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# THE VALUATION OF WITH-PROFITS BUSINESS AND THE ESTATE

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## 1. INTRODUCTION

The subject of valuation is one of the most important, and at the same time most difficult, of all the problems faced by actuaries.

The traditional approach to the subject has centred on discussion of the relative merits of the net premium and bonus reserve methods of valuation, and on the inherent conflict between valuation as a means of demonstrating solvency and valuation as a tool for ensuring a smooth emergence of surplus in accordance with the need to declare uniform reversionary bonuses. This approach is covered in the well-known papers by Redington (2) and Skerman (3) (4) and forms a significant part of the course of reading for examination subject 8 (formerly B2).

Bonus reserve methods are, typically, 'active' methods of valuation in which all relevant parameters, including expenses and future bonuses, are given values close to their expected values. The premium valued is the office premium, and assets are usually valued at market value or by discounting expected investment income. Because actual conditions are always changing, an active valuation method tends to lead to frequent changes of basis and this introduces an element of volatility to the results which can, in consequence, be difficult to interpret. Nevertheless, because of the emphasis on using realistic values for all parameters, it is commonly believed that bonus reserve methods are particularly appropriate for investigations into solvency, especially when external conditions are changing rapidly, and in determining appraisal values.

The net premium method, as traditionally applied to with-profits business, is a 'passive' method in which the basis reflects the premium bases of the business in force and changes in the valuation basis occur gradually. Apart, possibly, from a Zillmer adjustment, no explicit allowance is made for expenses or for future bonuses: it is assumed these will be met from the excess of the office premium over the net premium which is valued. The emergence of surplus may be delayed by making a reduction in the valuation rate of interest for with-profits policies. This increases the security of the office and helps to ensure that surplus emerges in a manner suitable for the declaration of reversionary bonuses. Assets are usually valued at book value, although this need not corespond to original cost as assets may be written down or up. The effect of infrequent changes in the valuation basis together with the use of a net premium formula is that successive valuation results tend to exhibit a stable trend, a feature which is desirable if stable uniform reversionary bonuses are to be declared.

These considerations may suggest that the conflict between valuation to investigate solvency and valuation to determine surplus is resolved by applying bonus reserve methods to solvency investigations and net premium methods to the determination of surplus. Needless to say, this view is too simple. Indeed, the current reality is that Life Offices are required to demonstrate solvency to the DTI (and to the public) using a net premium method of valuation for liabilities, or some other method at least as strong, together with a market valuation of assets, while investment in equities and property and the widespread use of terminal bonuses to distribute capital profits has resulted in the net premium method being incapable of determining an equitable value for distributable surplus.

This paper attempts to examine some of the issues which confront Life Office actuaries when valuing with-profits business. It also describes an approach to valuation which, it is hoped, will provide some insight into the development of an office transacting such business. This approach, which is illustrated in Sections 5 to 9, is a bonus reserve method which is primarily intended for use in internal investigations. Its main element is a cashflow projection which is used to determine future profits and losses incurred by each policy, where profit and loss are taken to represent transfers to and from the Estate respectively. Cashflow projection methods, or profit-testing as they are more commonly known, are widely used for both premium-rating and valuation and I do not claim any originality in applying such methods to the valuation of with-profits business, although the details of my approach may differ from those of others.

The main application of these methods in this paper is to the investigation of the circumstances under which any given level of maturity values for with-profits endowment business can be maintained in the future. In the numerical examples given, this level is set at about the average curent level for the market as a whole. This could, therefore, represent either an average office investigating its ability to maintain current bonuses or, perhaps, one of the better-performing offices investigating its ability to pay a lower level of bonuses than at present. In practice, an office using such methods would not restrict itself to one level of maturity values but would investigate a wider range of values.

Although it is obviously a very important topic, I do not discuss the problem of determining the appropriate current level of maturity values in the first place. This should be done by estimating the accumulated value of policyholders' net premiums, having regard to the actual investment performance and, perhaps, to miscellaneous items of profit or loss such as mortality, expenses and surrenders. In other words, retrospective methods are to be preferred in fixing actual maturity values. Nevertheless, prospective methods, whether net premium or bonus reserve, have traditionally been very important in determining the surplus arising each year and in analysing the sources of that surplus, and methods such as those discussed in this paper undoubtedly have a role to play in controlling an office's with-profits business from one year to the next. Indeed, whatever level of bonuses may be indicated by retrospective calculations as being appropriate, an

office will need to investigate its continuing ability to pay these bonuses to both existing and new policyholders in the future.

## 2. OBJECTIVES

A life office undertakes periodical valuations in order to assess its financial position and to monitor the progress it is making towards its overall financial objectives. Before discussing the details of the valuation method to be used, it is of interest to consider the nature of these financial objectives. If the office is a mutual office, its financial objectives may include any or all of the following:

(i) to ensure the solvency of the office;
(ii) to provide the best possible return to existing policyholders;
(iii) to maximize growth in new business;
(iv) to maximize the amount of capital in the business and obtain an adequate rate of return on that capital.

If the office is proprietary, it will also wish to provide the best possible return to its shareholders, although this can probably be done by achieving the above objectives. For most of this paper, I will assume that the office under consideration is mutual. However, the problems faced by mutual offices and proprietary offices are similar and I hope that the paper will be of general interest.

Although they are obviously very important, I have not included the maximization of investment returns or the minimization of expenses in the list of objectives as these may be regarded as means by which the above objectives are to be achieved. Most offices will also have other objectives, for example providing the best possible level of service to policyholders for a given level of expenses, so the above list is not intended to be exhaustive.

It may not be possible for an office to achieve all the objectives listed above. In particular, there is a conflict between (ii) and (iii), that is, between existing and new policyholders. For most offices, it is no doubt true that (ii) can be achieved by ceasing to write new business and distributing the entire Estate to the existing policyholders. This, however, would conflict with (iii) and I take it to be axiomatic that mutual offices have a duty to write new business on the best possible terms and in sufficient volumes to ensure the continuation of the business as an effective enterprise. That, after all, was the reason for setting up the office in the first place.

Some people may find it curious that (iv) has been included as an objective for mutual offices since it tends to conflict with (ii), although they would presumably expect proprietary offices to maximize the capital in the business as this is likely to be in the best interest of the shareholders. If, however, it is accepted that a mutual office should try to maximize new business growth, it will require capital to finance the resulting new business strain. Furthermore, increasing the office's capital will tend to increase its security. It follows, therefore, that effective capital management is an important aim for mutual, as well as proprietary, offices.

For a mutual office, the capital employed in the business, or the Estate as it is more commonly known, will have been built up by contributions from previous generations of policyholders. In effect, those policyholders leaving the fund will, in aggregate, have received less than the full value (including investment return) of their contributions. In other words, the aggregate payments made on deaths, surrenders and maturities will have been less than the aggregate asset shares of the respective policies.

These considerations suggest that it would be natural to measure the Estate by accumulating the contributions made by previous policyholders, that is, by a retrospective calculation. Unfortunately, the information required to undertake such a calculation is not likely to be available. Furthermore, reserves must be set up for existing and new policyholders and these will be calculated on a prospective basis. As a result, the Estate is generally taken to be the excess of the value of the office's assets over the value of its liabilities calculated prospectively.

In his 1952 paper, Redington defined the Estate as the sum of the margins in the assets, the margins in the liabilities and the central reserves, including shareholder's funds, if any. With this definition, the Estate is calculated by measuring the excess of assets over liabilities using actuarial bases which contain no margins. This is a simple enough definition and one which accords well with intuition, but actuarial bases are subjective and the assertion that any particular basis contains no margins is probably meaningless.

It is therefore more common to accept that the Estate is not a unique concept but is a function of the methods and bases used to determine the values of assets and liabilities. As a multivariate function, the Estate will be negative for certain values of its arguments, but the office will be regarded as solvent if these values correspond to conditions which are unlikely to occur, whether because of their inherent improbability or because the office can take steps, such as reducing bonuses, to prevent their occurrence.

As mentioned above, an office's Estate will have been built up from contributions made by previous generations of policyholders. If the office is to continue to grow, current and future generations will also be required to contribute in their turn and it should be an objective of the office to ensure that this happens. Under present conditions, however, it is likely that many offices either are or will in future be supporting the bonuses paid to existing policyholders by using their Estates to provide, in effect, a subsidy. This may be justified as a short-term necessity for those offices which, as a result of past underdeclarations, have acquired Estates which are larger than necessary, but it has obvious dangers if allowed to continue into the longer term. In these circumstances, offices need to be able to measure both the Estate and the strain on it caused by writing new business on increasingly competitive terms, and they also need to be able to assess the long-term consequences of present conditions for the development of the Estate. These matters are dealt with in some detail in Sections 9 and 10 of this paper.

## 3. SOME CURRENT ASPECTS OF WITH-PROFITS LIFE ASSURANCE BUSINESS

Any valuation method which is to be used to assist an office transacting with-profits business in achieving its financial objectives will need to take account of the following aspects of with-profits business which are important in current conditions:

(i) most with-profits business is endowment business which is primarily bought as a means of making regular savings; personal pensions business, where the policy funds for cash to provide a pension at retirement, is effectively a form of endowment business and may therefore be treated similarly;

(ii) a typical with-profits office invests the majority of its funds in equity-type investments (company shares or land);

(iii) because these investments are volatile, a significant part of the final maturity value is in the form of terminal bonuses which, unlike declared reversionary bonuses, are not guaranteed;

(iv) the role of reversionary bonuses, though not unimportant, has been diminished by (ii) and (iii) and by the strong rise of recent years in equity markets which has resulted in some very large terminal bonus levels;

(v) the statutory minimum valuation basis, being net premium, makes no allowance for future reversionary bonuses or for continuation of terminal bonuses and this, in conjunction with the valuation of assets at market value, gives an exaggerated view of an office's Estate;

(vi) increasing competition for savings has contributed to increased maturity values with the result that many offices are probably subsidizing existing business from interest earned on the Estate.

The combination of sustained high interest rates and strong equity markets, high maturity values, competition for new business and the existence of fairly large Estates has led to a situation in which many offices recognize that bonus levels may have to be reduced at some stage in the future but hope to defer that event for as long as possible by subsidy from the Estate.

This state of affairs appears to conflict with one of the basic concepts of U.K. actuarial practice, namely the concept of equity. Strict equity requires not only equivalence of treatment of different classes of assurance but also of different generations of policyholders. If this strict principle were to be followed, offices would not wish to subsidize existing, or future, generations of policyholders from their Estates which have been created by previous generations.

Nevertheless, there appear to be two reasons why offices are prepared to depart from this principle. The first is that many offices perceive that their Estates are currently larger than necessary for the continuation of their business. This has arisen because of past underdeclarations of bonus, so that previous generations of policyholders have contributed more to the Estate than, with hindsight, was strictly necessary. It is no longer possible to return this excess to the previous

generations and passing it on to present and future generations is therefore the only way in which the Estate can be distributed at all.

The second reason for departing from the principle of strict equity is competition. The degree of competition for with-profits business has increased greatly in recent years, although the market itself has also grown rapidly. The competition is not only amongst the with-profit offices themselves, but also comes from unit-linked life offices, unit trusts, banks and building societies, and the government through National Savings and privatizations. In these conditions, many offices believe that they are fully justified in using their financial reserves to withstand the pressures of competition as this will benefit the development of the office in the longer term. Indeed, it is reasonable to argue that one reason for building up large reserves is to assist an office through difficult periods, and this includes periods of intense competition as well as, for example, periods of economic difficulty.

One aspect of the principle of equity which should perhaps be considered here is that of equity between participating and non-participating policyholders. Although this paper is primarily concerned with investigations into with-profits business, any with-profits office is likely to have a certain volume of non-profit business and for some offices the volume may be substantial. What attitude should the office take to its non-profit business?

If the office is proprietary, it will probably seek to obtain a return on capital provided for non-profit business which is regarded as adequate for shareholders in view of the risk to their capital. Typically, the return will be of the order of 15% p.a.

If the office is mutual, however, it is not so obvious what its attitude should be. It could argue that the with-profits policyholders are putting up capital and that their role is similar to that of shareholders in a proprietary office so that they would expect a similar rate of return, i.e. around 15% p.a. However, it is strictly the Estate which puts up the capital and it would be wrong to assert that the Estate belongs to the present generation of with-profits policyholders. If the Estate is being used to subsidize existing with-profits business, should it not also subsidize equally non-profit business?

If this view of equity is adopted, then the rate of return obtained by the Estate on capital should, as far as possible, be the same for both with-profit and non-profit policyholders. Thus, for example, if the Estate could earn, say, 10% p.a. net from investments but instead earns, 6% p.a. by financing with-profits business, it would seem reasonable to require a return of 6% p.a. from non-profit business also. This would result in both classes of policy being provided with an equivalent subsidy, measured in terms of the reduction in yield to the Estate resulting from writing the business.

It is evidently important for a mutual office to decide what its attitude to its non-profit policyholders is as this will affect directly its non-profit premium rates. It seems likely that the current attitude of many offices is to maximize the return to the Estate from writing non-profit business, so that a rate of return of around

15% p.a. will be sought provided that this does not make premium rates uncompetitive. In this case, profits should accrue to the Estate and these can be used to enhance bonus payments to with-profits policyholders. In this situation, any investigation into the trend in profitability of the office will have to take into account the mix of business between participating and non-participating business, and the office will need to be able to control this mix if it is to meet its various financial objectives.

On the other hand, if a mutual office attempts to treat the two classes equally by obtaining the same rate of return on capital, say 6% p.a., then it will not need to be so concerned about the mix of its business, although it will have deprived itself of a source of miscellaneous profit which could otherwise be used to support bonuses.

In practice, the office will need to take into account other factors such as competition for the respective types of business and the need to provide adequate levels of remuneration for its agents or brokers. These factors might well lead it to provide a greater subsidy to participating than non-participating business and so controlling the mix of business will indeed be important.

Finally, it should be mentioned that, for a mutual office, the rate of return required on capital need not be the same as for a proprietary office. The office will, of course, need to provide for future growth but, assuming its Estate is already of adequate size, it should not require a rate of return on capital any greater than the expected rate of new business growth. Thus, for example, a mutual obtaining a return of 10% p.a. on free assets and a return of 6% p.a. from new business would, if it allocated half of its Estate from time to time to new business, earn an overall rate of return on its Estate of 8% p.a. As its Estate would be growing at 8% p.a. it would be able to support new business growth of 8% p.a. If the office considers this to be a reasonable target, it would seem to be fully justified in using its Estate to subsidize its existing business. Indeed, not to do so would result in the Estate growing faster than the business it is supporting and, in the long run, this would lead to further inequity between generations.

## 4. INTRODUCTION TO THE VALUATION METHOD

The mathematics of cash-flow projections, or 'profit-tests', has been discussed in several papers previously, notably by Smart (5) and Lee (1), and I do not wish to repeat familiar ground. However, there are a few elementary concepts which need to be emphasized and it is convenient to do this by considering the profit-testing of a single policy in isolation.

One of the themes of this paper is the relationship between liabilities, assets and the Estate and it is important to understand the interaction between these items in respect of each policy. In the case of individual policy reserves, the basic profit-test relationship is:

$$(PFT)_t = P_t + I_t - C_t - E_t - T_t - \Delta V_t \qquad (4.1)$$

where $(PFT)_t$ = profit arising in period $t$, i.e. between time $t$ and $t+1$;
$P_t$ = premium receivable in period $t$;
$I_t$ = investment return obtained on reserve, $V_t$, in period $t$;
$C_t$ = claim payments made in period $t$;
$E_t$ = expenses and commissions incurred in period $t$;
$T_t$ = tax payable in period $t$ in respect of investment income $I_t$;
$V_t$ = reserve required at time $t$;
$\Delta V_t = V_{t+1} - V_t$

Hence the policy reserves increase according to:

$$V_{t+1} = V_t + P_t + I_t - C_t - E_t - T_t - (PFT)_t \tag{4.2}$$

The basic asset share relationship for an individual policy is:

$$(AS)_{t+1} = (AS)_t + P_t + I'_t - C_t - E_t - T'_t \tag{4.3}$$

where $(AS)_t$ = asset share at time $t$;
$I'_t$ = investment return obtained on asset share, $(AS)_t$, in period $t$;
$T'_t$ = tax payable in period $t$ in respect of $I'_t$.

It will be seen that the asset share $(AS)_t$, is the accumulated balance of premiums and investment earnings over claims, expenses and tax i.e. the accumulated excess of income over outgo for the policy without any regard to the actual level of actuarial reserves. When summed over all policies (4.3), provides a model for the office's revenue account and for the asset side of the balance sheet, whereas (4.2) provides a model for the liability side of the balance sheet.

For an individual policy, the accumulated contribution to the Estate, $S_t$, at time $t$ is that part of $(AS)_t$ not required to cover $V_t$:

$$S_t = (AS)_t - V_t \tag{4.4}$$

When summed over all policies (4.4), simply states that the Estate is the excess of assets over liabilities. From (4.2)–(4.4), it follows that the Estate develops according to:

$$S_{t+1} = S_t + I''_t - T''_t + (PFT)_t \tag{4.5}$$

where $I''_t = I'_t - I_t$ is the investment return obtained on the Estate, $S_t$, in period $t$;
$T''_t = T'_t - T_t$ is the tax payable in period $t$ in respect of $I''_t$.

Equation (4.5) is important since, when summed over all policies, it indicates that the Estate is increased by investment earnings on the Estate plus net transfers to the Estate from the current generation of policyholders.

Given any with-profits endowment policy and a set of assumptions covering future investment and actuarial factors, including a reserve basis, equation (4.1) can be used to generate a series of future profits i.e. transfers to or from the Estate. In general, some of these profits will be negative, indicating a strain, and others positive. For example, if the reserve basis used is the statutory minimum basis, there will be a large negative profit at the maturity of the policy caused by

the terminal bonus payment for which no advance provision has been made in the statutory basis.

For a valuation method used for internal investigations into the office's continuing ability to maintain any particular level of maturity values, this failure to reserve adequately for expected payments merely highlights the shortcomings of the net premium method. To overcome this failure, equation (4.1) can itself be used to fix the reserve required at each time. This is done by requiring $V_t$ to be the smallest reserve, subject to a minimum of the statutory minimum reserve plus solvency margin, that is required at time $t$ to ensure that all future profits, $(PFT)_s$ for $s > t$, are non-negative. The details of how this is achieved in practice will be illustrated by an example in the next section.

## 5. EXAMPLE OF THE VALUATION METHOD

Consider a with-profits endowment policy which has the following characteristics:

Sum assured: £3,200;
Premium: £30·00, payable monthly;
Age at entry: 40 years;
Term: 10 years.

We will suppose that the office currently declares a compound reversionary bonus of $4\frac{1}{2}$% p.a. and a terminal bonus at maturity of 55% of the sum assured for a policy of term 10 years. The total amount payable at maturity, assuming that these rates of bonus are maintained, is therefore £6,729·50. This corresponds to a return on the gross premiums of about 12% p.a. For comparison, the leading office in Money Management's with-profits survey (May 1986) was giving a return on gross premiums of about 17·1% p.a. and the office lying 10th was giving about 14·5% although these were in respect of a policyholder aged 30 next birthday. Our example office would have been a fairly average performer and this should be remembered in the analysis that follows.

Assume statutory reserve and surrender value bases as follows:

| | | |
|---|---|---|
| *Statutory Reserve Basis*: | Interest: | 3·50% p.a.; |
| | Mortality: | A67/70 ultimate; |
| | Zillmer: | 1·50% of sum assured; |
| | Maximum net premium: | 95% of office premium. |
| *Surrender value basis*: | Interest: | 6·00% p.a. for sum assured; |
| | | 8·00% p.a. for bonuses; |
| | Mortality: | A67/70 ultimate −2 years; |
| | Zillmer: | 50·00% of premium. |

The statutory reserve is assumed to be calculated as a net premium reserve and

is included solely to ensure that the full bonus reserve is at least as great as the statutory reserve plus the required solvency margin.

Assume that claims on death or surrender receive a terminal bonus in accordance with the following scale:

| Duration (years): | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| TB (% of SA): | 0·0 | 1·5 | 4·0 | 7·5 | 12·6 | 19·6 | 28·8 | 40·5 |

For surrenders, the bonus is discounted in the same way as attaching reversionary bonus.

The above policy and bases are intended to be illustrative only and it should not be assumed that they are necessarily realistic.

It is now necessary to make assumptions about future financial and actuarial factors. Since the profit-testing method is to be used both for valuation and for making projections of expected experience it is convenient to distinguish between two separate bases:

### (i) *Anticipated Actual (AA) Basis*

This is the basis to be used for projections of expected experience. It should use best estimates of future experience and may be thought of as a basis with no margins, although, as mentioned before, this is necessarily a subjective concept. One way to think of the AA basis is as a basis which in practice is equally likely to prove too strong as too weak.

### (ii) *Bonus Reserve (BR) Basis*

Since the AA basis contains no margins, it would be unsound to use it as a reserving basis: something stronger is required to provide an element of actuarial prudence. Just how strong an office can afford to make this basis is an indication of the overall financial strength of that office. My inclination, although others may disagree, is to use as strong a basis as possible. This has the advantage that changes to the basis should only be required infrequently, when there is a major and permanent change in external factors, and this helps to introduce a degree of stability to the office's results from year to year. The effect of this is that the BR basis takes on the nature of a passive valuation basis while the AA basis is an active basis, reflecting changes in external factors.

The values assumed for these bases will be:

| *Item* | *AA Basis* | *BR Basis* |
|---|---|---|
| Gross interest | 16% p.a. | 12% p.a. |
| Tax on interest | 25% p.a. | 25% p.a. |
| Tax relief on expenses | 35% p.a. | 35% p.a. |
| Inflation | 5% p.a. | 6% p.a. |
| Mortality | 100% of A67/70 sel, −2 years; | 105% of A67/70 sel, −0 years |

| | | |
|---|---|---|
| Withdrawals: | | |
| Months 1–3 | 1% p.m. | 1% p.m. |
| Months 4–6 | 2% p.m. | 2% p.m. |
| Months 7–9 | 1½% p.m. | 1½% p.m. |
| Months 10–12 | 1% p.m. | 1% p.m. |
| Year 2 | 8% p.a. | 4% p.a. |
| Year 3 | 15% p.a. | 7½% p.a. |
| Year 4 | 9% p.a. | 4½% p.a. |
| Year 5 | 7% p.a. | 3½% p.a. |
| Years 6–8 | 5% p.a. | 2½% p.a. |
| Years 9–10 | 3% p.a. | 1½% p.a. |
| Per policy expenses: | | |
| 1st year | £50 | £60 |
| Every year | £20 | £24 |
| Per premium expenses: | | |
| 1st year | 10% | 12% |
| Every year | 5% | 6% |
| Per sum assured expenses: | | |
| 1st year | ·1% | ·1% |
| Every year | — | — |
| Commission (per premium): | | |
| 1st year | 27½% | 27½% |
| Every year | 2½% | 2½% |

The peak withdrawal rate in year 3 is caused by the assumption that a surrender value becomes payable after the policy has been in force 2 years. I have equated AA and BR withdrawal rates in year 1 as lapses may result in either profit or loss. Thereafter, the BR annual rates are half the AA rates. The combined effect of the mortality and withdrawal bases is that the ten-year survivorship rates are 60·66% on the BR basis and 44·31% on the AA basis.

Initial expenses are assumed to be incurred at the outset of the policy. Renewal expenses are incurred as premiums are paid, that is, monthly, and are subject to inflation. Commission is payable as premiums are paid and is not, of course, subject to inflation.

The net rates of interest assumed are 12% p.a. (AA basis) and 9% p.a. (BR basis). These represent total investment return, whether from interest and dividends or from capital appreciation. It may be thought that there is not much actuarial prudence in a net interest rate of 9% p.a., but this rate must be considered in conjunction with the levels of bonus assumed: the overall basis is quite strong.

Having set the bases to be used, the next stage is to perform the calculations. This has been done using a computer program which performs calculations using a time interval of one month. It is assumed that premiums are received and expenses incurred at the beginning of each month and that claims are paid at the end of each month. Interest is therefore earned on the reserve at the beginning of the month plus premiums received less expenses incurred and is credited, net of tax, at the end of the month.

The program is first run on the BR basis so that the bonus reserve can be calculated. In order to do this, a double decrement table is constructed and various per policy quantities are calculated at each duration: surrender value, death benefit, statutory reserve plus solvency margin, etc. The double decrement table and per policy quantities at each duration are then combined to produce a cash flow and profit projection for a group of policies (for convenience, I assume 1000 policies initially) which is summarized in Table 1. This table illustrates equation (4.1) where the reserve used is the statutory reserve plus solvency margin (remember that this is only included to provide a minimum for the bonus reserve which is being calculated). The initial strain of £9,600 is the initial solvency margin for this block of policies, 0·3% of the sum assured.

Table 1

| *Duration, t (years)* | $P_t$ | $I_t$ | $C_t$ | $E_t$ | $T_t$ | $V_t$ | $(PFT)_t$ |
|---|---|---|---|---|---|---|---|
| 0 | — | — | — | — | — | 9,600 | −9,600 |
| 1 | 333,268 | 15,623 | 3,138 | 249,704 | −80,722 | 301,411 | −115,040 |
| 2 | 298,718 | 53,968 | 3,927 | 48,659 | −3,486 | 628,836 | −23,839 |
| 3 | 281,516 | 89,012 | 47,565 | 48,178 | 5,103 | 912,593 | −14,075 |
| 4 | 263,658 | 121,401 | 44,436 | 47,440 | 13,141 | 1,204,620 | −11,985 |
| 5 | 252,408 | 154,722 | 48,195 | 47,765 | 21,041 | 1,501,252 | −6,503 |
| 6 | 244,085 | 189,081 | 46,668 | 48,595 | 29,020 | 1,811,909 | −1,774 |
| 7 | 237,287 | 224,471 | 58,822 | 49,720 | 37,152 | 2,124,441 | 3,532 |
| 8 | 230,588 | 260,086 | 72,843 | 50,871 | 45,324 | 2,439,038 | 7,039 |
| 9 | 225,030 | 297,367 | 58,973 | 52,288 | 53,809 | 2,784,130 | 12,235 |
| 10 | 220,723 | 337,082 | 4,154,624 | 54,032 | 62,767 | 0 | −929,488 |

The tax payments are slightly less than would be obtained if the assumed tax rates are applied directly to the total interest and expenses for the year. This arises because the program makes an interest adjustment for the early payment of tax implied in calculating tax monthly.

As might be expected, many of the profits arising are negative and there is a large negative profit at maturity resulting from the payment of terminal bonus. The profits arising are now discounted for each duration starting with the last, assuming decrements and net interest as in the bonus reserve basis, to obtain the additional reserve required at each duration to cover all negative profits which have emerged. This is divided between the survivors at each duration to arrive at a per policy additional reserve, and the per policy bonus reserve at each duration is then obtained by adding the additional reserve to the statutory reserve plus solvency margin. By definition, the additional reserve is non-negative so that the bonus reserve is never less than the statutory reserve plus solvency margin.

Once the bonus reserve per policy has been calculated in this way, the program can be rerun to provide a projection of the development of the block of policies. If this is done on the BR basis, the profits emerging in each period will be zero, which is only to be expected in view of the way in which the reserves were

constructed. If the program is run on the AA basis, positive profits emerge in each period, as illustrated in Table 2.

Table 2

| *Duration, t (years)* | $P_t$ | $I_t$ | $C_t$ | $E_t$ | $T_t$ | $V_t$ | $(PFT)_t$ |
|---|---|---|---|---|---|---|---|
| 0 | — | — | — | — | — | 549,093 | −549,093 |
| 1 | 333,304 | 93,767 | 2,402 | 225,149 | −53,093 | 768,309 | 33,397 |
| 2 | 293,128 | 138,440 | 2,952 | 40,542 | 19,314 | 1,066,741 | 70,328 |
| 3 | 259,961 | 175,725 | 84,459 | 37,419 | 29,219 | 1,256,190 | 95,140 |
| 4 | 227,469 | 205,008 | 71,682 | 34,101 | 37,283 | 1,471,254 | 74,347 |
| 5 | 208,678 | 237,269 | 73,763 | 32,590 | 45,454 | 1,693,419 | 71,975 |
| 6 | 195,571 | 271,633 | 67,514 | 31,829 | 53,873 | 1,936,930 | 70,477 |
| 7 | 185,386 | 307,626 | 83,567 | 31,449 | 62,549 | 2,176,672 | 75,705 |
| 8 | 175,680 | 343,014 | 101,322 | 31,073 | 71,082 | 2,411,966 | 79,923 |
| 9 | 168,015 | 381,118 | 75,807 | 30,996 | 80,159 | 2,693,983 | 80,154 |
| 10 | 162,463 | 423,553 | 3,073,268 | 31,266 | 90,151 | 0 | 85,314 |

Some interesting points emerge from this analysis. There is an initial strain of £549·09 per policy, equivalent to about 18 months' premiums. This is quite significant and is a reflection of the strength of the BR basis assumed. This strain must be borne by the Estate and is, in fact, a loan from the Estate to the group of policyholders. The policyholders obtain the benefit of all investment earnings on this loan and make repayments to the Estate in the form of the positive profits which emerge in subsequent periods.

The rate of return obtained by the Estate on this loan can be calculated and is 5·90% p.a. in this example. As this is less than the 12% p.a. net which the Estate would earn if this business was not written, the business is being subsidized using interest earned on the Estate. Indeed, if the asset share per policy at maturity is calculated, it will be found that there is a short-fall of £890·70 per policy, or about 13·2% of the maturity value.

In spite of the subsidy being provided by the Estate for the business, the Estate will still grow if the experience is as anticipated. The precise rate of growth will depend on the proportion of the Estate which is loaned to new policyholders and the proportion which is not and will therefore lie in the range 5·90% p.a. to 12% p.a. Whether this is considered adequate for the long-term viability of the office evidently depends on the rate of new business growth expected. This relationship between the Estate and new business is, I believe, of considerable interest and is therefore discussed in some detail in Sections 9 and 10 of this paper.

I need hardly emphasize that the figures given above depend very much on the bases assumed. In particular, changing the BR basis will give rise to a different initial strain and a different rate of return on that strain. For example, weakening the basis by increasing the net rate of interest to 10% p.a. will reduce the initial strain to £437·09 per policy but will also reduce the rate of return obtained on the capital loaned to 4·03% p.a.

There is, however, one quantity which can be calculated which is completely independent of the BR basis used. It therefore provides an absolute measure of the cost to the Estate of writing this business. This quantity is the initial value of all expected profits discounted at the AA net rate of interest and for the example under consideration its value is −£127,064. The negative sign indicates again that the business is being subsidized by the Estate. Alternatively, the eventual profit or loss accruing to the Estate is, as is well-known, independent of the reserving methods and bases used and its value is:

$$-£127{,}064 \times (1{\cdot}12)^{10} = -£394{,}641.$$

## 6. SOME REMARKS ON PREMIUM-RATING

Profit-testing methods have been widely used for premium-rating, particularly of unit-linked policies and, to some extent, non-profit temporary assurances. The technique described in the previous section can readily be applied to the premium-rating of with-profits policies. This will require some assumption concerning the rate of return required by the Estate. However, to determine an appropriate rate of return requires some analysis of the Estate and how it is likely to grow in the light of the new business being written. For with-profits business, therefore, premium-rating is to some extent subsidiary to valuation.

Indeed, if we consider a single with-profits policy in isolation, the premium to be charged is largely irrelevant provided that it is greater than the premium for a similar non-profit policy. Whether the with-profits premium is 20%, 30% or even 100% greater than the non-profit may be thought of as mainly a marketing problem. As the policy is essentially a savings product and its return is not guaranteed (apart from a 'non-profit' minimum which can be met out of the non-profit part of the premium, assuming suitable matching), any size of additional with-profit loading can be charged and future bonuses allocated according to the amount of surplus generated therefrom.

In practice, of course, premium-rating cannot be dismissed so easily. It is not possible to consider single policies in isolation and the uniform reversionary bonus system has been developed to bring some simplicity to the process of allocating investment return to policyholders. However, the practice of allocating bonuses in relation to sum assured, rather than in relation to the 'investment' part of the premium, means that the relationship between sum assured and premium must be consistent for different groups of policies since otherwise a uniform bonus system would lead to inequity. As a result premium-rating remains an important aspect of with-profits business. Nevertheless, it is probably fair to say that its importance is secondary to valuation and the problems of allocating surplus to policyholders. Indeed, if a with-profits office finds itself in financial difficulty, it is not likely to be because it has charged inadequate premiums but because it has declared overlarge bonuses.

## 7. THE VALUATION OF IN FORCE BUSINESS

The methods discussed in Section 5 can be extended to the valuation of in force business without any significant difficulty in principle. It is common practice to value a model of an office's in force business rather than attempting to value every policy which, in view of the complexity of the calculations involved, is impractical even with modern computers. It should be possible to contain any error resulting from using a simplified model to within 1 or 2%. This is quite acceptable when it is remembered that small changes in the BR basis can easily result in changes to liabilities of a similar order.

To illustrate this process, suppose that an office's in force business can be modelled by a collection of 10-year endowments having the same characteristics as the example in Section 5 and distributed as in Table 3.

Table 3

| *Duration in force* | | *Number of* | *Attaching Reversionary* | *Bonus Reserve* |
|---|---|---|---|---|
| *Years* | *Months* | *Policies* | *Bonus per policy (£)* | *per policy (£)* |
| 0 | 6 | 840 | 0 | 680·06 |
| 1 | 6 | 680 | 144 | 1115·23 |
| 2 | 6 | 555 | 294 | 1602·70 |
| 3 | 6 | 445 | 452 | 2135·61 |
| 4 | 6 | 375 | 616 | 2698·49 |
| 5 | 6 | 325 | 788 | 3304·15 |
| 6 | 6 | 280 | 967 | 3959·39 |
| 7 | 6 | 245 | 1,155 | 4674·62 |
| 8 | 6 | 215 | 1,351 | 5448·04 |
| 9 | 6 | 190 | 1,556 | 6284·83 |

The right hand column shows the bonus reserve per policy calculated as in Section 5. In practice, it would be incorrect to assume that each policy in the model has identical characteristics and so other items, such as sum assured and premium per policy, would need to be specified and each policy profit-tested independently.

From the above table, the total reserve for the office can be calculated. The result obtained is £9,874,592 which may be compared with the statutory reserve for this group of policies which is £6,765,444 (including the solvency margin requirement of £288,335).

To complete the picture, an independent model of the office's assets is required. For the purposes of this example, I shall assume that the assets may be represented by cash, but I shall make some remarks on the modelling of other types of asset in the next section. Let us suppose, therefore, that the office's assets total £11,000,000 so that its Estate on the bonus reserve basis is £1,125,408 or 11·4% of liabilities and its Estate on the statutory basis is £4,234,556 or 62·6% of liabilities. The office is therefore in a strong financial position and should be able to provide support for bonuses from the interest earned on its Estate for a number of years.

A projection of the in force business and the assets can now be carried out. The results are shown in Table 4.

Table 4

| *Time (years)* | $P_t$ | $I'_t$ | $C_t$ | $E_t$ | $T'_t$ | $V_t$ | $(PFT)'_t$ | $A_t$ | $S_t$ |
|---|---|---|---|---|---|---|---|---|---|
| 0 | — | — | — | — | — | 9,874,592 | — | 11,000,000 | 1,125,408 |
| 1 | 1,397,636 | 1,712,494 | 1,651,246 | 225,864 | 331,195 | 10,174,768 | 601,649 | 11,901,826 | 1,727,058 |
| 2 | 1,215,035 | 1,832,117 | 1,802,384 | 168,003 | 378,980 | 10,198,306 | 674,247 | 12,599,610 | 2,401,304 |
| 3 | 1,045,298 | 1,917,605 | 1,926,435 | 150,445 | 405,165 | 9,967,251 | 711,913 | 13,080,469 | 3,113,218 |
| 4 | 898,238 | 1,975,453 | 2,009,422 | 134,616 | 424,206 | 9,523,061 | 749,637 | 13,385,919 | 3,862,858 |
| 5 | 761,611 | 2,003,231 | 2,133,839 | 118,880 | 436,072 | 8,800,039 | 799,073 | 13,461,969 | 4,661,930 |
| 6 | 628,425 | 1,996,752 | 2,241,031 | 102,191 | 440,120 | 7,789,567 | 852,307 | 13,303,804 | 5,514,237 |
| 7 | 494,702 | 1,951,586 | 2,367,190 | 83,821 | 435,538 | 6,443,468 | 905,838 | 12,863,538 | 6,420,070 |
| 8 | 358,595 | 1,861,376 | 2,498,592 | 63,314 | 420,977 | 4,724,780 | 955,776 | 12,100,629 | 7,375,849 |
| 9 | 219,556 | 1,722,146 | 2,616,354 | 40,373 | 395,581 | 2,610,124 | 1,004,050 | 10,990,022 | 8,379,898 |
| 10 | 74,116 | 841,799 | 2,787,538 | 14,114 | 195,246 | 0 | 529,141 | 8,909,039 | 8,909,039 |

The table illustrates equations (4.3)–(4.5) with $A_t$ representing the total assets at time $t$ and $(PFT)'_t$ representing profit calculated using total interest and tax, $I'_t$ and $T'_t$, relating to those assets. It follows that $(PFT)'_t$ includes both interest on the Estate and profit transfers to the Estate from the in force policies:

$$(PFT)'_t = I'_t - T'_t + (PFT)_t \tag{7.1}$$

All unprimed quantities in Table 4 are calculated simply by summing the various items in respect of each model policy. The primed quantities cannot be obtained in this way as they are derived from the total assets rather than the total reserves. They are therefore calculated separately for the office as a whole, which is in accordance with the concept of treating assets independently of liabilities.

Table 4 shows that the Estate grows rapidly if no new business is written. According to (7.1), the growth arises from net interest on the Estate (at 12% p.a.) plus profit transfers which are positive since the AA basis is weaker than the BR basis. Overall, therefore, the Estate grows at a rate well in excess of 12% p.a., in fact, at an average rate of 23% p.a. However, once new business is taken into account a different picture will emerge.

As mentioned before, changing the BR basis will lead to a different pattern of emerging profits. For example, if the net rate of interest is increased to 10% p.a., the value of $V_0$ reduces to £9,320,698 and so $S_0$ increases to £1,679,302. The final value of the Estate is, of course, unchanged since $V_{10} = 0$ on either basis, and the average growth rate in the Estate is about 18·2% p.a.

## 8. SOME REMARKS ON ASSETS

In the previous section, assets were treated independently of liabilities but were limited to a single asset type, namely cash. In this section I will discuss some of the elementary points that arise when other types of asset are modelled. This topic is

not only of considerable interest within the general area of cashflow projections for life offices, but is also of importance in the calculation of the mismatching provisions which must now be made in accordance with Regulation 55 of the *Insurance Companies Regulations 1981*.

A comprehensive asset model requires:

(i) An initial asset distribution. For assets whose value may vary with time (gilts, equities, land), this distribution should be expressed in unitized form. This means that a distinction should be made between the number of units held and the average price per unit, as these will need to be modelled independently.

(ii) A description of the gross income derived from each asset. For example, cash may yield 10% p.a. and equities 4% p.a.

(iii) A description of the tax payable on the income in (ii). Ideally this should not only include rates of tax applicable to each type of income but should also take account of the incidence of payment. For example, where tax is deducted at source, as with equity dividends, the tax is effectively paid immediately but where investment income is received gross the tax may not actually be paid until after the end of the company's financial year.

(iv) A model of the unit prices of variable value assets. The simplest approach is to assume annual growth rates in the values of each asset, for example 8% p.a. for equities. Gilts will require different treatment as their market value is related to their gross redemption yield and this should be fixed to be consistent with (ii).

For mismatching calculations, a more detailed model is required. One possible approach is to assume that asset values follow some simple pattern (e.g. 8% p.a. growth) as long as the fund is producing a surplus of cash for investment but that asset values immediately fall by, say, 25% (the figure currently recommended by the Government Actuary for investigating the provision required under Regulation 55) as soon as the fund starts to disinvest. The mismatching provision is determined by obtaining the minimal set of assets which is required at the outset to ensure that all future liabilities can be met as they fall due under these circumstances. If the value of these assets exceeds the value of the liabilities, the excess should be regarded as the amount to be provided for mismatching.

A more ambitious approach to the treatment of asset values would be to use a stochastic model such as that proposed by Wilkie (16).

(v) A means of calculating CGT payable when assets are sold. The calculation will, of course, take account of the indexation allowance and it can be assumed that the tax will not actually be paid until the following financial year.

(vi) A calculation of the cash available for investment, or required from disinvestment, from time to time. The basis of this calculation is an

equation of the form of (4.3) which indicates the increase in the cash value of the assets held. However (4.3), is a revenue account equation and the true cash position may differ. In particular, as already mentioned, tax is not necessarily paid as incurred. Furthermore, where gilts are included as assets, it will be necessary to allow for the receipt of cash when they reach maturity.

(vii) Strategies for investment and disinvestment. It is necessary to specify the proportions of cash to be invested in each type of asset from time to time and, similarly, to specify the procedure by which disinvestment takes place.

In addition to each of the above, some consideration should be given to the accounting treatment of assets. If these are held at market value, changes in their values will be reflected in the total investment return obtained. This means that equation (4.3) takes on a new character. The left-hand side is no longer determined from the right-hand side but from independent calculations of cash available for investment and changes in the value of investments. Equation (4.3) then effectively determines $I'_t$, the gross investment return for the period.

For some purposes, such as projections on the statutory basis and DTI mismatching calculations under Regulation 55 it may, depending on the office's practice, be more appropriate to hold assets at book value and allow for them to be written up or down from time to time as appropriate. The write-up is determined as the balancing item in the revenue account where the fund, i.e. the assets at book value, is equal to or bears some other prescribed relationship to the liabilities.

Where assets are held at market value it is, of course, appropriate to include adequate provision for mismatching, and for any CGT liability, in the reserves before arriving at a value for the Estate and this must be done for internal investigations as well as published valuations. However, this has not been done explicitly in the numerical examples in this paper, partly because they take all assets to be cash. Furthermore, it could be argued that the BR basis used is sufficiently strong to include adequate provision for mismatching and CGT. In effect, the bonus reserve is sufficient to ensure that current maturity values can be maintained for existing business if future investment returns average only 9% p.a. net compared to the AA rate of 12% p.a. net. In other words, there is sufficient margin in the valuation basis to withstand a fairly large shortfall in expected investment return, whether arising from changes in yield or from reductions in capital values. Any shortfall beyond that implied by the BR net interest rate assumed would therefore have to be met by reductions in bonuses allocated to with-profits policyholders.

## 9. INCORPORATION OF NEW BUSINESS: THE FULL MODEL OFFICE

I now return to the example model office discussed in Section 7 in order to

consider the inclusion of new business. For simplicity I will continue to assume that all assets are cash and to ignore mismatching and CGT provisions (or to assume that margins in the BR basis are sufficient to cover any mismatching or CGT liability).

Assume that at time $t=0$ the business in force is as described in section 7. Assume that in the year to come, the office expects to write 1000 policies as described in Section 5 and that in future years the office expects to write increasing amounts of the same business. For simplicity, I will assume that all growth results from increases in numbers of policies, but in practice it is important to allow for growth in average premiums as well.

The mechanics of incorporating new business is quite straightforward. The new business is profit-tested as in Section 5 and the results obtained are added to the figures derived for the in force business, assuming that an appropriate level of new business enters in each subsequent time period. In the examples which follow, new business is assumed to enter monthly. Where increases in new business occur, these also are assumed to take place monthly. For any given rate of increase, the amount of business entering in the first month is set so that a total of 1000 policies enters during the first year.

If new business growth is assumed to occur at 10% p.a., the results obtained will be as in Table 5.

Table 5

| *Time (years)* | $P_t$ | $I_t'$ | $C_t$ | $E_t'$ | $T_t$ | $V_t$ | $(PFT)_t'$ | $A_t$ | $S_t$ |
|---|---|---|---|---|---|---|---|---|---|
| 0 | — | — | — | — | — | 9,918,377 | — | 11,000,000 | 1,081,623 |
| 1 | 1,580,677 | 1,714,836 | 1,652,568 | 389,574 | 276,949 | 10,857,783 | 37,016 | 11,976,422 | 1,118,639 |
| 2 | 1,725,168 | 1,866,791 | 1,806,518 | 431,310 | 299,073 | 11,878,081 | 34,760 | 13,031,479 | 1,153,398 |
| 3 | 1,883,354 | 2,030,065 | 1,974,507 | 479,193 | 321,830 | 12,987,448 | 28,522 | 14,169,369 | 1,181,921 |
| 4 | 2,060,985 | 2,208,469 | 2,141,047 | 531,659 | 346,647 | 14,218,142 | 19,407 | 15,419,474 | 1,201,332 |
| 5 | 2,257,550 | 2,402,351 | 2,351,752 | 588,860 | 373,557 | 15,555,489 | 8,385 | 16,765,208 | 1,209,719 |
| 6 | 2,475,216 | 2,613,624 | 2,551,433 | 651,292 | 402,843 | 17,044,169 | −5,408 | 18,248,480 | 1,204,311 |
| 7 | 2,716,265 | 2,845,490 | 2,784,602 | 719,461 | 435,106 | 18,688,664 | −21,909 | 19,871,070 | 1,182,406 |
| 8 | 2,982,483 | 3,098,520 | 3,050,650 | 793,761 | 470,340 | 20,497,045 | −42,129 | 21,637,325 | 1,140,280 |
| 9 | 3,277,102 | 3,376,800 | 3,311,680 | 874,842 | 509,302 | 22,521,466 | −66,343 | 23,595,397 | 1,073,931 |
| 10 | 3,602,455 | 3,682,315 | 3,874,248 | 963,172 | 552,309 | 24,511,620 | −95,113 | 25,490,440 | 978,820 |
| 11 | 3,956,462 | 3,999,099 | 3,983,231 | 1,058,560 | 595,630 | 26,962,781 | −133,021 | 27,808,578 | 845,797 |
| 12 | 4,352,106 | 4,362,669 | 4,381,555 | 1,164,416 | 646,560 | 29,659,060 | −174,035 | 30,330,821 | 671,761 |
| 13 | 4,787,318 | 4,758,232 | 4,819,709 | 1,280,858 | 701,545 | 32,624,966 | −222,468 | 33,074,254 | 449,288 |
| 14 | 5,266,049 | 5,188,469 | 5,301,684 | 1,408,945 | 760,874 | 35,887,462 | −279,481 | 36,057,274 | 169,812 |
| 15 | 5,792,653 | 5,656,258 | 5,831,849 | 1,549,838 | 824,831 | 39,476,208 | −346,353 | 39,299,666 | −176,542 |

The development of the Estate in this example is in marked contrast to the case where no new business is being written (see Table 4). A new business growth rate of 10% p.a. leads to the Estate starting to contract after 5 years and the office being insolvent (relative to the BR basis being used) after 15 years.

Before considering the remedies that an office finding itself in this position has

at its disposal, it is worth recalling the current characteristics of the office under consideration. It is financially very strong. Its maturity values are average. Its annual rate of return on investments is 16% gross or 12% net. Its expense ratio, as can be inferred from Table 5, is reasonable (about 25% initially). Other aspects of its performance are fairly typical of the with-profits life sector as a whole. Yet, nevertheless, a new business growth rate of 10% p.a. will lead to insolvency within 15 years.

To avoid this happening, the office has a number of alternatives to choose from. It could:

(i) Change its valuation basis. As mentioned earlier, the basis used in these examples is strong. Increasing the BR net interest rate to 10% p.a. will defer insolvency by a further 6 years. Alternatively, reverting to valuation on the statutory minimum basis will defer insolvency by a further 28 years. Weakening the valuation basis is evidently a useful short-term expedient, but it cannot change the underlying reality of the situation which is one of creeping insolvency.

(ii) Reduce maturity amounts by cutting bonuses. Reductions in bonuses have been rare in the past and have often been achieved only at times of national crisis, the last major industry-wide reduction having occurred at the outbreak of the second world war. Although, in principle, terminal bonuses can be reduced more easily than reversionary, a major cut in terminal bonuses might attract adverse publicity and many offices will try to defer such a decision for as long as possible. Unfortunately, if the decision to cut bonuses is deferred for too long, there is a risk that the DTI might be forced to act to prevent serious loss to policyholders.

(iii) Increase premiums for new business. This leads to inequity between generations of policyholders and may cause difficulty in obtaining new business, particularly in the competitive low-cost endowment market where policy sales are to some extent dependent on relative premium rates. It is likely, however, that this course of action would be adopted by a number of offices in the circumstances under discussion.

(iv) Reduce its rate of growth of new business. This is easier for offices which can switch to selling other types of business, for example unit-linked business, than for offices which are strongly dependent on sales of with-profits business for growth. Nevertheless, it can be a very effective remedy. It is evident that there must be a maximum rate of new business growth which the office in the above example can sustain without becoming insolvent since the office remains solvent if no new business at all is written.

The effect of different levels of new business growth on solvency can be investigated easily using the techniques described in this paper and I summarize some of the results obtained below:

| *Rate of NB growth (% p.a.)* | *Time taken to reach insolvency (years)* |
|---|---|
| 10 | 15 |
| 9 | 20 |
| 8 | 38 |

The maximum rate of new business growth which can be sustained without leading to inevitable insolvency appears to lie between 7% p.a. and 8% p.a.

## 10. DEVELOPMENT OF THE ESTATE

The examples in the previous section illustrate the close relationship which exists between the Estate and the rate of return available on the free assets of the Estate; the capital required to support new business and the rate of return obtained by the Estate on that capital; and the rate of new business growth. As this relationship is of some importance, I think it worthwhile to investigate it in general terms.

The development of any life office's Estate is described by equation (4.5). For the purposes of undertaking a general investigation into the Estate it is more convenient to express (4.5) in terms of a continuous, rather than a discrete, parameter $t$. This results in a differential equation which can be solved to provide a general description of the development of the Estate.

Let $i$ = net rate of investment return p.a.,
$j$ = rate of return p.a. earned by Estate on new business capital,
$g$ = rate of growth p.a. of new business,
$C(t)$ = density function describing flow of capital from Estate to new business at time $t$ years,
$R(t)$ = density function describing flow of capital and interest from in force business to Estate at time $t$ years.

Then equation (5.5) can be re-expressed in continuous form as:

$$\frac{dS(t)}{dt} = \delta_i S(t) - C(t) + R(t) \tag{10.1}$$

where $\delta_i$ is the force of interest at rate $i$. It will be noted that I have split the profit at time $t$ into negative flows, $-C(t)$, to new policyholders and positive flows, $R(t)$, representing repayments of capital and interest to the Estate.

To solve (10.1), some assumptions must be made about the form of $C(t)$ and $R(t)$. For any given distribution of new business at time $t$, assume that the initial capital $C(t)$ is repaid by a repayment schedule $r(t,s)$ representing repayments of capital and interest at time $t+s$, in respect of the initial capital $C(t)$. Then we can write:

$$C(t) = \int_0^n r(t,s) e^{-\delta_j s} ds \tag{10.2}$$

and
$$R(t) = \int_0^n r(t-s,s)ds \tag{10.3}$$

where $n$ is the period of years over which repayments to the Estate take place. Equation (10.2) is simply the definition of $j$, the rate of return earned by the Estate on $C(t)$. Equation (10.3) says that the total repayment of capital and interest at time $t$ is the sum of the individual repayments in respect of the business written in the last $n$ years.

It is reasonable to assume that $r(t,s)$ is in fact proportional to $C(t)$ and to re-express it as:

$$r(t,s) = C(t)r(s) \tag{10.4}$$

Then (10.2) and (10.3) become:

$$1 = \int_0^n r(s)e^{-\delta_j s}ds \tag{10.5}$$

and
$$R(t) = \int_0^n C(t-s)r(s)ds \tag{10.6}$$

If it is assumed that new business grows at a constant rate of $g$ p.a., then

$$C(t) = C_0 e^{\delta_g t} \tag{10.7}$$

where
$$C_0 = C(0) \tag{10.8}$$

Substituting (10.7) in (10.6) gives:

$$R(t) = C_0 e^{\delta_g t}\int_0^n r(s)e^{-\delta_g s}ds \tag{10.9}$$

Substituting (10.7) and (10.9) in the basic equation (10.1) gives:

$$\frac{dS(t)}{dt} = \delta_i S(t) + \alpha e^{\delta_g t} \tag{10.10}$$

where the constant $\alpha$ is given by:

$$\alpha = -C_0\left(1 - \int_0^n e^{-\delta_g s}r(s)ds\right) \tag{10.11}$$

It will be seen from (10.11) that $\alpha$ is the present value of all future strains or profits to the Estate from the initial tranche of new business, $C_0$, discounting being performed at the anticipated rate of growth of new business, $g$ p.a.

Equation (10.10) can now be solved to give:

$$S(t) = \left(S_0 + \frac{\alpha}{\delta_i - \delta_g}\right)e^{\delta_i t} - \frac{\alpha}{\delta_i - \delta_g}e^{\delta_g t} \tag{10.12}$$

This solution is very general, as the precise form of the profit flows to the Estate, $r(s)$, has not been specified. In fact, it is not even necessary for $C_0$ and $r(s)$ to be positive, so (10.12) can be applied, for example, to the statutory reserve basis. Whichever basis is used, the parameter $\alpha$ can be calculated readily using the profit-testing program and hence $S(t)$ can be obtained.

I should, however, point out that equation (10.6) contains an assumption that the initial distribution of business in force is consistent with constant new business growth of $g$ p.a. in the past. This, of course, is unlikely to be true although in some cases it may be a reasonable approximation to the truth. It is possible to modify (10.6) for $t<n$ to take account of the repayments made to the Estate by the initial in force business, but the resulting analysis is rather more complicated. As the theoretical analysis presented here is mainly intended to point to general principles, rather than to provide a means of modelling the future of the office (the methods described in earlier sections are more appropriate to this task), this is probably not a serious limitation. The solution (10.12) is appropriate where an office's business in force has reached, or approximates to, a stationary state. By stationary, I do not mean static, but unchanging in 'shape' with 'size' changing at a constant rate.

The behaviour of $S(t)$ as $t\to\infty$ can be investigated from (10.12) to determine whether a given course of action will lead to insolvency or not. There are two cases to consider:

(i) $g>i$ In this case, $e^{\delta_g t}$ dominates as $t\to\infty$ and so the behaviour of $S(t)$ is determined by the sign of $\alpha$, as follows:

$\alpha<0\Rightarrow S(t)$ becomes negative $\Rightarrow$ Insolvency occurs
$\alpha>0\Rightarrow S(t)$ becomes positive $\Rightarrow$ Estate grows at rate which asymptotically approaches $g$ p.a.

Now, from the definition of $\alpha$ (see (10.11) and recall (10.5)):

$$\alpha>0\Leftrightarrow g<j$$
$$\alpha<0\Leftrightarrow g>j$$

Hence, if the office is to remain solvent when $g>i$ it is necessary to have $j>g$. This is a restatement of the familiar rule (see, for example, Smart (5)) which says that "unless the rate of return earned on new business strain is at least as great as the long-term new business growth rate, the office will eventually run out of capital". As will be seen in (ii) below, this rule applies only when $g>i$, that is, when the rate of new business growth exceeds the net rate of return on assets. Although this may be desirable, it may not be attainable. Indeed, since it may not be possible to ensure $j>g$, it is probably not advisable to aim for $g>i$, except possibly in the short-term.

(ii) $g<i$ In this case, $e^{\delta_i t}$ dominates as $t\to\infty$. Evidently, if $\alpha>0$ (that is, $j>g$), then the coefficient of $e^{\delta_i t}$ is positive and the Estate continues to grow at a rate which asymptotically approaches $i$ p.a. Thus, the office will not encounter

problems if it can achieve a rate of return on new business strain which exceeds the rate of new business growth. So $j > g$ is a *sufficient* condition for a long-term growth.

It is not, however, *necessary* since the coefficient of $e^{\delta_i t}$ can still be positive when $\alpha < 0$ (that is $j < g$) provided that the initial Estate is large enough:

$$S_0 > -\frac{\alpha}{\delta_i - \delta_g} \tag{10.13}$$

Although it is not strictly correct to do so, it is of interest to apply (10.13) to the office described in Section 9. For this case:

$S_0 = 1{,}125{,}408$
$\alpha = -86{,}724$
$i = \cdot 12$
$g = \cdot 10$

Hence $$-\frac{\alpha}{\delta_i - \delta_g} = \frac{86{,}724}{\cdot 113329 - \cdot 095310} = 4{,}812{,}920$$

*Consequently, condition (10.13) is not satisfied for this case, which is consistent* with the results of Section 9.

Equation (10.13) can be used to determine the maximum value of $g$ for which continued growth is assured and this may be a useful guide to an office's long-term strategy. The result is a rate of around 7·7% p.a. The theoretical limitations of this analysis mean that this figure may be a bit high, but it is undoubtedly of the right order.

Where insolvency is expected to occur, equation (10.12) can be used to obtain an estimate of the time likely to elapse before insolvency occurs. Let this be $T$. Then $T$ is given by

$$S(T) = 0 \tag{10.14}$$

Hence: $$T = \frac{1}{\delta_i - \delta_g} \ln\left(\frac{\beta}{S_0 + \beta}\right)$$

where $$\beta = \frac{\alpha}{\delta_i - \delta_g}$$

For the case in Section 9:

$$T = \frac{1}{\cdot 113329 - \cdot 095310} \ln\left(\frac{-4812920}{1125408 - 4812920}\right)$$

$\Rightarrow$ $T = 14{\cdot}8$ years

This is consistent with the calculations of Section 9.

If it is assumed that the repayments of capital and interest are the same in each time period (i.e. $r(s)$ is constant), then $\alpha$ takes on a particularly simple form. This assumption, while obviously not strictly correct (see Table 2), is not a bad

approximation, especially for an office having a mix of business at different durations. From (10.5), the constant $r$ has the value

$$r = 1/\bar{a}^{j}_{\overline{n}|} \tag{10.15}$$

as would be expected for the level repayments of capital and interest on a loan. It follows by substitution in (10.10) that $\alpha$ takes the simple form:

$$\alpha = -C_0\left(1-\frac{\bar{a}^{g}_{\overline{n}|}}{\bar{a}^{j}_{\overline{n}|}}\right) \tag{10.16}$$

The parameter $C_0$ is the annualized initial strain on the new business being written and is given by 12 times the difference between the values of $V_0$ in Tables 4 and 5:

$$C_0 = 525{,}420$$

Hence

$$\alpha = -80{,}370$$

which is fairly close to the correct value given earlier.

In general, an office will issue new business for more than one original term. The simplified form of $\alpha$ is then given by an expression consisting of a weighted sum of terms of the form (10.16), the sum being over $n$, the original term, and the parameters $C_0$ providing the weights.

Finally, it is of interest to plot equation (10.12) graphically for various values of its parameters. I have done this for two cases, Figures 1 and 2, both of which are based on the example of Section 9.

Figure 1 illustrates the effect on the Estate of writing new business at various different rates of growth, 8% p.a., 9% p.a. and 10% p.a. Although the line for 8% p.a. suggests that the office is growing healthily after 25 years, this is deceptive. It will be recalled from Section 9 that in this case the office actually becomes insolvent after 38 years.

Figure 2 illustrates the effect of different initial levels of the Estate. The two cases illustrated are $S_0 = 1{,}125{,}408$ (corresponding to assets of £11 million) and $S_0 = 1{,}625{,}408$ (corresponding to assets of £11·5 million). In both cases, the rate of new business growth assumed in 9% p.a. The shape of the curve for $S_0 = 1{,}625{,}408$ is interesting as the office appears to be growing strongly for more than 25 years, only to be insolvent 10 years later. One is tempted to speculate what an office finding itself at the point $t = 25$ on this curve would think of its performance. Would it be congratulating itself on its good results to date, or would it be looking to the future and taking the necessary action to prevent itself stepping over the precipice?

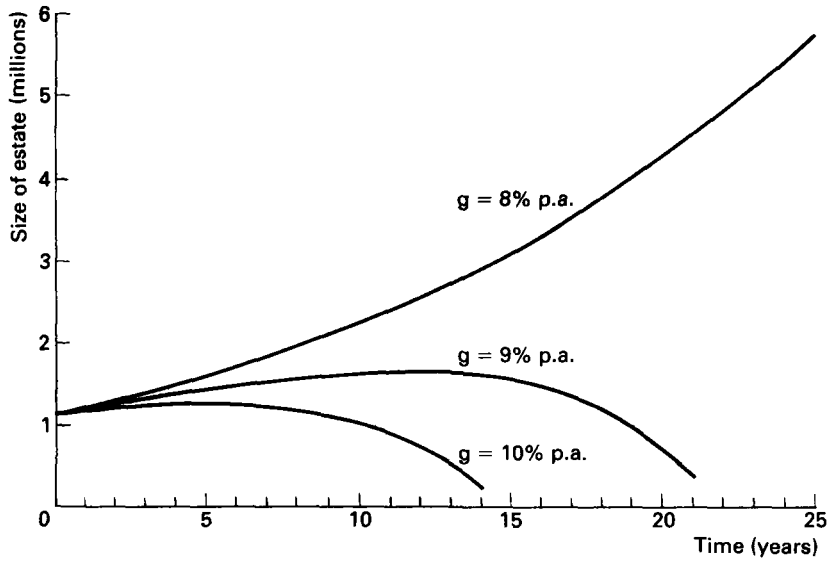


Figure 1. *Estate of hypothetical life office.*

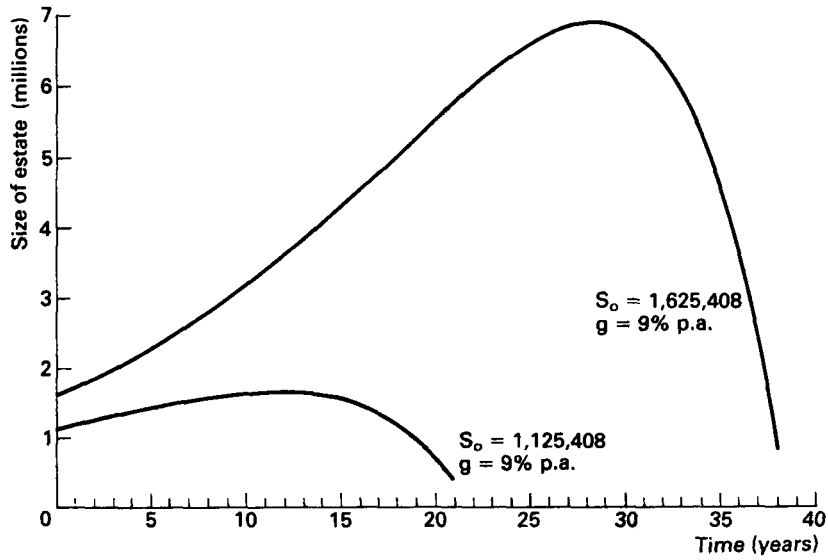


Figure 2. *Estate of hypothetical life office.*

## 11. CONCLUDING REMARKS

The original version of this paper, which was presented to meetings of both the Manchester Actuarial Society and the Staple Inn Actuarial Society, ended with the unresolved question posed at the end of the preceding section. It did not originally incorporate a concluding section mainly because I felt that the paper was by no means the final word on the subject and it would be premature to come to any definite conclusions. My feelings have not changed, but it became apparent during the discussions that several people would have welcomed a final section summarizing the paper and I have therefore decided to add these concluding remarks. In doing so, I will also attempt to clarify some of the points that have been raised, both in the discussions and subsequently.

The basic investigative tool employed in this paper is a gross premium bonus reserve method of valuation, described in Section 5. The method involves analysing the cashflows which are expected to emerge relative to the minimum statutory reserve, including solvency margin, from a block of business if the experience is in accordance with the reserving basis (the BR basis). If any of these cashflows are negative, an additional reserve (additional, that is, to the statutory reserve plus solvency margin) is calculated to ensure that sufficient funds will in future be available by drawing on this reserve to extinguish the negative cashflows. The additional reserve is therefore very similar to a sterling reserve for unit-linked policies but, as defined in Section 5, it cannot be negative so that the total reserve (the bonus reserve) is never less than the statutory reserve plus solvency margin.

Because of the way in which it is calculated, the additional reserve is in fact the *minimum* additional reserve required to ensure that no negative profits will emerge if the experience is in accordance with the BR basis. As a result, the profits that will emerge if the experience is indeed as in the BR basis will normally all be zero since if they were strictly positive at any duration it would follow that a smaller additional reserve could have been held. However, since the additional reserve cannot be negative, an exception to this occurs for durations (if any) at which the additional reserve is actually zero. In these cases, strictly positive profits will emerge, since the additional reserve cannot be made any smaller.

The rationale behind this approach to valuing with-profits business is that since the office hopes to pay a terminal bonus at the maturity of an endowment policy, it should, at least for internal investigations, ensure that the expected liability is adequately provided for. The statutory net premium basis is inadequate for this purpose as it gives rise to a negative profit to the office when the terminal bonus is paid, so an additional reserve is required to cover this negative profit. It is then a simple extension to require sufficient reserves to be held at each duration to extinguish all expected negative profits and so avoid the appearance of any future valuation strain.

The main application of the valuation method described in this paper is to investigations into the ability of an office to maintain a particular level of

maturity values for its with-profits policies and this leads naturally to a discussion of the office's Estate.

In discussing the Estate, it is important to remember that the Estate is not a unique concept. Redington's original definition took the Estate to be the excess of the assets over the liabilities calculated using bases which contain no margins. This would correspond to measuring the Estate on the AA basis, or perhaps on a variety of possible AA bases, but I have instead chosen to measure it on the more conservative BR basis. Provided that the Estate is regarded as a function of the methods and bases used to measure it, this approach is reasonable, if slightly unconventional. Indeed, I believe that the more conservative approach which I have adopted is appropriate for internal valuation, accounting and control, where some degree of actuarial prudence is required and where the increased stability which should result from less frequent changes of basis should prove to be an advantage.

Obviously it is possible to equate the two bases and for some purposes (for example, the estimation of appraisal values) this may be considered desirable, but more flexibility is retained if the two are allowed to differ. In particular, since the BR basis is a reserving basis and the AA basis a projection basis, it is possible to investigate what will happen if the actual experience is different from that assumed in the reserving basis.

The example discussed in the later sections of the paper is of an office which is financially strong, declaring bonuses at about the average current level for the market as a whole, with a respectable expense ratio and expecting to earn 12% per annum after tax on its investments. The analysis of this case shows that the future course of the office's Estate (and hence of its ability to remain solvent) depends critically on the rate of new business growth. A low level of new business growth will result in rapid growth in the Estate while a high level will lead eventually to insolvency unless appropriate remedial action, such as reducing bonuses, is taken.

The main characteristic of the model office considered is that it is writing business on terms which, if allowed to continue, will tend to diminish the size of the Estate relative to the liabilities. To put it another way, the Estate is being used to subsidize the business being written. As this may well be the current reality for some offices, it is important to understand the nature and extent of this subsidy. There are a number of equivalent criteria which may be used to determine whether or not an office's business is being subsidized by its Estate:

(i) Subsidy occurs if the amount paid to a policy on maturity exceeds that policy's asset share; conversely, the policy contributes to the Estate if the maturity value is less than the asset share.

(ii) Subsidy occurs if, and only if, the rate of return earned by the Estate on the capital provided to finance a policy is less than the actual rate earned by investments (in the notation of section 10, if and only if $j < i$).

(iii) Subsidy is expected to occur if, and only if, the present value on the AA

basis, at the time a policy is issued, of future strains and profits to the Estate is negative.

It should be mentioned that although the Estate is only defined relative to the methods and bases used in its calculation, each of the above criteria is independent of these methods and bases so that subsidy is an absolute concept.

Criterion (ii) emphasizes that the Estate may still be growing even when it is subsidizing current business, but that its rate of growth is reduced because of the existence of the subsidy. This may help to put the nature of the subsidy into perspective: part of the investment return earned on the Estate is transferred to maturing policies, while the remainder falls into the Estate. Since the Estate continues to grow, it will still be able to support growth in new business, although there will be an upper limit to the rate of new business growth that can be supported in the long term. Whether this upper limit is restrictive will depend on the office's intentions with regard to the development of its with-profits business. If it is content to expand such business relatively slowly while concentrating on developing other parts of its business (non-profit, unit-linked, etc.), it may be able to continue with unchanged bonus levels for many years even though investment returns are, strictly, insufficient to support such bonuses.

In this paper I have looked at some aspects of the valuation of with-profits business under current conditions. However, several interesting questions remain to be considered, such as the modelling of assets, and in particular the use of stochastic models, the costing of the guarantees implicit in with-profits business, the question of what is the appropriate contribution for a departing policyholder to make to the Estate, etc. At the discussion of this paper at Staple Inn, several contributors addressed the interesting question of the differences in philosophy between mutual and proprietary offices and this is a topic which could be explored further. A number of papers on some of these subjects have appeared recently, but these questions and others will no doubt provide fruitful ground for research and discussion for some time to come and it is hoped that the techniques outlined in this paper will be of assistance in tackling such problems.

## 12. ACKNOWLEDGEMENTS

My interest in the application of profit-testing methods to with-profits business originated from a conversation with Sidney Benjamin who first suggested the topic to me. My own office has subsequently provided me with much support, including access to computer facilities, and I would like to thank in particular Alan Sneddon for encouraging me to carry out this work.

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