denominator}} = a$. The column for finding the value of an increasing annuity is formed by the constant summation of the N column, hence the use of the letter S to represent the sum of N. Subsequently, when corresponding columns were formed for finding the assurance values, the preceding letters in the alphabet were used, namely, C, M, R. The open $\mathbb{N}$ has since been added to represent the numerator for an annuity-due, so that $\mathbb{N}_x \div \mathrm{D}_x = 1 + a_x$.

The notation in the paper will appear simple if the mental retina receives the fixed impression that—

$\frac{\mathbb{N}_x}{\mathrm{D}_x}$ is the present value of an annuity-due, and the present value of the amount of an annuity-due accumulated at compound interest when interest and discount are at the same rate.

$\frac{\mathrm{M}_x}{\mathrm{D}_x}$ is the present value of the assurance of 1 payable at the end of the year of death.

$\frac{\mathrm{R}_x}{\mathrm{D}_x}$ is the present value of the assurance of 1 increasing by 1 every year up to the year of death.

As the pension age is the *limiting age* in the working tables, it will be necessary sometimes to have a distinguishing symbol to represent the pension age. For this I propose that the pension age shall be placed in a half square as a left-hand suffix. Thus—

$$\left.\begin{array}{r} {}_{\overline{60|}}\mathbb{N}_x \\ {}_{\overline{60|}}\mathrm{M}_x \\ {}_{\overline{60|}}\mathrm{R}_x \\ \&c. \end{array}\right\} \text{ are the values of } \left.\begin{array}{c} \mathbb{N}_x \\ \mathrm{M}_x \\ \mathrm{R}_x \\ \&c. \end{array}\right\} \text{ when the pension age is 60.}$$

Throughout this work the tables are calculated on the assumption that all events occur at the end of the year of assurance; that is to say, that deaths, withdrawals, and retirements happen at the end of the year in which they occur, and that the yearly contributions, although due at the beginning of the year, are payable at the end of the year. A full year's contribution is therefore supposed to be received in the year of death, withdrawal, or retirement.

COMMUTATION SYMBOLS NOT INVOLVING SALARY.

Contributions.

$D_x = l_x v^x = \Sigma_x^\omega d_x \times v^x$. This is the common denominator for all columnar values not involving salary.

${}_{\overline{65|}}\mathbb{N}_x = v(\mathrm{N}_{x-1} - \mathrm{N}_{64}) = v(\mathrm{D}_x + \mathrm{D}_{x+1} + \ldots + \mathrm{D}_{64})$. A separate column has not been calculated for this value. This symbol is not used in the paper except in combination with salary.

$\dfrac{{}_{\overline{65|}}\mathbb{N}_x}{\mathrm{D}_x}$ = the present value, at age x, of contributions of 1 per annum, ceasing at the end of the year of death, withdrawal, early retirement, or retirement at the pension age, 65.

Columns are required for each of the various modes of exit, and these are distinguished by placing the letter representing the elementary function as a top prefix to the columnar symbol.

Return of Contributions with Interest.

$${}^{d}_{\overline{65|}}\mathrm{D}_x = v^{x+1}\Sigma_x^{64} d_x, \text{ or } l'_x v^{x+1} \text{ where } l'_x = \Sigma_x^{64} d_x$$

$${}^{d}_{\overline{65|}}\mathbb{N}_x = {}^{d}_{\overline{65|}}\mathrm{D}_x + {}^{d}_{\overline{65|}}\mathrm{D}_{x+1} + \ldots + {}^{d}_{\overline{65|}}\mathrm{D}_{64}$$

$${}^{r}_{\overline{65|}}\mathrm{D}_x = v^{x+1}\Sigma_x^{64} r_x$$

$${}^{r}_{\overline{65|}}\mathbb{N}_x = {}^{r}_{\overline{65|}}\mathrm{D}_x + {}^{r}_{\overline{65|}}\mathrm{D}_{x+1} + \ldots + {}^{r}_{\overline{65|}}\mathrm{D}_{64}$$

so that

$\frac{{}^{d}\mathbb{N}_x}{D_x} =$	The present value of an annuity-due of 1 per annum deferred one year; which is the same as	to the end of the year of death.
$\frac{{}^{r}\mathbb{N}_x}{D_x} =$	The present value of the accumulation of an annuity-due of 1 per annum, deferred one year, at compound interest at the same rate as used in the valuation,	to the end of the year of early retirement.

Return of Contributions without Interest.

$$^{d}C_x = v^{x+1}d_x$$

$$^{w}C_x = v^{x+1}w_x$$

$$^{r}C_x = v^{x+1}r_x$$

$$\overset{d}{\lvert\overline{65}}M_x = {}^{d}C_x + {}^{d}C_{x+1} + \ldots + {}^{d}C_{64}$$

$$\overset{w}{\lvert\overline{65}}M_x = {}^{w}C_x + {}^{w}C_{x+1} + \ldots + {}^{w}C_{64}$$

$$\overset{r}{\lvert\overline{65}}M_x = {}^{r}C_x + {}^{r}C_{x+1} + \ldots + {}^{w}C_{64}$$

$$\overset{d}{\lvert\overline{65}}R_x = \overset{d}{\lvert\overline{65}}M_x + \overset{d}{\lvert\overline{65}}M_{x+1} + \ldots + \overset{d}{\lvert\overline{65}}M_{64}$$

$$\overset{w}{\lvert\overline{65}}R_x = \overset{w}{\lvert\overline{65}}M_x + \overset{w}{\lvert\overline{65}}M_{x+1} + \ldots + \overset{w}{\lvert\overline{65}}M_{64}$$

$$\overset{r}{\lvert\overline{65}}R_x = \overset{r}{\lvert\overline{65}}M_x + \overset{r}{\lvert\overline{65}}M_{x+1} + \ldots + \overset{r}{\lvert\overline{65}}M_{64}$$

so that

$\frac{{}^{d}M_x}{D_x} =$	the present value at age x of the assurance of 1 at the end of the year of	death.
$\frac{{}^{w}M_x}{D_x} =$		withdrawal.
$\frac{{}^{r}M_x}{D_x} =$		early retirement.

and

$\frac{{}^{d}R_x}{D_x} =$	the present value at age x of the assurance of 1 increasing by 1 per annum; in other words, the present value of the assurance of the return of contributions of 1 per annum without interest	on death.
$\frac{{}^{w}R_x}{D_x} =$		on withdrawal.
$\frac{{}^{r}R_x}{D_x} =$		on early retirement.

Annuity on Early Retirement.

$${}^{ra}C_x = {}^{r}C_x \times (a'_{x+\cdot5} + \cdot5)$$

$${}^{ra}_{\overline{65}|}M_x = {}^{ra}C_x + {}^{ra}C_{x+1} + \ldots + {}^{ra}C_{64}$$

$${}^{ra}_{\overline{65}|}R_x = {}^{ra}_{\overline{65}|}M_x + {}^{ra}_{\overline{65}|}M_{x+1} + \ldots + {}^{ra}_{\overline{65}|}M_{64}$$

so that

$\dfrac{{}^{ra}C_x}{D_x}$ = the present value at age x of an annuity of 1 per annum on retirement before age $x+1$.

$\dfrac{{}^{ra}M_x}{D_x}$ = the present value at age x of the assurance of an annuity of 1 per annum on retirement before the pension age.

$\dfrac{{}^{ra}R_x}{D_x}$ = the present value at age x of the assurance of an annuity on retirement before the pension age of 1 increasing by 1 per annum; in other words, the present value of the assurance on early retirement of an annuity of the contributions of 1 per annum.

COMMUTATION SYMBOLS INVOLVING SALARY.

In all cases where salary is introduced into the values, the letter s is used as an index to the columnar symbol.

Contributions.

$D^s_x = D_x \times s_x$. This is the common denominator for all columnar values involving salary, when such values have to be equated to a salary of 1 at x.

$${}_{\overline{65}|}\mathbb{N}^s_x = v\left(D^s_x + D^s_{x+1} + \ldots + D^s_{64}\right).$$

so that

$\dfrac{\mathbb{N}^s_x}{D_x}$ = the present value at age x of total salary receivable until death, withdrawal, early retirement, or attainment of pension age.

$\dfrac{\mathbb{N}^s_x}{D^s_x}$ = the present value at age x of total salary receivable until death, withdrawal, early retirement, or attainment of pension age, equated to 1 of salary at age x.

(1) Benefits based on Average Salary.

Return of Contributions with Interest.

$$ {}^{d}_{|\overline{65}}D^{s}_{x} = {}^{d}_{|\overline{65}}D_{x} \times s_{x} = v^{x+1}\Sigma^{64}_{x} d_{x} \times s_{x} $$

$$ {}^{d}_{|\overline{65}}\mathbb{N}^{s}_{x} = {}^{d}_{|\overline{65}}D^{s}_{x} + {}^{d}_{|\overline{65}}D^{s}_{x+1} + \ldots + {}^{d}_{|\overline{65}}D^{s}_{64} $$

$$ {}^{r}_{|\overline{65}}D^{s}_{x} = {}^{r}_{|\overline{65}}D_{x} \times s_{x} = v^{x+1}\Sigma^{64}_{x} r_{x} \times s_{x} $$

$$ {}^{r}_{|\overline{65}}\mathbb{N}^{s}_{x} = {}^{r}_{|\overline{65}}D^{s}_{x} + {}^{r}_{|\overline{65}}D^{s}_{x+1} + \ldots + {}^{r}_{|\overline{65}}D^{s}_{64} $$

so that

$\dfrac{{}^{d}\mathbb{N}^{s}_{x}}{D^{s}_{x}} =$ }
$\dfrac{{}^{r}\mathbb{N}^{s}_{x}}{D^{s}_{x}} =$ } The present value of the accumulations of future salary compounded at the same rate of interest as used in the valuation, equated to a salary of 1 at age x { to the end of the year of death. / to the end of the year of early retirement.

Return of Contributions without Interest.

$$ {}^{d}_{|\overline{65}}M^{s}_{x} = {}^{d}_{|\overline{65}}M_{x} \times s_{x} $$

$$ {}^{w}_{|\overline{65}}M^{s}_{x} = {}^{w}_{|\overline{65}}M_{x} \times s_{x} $$

$$ {}^{r}_{|\overline{65}}M^{s}_{x} = {}^{r}_{|\overline{65}}M_{x} \times s_{x} $$

$$ {}^{d}_{|\overline{65}}R^{s}_{x} = {}^{d}_{|\overline{65}}M^{s}_{x} + {}^{d}_{|\overline{65}}M^{s}_{x+1} + \ldots + {}^{d}_{|\overline{65}}M^{s}_{64} $$

$$ {}^{w}_{|\overline{65}}R^{s}_{x} = {}^{w}_{|\overline{65}}M^{s}_{x} + {}^{w}_{|\overline{65}}M^{s}_{x+1} + \ldots + {}^{w}_{|\overline{65}}M^{s}_{64} $$

$$ {}^{r}_{|\overline{65}}R^{s}_{x} = {}^{r}_{|\overline{65}}M^{s}_{x} + {}^{r}_{|\overline{65}}M^{s}_{x+1} + \ldots + {}^{r}_{|\overline{65}}M^{s}_{64} $$

so that

$\dfrac{{}^{d}M^{s}_{x}}{D_{x}} =$ }
$\dfrac{{}^{w}M^{s}_{x}}{D_{x}} =$ } The present value, at age x, of 1 year's salary at age x, payable at the end of the year of { death. / withdrawal. / early retirement.
$\dfrac{{}^{r}M^{s}_{x}}{D_{x}} =$ }

$\dfrac{{}^{d}M^{s}_{x}}{D^{s}_{x}} = \dfrac{{}^{d}M_{x}}{D_{x}} =$ }
$\dfrac{{}^{w}M^{s}_{x}}{D^{s}_{x}} = \dfrac{{}^{w}M_{x}}{D_{x}} =$ } The present value, at age x, of 1 year's salary at age x equated to 1 of salary at age x; in other words, the present value of the assurance of 1 at the end of the year of { death. / withdrawal. / early retirement.
$\dfrac{{}^{r}M^{s}_{x}}{D^{s}_{x}} = \dfrac{{}^{r}M_{x}}{D_{x}} =$ }

$\frac{{}^{d}R^{s}_{x}}{D_x}=$, $\frac{{}^{w}R^{s}_{x}}{D_x}=$, $\frac{{}^{r}R^{s}_{x}}{D_x}=$ The present value, at age x, of the assurance of the total salary to the end of the year of death. / withdrawal. / early retirement.

$\frac{{}^{d}R^{s}_{x}}{D^{s}_{x}}=$, $\frac{{}^{w}R^{s}_{x}}{D^{s}_{x}}=$, $\frac{{}^{r}R^{s}_{x}}{D^{s}_{x}}=$ The present value, at age x, of the return of total salary, equated to salary of 1 at age x, to the end of the year of death. / withdrawal. / early retirement.

Pension on Early Retirement.

$${}^{ra}_{\overline{65}|}M^{s}_{x}={}^{ra}_{\overline{65}|}M_{x}\times s_{x}$$

$${}^{ra}_{\overline{65}|}R^{s}_{x}={}^{ra}_{\overline{65}|}M^{s}_{x}+{}^{ra}_{\overline{65}|}M^{s}_{x+1}+\ldots+{}^{ra}_{\overline{65}|}M^{s}_{64}$$

so that

$\frac{{}^{ra}M^{s}_{x}}{D_x}=$ the present value at age x of the assurance of an annuity of one year's salary at age x on retirement before the pension age.

$\frac{{}^{ra}M^{s}_{x}}{D^{s}_{x}}=\frac{{}^{ra}M_{x}}{D_x}=$ the above equated to a salary of 1 at age x; which is the assurance of an annuity of 1 per annum on retirement before the pension age.

$\frac{{}^{ra}R^{s}_{x}}{D^{s}_{x}}=$ the present value at age x of the assurance of an annuity, on retirement before the pension age, equivalent to the total salary received from age x until retirement equated to 1 of salary at age x.

(II) BENEFITS BASED ON LAST SALARY.

Return of Last Premiums.

$${}^{d}C^{s}_{x}={}^{d}C_{x}\times s_{x}$$

$${}^{w}C^{s}_{x}={}^{w}C_{x}\times s_{x}$$

$${}^{r}C^{s}_{x}={}^{r}C_{x}\times s_{x}$$

$$\overline{|65}{}^{d}M^{ls}_{x} = {}^{d}C^{s}_{x} + {}^{d}C^{s}_{x+1} + \ldots + {}^{d}C^{s}_{64}$$

$$\overline{|65}{}^{w}M^{ls}_{x} = {}^{w}C^{s}_{x} + {}^{w}C^{s}_{x+1} + \ldots + {}^{w}C^{s}_{64}$$

$$\overline{|65}{}^{r}M^{ls}_{x} = {}^{r}C^{s}_{x} + {}^{r}C^{s}_{x+1} + \ldots + {}^{r}C^{s}_{64}$$

so that

$$\left.\begin{array}{l} \dfrac{{}^{d}M^{ls}_{x}}{D^{s}_{x}} = \\[2ex] \dfrac{{}^{w}M^{ls}_{x}}{D^{s}_{x}} = \\[2ex] \dfrac{{}^{r}M^{ls}_{x}}{D^{s}_{x}} = \end{array}\right\} \left\{\begin{array}{l}\text{The present value, at age } x\text{, of} \\ \text{the assurance of the return} \\ \text{of the last year's full salary,} \\ \text{equated to a salary of 1 at} \\ \text{age } x\text{, receivable in the} \\ \text{year of} \end{array}\right\} \begin{array}{c} \text{death.} \\[2ex] \text{withdrawal.} \\[2ex] \text{early retirement.} \end{array}$$

Pension on Early Retirement.

$${}^{ra}C^{ls}_{x} = {}^{r}C^{s}_{x} \times (a'_{x+\cdot 5} + \cdot 5) = {}^{r}C_{x} \times s_{x} \times (a'_{x+\cdot 5} + \cdot 5)$$

$$\overline{|65}{}^{ra}M^{ls}_{x} = {}^{ra}C^{ls}_{x} + {}^{ra}C^{ls}_{x+1} + \ldots + {}^{ra}C^{ls}_{64}$$

$$\overline{|65}{}^{ra}R^{ls}_{x} = \overline{|65}{}^{ra}M^{ls}_{x} + \overline{|65}{}^{ra}M^{ls}_{x+1} + \ldots + \overline{|65}{}^{ra}M^{ls}_{64}$$

so that

$\dfrac{{}^{ra}M^{ls}_{x}}{D_{x}}$ = the present value, at age x, of the assurance of an annuity at the end of the year of early retirement, of the amount of the full last year's salary receivable in the year of retirement.

$\dfrac{{}^{ra}M^{ls}_{x}}{D^{s}_{x}}$ = the same as above equated to 1 of salary at age x.

$\dfrac{{}^{ra}R^{ls}_{x}}{D_{x}}$ = the present value, at age x, of the assurance of an annuity at the end of the year of early retirement, of the amount of the full last year's salary receivable in the year of retirement, multiplied by the number of years between age x and the age at the end of the year of retirement; that is, a pension of the last salary for every year of service.

$\dfrac{{}^{ra}R^{ls}_{x}}{D^{s}_{x}}$ = the same, equated to salary of 1 at age x.

APPENDIX II.

LIST OF CORRECTIONS AND ALTERATIONS.

Vol. xxxvi.

Page 220.—In the foot-note, for ${}_{64}^{d}\mathbb{N}_x$ read ${}_{\overline{65|}}^{d}\mathbb{N}_x$.

Page 221.—In all the formulas in Problem IIIA, for a'_{x+1} read $(a'_{x+\cdot5}+\cdot5)$.

Page 223.—In the first formula in Problem VIA, for $-\text{P}({}^{d}\text{R}_x+{}^{w}\text{R}_x)$ read $+\text{P}({}^{d}\text{R}_x+{}^{w}\text{R}_x)$.

Page 224.—In the second line in Problem IXA, for "benefits in Problem VIIA" read "benefits in Problem VIIIA."

Page 227.—In Problem XIIIA, 11th line, for ${}^{w}\text{R}_r$ read ${}^{w}\text{R}_x$; and in all the formulas for $({}^{r}\mathbb{N}_x-{}^{r}\mathbb{N}_{x+15}-15{}^{r}\text{D}_{x+15})$ read $({}^{r}\mathbb{N}_x-{}^{r}\mathbb{N}_{x+16}-a_{(16)}{}^{r}\text{D}_{x+15})$.

Page 233.—In the denominator of the two formulas at the bottom of the page, for ${}^{w}\text{D}_x^s$ read D_x^s.

Page 235.—In the formula at bottom of page, for—
$({}^{d}\text{M}_x^s+{}^{w}\text{M}_x^s+{}^{r}\text{M}_x^s)$ read $({}^{d}\text{M}_x^{ls}+{}^{w}\text{M}_x^{ls}+{}^{r}\text{M}_x^{ls})$ and refer to vol. xxxvii, page 193.

Pages 238–240.—In Problems XB and XIB, for a'_{x+1} read $(a'_{x+\cdot5}+\cdot5)$ throughout.

Page 241.—In 4 places; for $({}^{r}\mathbb{N}_x^s-{}^{r}\mathbb{N}_{x+15}^s-as_{\overline{15|}}{}^{r}\text{D}_{x+15})$ read $({}^{r}\mathbb{N}_x^s-{}^{r}\mathbb{N}_{x+16}^s-(as)_{(16)}{}^{r}\text{D}_{x+15})$.

Page 245.—Cancel "Section VI" and refer to vol. xxxvii, page 195.

Page 248.—Cancel whole page and refer to vol. xxxvii, page 196.

Page 269.—Cancel whole page and refer to vol. xxxvii, page 207.

Page 274.—Cancel whole page and refer to vol. xxxvii, page 207.

Page 275.—Heading to column 7, for ${}^{w}\text{R}^s\div\text{D}$ read ${}^{w}\text{R}_x^s\div\text{D}_x^s$.

Page 276.—Cancel columns 6, 7, and 8, and refer to vol. xxxvii, page 208.