Stop-loss and Investment Returns

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Abstract
This paper investigates how stop-losses affect the returns distribution. Firstly, we establish the stop-loss returns under the Brownian Random Walk with the assumption of drift. Secondly, we consider the profitability of a simple stop-loss strategy when there are dependencies in financial prices. Finally we review the profitability of stop-loss strategies in varied financial markets. We conclude by suggesting further research. Extensions to alternative money management techniques and take-profit strategies are discussed.

Many investors do not buy and hold assets over very long periods of time. Even the longest term players, pension funds, may not earn the buy and hold rate of return. They operate under constraints of various nature: legislative, demographic, ..., which force them to revise their portfolios constantly. Private investors either trade the markets actively or adopt some kind of asset allocation strategies.

One of the most widely adopted money management techniques is known as ‘the stop-loss strategy’. Stop-losses can be either a deliberate choice to protect past gains, further losses or a constraint imposed by senior management or legislators. For most independent traders, stop-loss orders are placed on purpose usually at a chart point, to avoid catastrophic losses. Many large houses have “stop-loss” limits which establish the maximum allowable losses of trading positions. Once these limits are reached, the position must be liquidated. Typically stop-loss limits are retrospective and cover cumulative losses for a day, month or year. Another example of an implicit stop-loss is a guaranteed product. Once the asset has lost more than the risk-free rate over the period, trading stops.

Those trading strategies, leading investors to liquidate portfolios exhibiting heavy losses, seem to have played a major role during the 1929 and 1987 crashes (Schiller, 1989), as well as during other periods of speculative attacks in foreign exchange markets (Krugman and Miller, 1993). Bensaid and de Bandt (1998) estimate that around 20 per cent of firms follow that kind of strategy in the French Treasury bond futures market. The same authors provide numerous explanations regarding the motive of stop-loss strategies. They suggest that traders may be psychologically more reluctant to realize losses than gains, hence it is recommended to impose stop-loss orders on them. Then they introduce a model with moral hazard based on a conflict of interest between an investment firm and its traders.

Our purpose is somewhat different. Using both well known theoretical results and recent empirical observations, we describe situations where the use of stop-losses could be counterproductive and others where it may happen to be useful. We concentrate on the statistical distribution of path-dependent active
strategies. Our study does not take into consideration slippage or transaction costs. Consequently, real trading might have triggered significantly different results. Nevertheless, establishing the distribution of returns generated by stop-loss (take-profit) strategies can be seen as a test of the random walk hypothesis. The characteristics of such an approach over conventional statistical tests is that it involves the joint distribution of Low (High), Open and Close over a given horizon. More precisely, this paper is organised as follows. Section 1 recalls the stop-loss returns under the Brownian Random Walk with the assumption of drift. Section 2 investigates the usefulness of conditional stops when there are price dependencies. Finally Section 3 reviews the profitability of both stop-loss and take-profit strategies in varied financial markets. Extensions to alternative money management techniques are discussed. We conclude by suggesting further research.

1) Random Walk and well-known theoretical results
Let's note X the underlying asset returns between time 0 and t. We assume that this follows a Wiener process with mean \( \mu \) and standard deviation \( \sigma \). \( X \sim N(\mu, \sigma\sqrt{t}) \). We will note H and L the corresponding high and low during the period \([0;t]\). Let's now assume that a percentage stop-loss \( a < 0 \), is put into place. The resulting pay-off is:

\[
S = \begin{cases} 
  a & \text{if } L < a \\
  X & \text{if } L > a
\end{cases}
\]

It can be shown (See Taleb\(^1\) and Appendix for sketched proofs) that:

\[
\text{Prob}[S = a] = \Phi\left(\frac{a - \mu}{\sigma\sqrt{t}}\right) + \text{Exp}[2\frac{a\mu}{\sigma^2}\frac{a + \mu}{\sigma\sqrt{t}}] \Phi\left(\frac{a + \mu}{\sigma\sqrt{t}}\right) \quad (1)
\]

\[
\text{Prob}[S < s] = \Phi\left(\frac{s - \mu}{\sigma\sqrt{t}}\right) + \text{Exp}[2\frac{a\mu}{\sigma^2}\frac{s + 2a + \mu}{\sigma\sqrt{t}}] \Phi\left(\frac{s + 2a + \mu}{\sigma\sqrt{t}}\right) \quad (2)
\]

Figure 1 illustrates the distribution of the cash flow assuming an asset mean of 10%, a stop-loss of –5% for three different volatility levels 5%, 10% and 20%. The probability of hitting the stop-loss is 1.68% for an asset volatility of 5% but 69.29% for an asset volatility of 20%. The expected returns are respectively 8.98% and 6.83%. Stop-losses affect many more assets exhibiting a low return to volatility ratio. Not only is the probability of hitting the stop-loss much higher but the expected gains are considerably lower. The results are not surprising since under the random walk hypothesis, the optimal strategy is buy and hold. As a consequence, path-dependent strategies such as the stop-loss rule, can only diminish earnings. In this situation, stop-losses are like premiums on an insurance policy and should be considered as a cost of doing business.

\(^1\) http://pw1.netcom.com/~ntaleb/stop.htm
Fig 1: Distribution of Cash Flow S with yearly stop-loss of −5% and annualised asset return of 10%

Similarly, we can establish the pay-off corresponding to a take-profit strategy (often called a lock-in strategy). Let’s now assume that a percentage take-profit $b > 0$ is put into place. The resulting cash flow is:

$$P = \begin{cases} 
X & \text{if } H < b \\
\ b & \text{if } H > b
\end{cases}$$

$$\Pr [P = b] = \Phi \left( \frac{\mu - b}{\sigma \sqrt{t}} \right) + \exp \left[ \frac{bu}{\sigma^2} \right] \Phi \left( \frac{-b - \mu}{\sigma \sqrt{t}} \right)$$

$$\Pr [P < p] = \Phi \left( \frac{b - \mu}{\sigma \sqrt{t}} \right) - \exp \left[ \frac{bu}{\sigma^2} \right] \Phi \left( \frac{p - 2b - \mu}{\sigma \sqrt{t}} \right)$$ with $p < b$,

In our example as in Dyvig (1988), the damage done by the take-profit strategy is far less costly than the damage done by the stop-loss strategy (Figure 2). Nonetheless, the cost of following a take-profit strategy is substantial and should not be ignored by practitioners.
A striking question is to measure the effect of frequency on stop-loss. Most risk managers affect stop-losses to their traders on a daily, weekly, monthly and/or yearly basis. However very little is known on the equivalence between these stops. Most often these stops are determined arbitrarily and revised as the traders make/lose money. Using the previous theoretical results, one may attempt to determine equivalences between stop-losses as a function of frequency. It remains to define the criteria to match when moving from one frequency to the next. We investigate here the most natural case of keeping constant the expected returns per unit of time.

Figure 3 assumes an annualised asset mean of 20%, volatility of 10% and yearly stop-loss of -5%. Integrating equations (1) and (2), this gives us an expected return of 17.95%. Now instead of using a yearly stop-loss of -5%, we could have implemented a daily stop-loss of -0.70% and achieves exactly the same expected return over a year of 252 days. However if the asset mean is only 5%, the daily stop-loss should be fixed at -0.40%.
2) Serial Dependencies and Conditional Stop-losses

The previous sections assumed that markets follow a (normal) random walk and that stop-losses are put into place to satisfy external constraints or risk preferences. The objective of the stop-loss has been there simply to reshape the distribution of returns and “forbids” extreme downside. The cost of the insurance can be measured as the loss in terms of expected returns.

In practise, dealers tend to believe that trading can be enhanced by systematically applying exit conditions. Most frequently, the stop would be placed close to a chart point or some level that chartists consider critical for the market direction. When chartists lose faith in their position, they want to be stopped. Indeed they believe that a down move is likely to follow an existing down move. By putting a conditional stop they hope to minimize their losses and maximize their gains. In other words, speculators refute the random walk hypothesis and believe strongly in trends and serial correlation in a market.

Establishing the exact distribution of stop-loss strategies in presence of autocorrelations is now difficult. Very little is known on the corresponding joint distribution of Low, Open and Close prices. Rather than proceeding to Monte-Carlo simulations, we prefer to slightly simplify our stop-loss strategy and work out its exact expected return assuming that the underlying markets follow an auto-regressive process of length one.

Our starting point is James and Thomas (2000). To test the usefulness of stop-loss strategies, they simulate a situation where trading positions are taken randomly. First they look at how the strategy with no stop loss would work. Every day they toss a coin. If it lands heads up, they buy the asset and if it tails up they sell it. This position is then held for N days and closed out at the end of the Nth day. They then introduce a stop-loss by adding a simple exit condition which forces the closure of any trade with a
negative marked to market value at the end of each day. If the market is random, then adding the stop-loss should make no difference at all, and the returns should still average to about zero. If however small losses lead to larger ones, then the stop-loss should prevent many of the large losses occurring and average returns should increase. Here we demonstrate that point by assuming that the market follows an auto-regressive process of length one and the holding period $N=2$. More specifically we suppose that the underlying market returns obey to the equation $X_t = \alpha X_{t-1} + \varepsilon_t$ where $\varepsilon_t$ is a white noise. Daily returns $X_t$ follow a normal distribution with zero mean and standard deviation $\sigma$. Let’s note $R$ and $Sc$ the strategies holding the asset over two days respectively without and with stop-loss. It is straightforward to show (See Appendix) that the expected returns are given by: $E(R)=0$ and $E(Sc) = \alpha \sigma \sqrt{\frac{2}{\pi}}$.

In fact the stop-loss strategy is nothing else than a simple trend-following rule which buys the asset after an up move and sells it after a down move. The only difference here is that one day is lost by inheriting a random position. The first holding day has zero expected value. Only the second day acts conditionally on past moves and therefore shows non-zero expected value. In the presence of trends (positive dependencies) and no drift (initial and final price identical), a conditional stop-loss strategy will outperform buy and hold. Similarly if the market is range-trading (negative dependencies), a conditional take-profit strategy will outperform buy and hold.

A more interesting case involves the presence of both drifts and autocorrelation. That is $X_t - \mu = \alpha (X_{t-1} - \mu) + \varepsilon_t$ where $\varepsilon_t$ is a white noise and daily return $X_t$ follows a normal distribution with mean $\mu$ and standard deviation $\sigma$. We no longer consider initial random positions but concentrate on the conditional stop-loss strategy $Sc$ selling the asset after a negative daily move and buying it otherwise. Acar (1998) shows that this has expected return: $E(Sc) = \sigma \alpha \sqrt{\frac{2}{\pi}} \exp(-0.5 (\frac{\mu}{\sigma})^2) + \mu (1 - 2\Phi[-\mu/\sigma])$.

Let’s quantify $E(Sc)$ when $\alpha = 0.05$, $\mu = 0.0397\%$ (10% annualised), $\sigma = 0.6299\%$ (10% annualised), then $E(Sc)=0.027\%$ (6.82% annualised).

In this case, the stop-loss strategy still underperforms buy and hold. In order to outperform the drift we need to satisfy the equation: $E(Sc) > \mu \iff \alpha > \sqrt{2\pi \frac{\mu}{\sigma}} \Phi[-\mu/\sigma] \exp(-0.5 (\frac{\mu}{\sigma})^2)$ \hspace{1cm} (3)

For $\mu = 0.0397\%$ and $\sigma = 0.6299\%$, this corresponds to $\alpha > 0.075$

Figure 4 highlights the required autocorrelation $\alpha$ given by (3) for the stop-loss strategy to beat buy and hold as a function of the annualised return to risk ratio $r = \sqrt{252 \frac{\mu}{\sigma}}$. Given the magnitude of daily autocorrelation observed in financial markets (mostly below 0.04), the stop-loss strategy should only be applied, if at all, to assets exhibiting low return to risk ratios and positive dependencies. The expected return may not be large enough to cover transaction costs, but such stop-loss strategies may still be of use to market makers and others who are “given” random positions and must decide how to handle them (James and Thomas, 2000).
3) **Empirical results in Financial Markets**

The conditional stop-loss strategies are in fact trend-following rules. It is therefore not surprising that both approaches work well in the foreign exchange markets (See for instance James and Thomas, 2000; Levich and Thomas, 1994; LeBaron, 1991; Taylor, 1994). The reader is referred to Silber (1994) for pan-market testing of the profitability of trend-following strategies. The general results tend to confirm Figure 4. Trend-following rules outperform Buy & Hold when the assets exhibit low return to risk ratios and positive dependencies (i.e. foreign exchange markets). Buy & Hold outperforms trend-following rules when the assets exhibit high return to risk ratios and little dependencies (i.e. equities markets). Here we focus our attention on absolute stop-loss strategies as described in Section 1. The latter have been rather untested in past literature. To assess empirically how stop-losses affect the distribution of returns, we need to restrict ourselves to the intra-day session during which markets trade continuously. Each morning we buy the asset at the opening price and square the position on the close of the market unless the stop-loss has been triggered. In the latter case, we assume that there is no slippage and that the fill has been done at the stop-loss price. The stop is left in percentage terms at one daily standard deviation of the intra-day price move. Further tests have been carried out at half and twice the daily standard deviation. They are available from the authors and are not reported here since they do not add much to this discussion. We have assumed prior knowledge of the daily mean and standard deviation of the logarithmic returns. Therefore the stop-loss strategy has no predictive power but is rather a test of departure from the normal random walk assumption.
Similarly a take-profit strategy has been tested at one daily standard deviation of the intra-day price move. Whereas the stop-loss strategy is a random walk test of the joint distribution of the open, close and low prices, the take-profit strategy is a random walk test of the joint distribution of the open, close and high prices. This may therefore bring some extra-information.

Our dataset covers stock indices, bonds and foreign exchange markets. More precisely we consider the S&P 500, T-Bonds and Yen Futures contracts from 26 December 1984 to 12 May 2000. There is no roll-over issue since our study looks only at the intra-day variation of the first available contract. Summary statistics are provided in Table 1. We checked that our stop-loss and take-profit strategies would not be affected by the presence of one or two very large outliers. In particular the month of October 1987 incurred both a crash (-26.54%) and a sharp recovery (+20.84%). In our analysis, the target price is placed one standard deviation away from the opening. Stop-losses and take-profits would have been triggered on both days therefore not affecting significantly the comparison with buy and hold over the total sample. Finally, one must remark that the freefall of the Dollar against the Yen during the month of October 1998 is not determinant in our analysis. Actually, this move happened mostly during the Asian and early European periods which are not included in our dataset which only reports the American session.

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P 500</th>
<th>[S&amp;P 500*]</th>
<th>T-Bonds</th>
<th>Yen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annualised Return</td>
<td>10.62%</td>
<td>[10.99%]</td>
<td>5.16%</td>
<td>0.73%</td>
</tr>
<tr>
<td>Daily Standard Deviation</td>
<td>1.10%</td>
<td>[0.96%]</td>
<td>0.60%</td>
<td>0.54%</td>
</tr>
<tr>
<td>Annualised Standard Deviation</td>
<td>17.51%</td>
<td>[15.27%]</td>
<td>9.51%</td>
<td>8.62%</td>
</tr>
<tr>
<td>T-statistic</td>
<td>5.24</td>
<td>[6.23]</td>
<td>4.69</td>
<td>0.74</td>
</tr>
<tr>
<td>Minimum</td>
<td>-26.54%</td>
<td>[-8.49%]</td>
<td>-2.82%</td>
<td>-4.16%</td>
</tr>
<tr>
<td>Maximum</td>
<td>20.84%</td>
<td>[7.35%]</td>
<td>3.50%</td>
<td>5.01%</td>
</tr>
<tr>
<td>Skewness</td>
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<td>[-0.56]</td>
<td>-0.13</td>
<td>0.49</td>
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<tr>
<td>Kurtosis</td>
<td>123.54</td>
<td>[9.67]</td>
<td>2.12</td>
<td>8.42</td>
</tr>
</tbody>
</table>

* excluding 19 and 22 October 1987

Figure 5 shows that the S&P 500 triggered a stop-loss of –1.10% (one standard deviation) in 18% of the cases. This is far less than what we would have expected from the normal assumption, 31%. Deviations could have been predicted from Table 1 which indeed indicates strong departure from normality. However the relationship between kurtosis (fat tails in the underlying returns distribution) and the profitability of stop-loss strategies is unclear. The focus on open to close returns is not sufficient to assess path-dependency and the relationships with high and low prices. The distributions of stop-loss returns exhibits a significantly different shape than the normal assumption would suggest. Nevertheless, the average profit generated by such a strategy is not far from its expected return.
Figure 6 demonstrates that take-profits are counter-productive since they are not triggered often enough and do not prevent very large losses which actually occur ten times more often than anticipated.

Given its mean and volatility, the T-Bonds contract seems to be more risky than the S&P contract. Indeed Figure 7 illustrates that stops at one standard deviation are triggered much more often with the T-bonds than with S&P (24% rather than 18%) whereas the normal assumption would have suggested a similar
number of occurrence (31%). Overall stop-losses have outperformed their expectations on the T-Bonds contract and surprisingly beaten buy and hold.

**Fig 7:** Intra-day T-Bonds; 26-Dec-1984 to 12-May-2000 with Stop-Loss of -0.60%

On the other hand, take-profit strategies have been performing poorly and below their expectation (Figure 8). Not letting run very large but rare profits during the day would have been costly in practise. Similarly to the S&P 500, large and unusual downside moves are not avoided.

**Fig 8:** Intra-day T-Bonds; 26-Dec-1984 to 12-May-2000 with Take-Profit of 0.60%
Figures 9 and 10 might indicate the presence of trends in the Yen Futures contract. The underlying market exhibits very little drift and as a consequence both stop-loss and take-profit strategies should not generate non-zero returns. In reality, stop-loss strategies are beneficial whereas take-profit strategies are detrimental. As for the T-bonds contract, these results could be consistent with the presence of positive serial dependencies.

**Fig 9:** Intra-day Yen; 26-Dec-1984 to 12-May-2000 with Stop-Loss of -0.54%

**Fig 10:** Intra-day Yen; 26-Dec-1984 to 12-May-2000 with Take-Profit of 0.54%
4) Conclusion and avenues for further research

Under the random walk assumption, stop-loss and take profits strategies are a cost for doing business. Such rules cannot beat buy and hold and can only be justified by specific risk preferences. When the markets trend, stop-loss strategies will outperform buy and hold whereas take-profit will underperform. Empirical observations tend to favour the trend hypothesis for both the T-bonds and Yen contracts whereas the results are less strong for the S&P 500. However, whereas take profit strategies may be a luxury, stop loss strategies are entirely necessary. An eclectic mix of both strategies is the trailing stop which, under certain conditions, may improve on buy and hold. The emphasis here lies in protecting gains and limiting losses close to zero as the length of time of holding a position increases, with the condition that one cancels the other. Further research is clearly needed. In addition we need to ascertain the confidence interval attached to the total stop-loss returns for given sample sizes. Whereas analytical results may be difficult to establish, one could use Monte-Carlo simulations. In the presence of fat tails, one may prefer using the Bootstrap approach similarly to Levich and Thomas (1994) to ascertain the statistical significance of stop-loss returns. However in both cases, this will turn out to be data intensive. Access to ticks by ticks data will be required to generate consistent Highs and Lows. As a consequence the number of runs will need to be phenomenal.
Appendix

Stop-loss

For the purpose of convenience and without loss of generality, all the forthcoming results involve normalized variables. Let's note X the underlying asset returns between time 0 and t. We assume that this follows a Wiener process with mean \( r = \frac{\mu}{\sigma} \) and standard deviation 1. \( X(r) \sim N(r,1) \). We will note H(r) and L(r) the corresponding high and low during the period[0;t]. Let's now assume that a stop-loss \( a < 0 \), is put into place. The resulting pay-off is:

\[
S = \begin{cases} \frac{a}{X} & \text{if } L < a \\ X & \text{if } L > a \end{cases}
\]

To derive the distribution of S, we need to recall the joint distribution of High H and Close X. This is a well known result which can be found for instance in Kwok (1998, p 266). We have in particular:

\[
\begin{align*}
\text{Prob}[H(r) < m] &= \Phi(m - r) - \text{Exp}[2mr] \Phi(-m - r) \text{ with } m > 0, \\
\text{Prob}[X(r) < x, H(r) < m] &= \Phi(x - r) - \text{Exp}[2mr] \Phi(x - 2m - r)
\end{align*}
\]

The joint density function of \( (X,H) \) is

\[
g(x,m) = \frac{\frac{2}{\sqrt{2\pi}}\exp\left(-\frac{(2m-x)^2}{2}\right)}{\Phi\left(-\frac{1}{2}\right)}, \ x < m, m > 0
\]

Then, we have:

\[
\begin{align*}
\text{Prob}[S = a] &= \text{Prob}[L(r) < a] = \text{Prob}[H(-r) > -a] = \Phi(a - r) + \text{Exp}[2ar] \Phi(a + r) \\
\text{with } s > a, \text{ Prob}[S > s] &= \text{Prob}[X(r) > s, L(r) > a] = \text{Prob}[X(-r) < -s, H(-r) < -a] \\
\text{Prob}[S < s] &= 1 - \text{Prob}[X(-r) < -s, H(-r) < -a] = \Phi(s - r) + \text{Exp}[2ar] \Phi(-s + 2a + r)
\end{align*}
\]

Take-Profit

Let's now assume that a take-profit \( b \) is put into place. The resulting pay-off is:

\[
P = \begin{cases} X & \text{if } H < b \\ b & \text{if } H > b \end{cases} \text{ with } b > 0
\]

\[
\begin{align*}
\text{Prob}[P = b] &= \text{Prob}[H(r) > b] = 1 - \text{Prob}[H(r) < b] = \Phi(r - b) + \text{Exp}[2br] \Phi(-b - r) \\
\text{with } p < b, \text{ Prob}[P < p] &= \text{Prob}[X(r) < p, H(r) < b] = \Phi(p - r) - \text{Exp}[2br] \Phi(p - 2b - r)
\end{align*}
\]

Conditional Stop-loss

Let’s note \( B_t \) the random position inherited at time t. \( B_t \) is established by the toss of a coin

\[
B_t = \begin{cases} -1 & \text{with probability } 50\% \\ 1 & \text{with probability } 50\% \end{cases}, \ X_t \text{ the buy and hold return from time } t-1 \text{ to } t. \text{ We suppose that the market } X_t = \alpha X_{t-1} + \epsilon_t \text{ where } \epsilon_t \text{ is a white noise and } X_t \text{ follows a normal distribution with zero mean and standard deviation } \sigma.
\]
The strategy without stop-loss, $R$, consists in holding the inherited position for two successive days. That is: $R = B_t(X_{i+1} + X_{i+2})$. Since $B_t$ is established by the toss of a coin which is independent on past moves, the strategy has zero expected value. $E(R) = E(B_t(X_{i+1} + X_{i+2})) = E(B_t)E(X_{i+1} + X_{i+2}) = 0$.

The strategy with stop-loss, $Sc$, closes the position after the first negative market to market value. That is:

$$Sc = \begin{cases} 0 & \text{if } B_t X_{i+1} \leq 0 \\ B_t X_{i+2} & \text{if } B_t X_{i+1} > 0 \end{cases}$$

Then $E(Sc) = E(B_t X_{i+1}) + E(B_t X_{i+2}, B_t X_{i+1} > 0) = 0 + E(X_{i+2}, X_{i+1} > 0) + E(-X_{i+2}, -X_{i+1} > 0)$.

For symmetry reasons $E(Sc) = 2 E(X_{i+2}, X_{i+1} > 0) = \frac{\sqrt{2}}{\sqrt{\pi}} \sigma \alpha$ given the multinormal moments which can be found in Kamat (1953).
Bibliography


